Crime, wealth, and protection: a theory and Canadian evidence

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Abstract: In Canada, rich neighborhoods are less victimized than poor ones, rich people are more victimized at given neighborhood, and households in rich neighborhoods invest more in private protection. We provide a theoretical explanation to these facts. Our model emphasizes the notion of imperfect observability of private protection. A fixed supply of criminals decide which neighborhood to enter, and how much they spend to compare potential victims. Households differ in wealth and choose the amount of self-protection. The model features strategic complementarity between criminals' efforts and households' protection investments. In rich neighborhoods, households may enter a rat race to protection, which drives criminals towards poorer areas. The implications of our model are tested using data from the Canadian Victimization Survey.

Keywords: Economics of crime; Search frictions; Private protection; Inequality

J.E.L.: K14; K42

1 Introduction

The purpose of this paper is to provide a theoretical explanation to a set of four Canadian facts relating property crime, wealth, and protection. Such facts are displayed by Figure 1. First, there is no correlation between income and victimization. Second, richer neighborhoods are less victimized than poorer ones. Third, rich people are more victimized than their neighbors. Finally, people in rich neighborhoods invest more in individual protection. The explanation hinges on imperfect observability of individual protection effort in a context where criminals choose which neighborhood they prospect. Of course, criminals are more attracted to rich neighborhoods. However, to divert criminals' attention towards their neighbors, rich people enter a rat race to private protection. This rat race is more intense in richer neighborhoods. It may become so intense that criminals actually prefer to enter poorer neighborhoods. We propose a formal model of this scenario, and test its main predictions on Canadian micro data.

Fig.1

Although our attention is restricted to Canadian data, our paper discusses ideas with a more widespread application. The correlation between individual wealth and individual victimization is indeed very volatile throughout the world. Figure 2 uses data from the xxx survey for a selection of thirteen OECD countries. It shows the ratio of the victimization probability for more-than-median income to the victimization probability for less-than-median income. This ratio goes from 80% in Sweden to 190% in Portugal. For seven countries, the ratio lies between 93% and 108%. One possible rationale behind such volatility is that criminals are imperfectly mobile between neighborhoods, whereas they are over-represented in poorer neighborhoods. This argument, however, does not account for the massive reduction of mobility costs over the past decades. It also fails to explain why rich neighborhoods are more protected. In our approach, criminals are not attached to a particular location, and non-cooperative protection investments alter the geography of property crime.

Fig.2

Section 2 describes a model where a fixed number of criminals allocate across heterogenous neighborhoods, whereas households make protection investments. Our model is based on two fundamental principles. First, protection investments are decided in a non-cooperative way. Second, protection investments are imperfectly observable. More particularly, we assume that there are search frictions. Once in a neighborhood, criminals randomly select one of the households. They can also make a search effort to draw an additional potential victim. In such a case, they compare both theft opportunities and select the one with the highest payoff. In turn, criminal effort motivates protection investment. When criminals search intensively, each household has a high chance to be compared to a neighbor, which increases the return to self-protection. Our model thus features strategic complementarity between criminals' search efforts and households' protection investments. This form of strategic complementarity has several implications. On the one hand, similar households make different protection investments in equilibrium. We show there is a continuous distribution of investment by income level. On the other hand, there may be multiple equilibria. High criminal effort / high protection equilibria coexist with low criminal effort / low protection equilibria.

The most important prediction of the model is the following. In the absence of protection investments, richer neighborhoods are more attractive to criminals. However, residents in such neighborhoods also invest more in protection, which tends to repulse criminals. The reason why rich neighborhoods invest more in protection relies on criminals' incentive to make search effort. Criminals in rich neighborhoods are more willing to spend resources to compare potential victims. Thus rich households expect to be frequently compared to their neighbors. The dispersion of protection investment increases as a result, which encourages criminals to search even more. That richer neighborhoods invest more in protection implies lower returns to crime, and therefore criminals are attracted to poorer neighborhoods. Put otherwise, our model can predict 1) within a neighborhood, wealthier households are more victimized than poorer households, 2) rich and diversified neighborhoods may be less victimized than poor and less diversified ones, and 3) private protection is higher and more dispersed in wealthier neighborhoods.

In Section 3, we test the three main predictions that we just mentioned. We use three waves of the Canadian Victimization Survey, which we match with Census Data. We jointly estimate the probability of making protection investment, and the probability of being victimized. We show that...

The ambiguity of the relationship between wealth and protection is entirely driven by *imperfect observability* of *private protection.* It is not due to black box technological assumptions. For instance, we assume that protection costs are proportional to property value, which rules out cases where rich households find it easier to invest in protection (this would happen with fixed costs of protection). Similarly, it is not due to cooperative behavior at neighborhood level. Individual households do not account for the fact that protection investments deter criminals from prospecting the neighborhood. This is only the protection game played by neighbors that leads to an escalation in protection investments, which ultimately drives criminals out of the neighborhood. Of course, it is true that such cooperative behavior exists in the US. However, it is far from being universal. For instance, it is much less widespread in Canada.

Our way to model private protection is standard in the literature. Private protection has three main effects. The first effect is to make one's belongings less attractive to criminals; this is known as the crime diversion effect. The second effect is to reduce what is stolen in case of victimization. This is known as the theft reduction effect. Last is the deterrence effect: When private protection increases, crime is less lucrative, and therefore the entry into criminal activity drops. In our framework, the total amount of crime is unchanged, but criminals choose a different neighborhood. The deterrence effect is not taken into account by individual households. It is already known that the crime diversion effect can lead to an over-provision of private protection when private protection is observable (see Shavell, 1992, for an early discussion, and Hotte and van Ypersele, 2008, for a normative analysis). In this literature, the degree of observability of private protection is exogenous. In the current paper, the degree of observability is endogenous, and this endogeneity is key to obtain strategic complementarity between criminals' search effort and households' protection investments.

Our paper also contributes to the explanation of the volatility of crime across geographic locations. According to Glaeser, Sacerdote and Scheinkman (1996), "the high degree of variance of crime rates across space and time is one of the oldest puzzles in the social sciences". Different determinants of crime have been analyzed: Glaeser and Sacerdott (1999) look at the impact of the size of a city on victimization. They show that big cities have a higher crime rate. Levitt (1999) shows that inner city is more victimized than the suburbs. However, Glaeser et al (1996) argue that the variance of crime rates is way too important to be explained by observed or unobserved geographical attributes. They put forward a social multiplier: individuals are more prone to engage in criminal activities when their peers also adopt a criminal behavior (see also Zenou, 2003, who also discusses the role played by the distance to jobs). We propose an original multiplier effect based on the interaction between private protection and criminals' searching and sorting activities.

