Mathematical modeling of choice behavior: from theory to practice

Michel Bierlaire

Transport and Mobility Laboratory
School of Architecture, Civil and Environmental Engineering
Ecole Polytechnique Fédérale de Lausanne

Outline

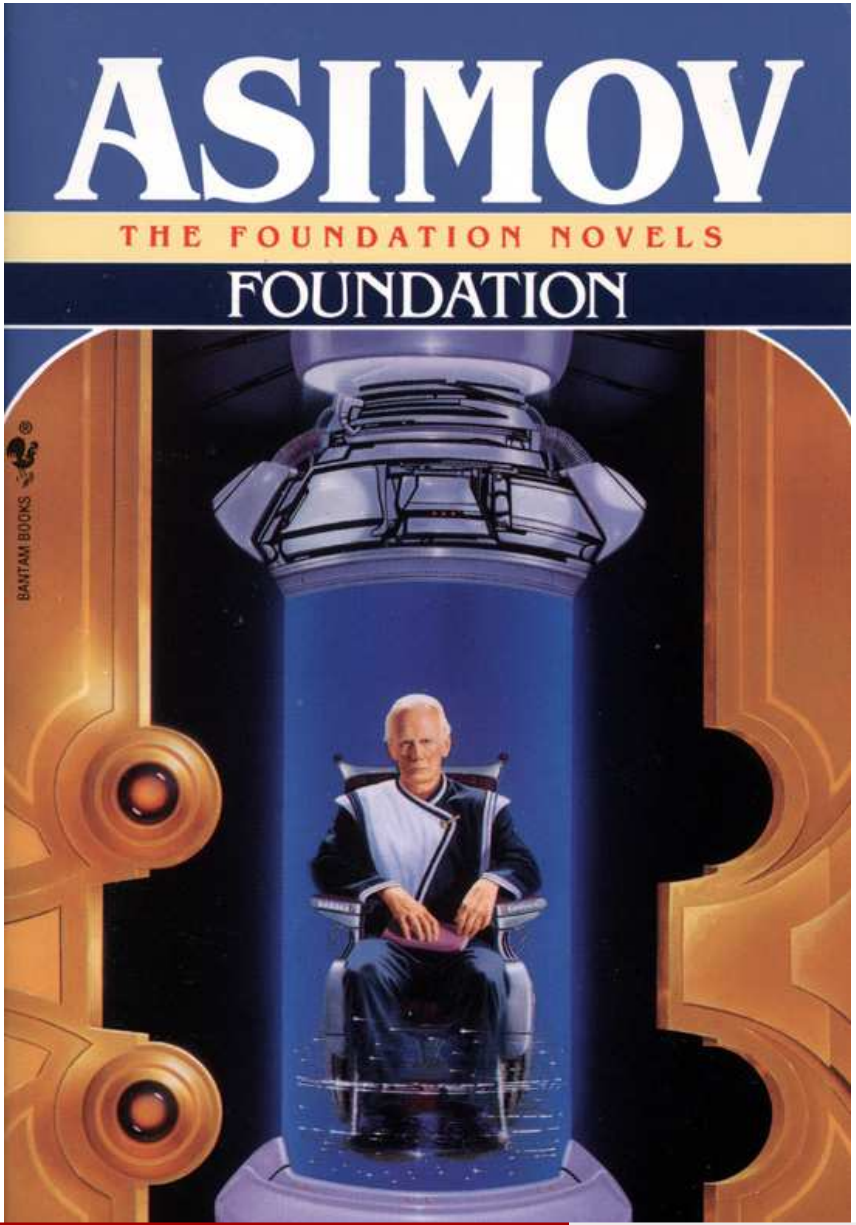
Literature review

Psychohistory

Branch of mathematics which deals with the reactions of human conglomerates to fixed social and economic stimuli. The necessary size of such a conglomerate may be determined by Seldon's First Theorem.

Encyclopedia Galactica, 116th Edition (1020 F.E.)
Encyclopedia Galactica Publishing Co., Terminus

Literature review



Asimov, I. (1951) *Foundation*, Gnome Press

Motivation: shorten the period of barbarism after the Fall of the Galactic Empire

Literature review

Albus Dumbledore

It is our choices that show what we truly are, far more than our abilities

Jean-Paul Sartre

We are our choices

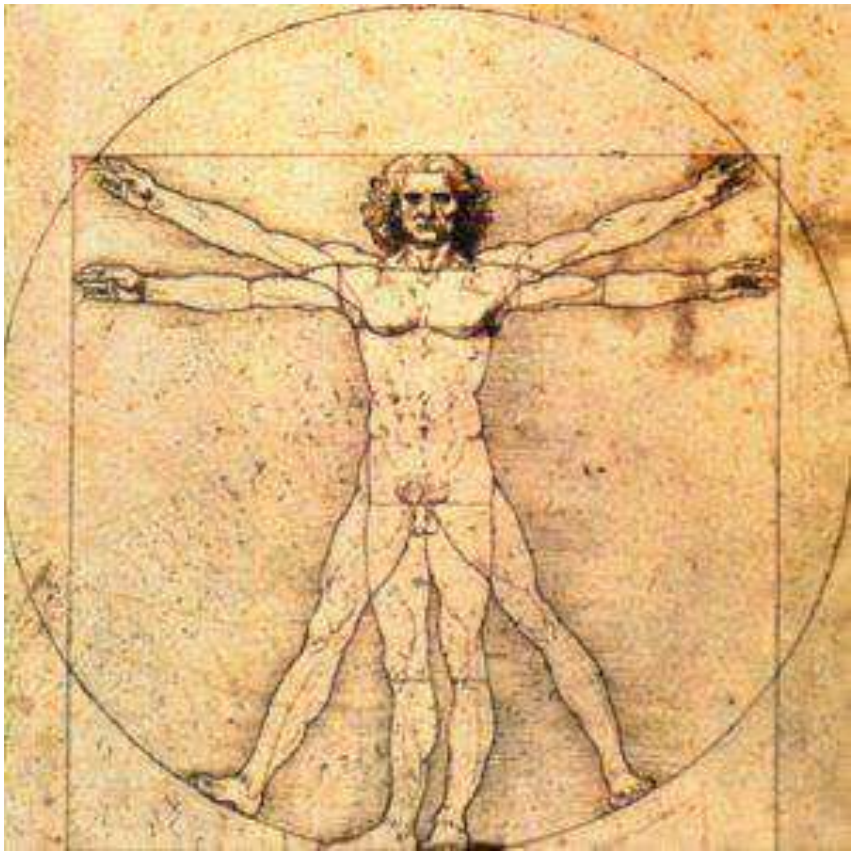
Ken Levine

- We all make choices, but in the end our choices make us.
- In the end what separates a man from a slave? Money? Power? No, a man chooses... a slave obeys.

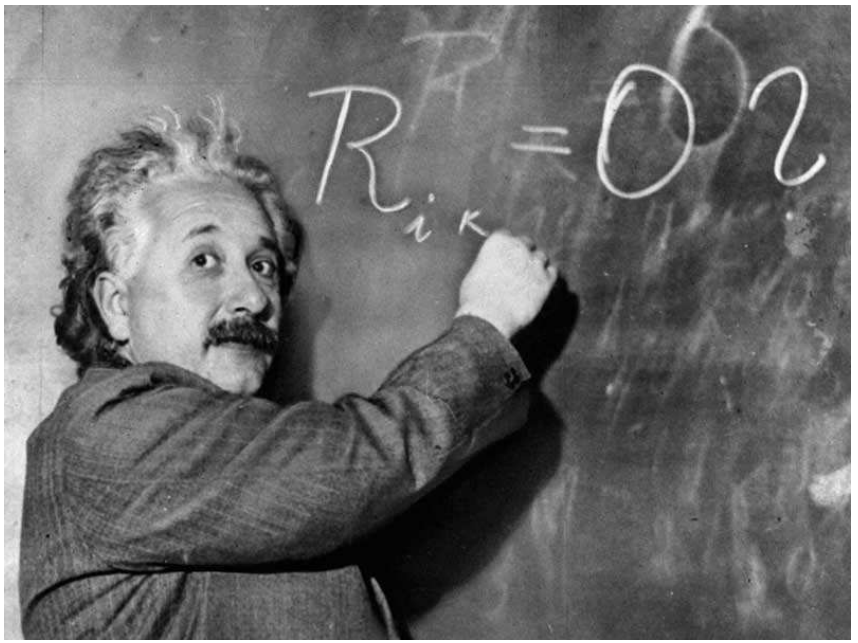
Motivation

Human dimension in

- engineering
- business
- marketing
- planning
- policy making



Theories and methods



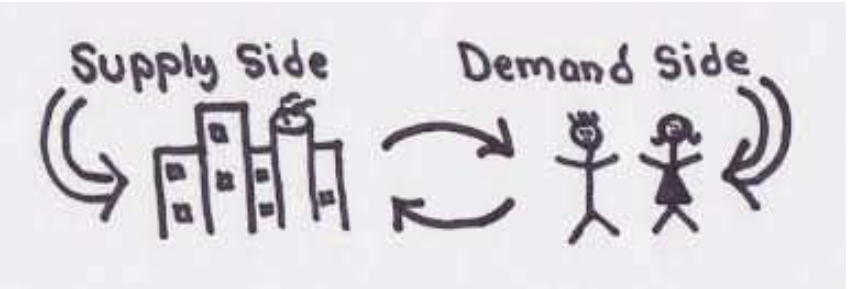
Need for

- behavioral *theories*
- quantitative *methods*
- operational mathematical *models*

