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ABSTRACT

Currency Risk in Currency Unions

Sovereign yield spreads within currency unions may reflect the risk of outright default. Yet, if exit from the currency union is possible, spreads may also reflect currency risk. In this paper, we develop a New Keynesian model of a small member country of a currency union, allowing both for default within and exit from the union. Initially, the government runs excessive deficits as a result of which it lacks the resources to service the outstanding debt at given prices. We establish two results. First, the initial policy regime is feasible only if market participants expect a regime change to take place at some point, giving rise to default and currency risk. Second, the macroeconomic implications of both sources of risk differ fundamentally. We also analyze the 2009--2012 Greek crisis, using the model to identify the beliefs of market participants regarding regime change. We find that currency risk accounts for about a quarter of Greek yield spreads.

JEL Classification: E62 and F41
Keywords: currency risk, currency union, default, euro, exit, fiscal deficits, greek crisis, irreversibility and spreads

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1 Introduction

The European sovereign debt crisis started to unfold in early 2010 after a number of countries experienced larger-than-expected fiscal deficits (see, e.g., Lane 2012). Sovereign bond yields started to take off, notably relative to Germany, as illustrated by Figure 1. It displays monthly yield spreads for Italy, Spain, Ireland, and Greece over the period 1993–2012. Such spreads may reflect the risk of an outright sovereign default. At the same time, they may also reflect currency risk, that is, they may be driven by expectations of an exit of a member state from EMU, coupled with a depreciation of the new currency. Indeed, as spreads have reached levels comparable to those prior to the introduction of the euro, "unfounded fears of the reversibility of the euro" have been suggested by the ECB as the main culprit (see, e.g., ECB 2013). Yet to what extent sovereign yield spreads in the euro area are actually driven default and currency risk is an open question.[1]

In this paper we put forward a model which allows us identify the distinct contribution of default and currency risk to sovereign yield spreads. We build on the New Keynesian small open economy framework developed by Galí and Monacelli (2005) and others, but allow for policy regimes to change within a Markov-Switching Linear Rational Expectations Model. Policy regimes are captured by simple feedback rules for monetary and fiscal policy. Initially, there is no independent monetary policy, as the economy is assumed to be part of a currency union. In terms of fiscal policy, we assume that the government is running excessive deficits, that is, under the fiscal rule in place the government will lack the resources to service the outstanding debt at given prices. Put differently by Leeper (1991), fiscal policy is "active" as is it does not adjust (sufficiently) in a "passive" manner to stabilize debt. In principle, an active fiscal policy is sustainable as long as the price level is free to adjust in order to bring about a change in the value of of government debt (Sims 1994, Woodford 1995, and Cochrane 2001). Yet, in a small open economy which is a member of a currency union, purchasing power parity ties down the domestic price level in the long-run.

Against this background we establish a first result, similar in spirit to Davig and Leeper (2007a). Namely, given the initial regime, an equilibrium may obtain only to the extent that market participants expect a regime change to take place at some point.[2] We allow for two

[1] That said, the integrity of the euro area is not beyond doubt. Shambaugh (2012) presents evidence from the online betting market Intrade according to which prices in March 2012 were consistent with a 40 percent probability of an exit of at least one country within 2013. In February 2012 Buiter and Rahbari (2012) use the term "Grexit" and suggest a "likelihood of a Greek exit to 50% over the next 18 months". In May 2012 the German Ifo-think tank published a report on "Greece's exit from European Monetary Union: historical experience, macroeconomic implications and organisational implementation", see Born et al. (2012).

[2] Davig and Leeper (2007a) generalize the Taylor principle by showing that equilibrium determinacy obtains under a policy rule which would give rise to equilibrium indeterminacy in a fixed-regime model, provided there are expectations of a switch to a policy rule which is sufficiently aggressive towards inflation. In contrast, in

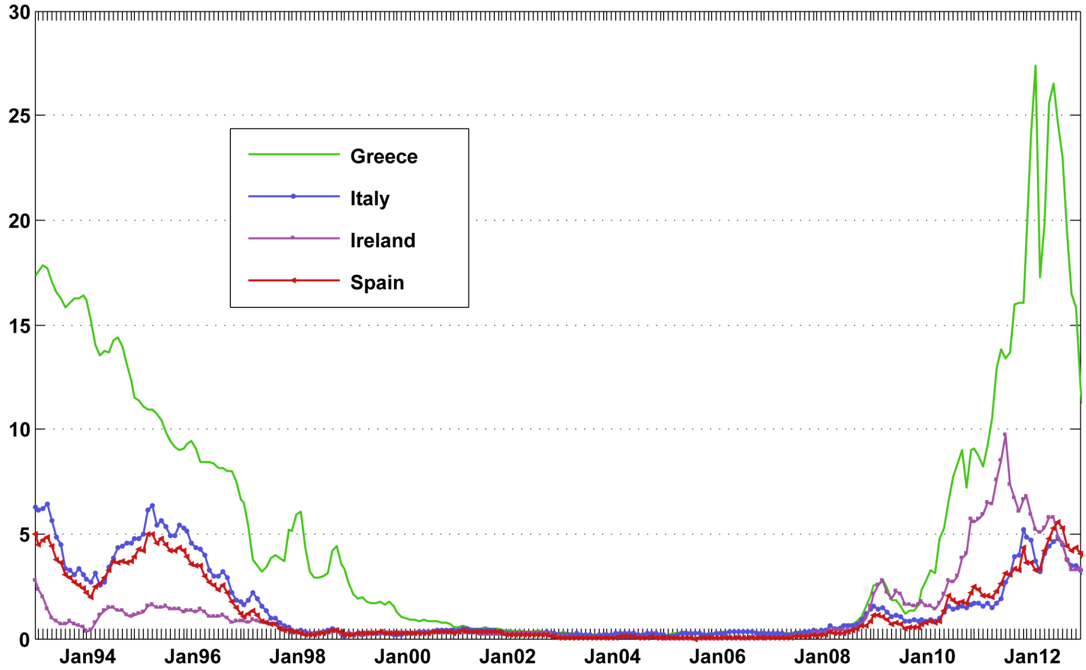

Figure 1: Interest rate spread vis-à-vis Germany. Notes: 10 year bond yields relative to German bond yields, monthly observations 1993–2012; source: ECB, long-term interest rate for convergence purposes.