Our model is an application of search theory to crime. Thus our results find echoes in search papers devoted to other market situations. Our way to model criminal activities borrows from Stigler (1965) who focuses on the good market. In his model, customers decide on the number of buy offers they sample from the distribution of offer prices. Stigler shows that there is price dispersion as a result (see also Acemoglu and Shimer, 2000, for an application to the labor market). That there is a rat race to protection investment in our framework is close to Moen (1999) who considers a matching model with ex-ante human capital investments. Firms may receive multiple application and select the most educated workers. This gives a positional good component to education, and Moen shows that there is equilibrium dispersion in educational investment. One of the differences a crime model has with labor market models is that a household may be victimized by several criminals (which occurs in our model), whereas a job can only be occupied by a single worker. Congestion is more natural in the labor market: increasing the number of job-seeker reduces the chances of finding a job for each of them. In our model, there is no congestion per se. However, protection investments increase with the number of criminals, which reduces the return to property crime in a given neighborhood.

Finally, there already exist search models of crime, but they do not focus on protection. Burdett and Wright provide a model in which the entry into criminal activity and the distribution of wages are jointly determined... **To be followed.**

2 Property crime and protection: theory

2.1 The model

Model assumptions.—The model is static. There are N neighborhoods indexed by $j \in \{1, ..., N\}$. Each neighborhood is composed of K_j residents. The proportion $\mu_j^P = 1 - \mu_j$ are poor, and have belongings worth V_j^P, whereas the proportion $\mu_j^R = \mu_j$ of these residents are rich, and have belongings worth $V_j^R = V_j^P + \Delta_j > V_j^P$. The wealth difference Δ is sufficiently large, a statement that will be qualified later.

There are also P criminals. They have limited mobility across neighborhoods. Each criminal must choose a neighborhood and then commits one crime in the neighborhood. The number of criminals in neighborhood j is C_j. Thus the crime rate in neighborhood j is $c_j = C_j/K_j$. By definition, we have $\sum_{j=1}^{N} C_j = P$.

If visited by a criminal, a resident of wealth V endures the cost γV. However, residents choose the level of self-protection $\theta \geq 0$. This comes at the cost θV. In exchange for such a cost, they inflict the damage θV to the potential criminal. This one-for-one technology is chosen to save on the number of parameters. We denote by H_j^i the equilibrium distribution of protection efforts in the type-i population of neighborhood j. We also denote by $\Theta^i \subset \mathbb{R}_+$ the support of this distribution.

A criminal who visits a resident of wealth V and protection θ obtains $(\alpha - \theta)V$. The cost of a robbery for a resident may be lower or larger than what is actually stolen by the criminal, i.e. α may be lower or larger than γ. Households may be insured, and consequently they suffer smaller losses. They may also suffer larger losses because part of the wealth is destroyed during the robbery, or because of the psychological cost associated with the crime. Note that under all these interpretations, total losses can be larger than initial wealth.

Wealth and protection are imperfectly observable. A criminal who plans a robbery in neighborhood j is presented with one theft opportunity at random. S/he then observes the value of the potential victim's belongings V, and the protection level θ. S/he may also benefit from a second opportunity, but this depends on search effort q. This effort comes at the cost $s(q)$. In exchange, the criminal benefits from the probability q of having another theft opportunity. The search cost function is strictly increasing, twice differentiable, with $s'(\cdot) > 0$, $s''(\cdot) > 0$, $s(0) = 0$, $s'(0) = 0$ and $s'(1) \to \infty$.

We denote by C_{j1} the expected number of criminals in neighborhood j who have a single theft option, and by C_{j2} the expected number of criminals who have two options. Similarly, c_{j1} and c_{j2} denote the ratio of expected number of criminals to households.

Model agenda.—The timing is as follows.

Stage 1. Mobile criminals choose which neighborhood $j \in \{1, ..., N\}$ they enter.

Stage 2. All criminals set their search effort q, whereas all households choose their protection level θ.

Stage 3. Theft occurs. Criminals with two theft opportunities select the most interesting one. If indifferent, criminals select one of the two options with probability one half.

The model is solved backward. We start with the final stage, and then go back to the first stage. To simplify notations, we neglect the neighborhood index j until we discuss the mobile criminals' location choices.

Agents' payoffs.—Let Ω denote the expected payoff of a given criminal. We have:

$$\Omega = (1-q)\mathbb{E}[(\alpha-\theta)V] + q\mathbb{E}\max[(\alpha-\theta)V, (\alpha-\theta')V'] - s(q). \tag{1}$$

With probability $1-q$, the criminal has a single theft opportunity, which consists of a random draw within the households' set. With probability q, s/he has two options, and s/he selects the highest reward.

We now define the expected payoff W^i of a household of wealth V^i and protection θ. This payoff depends on the expected number of visits by criminals. Let η_1 and η_2 denote the expected number of visits by single-option criminals and double-option criminals. We have

$$W^i = V^i - [\eta_1 + \eta_2^i]\gamma V^i - V^i\theta. \tag{2}$$

Wealth is reduced by theft, and by the protection cost.

The following Lemma details expected number of visits.

Lemma 1 *The expected number of visits are given by*

$$\eta_1 = c_1; \tag{3}$$

$$\eta_2^R(\theta) = 2c_2\left[1 - \mu H^R(\theta) - (1-\mu)H^P\left(\frac{\theta(V+\Delta)-\alpha\Delta}{V}\right)\right]; \tag{4}$$

$$\eta_2^P(\theta) = 2c_2\left[1 - \mu H^R\left(\frac{\theta V+\alpha\Delta}{V+\Delta}\right) - (1-\mu)H^P(\theta)\right]. \tag{5}$$

Single-opportunity criminals randomly sample within the household set. Thus η_1 is equal to the ratio of such criminals to households. To compute the expected number of visits by double-opportunity criminals, we must distinguish two cases. When a given household is compared to the same type of potential victim, only the level of private protection matters. This is why we find the term $\mu H^R(\theta)$ in equation (4), and the term $(1-\mu)H^P(\theta)$ in equation (5). When the household is compared to a potential victim of a different type, both protection and wealth matter. Consider for instance a rich household. A criminal prefers a randomly selected poor household of protection θ' when $\theta' V + \alpha\Delta < \theta(V+\Delta)$. The right-hand side is the rich-household theft cost. The left-hand side is the sum of the poor-household theft cost plus the opportunity cost of selecting the poor household, i.e. the differential theft benefit $\alpha\Delta$. This event occurs with the following probability $\Pr[\theta' V + \alpha\Delta < \theta(V+\Delta)] = H^P\left((\theta(V+\Delta) - \alpha\Delta)/V\right)$. A similar reasoning holds in the case of a poor household.