Economic approach

Concept of demand

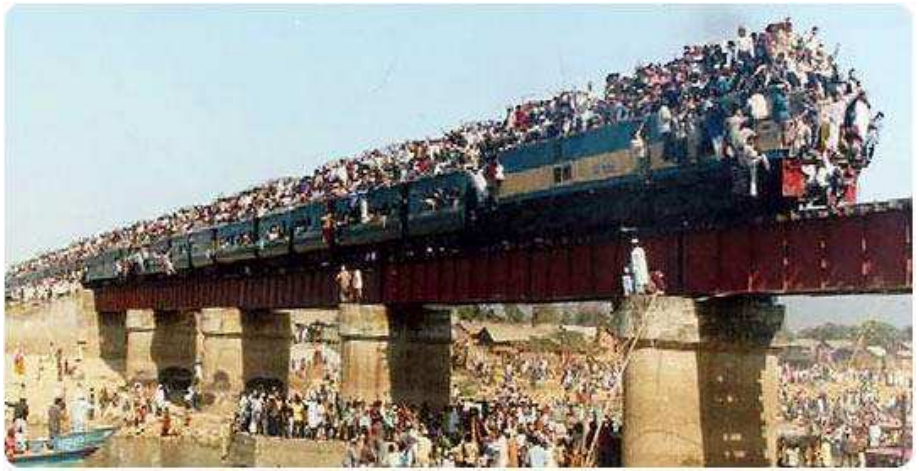
- marketing
- transportation
- energy
- finance



Transportation

- Supply = infrastructure
- Demand = behavior, choices
- Congestion = mismatch

Transportation



- Usually in operations research:
- optimization of the supply
- for a given (fixed) demand

Aggregate demand

- Homogeneous population
- Identical behavior
- Price (P) and quantity (Q)
- Demand functions: $P = f(Q)$
- Inverse demand: $Q = f^{-1}(P)$

Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.

Choices

Concept of choice

- Marketing:brand, product
- Transport: mode, destination
- Energy: type, usage
- Finance: buy/sell, product

Applications

Willingness to pay for travel time savings

- Swiss Federal Road Office
- Compute the Swiss value of time

Applications

Route choice

- How do traveler select an itinerary?
- Impact of information and guidance
- Data: Nokia

Applications

Market share of electrical vehicles

- Renault Suisse
- Forecasting of market shares

Applications

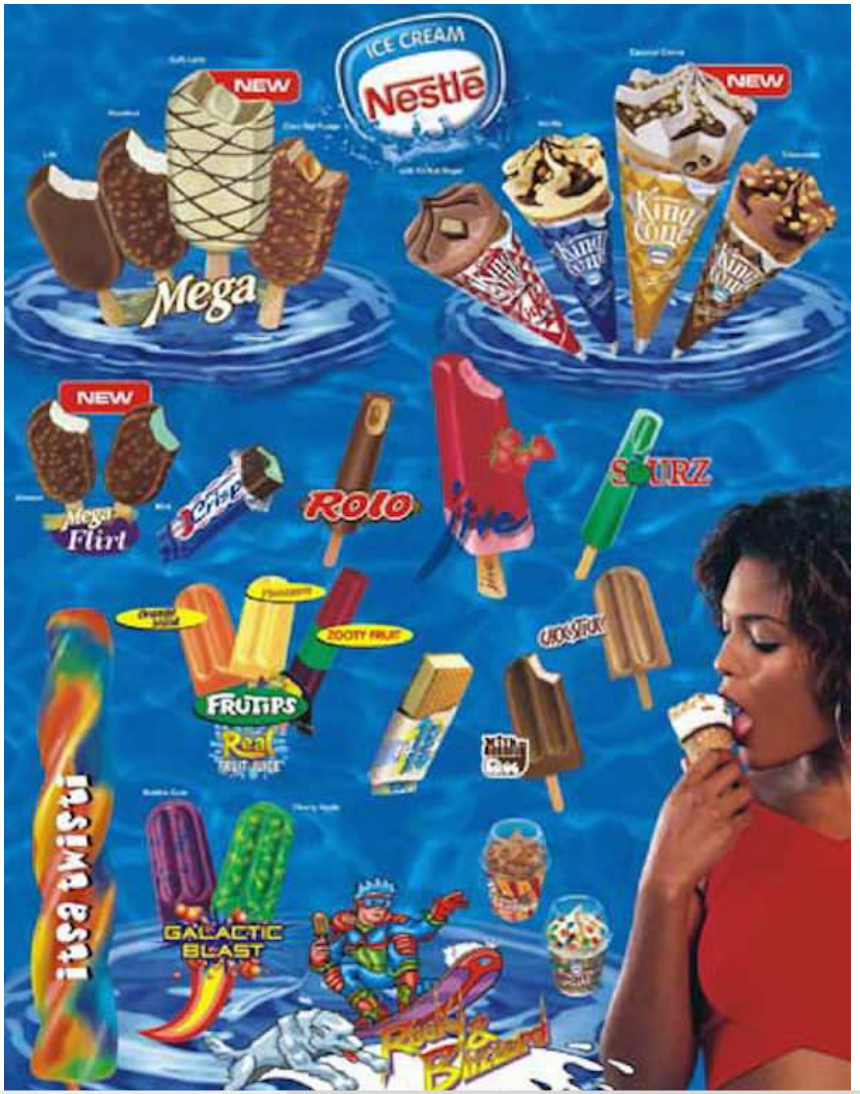
Dynamics of vehicle ownership

- PSA Peugeot Citroën
- Vehicle transactions model
- Changes in households vehicle ownership

Applications

Path to purchase: the case of ice creams

- Nestlé Research Center
- Impact of the design of the poster
- on the choice of ice cream



Applications

Automatic analysis of facial expressions

- Images and videos
- Signal Processing Lab
- Classification algorithm

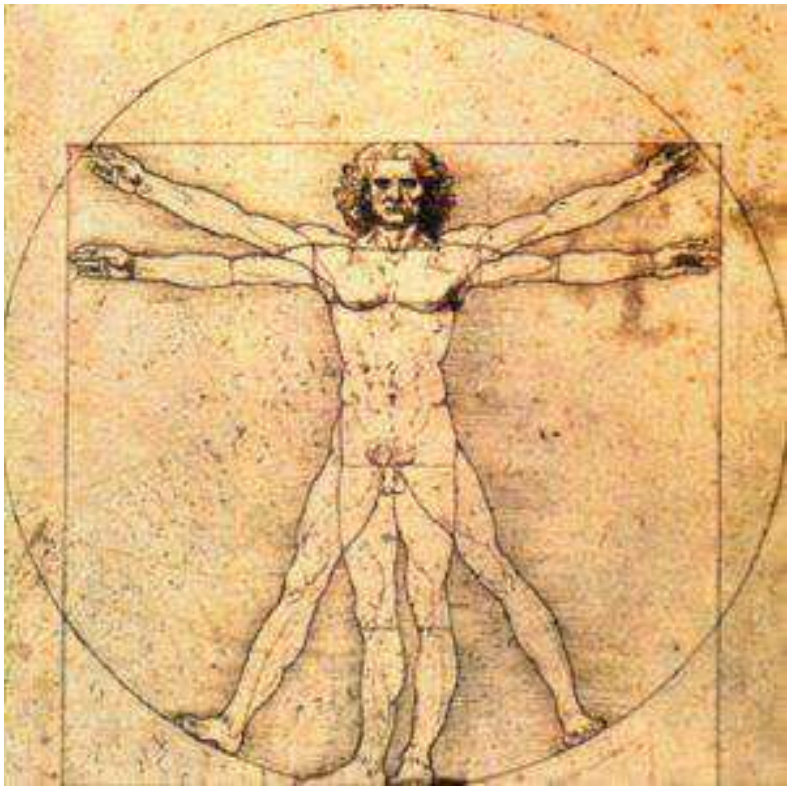
Importance

Daniel L. McFadden

- UC Berkeley 1963, MIT 1977, UC Berkeley 1991
- Laureate of *The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2000*
- Owns a farm and vineyard in Napa Valley
- "Farm work clears the mind, and the vineyard is a great place to prove theorems"

Outline

Homo economicus



Decision rule

Homo economicus

Rational and narrowly self-interested economic actor who is optimizing her outcome

Utility

$$U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a)$$

- captures the attractiveness of an alternative
- measure that the decision maker wants to optimize

Behavioral assumption

- the decision maker associates a utility with each alternative
- the decision maker is a perfect optimizer
- the alternative with the highest utility is chosen

Simple example: mode choice

Attributes

Alternatives	Attributes	
	Travel time (t)	Travel cost (c)
Car (1)	t_1	c_1
Bus (2)	t_2	c_2

Simple example: mode choice

Utility functions

$$\begin{aligned} U_1 &= -\beta_t t_1 - \beta_c c_1, \\ U_2 &= -\beta_t t_2 - \beta_c c_2, \end{aligned}$$

where $\beta_t > 0$ and $\beta_c > 0$ are parameters.