scenarios which give rise to default and currency risk, respectively. First, the country remains part of the currency union, but alters its fiscal regime as in Davig and Leeper (2007b). The new fiscal regime is passive and ensures sufficient budget surpluses to service the outstanding debt at given prices. In addition, we assume that at the time of the switch there is partial default on the outstanding government debt as in Uribe (2006). Second, the fiscal regime remains unchanged, but the country exits the currency union. Upon exiting, the country starts operating an independent monetary policy which adjusts interest rates less than one-for-one to inflation, thereby accommodating excessive fiscal deficits.

As a second result, we show that the macroeconomic implications of default and currency risk differ fundamentally. Consider a deficit shock which arises as a result of a lump-sum tax cut under the initial policy regime. In the presence of currency risk, the shock is recessionary. Intuitively, as government debt starts to build up under the current policy regime, agents expect inflation and a nominal depreciation in case of an exit. Domestic effective interest rates rise immediately, reflecting the currency risk. Private consumption and output fall, while inflation takes off already before the actual exit takes place due to forward-looking

our setup, the expected regime change ensures a mean square stable equilibrium as defined by Farmer et al. (2009) rather than determinacy.

price-setting decisions. Higher prices in turn crowd out net exports, which leads to a further decline in domestic output. In contrast, if we rule out an exit from the currency union, deficits have no allocative consequences. In this case, the accumulated debt is known to be serviced eventually, once the new fiscal regime is put in place. Ricardian equivalence thus obtains already under the initial regime of excessive deficits. This neutrality result holds up despite the haircut on outstanding debt at the time of the switch to the new fiscal regime. As investors ask for a premium relative to the riskless interest rate, up to first order, the effective interest rate remains unchanged and the allocation unaffected.[3]

We also interpret the European sovereign debt crisis through the lens of our model, notably the macroeconomic developments in Greece between late 2009 and early 2012. The upward revision of the fiscal deficit at the beginning of this period presumably supported the notion that deficits were indeed excessive. In due course, the macroeconomic outlook deteriorated further, fueling speculation of a Grexit. Eventually, debt was restructured in early 2012, as fiscal reform was supposedly under way. We calibrate the model to account for these developments, while exposing it to the time series of actual primary deficits in Greece. We use Greek time-series data for yield spreads and consumer prices to identify the beliefs of market participants regarding regime change. We find that currency risk explains half of the output decline and accounts for a quarter of the yield spread during the period under consideration.

A number of papers have analyzed the conduct of fiscal policy in currency unions from the perspective of the fiscal theory of the price level. Their focus, however, are the implications of the fiscal regime in one or several member countries for the entire union (Woodford 1996, Sims 1997, Bergin 2000). In contrast to these contributions, we analyze the case of a small open economy and abstract from developments in the rest of the union. This allows us to analyze the implications of an expected exit from the union. In this regard we find that ongoing excessive deficits give rise to expectations of a large devaluation after exit, that is, "a fiscal theory of currency risk".

The remainder of the paper is organized as follows. Section 2 presents the New Keynesian small open economy framework, on which our regime-switching model builds. It is fully developed in Section 3 which also presents results regarding the existence of (mean-square) stable equilibria and the effects of deficit shocks. In Section 4, we apply the model to Greek data and decompose the Greek yield spread into default and currency risk. Section 5 concludes.

[3]Results differ in the presence of a sovereign risk channel as a result of which there are spillovers of sovereign spreads on private borrowing conditions (Corsetti et al. 2013a) or in case taxes are assumed to be distortionary (Bi 2012).

2 The small open economy framework

Our analysis builds on the New Keynesian small open economy framework developed by Galí and Monacelli (2005) and others. We augment it by a government sector issuing one period risky debt in order to finance lump-sum transfers. In the following, we outline the decision problems of firms and households and discuss the specification of monetary and fiscal policy, both captured by simple feedback rules. In this section we consider alternative, but time-invariant specifications, while section 3 introduces the possibility of policy-regime change. Our exposition draws on Corsetti et al. (2013b), focusing on the domestic economy and its interaction with the rest of the world, ROW, for short.

2.1 Final Good Firms

The final consumption good, C_t, is a composite of intermediate goods produced by a continuum of monopolistically competitive firms both at home and abroad. We use $j \in [0,1]$ to index intermediate good firms as well as their products and prices. Final good firms operate under perfect competition and purchase domestically produced intermediate goods, $Y_{H,t}(j)$, as well as imported intermediate goods, $Y_{F,t}(j)$. Final good firms minimize expenditures subject to the following aggregation technology

$$C_t = \left[(1-\omega)^{\frac{1}{\sigma}} \left(\left[\int_0^1 Y_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \right)^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} \left(\left[\int_0^1 Y_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (2.1)$$

where σ measures the trade price elasticity. The parameter $\epsilon > 1$ measures the price elasticity across intermediate goods produced within the same country, while ω measures the weight of imports in the production of final consumption goods—a value lower than 1/2 corresponds to home bias in consumption.