2.2 Equilibrium without crime mobility

We first leave aside stage 1, i.e. criminal location choices. Thus we focus on protection effort and search effort at given number of criminals C and corresponding crime rate $c = C/K$. We start with equilibrium definition and structure. We then examine existence and uniqueness. We finally turn to comparative statics.

Definition 1 An *equilibrium without crime mobility* is a search effort q^* and two cdf H^R and H^p such that

(i) $\theta \in \Theta^i$ if and only if (iff) $\theta \in \arg\max_{\theta' \geq 0} W^i(\theta', q^*)$;

(ii) $q^* \in \arg\max_{q \in [0,1]} \Omega(q, H^R(.), H^P(.))$.

In equilibrium, protection efforts must maximize households' well-being, whereas search effort maximizes criminals' payoffs. If there is a unique value of θ that maximizes individual well-being, then the distribution H^R and H^P are degenerate. Otherwise, the exact distributions result from the equality of payoffs $W^i(\theta, .)$ over the equilibrium support of the distribution.

Two types of equilibrium are possible: a first one where the rich are always preferred to the poor, and another one, where some of the poor are actually preferred to some of the rich. Without protection, rich households would always be preferred to poor households. However, the rich have the incentive to invest more in protection than the poor, and this may reverse the initial intuition. We focus one the type of equilibrium where the rich are always preferred to the poor. Thus we assume that the wealth difference is sufficiently large to overcome the potential difference in protection level θ. Formally,

$\Delta/V^R \geq 2\mu\gamma c/\alpha$. Appendix C focuses on the alternative equilibrium configuration where some of the poor are preferred to some of the rich.

Proposition 1 *Let* $\mathrm{MAD}(\Delta) = \mu(1-\mu)\Delta$ *be the Mean Absolute Deviation of wealth, and* $\mathrm{R}(V,\Delta)$ *be such that*

$$\mathrm{R}(V,\Delta) = \frac{[1-6\mu^2(1-\mu)]V + \mu^2(-3+4\mu)\Delta}{3}. \tag{6}$$

In an equilibrium without crime mobility, $\Theta^i = [0, 2\mu_i\gamma cq]$ *for* $i = R, P$, *and*

$$H^i(\theta) = \frac{\theta}{2\mu_i\gamma cq} \tag{7}$$

$$\alpha\mathrm{MAD}(\Delta) + \gamma\mathrm{R}(V,\Delta)cq = s'(q). \tag{8}$$

There is a continuous equilibrium distribution of protection investment by household type. The reason why similar households invest differently is because protection is a positional good. If the distribution was not continuous, investing a little bit more would achieve a mass gain. The household would only pay slightly more, but they would not be chosen each time they would be compared with households of the same type.

Zero protection is always in the interval played by both types of households. On the contrary, suppose that the lower bound is strictly positive. Investing less in protection would cost less and would not increase the probability of being visited. This contradicts the fact that the lower bound is strictly positive.

Both distributions are uniform. **Say why.**

Equilibrium search effort is described by equation (8). The left-hand side is the marginal return to search effort. It increases with dispersion in robbery outcomes. Thus it increases with wealth dispersion, as well as with dispersion in robbery costs inflicted by protection investments. The right-hand side is the marginal cost of search effort.

Proposition 2 (i) *There exists an equilibrium without crime mobility. (ii) There may be multiple equilibria. (iii) The equilibrium is unique if* $s'''(.) > 0$.

There is an externality associated with individual search effort. This externality implies that there is strategic complementarity between criminals. Increasing individual effort in a proportion of criminals raises the number of double-opportunity criminals. Protection dispersion increases as a result. This, in turn, magnifies the return to search effort.

When strategic complementarity is sufficiently strong, there may be multiple equilibria. High search effort and high protection equilibria coexist with low search effort and low protection equilibria. In high search effort equilibria, rich households are more likely to be stolen.

Thus the model can predict that similar neighborhoods with similar crime rates may display different profiles of victims. In some of the neighborhoods, burglaries are concentrated on rich households, and so the value of the stolen belongings is high. In other neighborhoods, victims are more randomly selected, and so the total value of theft is lower.

Properties of equilibria.—We examine how equilibrium outcomes vary the crime rate, with mean wealth, and with wealth dispersion. In this purpose, we assume that there is a unique equilibrium without crime mobility.

Increasing the number of criminals leads to an increase in protection investment, an increase in the dispersion of investment, and an increase in search effort. Appendix C shows that these properties are also true in the alternative configuration where some of the poor are preferred to some of the rich.

Increasing the wealth level V leads to an increase in equilibrium search effort. Richer neighborhoods invest more in protection; they also make more dispersed protection investments. The return to search effort increases as a result. Formally, the derivative $\mathrm{R}_V > 0$.

When income dispersion increases, two effects operate in opposite directions. On the one hand, having a second chance to draw a rich household is more valuable. This benefit is proportional to the proportion of stolen wealth, α. On the other hand, criminals are willing to accept a higher robbery cost in exchange for targeting a rich household instead of a poor one. Since average private protection increases with the damage caused by a robbery, this effect is stronger for high value of γ. Overall,

$$\frac{dq^*}{d\Delta} \overset{\text{sign}}{=} \alpha(1-\mu) + \gamma \frac{\mu(-3+4\mu)}{3}. \tag{9}$$

It is positive when the loss factor γ is lower than the gain α.