Equivalent specification

$$\begin{aligned} U_1 &= -(\beta_t/\beta_c) t_1 - c_1 &= -\beta t_1 - c_1 \\ U_2 &= -(\beta_t/\beta_c) t_2 - c_2 &= -\beta t_2 - c_2 \end{aligned}$$

where $\beta > 0$ is a parameter.

Choice

- Alternative 1 is chosen if $U_1 \geq U_2$.
- Ties are ignored.

Simple example: mode choice

Choice

Alternative 1 is chosen if

$$-\beta t_1 - c_1 \geq -\beta t_2 - c_2$$

or

$$-\beta(t_1 - t_2) \geq c_1 - c_2$$

Alternative 2 is chosen if

$$-\beta t_1 - c_1 \leq -\beta t_2 - c_2$$

or

$$-\beta(t_1 - t_2) \leq c_1 - c_2$$

Dominated alternative

- If $c_2 > c_1$ and $t_2 > t_1$, $U_1 > U_2$ for any $\beta > 0$
- If $c_1 > c_2$ and $t_1 > t_2$, $U_2 > U_1$ for any $\beta > 0$

Simple example: mode choice

Trade-off

- Assume $c_2 > c_1$ and $t_1 > t_2$.
- Is the traveler willing to pay the extra cost $c_2 - c_1$ to save the extra time $t_1 - t_2$?
- Alternative 2 is chosen if

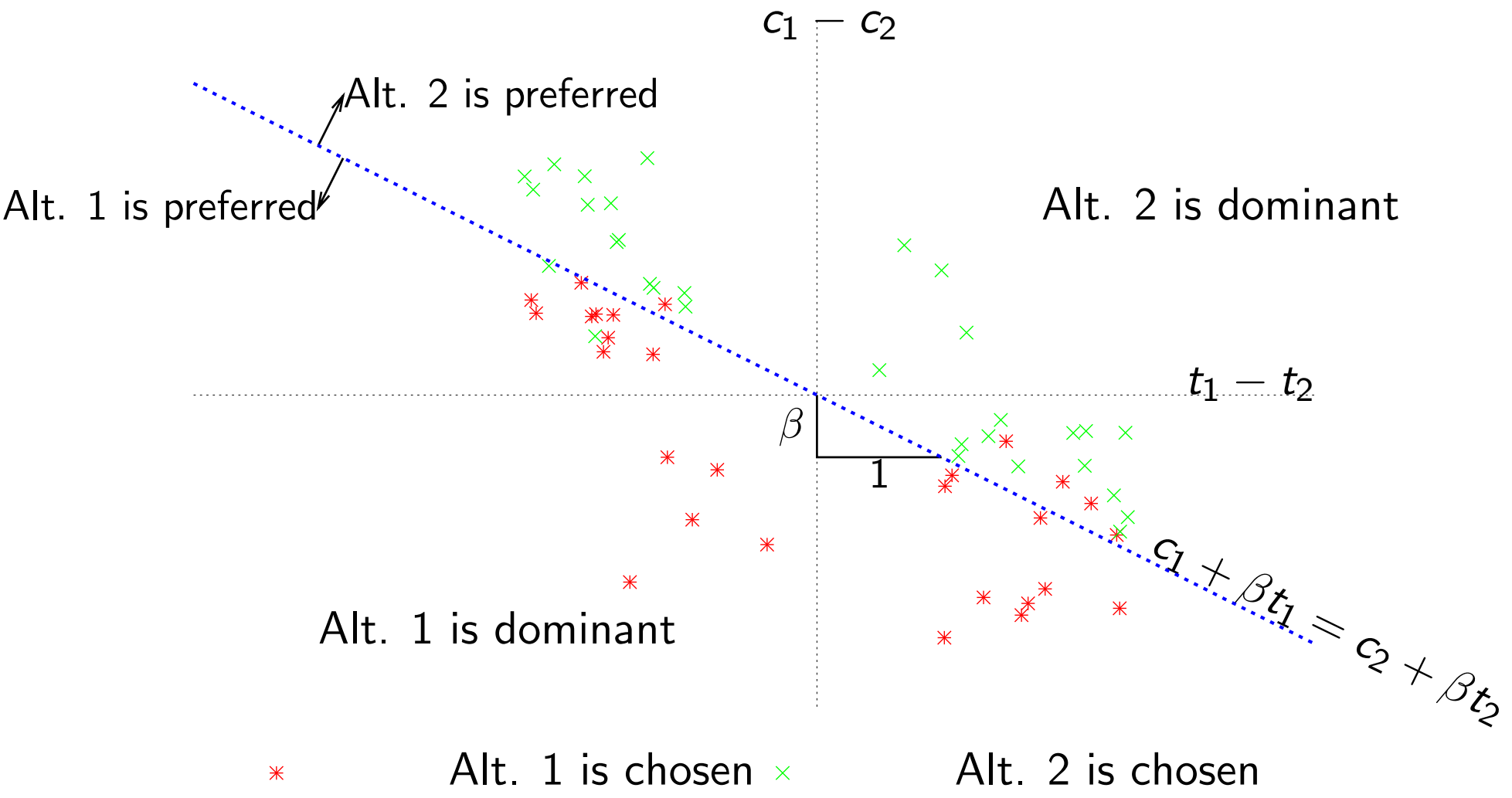
$$-\beta(t_1 - t_2) \leq c_1 - c_2$$

or

$$\beta \geq \frac{c_2 - c_1}{t_1 - t_2}$$

- β is called the *willingness to pay* or *value of time*

Simple example: mode choice



Random utility model

Random utility

$$U_{in} = V_{in} + \varepsilon_{in}.$$

The logit model

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in}}}{\sum_{j \in \mathcal{C}_n} e^{V_{jn}}}$$

Outline

Homo economicus?

Motivation

Rationality?

- Standard random utility assumptions are often violated.
- Factors such as attitudes, perceptions, knowledge are not reflected.

Example: pain lovers

Kahneman, D., Fredrickson, B., Schreiber, C.M., and Redelmeier, D., When More Pain Is Preferred to Less: Adding a Better End, *Psychological Science*, Vol. 4, No. 6, pp. 401-405, 1993.

- Short trial: immerse one hand in water at 14° for 60 sec.
- Long trial: immerse the other hand at 14° for 60 sec, then keep the hand in the water 30 sec. longer as the temperature of the water is gradually raised to 15°.
- Outcome: most people prefer the long trial.
- Explanation:
  - duration plays a small role
  - the peak and the final moments matter

Example: *The Economist*

Subscription to *The Economist*

Web only	@ $59
Print only	@ $125
Print and web	@ $125

Example: *The Economist*

Subscription to *The Economist*

Experiment 1	Experiment 2
Web only @ $59	Web only @ $59
Print only @ $125	
Print and web @ $125	Print and web @ $125

Example: *The Economist*

Subscription to *The Economist*

	Experiment 1	Experiment 2	
16	Web only @ $59	Web only @ $59	68
0	Print only @ $125		
84	Print and web @ $125	Print and web @ $125	32

Source: Ariely (2008)

- Dominated alternative
- According to utility maximization, should not affect the choice
- But it affects the perception, which affects the choice.

Example: good or bad wine?

Choose a bottle of wine...

	Experiment 1	Experiment 2
1	McFadden red at \$10	McFadden red at \$10
2	Nappa red at \$12	Nappa red at \$12
3		McFadden special reserve pinot noir at \$60
	Most would choose 2	Most would choose 1

- Context plays a role on perceptions

Example: live and let die

Population of 600 is threatened by a disease. Two alternative treatments to combat the disease have been proposed.

Experiment 1 # resp. = 152	Experiment 2 # resp. = 155
Treatment A: 200 people saved	Treatment C: 400 people die
Treatment B: 600 people saved with prob. 1/3 0 people saved with prob. 2/3	Treatment D: 0 people die with prob. 1/3 600 people die with prob. 2/3

Example: live and let die

Population of 600 is threatened by a disease. Two alternative treatments to combat the disease have been proposed.

	Experiment 1 # resp. = 152	Experiment 2 # resp. = 155	
72%	Treatment A: 200 people saved	Treatment C: 400 people die	22%
28%	Treatment B: 600 people saved with prob. 1/3 0 people saved with prob. 2/3	Treatment D: 0 people die with prob. 1/3 600 people die with prob. 2/3	78%

Source: Tversky & Kahneman (1986)

Example: to be free

Choice between a fine and a regular chocolate

	Experiment 1	Experiment 2
Lindt	\$0.15	\$0.14
Hershey	\$0.01	\$0.00
Lindt chosen	73%	31%
Hershey chosen	27%	69%

Source: Ariely (2008) *Predictably irrational*, Harper Collins.