Expenditure minimization implies the following price indices for domestically produced intermediate goods and imported intermediate goods, respectively,

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}, \qquad P_{F,t} = \left(\int_0^1 P_{F,t}(j)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \quad (2.2)$$

By the same token, the consumption price index is

$$P_t = \left((1-\omega) P_{H,t}^{1-\sigma} + \omega P_{F,t}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (2.3)$$

Regarding the ROW, we assume an isomorphic aggregation technology. Further, the law of one price is assumed to hold at the level of intermediate goods such that

$$P_{F,t} \mathcal{E}_t = P_t^*, \quad (2.4)$$

where $\mathcal{E}_t$ is the nominal exchange rate (the price of domestic currency in terms of foreign currency). P_t^* denotes the price index of imports measured in foreign currency. It corresponds to the foreign price level, as imports account for a negligible fraction of ROW consumption. For future reference we define the terms of trade and the real exchange rate as

$$S_t = \frac{P_{H,t}}{P_{F,t}},\ Q_t = \frac{P_t \mathcal{E}_t}{P_t^*} \tag{2.5}$$

respectively. Note that while the law of one price holds throughout, deviations from purchasing power parity (PPP) are possible in the short run, due to home bias in consumption.

2.2 Intermediate Good Firms

Intermediate goods are produced on the basis of the following production function: $Y_t(j) = H_t(j)$, where $H_t(j)$ measures the amount of labor employed by firm j. Intermediate good firms operate under imperfect competition. We assume that price setting is constrained exogenously à la Calvo. Each firm has the opportunity to change its price with a given probability $1-\xi$. Given this possibility, a generic firm j will set $P_{H,t}(j)$ in order to solve

$$\max E_t \sum_{k=0}^{\infty} \xi^k \rho_{t,t+k} \left[Y_{t,t+k}(j) P_{H,t}(j) - W_{t+k} H_{t+k}(j)\right], \tag{2.6}$$

where $\rho_{t,t+k}$ denotes the stochastic discount factor and $Y_{t,t+k}(j)$ denotes demand in period $t+k$, given that prices have been set optimally in period t. E_t denotes the conditional expectations operator.

2.3 Households

The domestic economy is inhabited by a representative household that ranks sequences of consumption and labour effort, $H_t = \int_0^1 H_t(j)dj$, according to the following criterion

$$E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\gamma}}{1-\gamma} - \frac{H_{t+k}^{1+\varphi}}{1+\varphi} \right). \tag{2.7}$$

We assume that the household trades a complete set of state-contingent securities with the rest of the world. Letting Ξ_{t+1} denote the payoff in units of domestic currency in period $t+1$ of the portfolio held at the end of period t, the budget constraint of the household is given by

$$W_t H_t + \Upsilon_t - T_t - P_t C_t = E_t \left\{\rho_{t,t+1} \Xi_{t+1}\right\} - \Xi_t, \tag{2.8}$$

where T_t and Υ_t denotes lump-sum taxes and profits of intermediate good firms, respectively.

2.4 Monetary and Fiscal Policy

In case the economy is not part of a currency union, domestic monetary policy is specified by an interest rate feedback rule. Defining the riskless one period interest rate as $R_t \equiv 1/E_t(\rho_{t,t+1})$, we assume

$$\log(R_t) = \log(R) + \phi_\pi(\Pi_{H,t} - \Pi_H), \tag{2.9}$$

where $\Pi_{H,t} = P_{H,t}/P_{H,t-1}$ measures domestic inflation and (here as well as in the following) variables without a time subscript refer to the steady-state value of a variable. In this case, the nominal exchange rate is free to adjust in accordance with the equilibrium conditions implied by the model.

Conversely, if the country is part of a currency union the exchange rate is exogenously fixed at unity, $\mathcal{E}_t = 1$, while the interest rate is free to adjust in equilibrium. We specify explicitly the exchange rate also in this case, as our analysis is concerned with the possibility of an exit from the union. We thus keep track of a "shadow" exchange rate, that is, the level of the exchange rate should the country exit the currency union. We also distinguish between securities denominated in foreign currency and in home currency. The latter are assumed to be issued under domestic jurisdiction and to be converted at par into the new currency upon exit.

As regards fiscal and budget policy, we posit that the government levies lump sum taxes, T_t, and issues one-period risky debt, D_t. Debt becomes risky as in any period, the government may default on a fraction $\delta_t \in [0, 1]$ of its outstanding liabilities (see, e.g., Uribe 2006 or Bi 2012). The period budget constraint of the government then reads as follows:

$$I_t^{-1} D_t = (1 - \delta_t) D_{t-1} - T_t, \tag{2.10}$$

where I_t denotes the gross interest rate which the government pays on newly issued debt. For further reference we explicitly state the following no-arbitrage condition which holds in equilibrium:

$$I_t^{-1} = E_t(\rho_{t,t+1}(1 - \delta_{t+1})). \tag{2.11}$$

It links the interest rate to the expected loss due to default.

Next, defining $D_t^r := D_t/P_{H,t}$ and $T_t^r := T_t/P_{H,t}$ as a measure of real debt and tax revenues, we posit that

$$T_t^r - T^r = \psi(D_{t-1}^r - D^r) - \varepsilon_t^d. \tag{2.12}$$

ε_t^d measures an exogenous iid shock to taxes, or, equivalently a "deficit shock". It is the sole driving force in our model. The parameter ψ captures the responsiveness of taxes to the level of debt. It determines whether fiscal deficits are excessive or not and thus serves to characterize alternative fiscal regimes in our analysis below.

2.5 Market clearing

At the level of each intermediate good, supply equals demand stemming from final good firms and the ROW:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \left((1-\omega)\left(\frac{P_{H,t}}{P_t}\right)^{-\sigma} C_t + \omega \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\sigma} C^*_t\right), \tag{2.13}$$

where $P^*_{H,t}$ and C^*_t denote the price index of domestic goods expressed in foreign currency and ROW consumption, respectively. It is convenient to define an index for aggregate domestic output: $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$. Substituting for $Y_t(j)$ using (2.13) gives the aggregate relationship

$$Y_t = (1-\omega)\left(\frac{P_{H,t}}{P_t}\right)^{-\sigma} C_t + \omega \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\sigma} C^*_t. \tag{2.14}$$

We also define the trade balance in terms of steady-state output as follows:

$$\frac{1}{Y}\left(Y_t - \frac{P_t}{P_{H,t}} C_t\right). \tag{2.15}$$

3 An MS-LRE model of a small open economy

The framework outlined in the previous section is suitable to analyze the properties of alternative policy regimes, including the exchange rate regimes as well as different fiscal rules (see, for instance, Galí and Monacelli 2005 and Corsetti et al. 2013b). Yet, as stressed by Davig and Leeper (2007a), once it is recognized that policy regimes may differ across time, it is desirable to endow agents in the economy with this very insight. In the following we thus allow for changes in policy regimes, such as membership to a currency union or not, in a way consistent with agents' expectations. Specifically, to keep the analysis tractable, we employ a Markov-Switching Linear Rational Expectations (MS-LRE) model.