2.3 Criminals' location decisions

Criminals choose a neighborhood based on their expected payoff. In a given neighborhood, the criminal's payoff is

$$\Omega = \alpha \mathrm{E}(V) - \mathrm{E}(\theta V) + q\alpha \mathrm{MAD}(\Delta) + \gamma \mathrm{R}(V, \Delta) cq - s(q). \tag{10}$$

The first two terms represent the net benefit accruing to random search, whereas the next two ones represent the additional benefit induced by the possibility of selecting a victim between two randomly sampled households. Under random search, the criminal obtains the fraction α of the mean wealth $\mathrm{E}[V]$, whereas s/he pays the expected cost $\mathrm{E}[\theta V]$. The third term represents the fact that having two options (with probability q) improves the chance of selecting a rich household. This term increases with the mean absolute deviation of wealth. The fourth term represents the fact that criminals with two options choose the best opportunity. The term $\mathrm{R}(V, \Delta)$ is the reduction in robbery cost generated by having two options to choose from. This term is positive because criminals always select the lowest theft cost.[1] The last term is the search cost.

Definition 2 An *equilibrium with crime mobility* is a vector of search efforts $(q_1^*, ..., q_N^*)$, a collection of cdf $\{H_1^R, ..., H_N^R\}$ and $\{H_1^P, ..., H_N^P\}$, and an allocation of criminals across neighborhoods $(C_1^*, ..., C_N^*)$ such that

(i) $\theta \in \Theta_j^i$ iff $\theta \in \arg\max_{\theta' \geq 0} W_j^i(\theta', q_j^*)$;

(ii) $q_j^* \in \arg\max_{q \in [0,1]} \Omega_j(q, H_j^R(.), H_j^P(.))$;

(iii) $C_j^* > 0$ iff $j \in \arg\max_{j \in \{1,...,N\}} \Omega_j(q_j^*, H_j^R(.), H_j^P(.))$;

(iv) $\sum_{j=1}^N C_j = P$.

An equilibrium with crime mobility is composed of N equilibria without crime mobility (parts (i) and (ii)) with the additional requirement that the number of criminals by location is endogenous. Thus part (iii) requires that criminals enter neighborhoods with the highest payoffs. Finally, part (iv) states that the total supply of criminals is fixed.

[1] In the equilibrium described in appendix C, however, this term may be negative. Criminals may select a rich household who invested more in private protection instead of a poor household with lower private protection investment.

Proposition 3 *(i) There is an equilibrium with crime mobility. (ii) Under condition C, there are at least two neighborhoods with a positive number of criminals.*

Unless one of the neighborhoods is dramatically richer and more populated than the others, there is an equilibrium with at least two victimized neighborhoods. The reason why all criminals do not enter the same neighborhood is because location choices convey a congestion externality. However, this externality is not due to direct congestion between criminals. Indeed, in line with the phenomenon of multiple victimization, we assume that a household can be stolen as many times as they are visited by criminals. Congestion operates through protection investments. In each neighborhood, the distribution of protection investments widens with the number of criminals. Theft is more costly for criminals, which tends to discourage further entry into this particular neighborhood.

The next result examines the impacts of wealth on the number of criminals in a given neighborhood.

Proposition 4 *Suppose there is a unique equilibrium with crime mobility where* $C_j^* > 0$*. A marginal increase in* V_j *reduces the number of criminals iff*

$$\epsilon_{qc} > -\frac{\epsilon_{\Omega V}\big|_{\text{at } q(e)c \text{ constant}}}{\epsilon_{\Omega c}}. \tag{11}$$

Wealth does not necessarily make a neighborhood more attractive for criminals. The reason is due to the interaction between protection investments and search efforts in a given neighborhood. Higher wealth tends to increase protection investments; in turn, criminals make more efforts, which further motivates protection investments. Overall, entering such neighborhoods may become so costly that it overcomes the direct positive effect of wealth on criminals' expected payoffs.

The last question is when the condition stated in Proposition 5 can be satisfied. To answer this question, this we will at the following example.

Example: Imagine that μ is equal to zero or one, and that $s(q) = \frac{s^{2+1/a}}{2+1/a}$. With homogeneity inside the neighborhood, we can explicitly solve for $q(c)$, and with the assumption on the $s(q)$, we get that $q(c)$ is iso-elastic in V with a elasticity of a. REST TO COME

3 Property crime and protection: empirical analysis

By looking at the intra-neighbourhood versus across neighbourhoods incentives our model can predict patterns that may be difficult to generate with competing models. Our model predicts that wealthier individuals inside a neighborhood face higher victimization rates, but that wealthier neighborhoods support lower crime rate. At the same time, individuals in wealthier neighbourhoods invest more in private protection. Moreover, our model predicts that more heterogeneous neighbourhood may support lower crime rate, and higher private protection.

Before even looking at the data, we should reflect on the types of patterns competing models would generate. The simplest model where only the direct elasticity of private protection matter, would have a hard time generating different intra-neighbourhood versus across neighbourhoods differences. If the elasticity of private protection with respect to wealth were to be lower than one, wealthier individuals and wealthier neighbourhood would be more attractive. The opposite would be true with an elasticity higher than one. A political economy model with public protection could easily generate lower crime rates in wealthier neighbourhood, but the same rational for heterogonous neighbourhood would require many additional assumptions. More importantly, in such model, it could be difficult to explain why private protection is higher in wealthy neighbourhoods since they would benefit from much more public protection. Another competing argument that can be made is that the supply of criminals is higher in less wealthy neighbourhood. This poses the question, why criminals dont move. Never the less, if it were to be the case the incentives to invest in private protection in wealthier neighbourhood would be much smaller.

To test if inside a neighbourhood wealthier households are more victimized than poorer households, we use the respondent placement into the neighbourhood income distribution (2, 4 and 10 Quantiles). To assess whether wealthy and heterogeneous neighbourhoods may be less victimized than poorer and less diversified ones, we will use average income and standard deviation of income inside a neighbourhood. The same exercise will be done for investment in private protection.

In our model two variables - the victimization probability and the protection effort simultaneously are simultaneously determined. A higher victimization probability leads to more private protection. More private protection leads to lower victimization probability.