Operational models

Behavioral aspects

- Attitudes
- Habits
- Perceptions
- Social norms
- etc.

Modeling framework

- Random utility
- Latent variables

Data

- Choice data
- Psychometrics

Outline

Choice data

Revealed preferences

- actual choice observed
- in real market situations
- Example: scanner data in supermarkets

Stated preferences

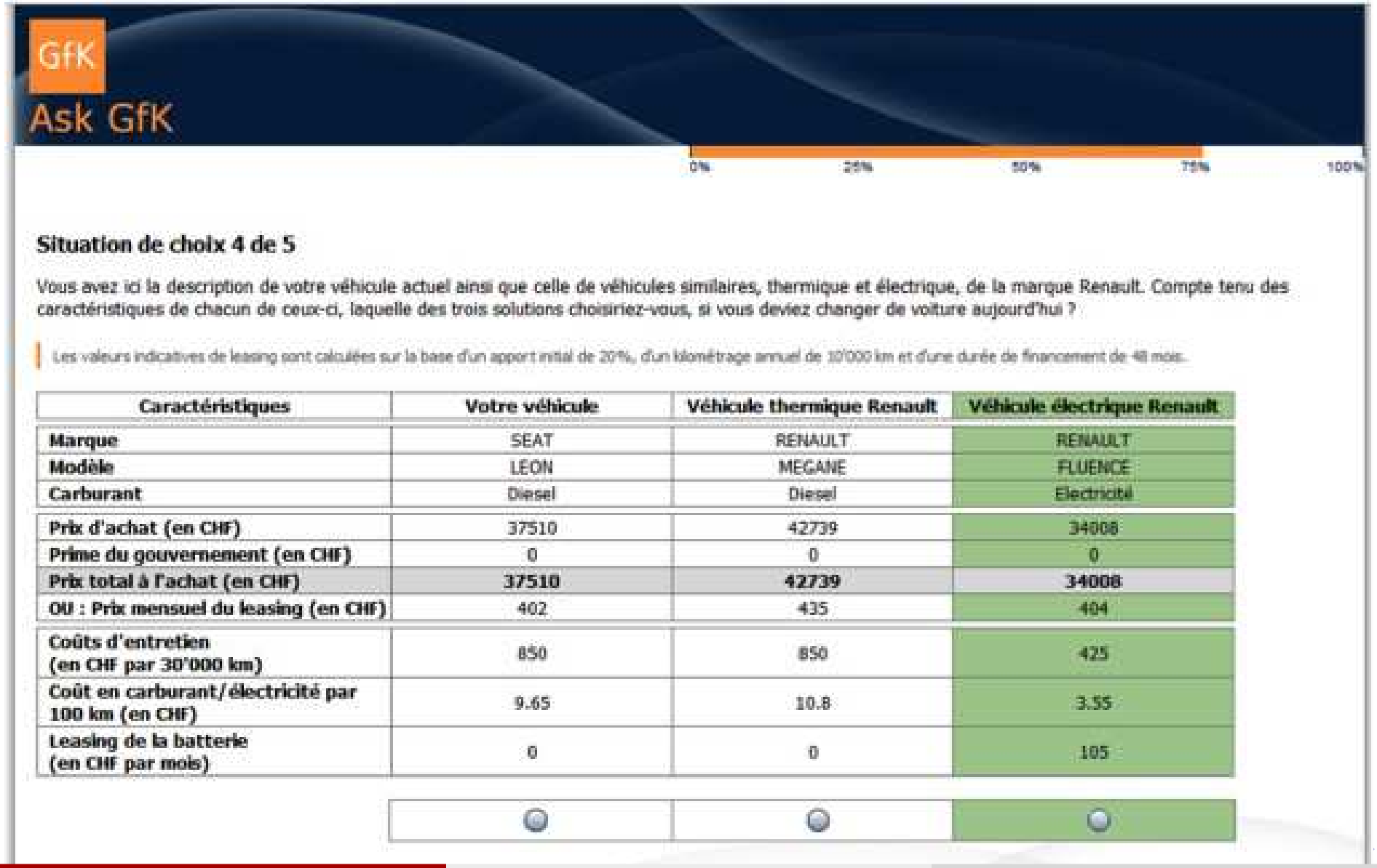
- hypothetical situations
- attributes defined by the analyst

Data

Questionnaires

- Data about the respondent
- Choice data
- Revealed preferences
- Stated preferences

Data: example of a questionaire



GfK

Ask GfK

0% 25% 50% 75% 100%

Situation de choix 4 de 5

Vous avez ici la description de votre véhicule actuel ainsi que celle de véhicules similaires, thermique et électrique, de la marque Renault. Compte tenu des caractéristiques de chacun de ceux-ci, laquelle des trois solutions choisiriez-vous, si vous deviez changer de voiture aujourd'hui ?

Les valeurs indicatives de leasing sont calculées sur la base d'un apport initial de 20%, d'un kilométrage annuel de 10'000 km et d'une durée de financement de 48 mois.

Caractéristiques	Votre véhicule	Véhicule thermique Renault	Véhicule électrique Renault
Marque	SEAT	RENAULT	RENAULT
Modèle	LEON	MEGANE	FLUENCE
Carburant	Diesel	Diesel	Electricité
Prix d'achat (en CHF)	37510	42739	34008
Prime du gouvernement (en CHF)	0	0	0
Prix total à l'achat (en CHF)	37510	42739	34008
OU : Prix mensuel du leasing (en CHF)	402	435	404
Coûts d'entretien (en CHF par 30'000 km)	850	850	425
Coût en carburant/électricité par 100 km (en CHF)	9.65	10.8	3.55
Leasing de la batterie (en CHF par mois)	0	0	105

Data

Smartphones

- GSM, GPS
- Accelerometer
- WiFi
- Bluetooth
- Ambient sound
- And more...

Data

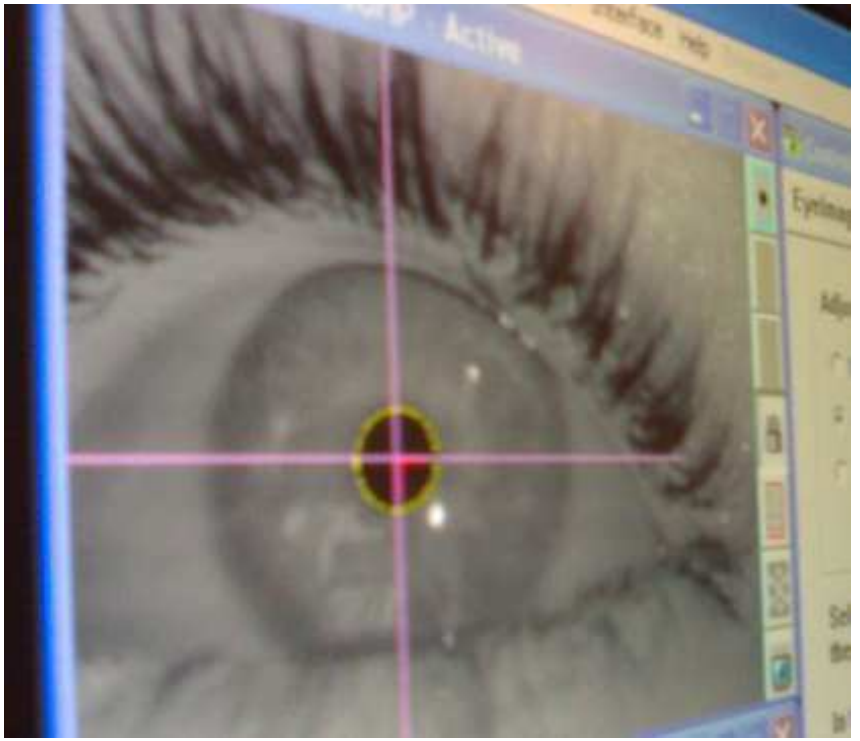
Scanner data

- Detailed purchase information
- Personalized

Data

Eye tracking

- Where do people look?
- Used in marketing research
- Used in driving safety research
- Relevant for pedestrian models



Data: eye tracking

Movie: Nestlé data collection

Outline

Market shares of electrical vehicles

Glerum, A., Stankovikj, L., Thmans, M., and Bierlaire, M. (to appear)

Forecasting the demand for electric vehicles: accounting for attitudes and perceptions, *Transportation Science* (accepted for publication on May 29, 2013)

Objectives

Demand analysis for two electrical vehicles: Zoe & Fluence (Renault)

Sample

Target groups

Sampling from

- Recent buyers
- Prospective buyers
- Renault customers

Everybody from

- Pre-orders
- Z. E. newsletter

Sampling protocol: representative for

- 3 language regions of Switzerland (German, French, Italian)
- Gender
- Age category (18–35, 36-55, 56–74)