3.1 Equilibrium

We set up our MS-LRE model by considering a first-order approximation to optimality conditions implied by the optimization problems and market clearing conditions, as well as to the policy rules outlined in section 2. The approximation is valid around a deterministic steady state, which is the same for every policy regime, with balanced trade, zero inflation, zero default and purchasing power parity. In what follows, small letters denote deviation from this steady state. Note also that we consider only domestic deficits shocks which do not impact the ROW.

A first set of equilibrium conditions is *invariant across policy regimes.* The dynamic IS equation and the open-economy New Keynesian Phillips curve are, in turn, given by

$$y_t = E_t y_{t+1} - \frac{\varpi}{\gamma}(r_t - E_t \pi_{H,t+1}), \tag{3.1}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa \left(\varphi + \frac{\gamma}{\varpi} \right) y_t. \tag{3.2}$$

Here $\pi_{H,t}$ denotes domestic (producer price) inflation, with $\varpi := 1 + \omega(2-\omega)(\sigma\gamma - 1)$ and $\kappa := (1-\beta\xi)(1-\xi)/\xi$. Combining the good market clearing condition and the risk sharing condition ties output to the terms of trade:

$$y_t = -\frac{\varpi}{\gamma} s_t, \tag{3.3}$$

$$s_t = p_{H,t} + e_t, \tag{3.4}$$

where the second equation defines the terms of trade. Real public debt and tax receipts are both stated in terms of steady-state output, and their evolution is measured in percentage point deviation from steady state (indicated by a hat). Bond yields in turn are related to the nominal interest rate and to the expected fraction of default:[4]

$$\beta \hat{d}^r_t = \hat{d}^r_{t-1} + \zeta(\beta i_t - \pi_{H,t} - \delta_t) - \hat{t}^r_t, \tag{3.5}$$

$$i_t = r_t + E_t(\delta_{t+1}). \tag{3.6}$$

Here $\zeta := \frac{D}{PY}$ denotes the debt-to-GDP ratio in steady state.

A second set of equilibrium relationships *vary across policy regimes.* Specifically, regarding tax collections and default we posit the following:

$$\hat{t}^r_t = \psi_{\varsigma_t} \hat{d}^r_{t-1} - \varepsilon^d_t, \tag{3.7}$$

$$\delta_t = \zeta^{-1} \delta_{\varsigma_t} \, \hat{d}^r_{t-1}, \tag{3.8}$$

where the ς_t indicates that the two parameters ψ and δ follow a discrete-time Markov chain $\{\varsigma_t\}$ which determines the evolution of policy regimes over time. Monetary policy also differs across regimes. In case of membership in the currency union we impose $e_t = 0$ while under a float there is an independent central bank characterized by an interest rate feedback rule.

[4]We note that bond yields expressed in terms of deviation from steady state corresponds to *spreads* relative to ROW, as we consider only domestic shocks which leave the sovereign yield in the ROW unaffected.

Altogether we consider four different regimes:

$$\text{Union AF:} \qquad e_t = 0, \qquad \delta = 0\ ,\ \psi < 1 - \beta \tag{3.9 - 1}$$

$$\text{Default (Union PF):} \qquad e_t = 0, \qquad \delta > 0\ ,\ \psi > 1 - \beta \tag{3.9 - 2}$$

$$\text{Union PF:} \qquad e_t = 0, \qquad \delta = 0\ ,\ \psi > 1 - \beta \tag{3.9 - 3}$$

$$\text{Exit AF:} \qquad r_t = \phi_\pi \pi_{H,t}, \qquad \delta = 0\ ,\ \psi < 1 - \beta \tag{3.9 - 4}$$

In the first and fourth regime, ψ is small such that taxes adjust not sufficiently to stabilize outstanding debt, a situation of excessive deficits or active fiscal policy (AF). Instead, given the specific assumptions on the Markov chain that we impose below, tax collections suffice to stabilize the level of outstanding debt at given prices in regimes two and three, a situation of passive fiscal policy (PF). The "AF/PF" suffix thus characterizes the fiscal regime. Regimes one to three are associated with membership in a currency union, whereas in regime four the country operates an independent monetary policy. There is default only in regime Default.

We are now in a position to define an *equilibrium*, following Farmer et al. (2011). First, we restate equations (3.1) - (3.9) more compactly:

$$\Gamma_{\varsigma_t} x_t = E_t x_{t+1} + \Psi_{\varsigma_t} \varepsilon_t^d, \quad \varsigma_t \in \{\text{Union AF, Default, Union PF, Exit AF}\}, \tag{3.10}$$

where $x_t = (y_t, r_t, i_t, \pi_{H,t}, p_{H,t}, e_t, s_t, \hat{t}_t^r, \hat{d}_t^r, \delta_t)'$ and $\pi_{H,t} = p_{H,t} - p_{H,t-1}$; ε_t^d is a deficit shock. The matrices Γ_{ς_t} and Ψ_{ς_t} contain the model's deep parameters and ς_t indicates that they are regime dependent. Regime transitions are governed by a matrix $P = [p_{ij}] = [Prob(\varsigma_t = j; \varsigma_{t-1} = i)]$ specified below.

Definition: A rational expectations equilibrium is a mean square stable (MSS) stochastic process that, given the Markov chain $\{\varsigma_t\}$, satisfies equation (3.10).