None of them are directly observed. What is observed are dummies that take a value of 1 when the agent is victimized or when the agent undertakes more than a certain level of protection and zero otherwise. Our estimation strategy is based on Maddala (1983). More precisely the process we describe corresponds to the model 6 presented page 246–247 of that book. We will follow the estimation procedure and the correction techniques. To do this, we need at least one exclusion variable that affects only one of the endogenous variables and not the other. We propose that the number of young children in the household would affect the probability to exert a protection effort and not the probability of victimization. Take V_{ij} the victimization dummy of agent i in location j and the P_{ij} the protection dummy with respectively $\hat{V}_{ij}$ and $\hat{P}_{ij}$ their latent variables that are the victimization probability and the protection effort. We want to identify the direct impact of I_{ij} the income of individual i, $\bar{I}_j$ the average income of location j, σ_j the standard deviation of the income distribution in location j on both the protection effort and on the victimization probability. Our model predicts that victimization and protection depend on each other:

$$\hat{V}_{ij} = a_0 + a_1 I_{ij} + a_2 \bar{I}_j + a_3 X_j + a_4 \hat{P}_{ij}; \tag{12}$$

$$\hat{P}_{ij} = b_0 + b_1 I_{ij} + b_2 \bar{I}_j + b_3 \sigma_j + b_4 n_{ij} + b_5 \hat{V}_{ij}. \tag{13}$$

Mallar (1977) and Maddala (1983) showed that in terms of parameter restrictions, criteria for identification are identical to those for linear simultaneous equations systems.

This specification enables us to identify the direct effect of income on the victimization and the indirect effect that transmits via the protection effort. Our model predicts that a_1 , b_1, b_2, b_3, b_4, and b_5 are all positive, while a_2, a_3, and a_4 should be negative.

We will also estimate an equation including some interaction variables like $I_{ij}(\bar{I} - \bar{I}_j)$ that would indicate whether being rich in a richer than average localization has a positive impact on victimization and on protection.

3.1 Data

The first step will be to construct our measure of a neighbourhood. We will concentrate only on urban area. Among the geographic units as defined by Statistic Canada the one that corresponds most to our model is the Census Tract. Other geographic units will be used as a measure of comparison, such as, the dissemination Area for a smaller definition of a neighbourhood, and the Federal Electoral District for a larger definition of a neighbourhood. We are using the three cycles of the GSS-Victimization (23, 18 and 13) to maximize the number of observations. Each of these cycles needs to be matched with the appropriate years of the Census. Since victimization happens before the GSS 1999, 2004 and 2009, we will use the Census from 1996, 2001, and 2006 respectively.

The main variable of interest is about victimization incidents, more specifically about attempt to break in or break and enter. To test our first prediction pertaining to whether within a neighbourhood the wealthier are more victimized than those who are poorer, we plan to use household incomes in the GSS, and the income distribution in the neighbourhood using the corresponding Census.

To test our second prediction about whether rich and heterogeneous neighbourhoods may be less victimized than poor and less diversified ones we will need to match the geographic units of a B&E in the GSS with the average and standard deviation of income from the corresponding Census

To test our last prediction about whether the heterogeneity in private protection is higher in the wealthy and more heterogeneous neighbourhoods, we will need information about private protection. The GSS-Victimization asks the some questions about whether the respondent has ever done (and in the last 12 months) any of the following things to protect oneself or ones property from crime. We are interested in variables about locks and bars, about alarms and about dogs.

4 Conclusion

5 References

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6 Appendix A: Proofs

Proof of Proposition 1: We have $H^P\left((\theta V^R - \alpha\Delta)/V^P\right) = 0$ and $H^R\left((\theta V^P + \alpha\Delta)/V^R\right) = 1$. Thus,

$$W^R(\theta) = \left[1 - \gamma c_1 - 2\gamma c_2\left[1 - \mu H^R(\theta)\right] - \theta\right] V^R, \tag{14}$$

$$W^P(\theta) = \left[1 - \gamma c_1 - 2\gamma c_2(1-\mu)\left[1 - H^P(\theta)\right] - \theta\right] V^P. \tag{15}$$

We first show that there cannot be a degenerate distribution of protection investment. We proceed by contradiction. Consider the rich households. Suppose that the equilibrium distribution of protection investment is degenerate, and so the equilibrium investment is θ^R. This implies that

$$W^R(\theta, .) \leq W^R(\theta^R, .) \text{ for all } \theta \geq 0.$$

Using (2), we rewrite the payoff of a rich victim as:

$$\begin{aligned} W^R(\theta) = & V^R - \theta V^R - \gamma\eta_1 V^R \\ & -2\gamma V^R cq\left[1 - \mu\Pr(\theta < \theta^R) - (1-\mu)\Pr([\alpha - \theta]V^R > [\alpha - \theta^P]V^P)\right]. \end{aligned} \tag{16}$$

Setting $\theta = \theta^R + \varepsilon$, ε being very small, generates a discrete gain $\mu\gamma V^R cq$ at an infinitely small cost εV^R. It follows that the distribution H^R cannot be degenerate. The reasoning is similar for poor households.

In a mixed-strategy equilibrium, households must be indifferent between all strategies. The cdf $H^i(\theta)$ is obtained by solving $W^i(\theta, .) = W^i(0, .)$. This gives equation (7). At the upper bound of the support, the cdf must be equal to one, e.g. $H^R(\bar{\theta}^R) = 1$. This implies that $\Theta^i = [0, 2\mu_i\gamma cq]$.

Now, consider the criminals. For any positive search effort level q, a criminal is matched with two households with probability q. In such a case, the criminal may be matched with two poor households with probability $(1-\mu)^2$, with two rich households with probability μ^2 and with one rich and one poor households with probability $2\mu(1-\mu)$. If matched with two rich households, the expected payoff is given by:

$$\begin{aligned} \mathbb{E}\max[\alpha V^R - \theta, \alpha V^{R'} - \theta'] &= \alpha V^R - 2\int_0^{\bar{\theta}^R} \theta[1 - H^R(\theta)]dH^R(\theta) \\ &= \alpha V^R - \frac{2}{3}\mu\gamma c_2 V^R. \end{aligned} \tag{17}$$

If matched with two poor households, the expected payoff is given by:

$$\begin{aligned}\mathbb{E}\max[\alpha V^P-\theta,\alpha V^{P'}-\theta'] &= \alpha V^P - 2\int_0^{\bar{\theta}^P}\theta[1-H^P(\theta)]dH^P(\theta)\\ &= \alpha V^P-\frac{2}{3}(1-\mu)\gamma c_2 V^P. \end{aligned} \tag{18}$$