Sample

High response rate - possibility to segment

Group name	Sent	Phase I		Phase II		Phase I vs phase II
		Number	Rate	Number	Rate	Rate
Recent buyers	3006	150	10.0%	141	9.4%	94.0%
Prospective buyers		151		141		93.4%
Renault customers	1000	145	14.5%	120	12.0%	82.8%
Pre-orders	42	23	54.8%	19	45.2%	82.6%
Z.E. newsletter	656	197	30.0%	172	26.2%	87.3%
Total	4704	666	14.2%	593	12.6%	89.0%

Sample

Unbalanced sample (gender): need for corrections

Variable	Level	Targeted rate	Rate phase I	Rate phase II
Language	German	72.5%	67.3%	67.8%
	French	23.0%	27.2%	26.6%
	Italian	4.5%	5.6%	5.6%
Gender	Male	49.4%	74.0%	74.2%
	Female	50.6%	26.0%	25.8%
Age category	18-35 years	33.6%	23.0%	21.8%
	36-55 years	41.6%	51.8%	52.6%
	56-74 years	24.8%	25.2%	25.6%

Survey

Phase I

- Characteristics of car(s) of respondents household
- Socio-economic information
- Mobility habits

Phase II

- Opinions and perceptions on topics related to EV
- Choice situations
- Willingness-to-pay
- Interest in additional services

Design of the choice experiment

EV variable		Level 1	Level 2	Level 3	Level 4
Purchase price	< 55 KCHF	$(P_{gasoline} + 5'000) * 0.8$	$(P_{gasoline} + 5'000) * 1$	$(P_{gasoline} + 5'000) * 1.2$	-
	≥ 55 KCHF	$(P_{Mégane} + 5'000) * 0.8$	$(P_{Mégane} + 5'000) * 1$	$(P_{Mégane} + 5'000) * 1.2$	-
Governmental incentive		- 0 CHF	- 500 CHF	- 1'000 CHF	- 5'000 CHF
Cost of fuel/electricity for 100 km		1.70 CHF	3.55 CHF	5.40 CHF	-
Battery lease		85 CHF	105 CHF	125 CHF	-

Segmentation

A priori higher interest for EV and/or Renault

- Pre-orders (1)
- Subscribers of the Z.E. newsletter (2)

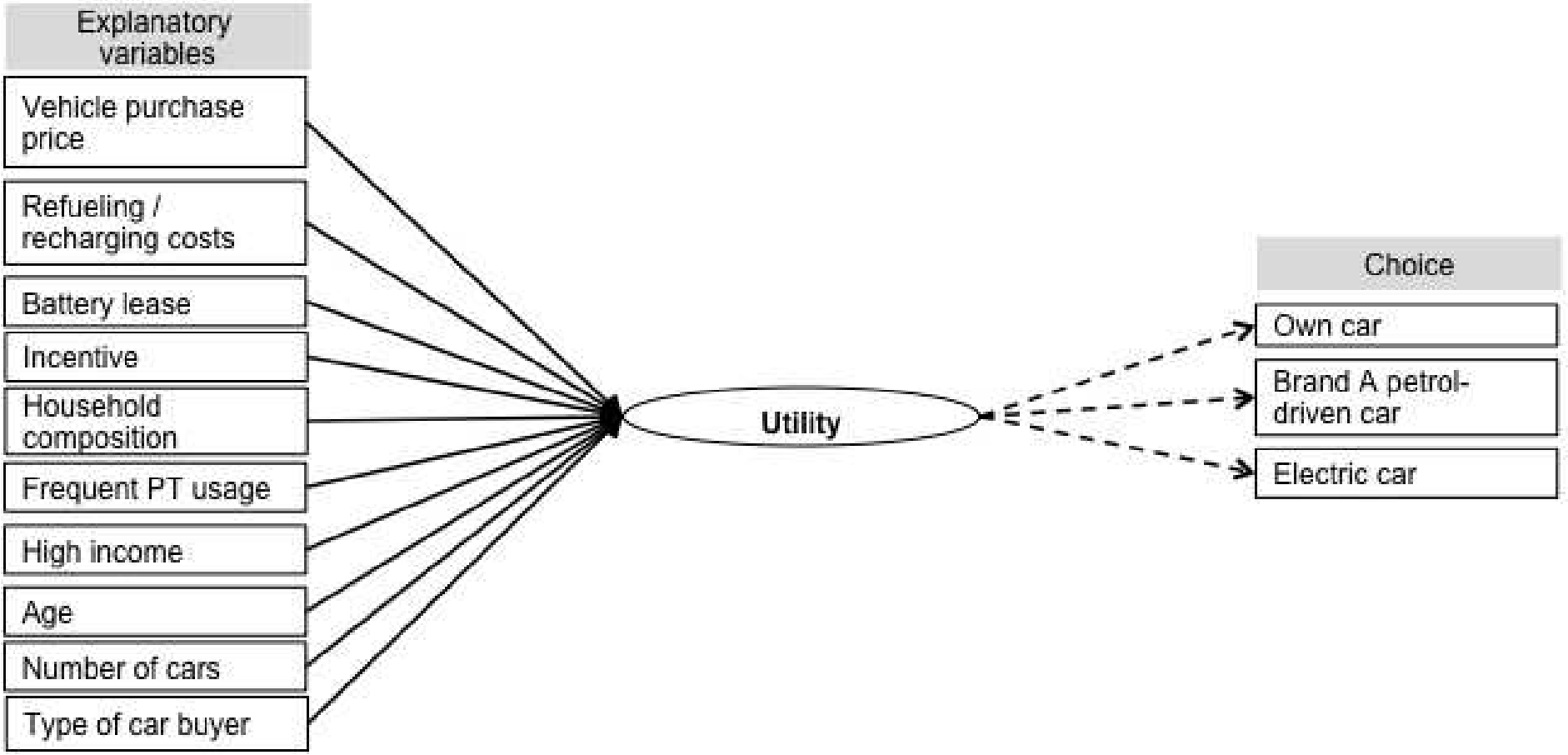
A priori interest in Renault

- Renault customers (3)

No a priori interest for EV and/or Renault

- Recent buyers (4)
- Prospective buyers (5)

Model specification



Parameter estimates

Utilities	Competitor – Gasoline (CG)	Renault – Gasoline (RG)	Renault – Electric (RE)
-0.0212**	Prix CG	-	-
-0.211	-	Price RG · TG1245	-
-0.598	-	Price RG · TG3	-
-0.404	-	-	Price RE · TG12
-1.00	-	-	Price RE · TG3
-0.628	-	-	Price RE · TG45
-0.049**	Operating cost gasoline	Operating cost gasoline	-

Utilities	Competitor – Gasoline (CG)	Renault – Gasoline (RG)	Renault – Electric (RE)
-0.252	-	-	High operating cost · Fluence
-0.778	-	-	High operating cost · Zoé
-0.447	-	-	Medium operating cost · Zoé
-0.205*	-	-	High battery lease
-0.0539**	-	-	Medium battery lease
0.73	-	-	High incentive
0.0803**	-	-	Medium incentive
-0.00224**	-	-	Low incentive