Definition: An $n-$dimensional process $\{x_t\}$ is MSS if there exists an $n-$vector μ and an $n \times n$ matrix Σ such that

- $\lim_{n\to\infty} E_t[x_{t+n}] = \mu$
- $\lim_{n\to\infty} E_t[x_{t+n}\ x_{t+n}'] = \Sigma.$

Note that the concept of *stability* as defined above thus differs from stability as it is commonly applied in fixed-regime models. Intuitively, unstable roots in some regimes are not an issue, if the economy does not stay in these regimes for too long. What matters is that agents' decision rules are well defined at any time, which is guaranteed by MSS. The expected duration of a regime is thus key for stability. It is governed by the transition matrix on which we impose a specific structure:

$$P = \begin{pmatrix} \mu & (1-\mu)\lambda & 0 & (1-\mu)(1-\lambda) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

It implies that regimes three and four are absorbing states, while regime two is a regime which the economy leaves immediately in order to settle on regime three. Graphically, our Markov chain prescribes the following sequence of regime transitions:

$$\text{Union AF}_{\circlearrowleft\mu} \qquad \longrightarrow_{1-\mu} \qquad \begin{cases} \lambda & \text{Default} \quad \longrightarrow_{1} \quad \text{Union PF}_{\circlearrowleft 1} \\ 1-\lambda & \text{Exit AF}_{\circlearrowleft 1} \end{cases}$$

Initially, there is thus membership in a monetary union paired with excessive deficits. In any period, the economy stays in Union AF with probability μ, and leaves this regime with probability $1-\mu$. λ, in turn, is the probability weight of a change in the fiscal regime. In this case, there will be a one time default on some fraction $\delta > 0$ of outstanding debt. No default will occur thereafter, which is modelled as another switch (with probability 1) to the final Union PF regime, that is, the economy stays in the monetary union but operates a passive fiscal rule. By contrast, a change in the exchange rate regime, that is, exit from the monetary union, is expected with a probability of $1-\lambda$. In this case, the fiscal regime is assumed to remain unchanged, which leads to "default by inflation", associated with a nominal depreciation. Importantly, both Union PF and Exit AF are absorbing states, in the sense that the regimes will remain in place indefinitely.

Generally, the solution of MS-LRE models is obtained through specific algorithms (Farmer et al. 2011). Under our assumptions on the transitions probabilities, the problem simplifies considerably. Since the two target regimes are absorbing, we are able to solve the model backwards using the method of undetermined coefficients. This is particularly welcome, because we can thereby ensure the *uniqueness* of our solution, as the method of undetermined coefficients always delivers all candidate solutions. For the numerical specifications which we consider, we find that at most one of the candidate solutions satisfies MSS.[5] The appendix provides a detailed derivation of the model solution.

[5]Note that in general MS-LRE models may have multiple fundamental equilibria, see Farmer et al. (2011) for an example.

3.2 Stability

In what follows, we briefly discuss the stability properties of each regime in isolation (assuming that regimes remain in place forever), as well as the stability of the model while allowing for regime change. We first note that the regimes Union PF and Exit AF on which the economy will eventually settle for good are stable.[6] In the first case, there is no independent monetary policy and fiscal policy is passive, ensuring the stability of debt. In the second case, fiscal policy is active, coupled with passive monetary policy, that is, the scenario analyzed by Leeper (1991).[7]

In contrast, regarding the initial regime (Union AF), we now show that—absent regime change—there is no stable equilibrium. Combining the equations for debt (3.5) and taxes (3.7) gives

$$\beta \hat{d}_t^r = (1-\psi)\hat{d}_{t-1}^r + \zeta(\beta r_t - \pi_{H,t}) + \varepsilon_t^d, \tag{3.11}$$

where $i_t = r_t$ and $\delta_t = 0$ because default is ruled out by assumption. Remember that deficits are assumed to be excessive, that is, $1-\psi > \beta$. In this case there is an explosive root in equation (3.11) if r_t and $\pi_{H,t}$ are independent of outstanding debt.[8] We show that this is indeed the case as monetary policy independence has been surrendered. Specifically, consider the following two equilibrium conditions:[9]

$$r_t = -E_t(\Delta e_{t+1}) \tag{3.12}$$

$$\beta E_t(p_{H,t+1}) = (1+\beta+\frac{\kappa\varphi\varpi}{\gamma}+\kappa)p_{H,t} - p_{H,t-1}. \tag{3.13}$$

The first equation is the uncovered interest parity (UIP) condition. If staying in the monetary union is perfectly credible, $r_t = 0$, that is, the interest rate is fixed at its steady-state level. The second equation, in turn, shows that under Union AF, prices and thus inflation are fully isolated of the level of outstanding debt and of deficit shocks, but solely governed by long run PPP.[10]

This result offers a theoretical perspective on why expectations of a regime change should arise in the first place. If, under Union AF, fiscal policy induces explosive debt dynamics,

[6] If the current regime is not expected to change, the notion "stable" encompasses both mean square stability and stability in the common sense (no explosive roots in equilibrium), which are then equivalent concepts.

[7] While we consider a small open economy model, its canonical representation is identical to that of a closed economy model (Galí and Monacelli 2005). The regime Default has the same stability properties as Union PF.

[8] As a result the debt level explodes. We note that the private sector's optimality conditions do not rule out an explosive path for debt. Still, it is reasonable to assume that the member state's government is constrained by a no-Ponzi condition on its liabilities, so we discard equilibria where debt is expected to be rolled over forever. See Bergin (2000) and also Sims (1999) for further discussion.

[9] To obtain (3.12) and (3.13), combine equations (3.1),(3.3),(3.4) and (3.2),(3.3),(3.4), respectively.