If matched with one rich and one poor households, the criminal opts for the rich household, and the payoff is given by:

$$\begin{aligned}\mathbb{E}\max[\alpha V^R-\theta,\alpha V^P-\theta] &= \alpha V^R - \int_0^{\bar{\theta}^R}\theta h^R d\theta\\ &= \alpha V^R-\mu\gamma c_2 V^R. \end{aligned} \tag{19}$$

On top of functions MAD and R, we define $\mathrm{E}[V]=\mu V^R+(1-\mu)V^P$ the mean wealth, and $\mathrm{E}[\theta V]=[\mu^2 V^R+(1-\mu)^2 V^P]\gamma c_2$ the mean robbery cost. Then, the expected payoff for a criminal is

$$\Omega=\alpha\mathrm{E}[V]-\mathrm{E}[\theta V]+q\alpha\mathrm{MAD}[V]+q\gamma R(V^R,V^P)c_2-s(q). \tag{20}$$

The optimal search investment maximizes the payoff function Ω. The first-order condition gives

$$\alpha\mathrm{MAD}[V]+\gamma R(V^R,V^P)c_2=s'\Big(q\Big). \tag{21}$$

Imposing $c_2=cq$ gives equation (8).

Proof of Proposition 2: Part (i). To prove existence, it is sufficient to show that equation (8) has a solution $q^*\in[0,1]$. Both the left-hand side and the right-hand side of the equation are increasing in q. The conditions $s'(0)=0$ and $\lim_{q\to 1}s'(q)=\infty$ guarantee there is a solution.

Part (ii). As indicated above, both sides of equation (8) are increasing in q. It is easy to conceive cases where $s''(.)$ is sufficiently non-monotonic (but positive) to have multiple equilibria.

Part (iii). The condition $s'''(\cdot)>0$ guarantes that the right-hand side is cuts the left-hand side only once.

Proof of Lemma 3: Comparative static on $q(c)$ and V, reveals that

$$\frac{\partial q(c)}{\partial V}=-\frac{\gamma R_V qc}{\gamma R(V,\Delta)c-\lambda s''(q)}\geq 0. \tag{22}$$

The numerator is positive since $1-6\mu^2(1-\mu)$ is positive, and the denominator is negative since our unique equilibrium is stable. Similarly, comparative static on $q(c)$ and Δ, reveals that

$$\frac{\partial q(c)}{\partial \Delta} = -\frac{\alpha\mu(1-\mu)+\gamma R_{\Delta} qc}{\gamma R(V,\Delta)c - \lambda s''(q)}. \tag{23}$$

The numerator is definitively positive when $\gamma < 4\alpha$ for all value of c, q and μ including one. QED

Proof of Proposition 4: First, we will define the search effort elasticity ϵ_{qc} with respect to the number of criminal in the neighbourhood by:

$$\epsilon_{\{q,c\}} = -\frac{\gamma R(V,\Delta)c}{\gamma R(V,\Delta)c - \lambda s''(q)} \in [-1,\infty].$$

We now show that $\Omega(c)$ is always decreasing in c. Using the first order condition on $q(c_2)$, we can see:

$$\frac{\partial \Omega(c)}{\partial c} = -q(c)\gamma\left[[\mu^2+(1-\mu)^2]V + \mu^2\Delta - q(c)R(V,\Delta)\right]\left[1+\frac{c}{q(c)}\frac{\partial q(c)}{\partial c}\right] < 0 \tag{24}$$

The term in bracket is always positive, and since $\frac{c}{q(c)}\frac{\partial q(c)}{\partial c} > -1$, the expected benefit is always decreasing with c. As long as $\Omega^B(n_B) > \Omega^A(n_a+m)$ the two benefit (in A and B) will be equalized when criminals locate in both regions. QED

Proof of Proposition 4: The effect of V on Ω using the first order condition on $q(c_2)$ is given by:

$$\frac{\partial \Omega(c)}{\partial V} = \left.\frac{\partial \Omega(c)}{\partial V}\right|_{\text{at } q(e)c \text{ constant}} + \frac{\partial \Omega(c)}{\partial c}\frac{\partial q(c)}{\partial V} \tag{25}$$

We can see that Ω is decreasing with V if only if:

$$\frac{\partial q(c)}{\partial V} > -\frac{\left.\frac{\partial \Omega(c)}{\partial V}\right|_{\text{at } q(e)c \text{ constant}}}{\frac{\partial \Omega(c)}{\partial c}} \tag{26}$$

We can easily re-arrange the expression in term of elasticity. QED

7 Appendix B: Number of visits

We now compute the expected number of robberies against individuals of type i from the c_{j2} criminals with two options. With probability

$$\Pi(t) = \binom{C_{j2}}{t} \left(\frac{2}{K_j}\right)^t \left(1 - \frac{2}{K_j}\right)^{C_{j2}-t}, \tag{27}$$

the victim is matched with t other criminals. With probability $\Pi(t^R;t)$ out of those t criminals t^R are also matched with another rich households, and $t^P = t - t^R$ are matched with a poor one. Consequently:

$$\Pi(t^R;t) = \binom{t}{t^R} \mu^{t^R}(1-\mu)^{t-t^R}. \tag{28}$$

Out of those potential matches, only the cases where the household is matched with a less profitable victim will trigger a robbery against agent i. A victim of type i is less profitable to a criminal if than another agent i if $\theta < \theta^i$, while a type $-i$ is less profitable if $\theta < \alpha(V_i - V_{-i})/V_i + \theta^{-i}V_{-i}/V_i \Leftrightarrow \theta^{-i} > \theta V_i/V_{-i} - \alpha(V_i - V_{-i})/V_{-i} = \tilde{\theta}^{-i}(\theta)$.