Parameter estimates

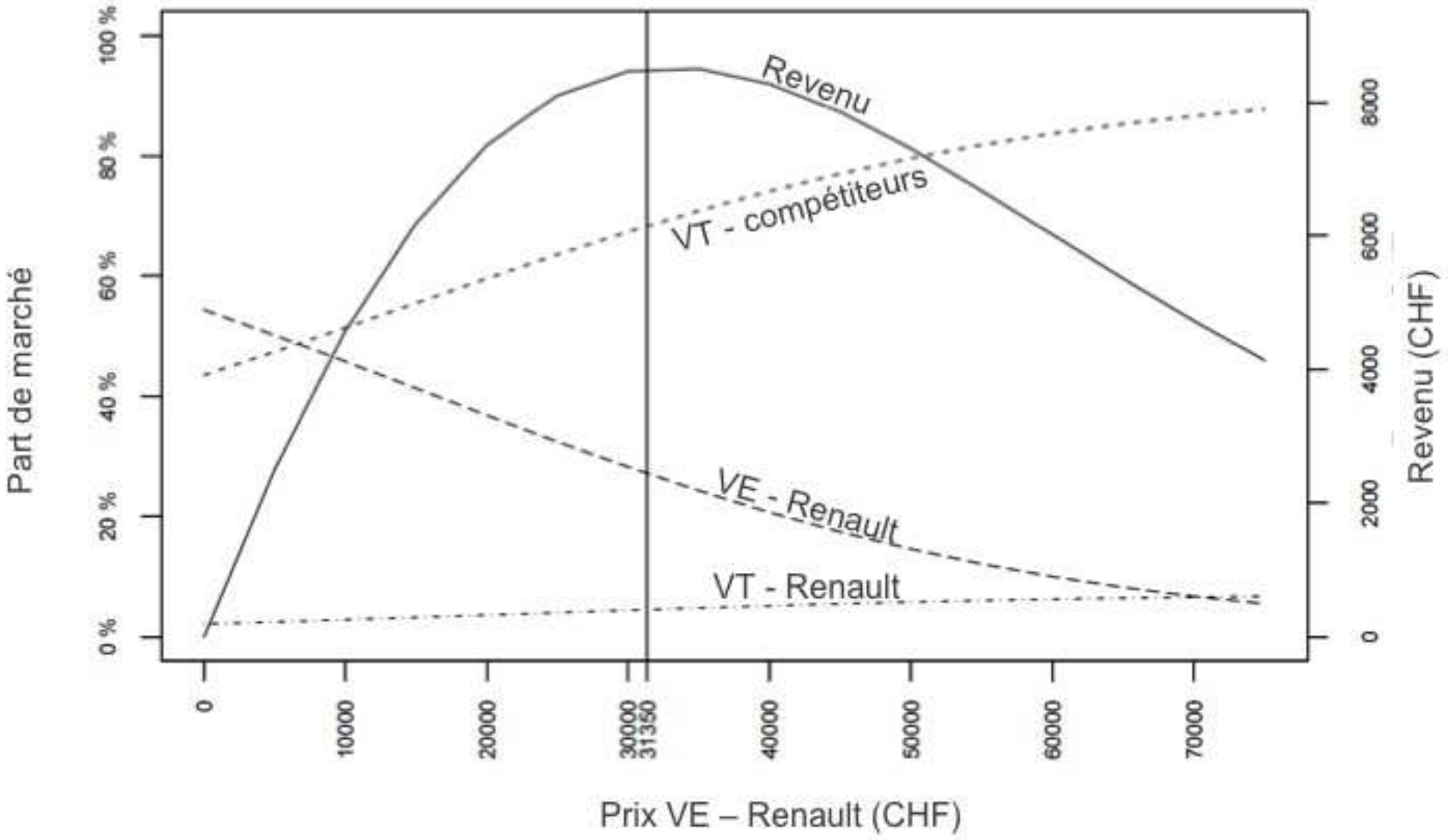
Utilities	Competitor – Gasoline (CG)	Renault – Gasoline (RG)	Renault – Electric (RE)
-0.279	PT · TG1245	-	-
-0.552	-	PT · TG1245	-
-1.85	PT · TG3	-	-
-1.07	-	PT · TG3	-
-0.217	Family with children	-	-
0.0454**	-	Family with children	-
-0.25	Income	-	-
-0.297	-	Income	-

Utilities	Competitor – Gasoline (CG)	Renault – Gasoline (RG)	Renault – Electric (RE)
-0.172	Nb cars · TG1245	-	-
-0.157	-	Nb cars · TG1245	-
-0.384**	Nb cars · TG3	-	-
-0.729	-	Nb cars · TG3	-
0.335	French	-	-
0.0876**	-	French	-
0.0124	Age	-	-
-0.00187**	-	Age	-

Parameter estimates

Utilities	Competitor – Gasoline (CG)	Renault – Gasoline (RG)	Renault – Electric (RE)
1.97	TG12	-	-
1.04	-	TG12	-
-0.635	TG3	-	-
2.45	-	TG3	-
-2.12	1	-	-
-1.67	-	1	-

Market shares and revenues



Outline

Value of time in Switzerland

Axhausen, K., Hess, S., Koenig, A., Abay, G., Bates, J., and Bierlaire, M. (2008)

Income and distance elasticities of values of travel time savings: new Swiss results, *Transport Policy* **15**(3):173-185.

Data collection

- Source for recruitment: survey "Kontinuierliche Erhebung zum Personenverkehr" (KEP) by SBB/CFF
- Stated preferences
- Questionnaire designed based on a real reference trip
- Three parts:
  - SP mode choice (car / bus or rail)
  - SP route choice (current mode or alternative mode)
  - Socio-demographics and information about the reference trip

Value of time in Switzerland

Mode choice car – rail (main study version)

Travel costs:	18 Fr.	Travel costs:	23 Fr.
Total travel time:	40 minutes	Travel time:	30 minutes
... congested:	10 minutes	Headway:	30 minutes
... uncongested:	30 minutes	No. of changes:	0 times

☐ ← Your choice → ☐

Route choice rail (main study version)

Travel costs:	20 Fr.	Travel costs:	23 Fr.
Travel time:	40 minutes	Travel time:	30 minutes
Headway:	15 minutes	Headway:	30 minutes
No. of changes:	1 times	No. of changes:	0 times

Value of time in Switzerland

Number of observations (1225 individuals)

	Business	Commuters	Leisure	Shopping	Total
Mode : car/bus	6	162	186	126	480
Mode : car/rail	426	1716	2538	1104	5784
Route : bus for bus users	9	405	450	342	1206
Route : car for car users	156	846	1176	660	2838
Route : rail for car users	126	594	837	504	2061
Route : rail for rail users	324	1008	1881	288	3501
Total	1047	4731	7068	3024	15870