[10] Note that (3.11)-(3.13) also characterize the equilibrium under Union PF. The solution of (3.13) is given by $p_{H,t} = \phi p_{H,t-1}$, with $\phi = \phi_{aux}/2\beta - \sqrt{4\phi_{aux}^2/\beta^2 - 1/\beta} < 1$, where $\phi_{aux} = 1+\beta+\kappa\varphi\varpi/\gamma+\kappa$. Thus, long run PPP holds in equilibrium. (3.11) ensures that debt does not explode, as under PF, $1-\psi<\beta$. This establishes that Union PF is indeed stable, as claimed in the beginning of the section.

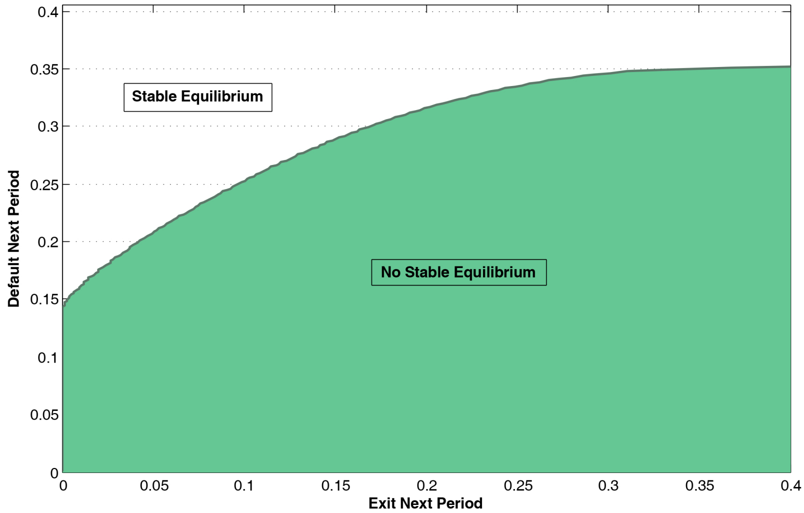

Figure 2: Stability and instability regions. Notes: The x-axis displays the probability of exiting the union next period, $(1-\mu)(1-\lambda)$, whereas the y-axis displays the probability of switching to Default in the next period, $(1-\mu)\lambda$.

market participants know that the policy can be sustained only for a limited period of time.[11] As economic fundamentals continue to deteriorate, at some point the country will be forced into either default (coupled with fiscal reform) or into exit from the currency union.

We therefore investigate whether a stable equilibrium exists once we allow for regime change (formally, this corresponds to $\mu < 1$), where "stable" now and in the following always refers to mean square stability, which is the relevant metric in regime switching models. Intuitively, decision rules in Union AF may be well defined—agents do not expect explosive trajectories—if there is a possibility of regime change in the future.

First, we note that government debt is still on an explosive path (as are other variables) as long as the economy operates under regime Union AF.[12] Second, it turns out that allowing for switches *per se* is not a sufficient condition for a rational expectations equilibrium to exist. Instead, its existence depends on the specific parameter values of the model. Thus, even though for *any* value of $\mu < 1$ agents know they will settle on Union PF or Exit AF eventually (both of which are sustainable regimes), mean square stability might not obtain. Intuitively, if the initial regime is highly persistent, agents still expect explosive trajectories. To establish this result formally, we evaluate numerically whether a rational expectations equilibrium exists or not. In terms of parameter values we adjust μ and λ such that the

[11] Instead, in a large member state of a currency union an active fiscal policy may be sustained indefinitely, as it triggers inflationary consequences for the entire union (Sims 1997).

[12] Technically, the recursive solution in regime Union AF displays an explosive root, i.e. an eigenvalue strictly greater than one.

probabilities of an exit or a default in the next period range between 0 and 0.4. All other parameter values are set according to the values established in Section 4.2, where we calibrate the model to Greece (see Table 1 below). Results are shown in Figure 2. The horizontal axis displays the probability of exiting the union next period, while the vertical axis displays the probability of a default in the next period. The origin corresponds to the case where no regime change is expected to take place in the next period ($\mu = 1$). The area below the curve (including the curve itself) represents the instability region, that is, the combinations of belief-parameters for which no stable equilibrium exists. Note that for a given probability of defaulting and changing the fiscal regime in the next period, raising the exit probability tends to make the equilibrium less stable, up to the point where no equilibrium exists. Intuitively, apart from its impact on risk premia and government solvency, currency risk has additional adverse effects on the economy, an issue which we will take up in the following section.

3.3 Transmission mechanism

Throughout the paper, we focus on deficit shocks as an exogenous source of cyclical fluctuations. In this section, we provide details on the transmission mechanism. We focus on how the possibility of regime change impacts the transmission mechanism while the economy operates under the Union AF regime. Quite generally, deficit shocks may impact the economy via their effect on interest rates. We show that depending on the nature of the expected regime change, the adjustment differs fundamentally.
Recall that government bond yields are determined according to equation (3.6):

$$i_t = r_t + E_t(\delta_{t+1}),$$

that is, the government pays the nominal interest rate plus a premium which depends on the expected haircut. Nominal interest rates in turn are determined through the UIP condition, as established by equation (3.12). Apply the law of iterated expectations ($\mu < 1$) to obtain

$$i_t = -(1-\mu)(1-\lambda)E_t(e_{t+1}|\text{Exit AF}) + (1-\mu)\lambda E_t(\delta_{t+1}|\text{Default}),$$

where we use that $e_t = 0$ today and that the exchange rate fluctuates only in case of an exit from the currency union. Analogously, $\delta \neq 0$ only if there is a switch to the default regime. Note that in case market participants expect the newly created currency to depreciate in the event of exit, we have $e_{t+1} < 0$, such that today's bond yields rise through an increase in the nominal interest rate.[13] The above expression thus decomposes sovereign yield spreads (l.h.s.) into currency risk (r.h.s.: first term) and default risk (r.h.s.: second term).

[13]Here, we impose that government debt is issued under the jurisdiction of the domestic economy and would be converted into local currency upon exit. We verify below that this is indeed justified for Greece in the time span under consideration.