Let write $(\theta^i_1, ...\theta^i_{t^i})$ the t^i order of that sample i.e. $\theta^i_1 < \theta^i_2 ... < \theta^i_{t^i}$. A particular household will be visited by $t^i - g$ of the criminals matched with another type i household if $\theta \in [\theta^i_g, \theta^i_{g+1}]$. For any cumulative distribution $F(\cdot)$, the joint density distribution (x,y) of $(g, g+1)$ order statistics is given by

$$\frac{t^i!}{(g-1)!(t^i-g-1)!} F^i(x)^{g-1} \left(1 - F^i(y)\right)^{t-g-1} f^i(x) f^i(y). \tag{29}$$

Therefore the probability that $\theta \in [\theta^i_g, \theta^i_{g+1}]$ is given by

$$\begin{aligned} p^i(g) =& \int_\theta^{\bar{\theta}^i} \int_0^\theta \frac{t^i!}{(g-1)!(t^i-g-1)!} F^i(x)^{g-1} \left(1 - F^i(y)\right)^{t-g-1} f^i(x) f^i(y)\ dx\ dy; \\ =& \frac{t^i!}{(g-1)!(t^i-g-1)!} \frac{F^i(\theta)^g}{g} \frac{\left(1-F^i(\theta)\right)^{t^i-g}}{t^i-g}; \\ =& \frac{t^i!}{g!(t^i-g)!} F^i(\theta)^g (1-F^i(\theta))^{t^i-g}. \end{aligned} \tag{30}$$

as $\int F(X)^{g-1} f(x) dx = \frac{F(X)^g}{g}$ and $\int (1-F(y))^{t-g-1} f(y) dy = \frac{(1-F(y))^{t-g}}{g-t}$.

We also write $(\theta_1^{-i}, ...\theta_{t^{-i}}^{-i})$ the t^{-i} order of that sample $i.e.\theta_1^{-i} < \theta_2^{-i} ... < \theta_{t^i}^{-i}$. My victim will be visited by $t^i - l$ of the other criminals matched with type i victim if $\theta - \alpha(V_i - V_{-i}) \in [\theta_l^{-i}, \theta_{l+1}^{-i}]$.

The probability that $\theta - \alpha(V_i - V_{-i}) \in [\theta_l^{-i}, \theta_{l+1}^{-i}]$. is given by

$$\begin{aligned} p^{-i}(l) &= \int_{\tilde{\theta}^{-i}(\theta)}^{\bar{\theta}^{-i}} \int_0^{\tilde{\theta}^{-i}(\theta)} \frac{t^{-i}!}{(l-1)!(t^{-i}-l-1)!} F^{-i}(x)^{l-1} \left(1 - F^{-i}(y)\right)^{t-l-1} f^{-i}(x) f^{-i}(y)\, dx\, dy; \\ &= \frac{t^{-i}!}{(l-1)!(t^{-i}-l-1)!} \frac{F^{-i}(\tilde{\theta}^{-i}(\theta))^l}{l} \frac{\left(1 - F^{-i}(\tilde{\theta}^{-i}(\theta))\right)^{t^i-l}}{t^i - l}; \\ &= \frac{t^{-i}!}{l!(t^{-i}-l)!} F^{-i}(\tilde{\theta}^{-i}(\theta))^l \left(1 - F^{-i}(\tilde{\theta}^{-i}(\theta))\right)^{t^{-i}-l}. \end{aligned} \tag{31}$$

We can therefore compute the joint density of the distribution of the (g, l) for an agent of type i that is $p^i(g)p^{-i}(l)$. The expected number of visits is given by

$$\begin{aligned} n_1^i &= \sum_{t=0}^{c_2} \Pi(t) \sum_{t^i=0}^{t} \Pi(t^i; t) \sum_{l=0}^{t-t^i} \sum_{k=0}^{t^i} (t - g - l) p^i(g) p^{-i}(l); \\ &= 2c_2 \left[1 - \mu_i F^i(\theta) - \mu_{-i} F^{-i}(\tilde{\theta}^{-i}(\theta))\right]. \end{aligned} \tag{32}$$

8 Appendix C: Some poor households are preferred

In any equilibria of that type, all poor households provide protection effort according to an overall cumulative distribution function $H^P(\theta)$ on the full support $\left[0, \bar{\theta}^P\right]$. As in the equilibrium described earlier, some rich households with low level of private protection will be selected when compared to any poor households; a rich household with zero effort for example. More precisely, any rich households with protection on the support $\left[0, \frac{\alpha\Delta}{V^R}\right]$, will always be selected against any poor households. Rich households who play above $\frac{\alpha\Delta}{V^R}$, on the other hand, will not necessarily be selected. Consequently, there exists two different cumulative distribution functions for rich individuals. Private protection will be selected according to a cumulative distribution $H_\ell^R(\theta)$ on the lower par of the support $\left[0, \frac{\alpha\Delta}{V^R}\right]$, and according to $H_u^R(\theta)$ on the upper par of the support $\left[\frac{\alpha\Delta}{V^R}, \bar{\theta}^R\right]$. The expected payoff for a rich household over the lower and upper supports are given by:

$$W_\ell^R(\theta) = \left[1 - \gamma c_1 - 2\gamma c_2 \left[1 - \mu H_\ell^R(\theta)\right] - \theta\right] V^R; \tag{33}$$

$$W_u^R(\theta) = \left[1 - c_1\gamma - 2\gamma c_2\left[1 - \mu H_u^R(\theta) - (1-\mu)H^P\left(\frac{\theta V^R - \alpha\Delta}{V^P}\right)\right] - \theta\right]V^R. \quad (34)$$

Similarly, we can solve for the expected payoff for poor households over the unique range. Denote by $W^P(\theta)$ the expected payoff of a poor household where:

$$W^P(\theta) = \left[1 - c_1\gamma - 2\gamma c_2\left[1 - \mu H_u^R\left(\frac{\theta V^P + \alpha\Delta}{V^R}\right) - (1-\mu)H^P(\theta)\right] - \theta\right]V^P. \quad (35)$$

Since in a mixed strategy equilibrium, players must be indifferent between all their strategies, we can find $H_\ell^R(\theta)$ for $\theta < \frac{\alpha\Delta}{V^R}$, by solving for $W_\ell^R(\theta) = W^R(0)$:

$$H_\ell^R(\theta) = \frac{\theta}{2\mu\gamma c_2}. \quad (36)$$

To guarantee that $H_\ell^R(\theta) < 1$ for all $\theta \in \left[0, \frac{\alpha\Delta}{V^R}\right]$, it requires that $\Delta/V^R < 2\mu\frac{\gamma}{\alpha}c_2$. Similarly, we can find $H_u^R(\theta)$ for $\frac{\alpha\Delta}{V^R} < \theta < \bar{\theta}^R$, by solving for $W_u^R(\theta) = W^R(0)$:

$$H_u^R(\theta) = \frac{\theta}{2\mu\gamma c_2} - \frac{(1-\mu)}{\mu}H^P\left(\frac{\theta V^R - \alpha\Delta}{V^P}\right) \quad (37)$$

There are many distribution functions that may satisfy equation (37) above.