Value of time in Switzerland

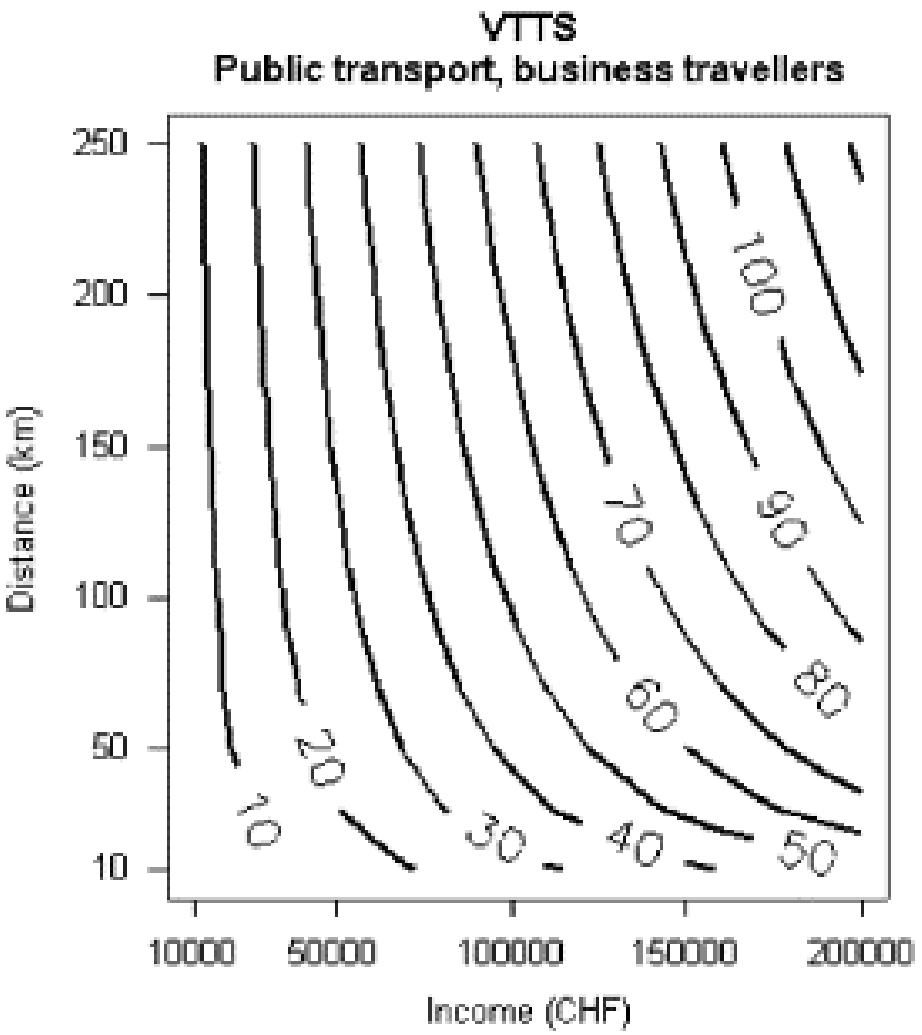
Explanatory variables

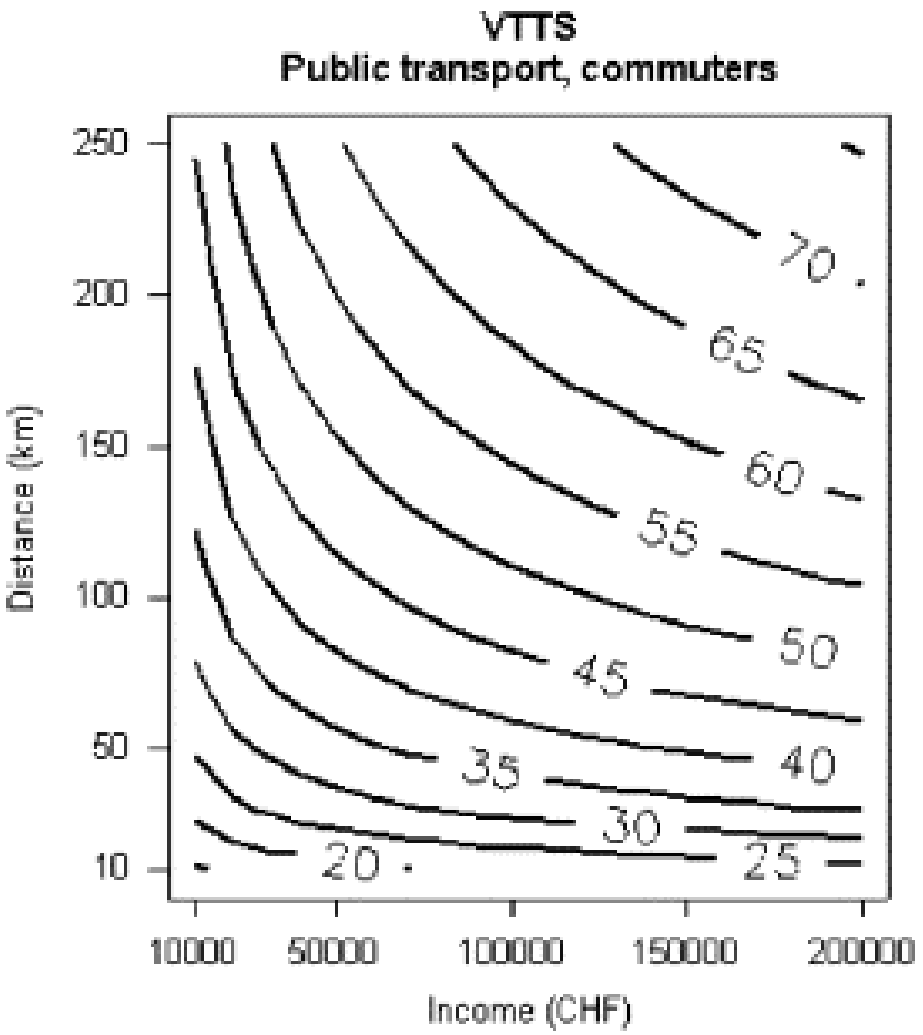
- travel time
- travel cost
- level of congestion (car)
- frequency (TC)
- number of transfers (TC)
- trip length
- income
- inertia
- car availability
- sex
- 1/2-fare CFF
- general subscription
- trip purpose

Value of time in Switzerland

	Business	Commute	Leisure	Shopping
Time TC (CHF/h)	49.57	27.81	21.84	17.73
Time car (CHF/h)	50.23	30.64	29.20	24.32
Headway (CHF/h)	14.88	11.18	13.38	8.48
CHF/transfer	7.85	4.89	7.32	3.52

Value of time in Switzerland





Value of time in Switzerland

Value of time varies (namely) with

- transportation mode,
- trip purpose,
- income,
- trip length.

Outline

Path to purchase: the case of ice-cream

Collaboration Nestlé-EPFL

- 2006–2008
- Nestlé
  - Nestlé Research Center
  - Ice cream Business Unit
- EPFL
  - Transport and Mobility Laboratory (Prof. Bierlaire)
  - Signal Processing Laboratory (Prof. Thiran)

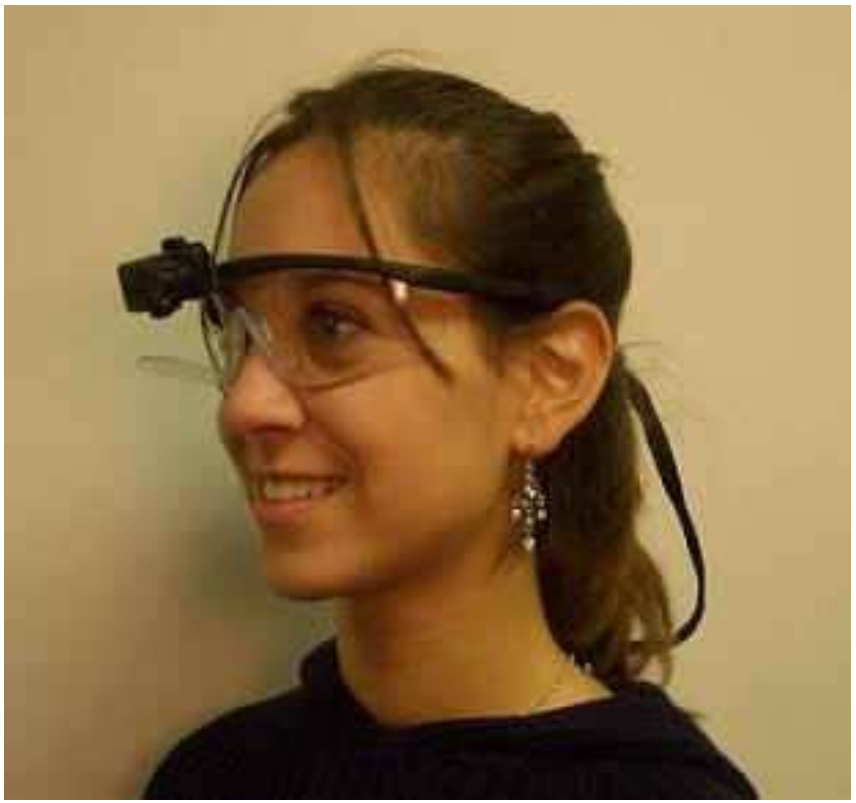
Path ro purchase

Project

- Impact of the stimuli on the consumers behavior
- Example: design of an ice cream board