Currency risk driving nominal interest rates reflects a no-arbitrage condition implicit in the UIP. Specifically, market participants trade state contingent claims denominated either in home or foreign currency. If the home currency is expected to depreciate, the domestic-currency return on foreign-currency claims increases in those states of the world where depreciation takes place. Thus, in equilibrium, the return on claims denoted in home currency will also be higher on average. Besides its effect on government bond yields, currency risk therefore also affects the economy's private sector through its impact on effective borrowing conditions. Default risk, by contrast, only raises the refinancing costs of the government, but does not change the economy-wide nominal interest rate. Investors hold the government bond only if they are paid a higher notional return which compensates for the risk of default. Up to first order, the *effective* interest rate thus remains unaffected by default risk.[14]

These considerations are key to understand the macroeconomic effects of a one-time deficit shock while the economy operates in regime Union AF. Figure 3 displays impulse response functions computed for the calibration obtained in Section 4.2 below (except for μ, λ and δ). Solid lines represent the case where there is only default risk ($\lambda = 1$). Dashed lines, in turn, represent the case where there is only exit risk ($\delta = 0$ in regime Default).[15] In each instant, we fix the belief parameter μ at 0.8.

For the default scenario (solid lines) we assume $\delta = 0.5$, implying an expected default on 50% of excess debt upon changing regimes. We find that in this case a deficit shock does not affect any variables, except for government debt, the deficit and government bond yields (shown in the third row of Figure 3). Importantly, as discussed above, private sector borrowing rates are unaffected by default risk. Debt is known to be serviced eventually, once the switch to Union PF has taken place. Ricardian equivalence thus obtains even under the current Union AF regime: deficits are neutral in the sense that they have no allocative consequences.[16] Note also that bond yields and thus default risk rise endogenously with the level of outstanding debt.

For the exit scenario we set $\lambda = 0.5$. Agents thus attach a probability of 50% of exit at some point in time, and of $(1 - 0.8)0.5 = 10\%$ within the next quarter.[17] In this case, a

[14]Through a sovereign risk channel (Corsetti et al. 2013a) sovereign default risk may affect the effective borrowing conditions in the private sector. We also note that in our complete-markets setup there are no distributional effects associated with government default.

[15]Note that although market participants attach some probability on leaving the regime Union AF in any period, in this experiment the regime is held fixed, which continually surprises agents and induces a series of forecast errors. A similar experiment is conducted in for example Davig and Leeper (2011).

[16]As the government's financing costs rise, this neutrality result would break down if taxes were distortionary (Bi 2012).

[17]The unconditional probability of exiting the union is given by $\text{Prob(Exit)} = \sum_{i=0}^{\infty} \mu^i (1 - \mu)(1 - \lambda) = 1 - \lambda$.

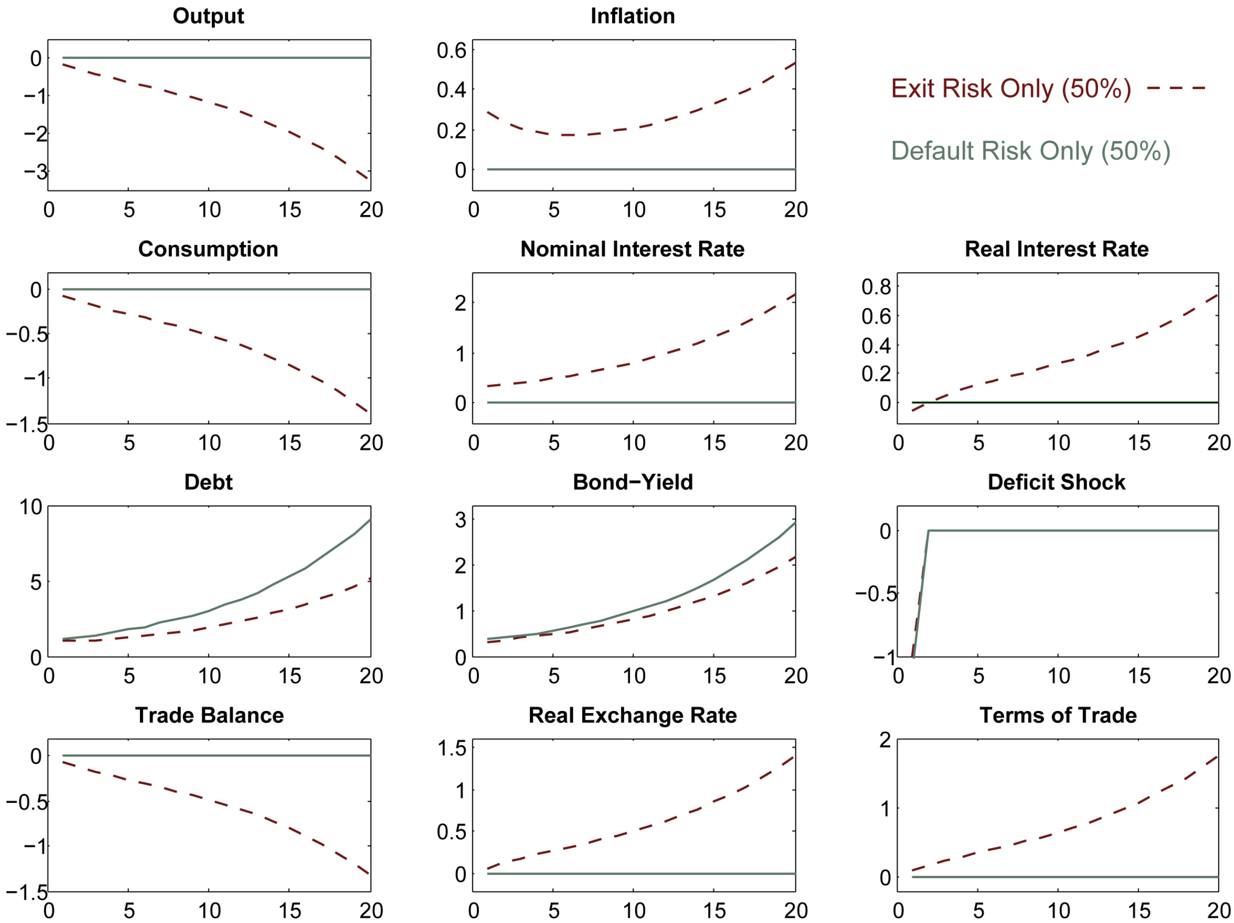

Figure 3: Impulse responses to deficit shock conditional on remaining in regime Union AF. Notes: deficit shock equal to one percent of steady-state output. Horizontal axes measure quarters. Vertical axes measure deviations from steady state in percent, and percentage points in case of debt. Interest rates and inflation are annualized.