Lemma C1: *In any mixed strategy equilibrium, it must be the case that* $\bar{\theta}_R^{II} = 2\gamma c_2$ *and* $\bar{\theta}^P = \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$.

Proof of Lemma C1: We prove this statement by contradiction. First assume that $\bar{\theta}^P < \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$, this implies that a rich household by announcing the maximal level of protection is never chosen when compared to a poor household. Therefore,

$$\begin{aligned} W_u^R(\bar{\theta}^R) &= W_\ell^R(0); \\ V^R\left[1 - c_1\gamma - \bar{\theta}^R\right] &= V^R\left[1 - c_1\gamma - 2\gamma c_2\right]; \\ \bar{\theta}^R &= 2\gamma c_2. \end{aligned} \quad (38)$$

Since $\bar{\theta}^P \leq \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$, a poor household may be chosen in some cases, so:

$$\begin{aligned} W^P(\bar{\theta}^P) &= W^P(0); \\ 2c_2\gamma\left[1 - \mu H_u^R\left(\frac{\bar{\theta}^P V^P + \alpha\Delta}{V^R}\right) - (1-\mu)\right] + \bar{\theta}^P &= 2c_2\gamma. \end{aligned} \quad (39)$$

Using equation (38), we get that:

$$\bar{\theta}^P = (1-\mu)\bar{\theta}^R + \mu\bar{\theta}^R H_u^R\left(\frac{\bar{\theta}^P V^P + \alpha\Delta}{V^R}\right). \tag{40}$$

This implicitly defines $\bar{\theta}^P$.The LHS and RHS are linearly increasing in $\bar{\theta}^P$. To see the second note that by (37), the RHS has a slope of $\frac{1}{2\mu\gamma c_2}$. This is enough to show that when $\bar{\theta}^P < \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$, the only possible solution is that $\bar{\theta}^P = \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$. Consequently. $\bar{\theta}^P$ can not be strictly smaller than $\frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$

Second, assume that $\bar{\theta}^P > \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$. A poor households by investing the maximal level of private protection is never chosen when compared to a rich household. Therefore,

$$\begin{aligned} W^P(\bar{\theta}^P) &= W^P(0); \\ V^P\left[1 - c_1\gamma - \bar{\theta}^P\right] &= V^P\left[1 - c_1\gamma - 2c_2\gamma\right]. \end{aligned} \tag{41}$$

Consequently,

$$\bar{\theta}^P = 2\gamma c_2. \tag{42}$$

On the other hand, a rich household may be chosen, so

$$\begin{aligned} W_\ell^R(0) &= W_u^R(\bar{\theta}^R); \\ \gamma 2c_2 &= \gamma 2c_2(1-\mu)\left[1 - H^P\left(\frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}\right)\right] - \bar{\theta}^R. \end{aligned} \tag{43}$$

This is to say,

$$\bar{\theta}^R = \left[\frac{\bar{\theta}^P V^P + \alpha\Delta}{V^R}\right]\left[1 - (1-\mu)\left(1 - H^P\left(\frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}\right)\right)\right]. \tag{44}$$

As above, the RHS and LHS are linearly increasing in θ, and generate a unique solution when $\bar{\theta}^P = \frac{\bar{\theta} = ^R V^R - \alpha\Delta}{V^P}$. This implies that $\bar{\theta}^P$ can not be strictly larger than $\frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$. QED

Among the interesting properties of such equilibria, both $\bar{\theta}^P$ and $\bar{\theta}^R$ are increasing with c_2; more criminal who are searching implies that all households invest more in private protection. Moreover, since all rich households with protection under $\frac{\alpha\Delta}{V^R}$ are always selected, rich households face higher probability of being victimized.

In the proposition bellow, we restrict our attention to mixed strategy equilibria with uniform distribution.

Proposition C1: *There exist a mixed strategies equilibrium where rich households invest in private protection on the supports* $[0, \frac{\alpha\Delta}{V^R}]$ *and* $[\frac{\alpha\Delta}{V^R}, \bar{\theta}^R]$*, according to the cumulative distribution functions:*

$$H_\ell^R(\theta) = \frac{\theta}{2\mu\gamma c_2}, \quad \text{and}$$

$$H_u^R(\theta) = H_\ell^R\left(\frac{\alpha\Delta}{V^R}\right) + \frac{\mu\bar{\theta}^R V^R - \alpha\Delta}{\bar{\theta}^R V^R - \alpha\Delta}\frac{1}{\mu\bar{\theta}^R V^R}[\theta V^R - \alpha\Delta],$$

$$\text{where} \quad \bar{\theta}^R = 2\gamma c_2.$$

Similarly, poor households invest in private protection on the support $[0, \bar{\theta}^P]$*, according to the cumulative distribution function:*

$$H^P(\theta) = \frac{\theta}{2\gamma c_2 - \alpha\Delta}, \quad \text{where} \quad \bar{\theta}^P = 2\gamma c_2 - \alpha\Delta.$$

Proof of Proposition C1: We will propose two uniform distributions for $H_u^R(\theta)$ and $H^P(\theta)$. Note that $W^P(0)$ is given by:

$$W^P(0) = [1 - c_1\gamma - 2c_2\gamma]\, V^P, \tag{45}$$

We can now derive the following uniform distribution for poor households:

$$H^P(\theta) = \frac{\theta}{\bar{\theta}^P}, \tag{46}$$

From equation (37), we get that:

$$H_u^R(\theta) = H_\ell^R\left(\frac{\alpha\Delta}{V^R}\right) + \frac{\mu\bar{\theta}^R V^R - \alpha\Delta}{\bar{\theta}^R V^R - \alpha\Delta}\frac{1}{\mu\bar{\theta}^R}[\theta V^R - \alpha\Delta]$$

Finally, we can see that when $c_2 > \frac{1}{2\mu}\frac{\alpha}{\gamma}\Delta V^R$, then $H_\ell^R(\theta) < 1$ for all $\theta \in [0, \alpha\frac{\Delta}{V^R}]$. QED