Data collection

Eye tracking

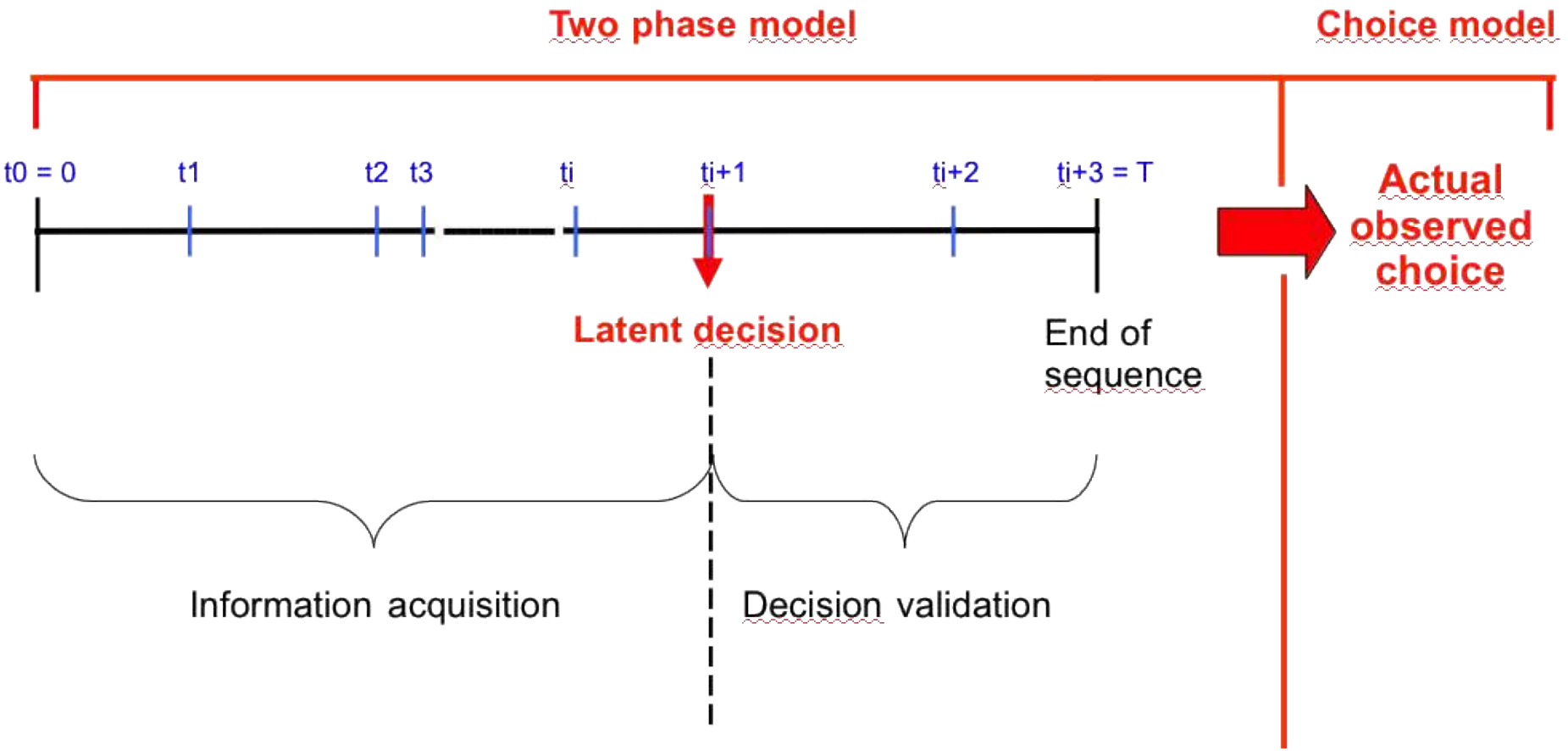


Data processing

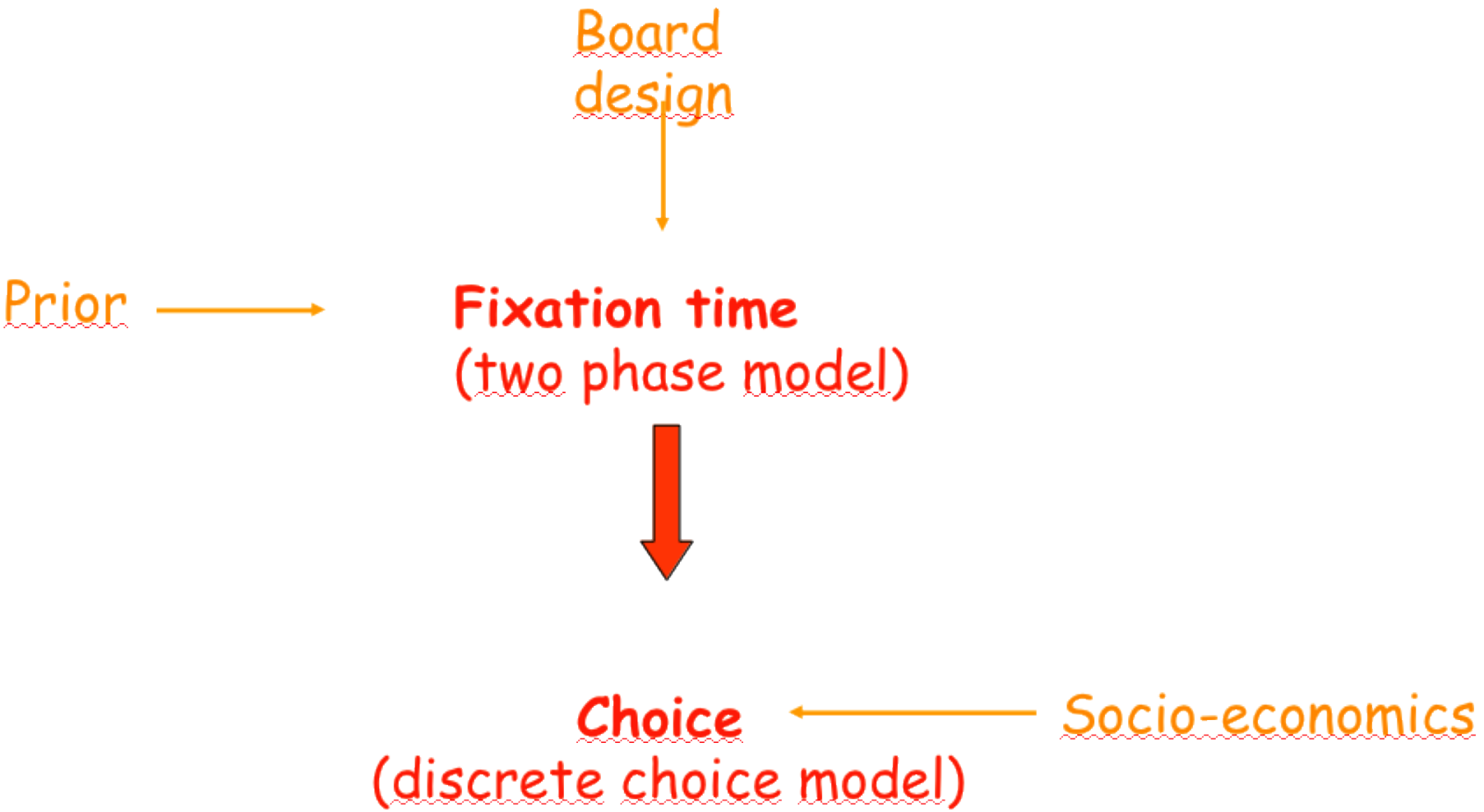
From raw video to numerical data

- Movie: Original video
- Movie: Correct distortions
- Identify locations

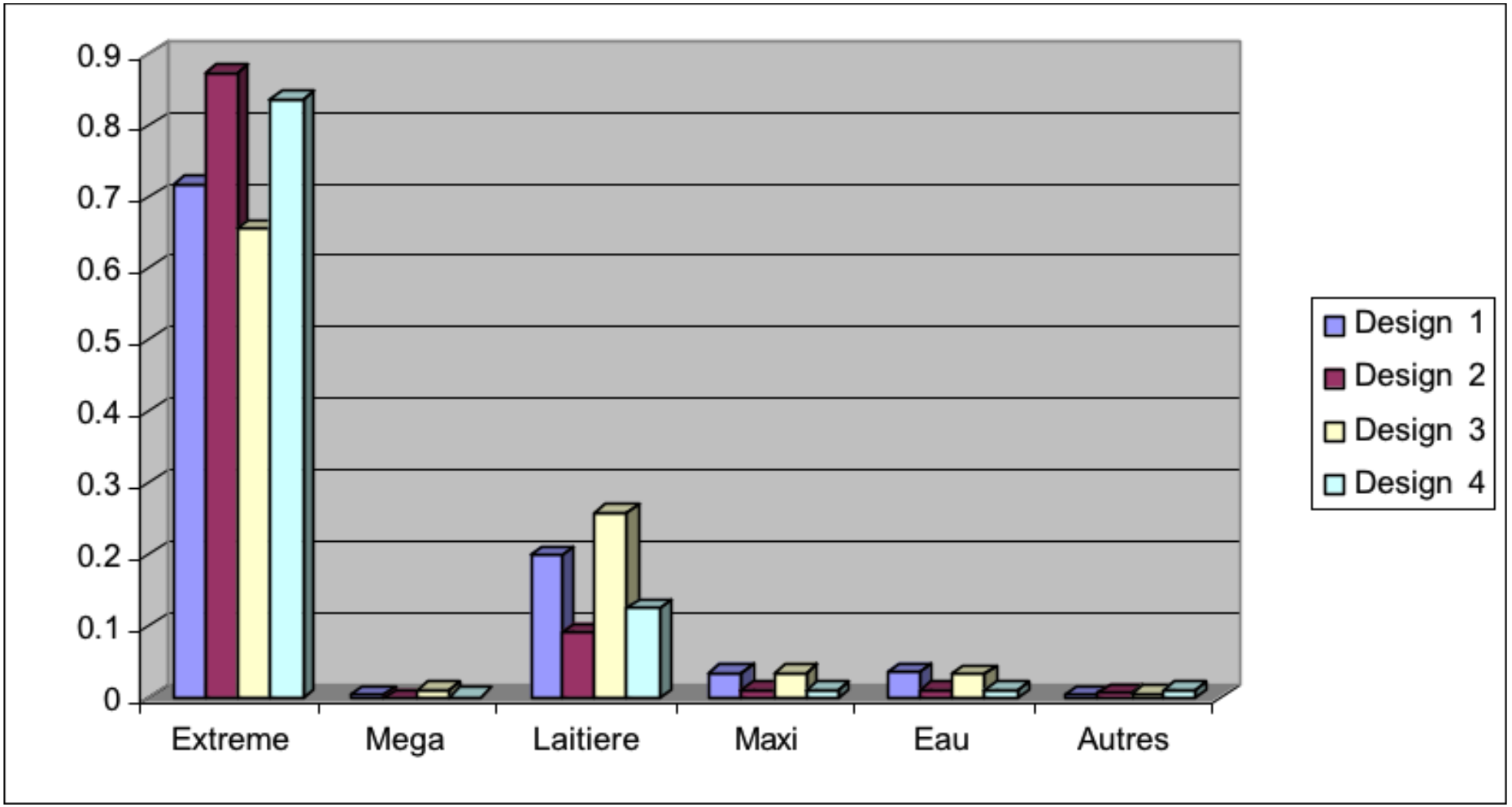
The model


Two phase model
Choice model
t0 = 0
t1
t2
t3
ti
ti+1
ti+2
ti+3 = T
Actual observed choice
Latent decision
End of sequence
Information acquisition
Decision validation

The model


Board design
Prior
Fixation time
(two phase model)
Choice
(discrete choice model)
Socio-economics

Results


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Extreme
Mega
Laitiere
Maxi
Eau
Autres
Design 1
Design 2
Design 3
Design 4

Outline

Conclusion

Behavioral models

- Individual choice model
- Disaggregate market segments
- Flexible specification
- Quantitative and qualitative variables
- Usage of revealed and stated preferences data
- Wide range of applications
- Can account for subjectivity (attitudes and perceptions)

Short course: Discrete Choice Analysis: Predicting Demand and Market Shares

March 23 – 27, 2014

- Ecole Polytechnique Fédérale de Lausanne
- Prof. Ben-Akiva (MIT)
- Prof. Bierlaire (EPFL)
- Prof. McFadden (UC Berkeley)
- Prof. Walker (UC Berkeley)
- `transp-or.epfl.ch/dca`