deficit shock is non-neutral. In fact, we find that a one-time deficit shock induces long-lasting effects—the model generates substantial internal propagation. Moreover, the deficit shock is inflationary. The inflationary consequences of a deficit shock are due to forward-looking price-setting behavior. All else equal, firms tend to raise prices given that they expect inflation and devaluation upon exit which, in turn, will raise future marginal costs.[18]

Finally, we find the deficit shock to be recessionary provided that market participants expect an exit to take place at some point. To understand this result, note first that higher inflation induces a loss of competitiveness and thus a crowding out of net exports. Under our calibration this accounts for about half the output decline. Importantly, however, output also declines, because domestic consumption declines reflecting the rise in real interest rates due to currency risk. The response of consumption thus differs fundamentally from what related analyses have documented for closed economy models (see Kim 2003 for a fixed-regime model and Davig and Leeper 2007b for a regime-switching model). Assuming an active fiscal policy, deficit shocks typically raise consumption, as real interest rates decline. The decline of real rates, in turn, is due to a passive monetary policy, that is, the central bank adjusts nominal rates less than one for one with inflation.

Figure 3 also illustrates that default and currency risk are a function of the state of the economy. As debt levels increase and economic fundamentals deteriorate, yield spreads also increase. Intuitively, the longer the government procrastinates, the more severe the consequences will be. In the case of default, the amount of obligations it is expected to default on rises as time elapses. In the case of an exit, the level of inflation and the level of depreciation which are necessary to stabilize debt upon exit increases over time. In this sense procrastination is counterproductive under both scenarios, as stressed in Uribe (2006). Note that once both types of risk are present simultaneously, their effects will reinforce each other, as both impact adversely on public finances. For instance, in the presence of currency risk, a rise in default risk will, all else equal, deepen the recession which results from a deficit shock.

Finally, note that even though expectations of regime change are fundamentally justified under Union AF, they also raise the government's financing costs provoking a further deterioration of fundamentals. Thus, there is the possibility that an autonomous shift in expectations regarding regime change causes fiscal deficits to become excessive, even if they are not in the absence of such a shift. We do not analyze this possibility in the present paper.[19]

[18] A similar channel operates in a closed economy model: Davig and Leeper (2007b) find that deficit shocks are inflationary in a regime of passive fiscal policy, if agents anticipate a switch to a regime of active fiscal policy, where the latter regime is associated with high levels of inflation.

[19] Specifically, in case of a membership in the currency union, the condition $\psi > 1-\beta$ is generally not sufficient for debt to be stable, if speculation about default or exit arises. The latter induces fears of a devaluation if debt is expected to be financed through inflation upon exit. Because of the resulting risk premia, the initial

4 The case of Greece 2009–2012

In this section we use the model to analyze key aspects of the macroeconomic developments in Greece during the period 2009–2012. The model, in particular, allows us to gauge to which extent bond yield spreads have been driven by expectations of default and to which extent by expectations of an exit. As discussed in the introduction, the Greek government faced spiralling financing costs starting in 2010, as did several other governments in the euro area (see Figure 1 above). Yet the experience of Greece is most dramatic in terms of spreads. Moreover, the scenario of an exit from the euro area was arguably most plausible in the case of Greece—reflected in widely used neologism "Grexit".[20] Finally, the size and persistence of fiscal deficits support the notion of some investors that deficits have indeed been excessive, both prior to and during the crisis period under consideration. This makes the case of Greece particularly suitable to be studied through the lens of our model. In what follows we provide a short summary about the developments in Greece which are particular relevant for our analysis, calibrate the model to capture key features of the data and perform a decomposition of yield spreads.

4.1 The Greek crisis

By late 2009, the Greek crisis escalated as the newly elected government of George Papandreou announced a substantial overshooting in the previous government's projection for Greece's 2009 deficit, from 6 to 12.7 percent of GDP (Gibson et al. 2012). Following the fast rise of yields and the debt-to-GDP ratio, rating agencies downgraded Greek debt obligations to junk-bond status in April 2010. At this point, Greece had lost access to international financial markets.

In May 2010, official lending by the EU and the IMF provided a substitute, as a support package amounting to 110 billion euros (or about 50 percent of Greek GDP) was agreed upon. At the same time, austerity measures and various structural reforms were initiated in order to stabilize fiscal imbalances. Yet the success of these measures has been limited—at least to the extent that sovereign yield spreads relative to Germany continued to widen over the course of 2011.

In July 2011 a second support package for Greece was discussed, and eventually ratified in the beginning of 2012. It involved a substantial restructuring of privately held debt. In March

regime may become unsustainable, confirming fears about default or exit—the classic scenario of a self-fulfilling currency crisis (see, e.g., Obstfeld 1996). Above, however, we assume that the initial regime is unsustainable independent of expectations regarding regime change.

[20] See footnote 1. Occasionally, commentators also contemplate a "Spexit", although at the time of writing the term is certainly less present in the policy discourse.

2012, 200 billion euros, about 56% of the end-2011 debt total, were renegotiated, reaching an initial net decline of the debt-to-GDP ratio from 170% to about 120% (Zettelmeyer et al. 2012). However, in order to recapitalize Greek banks which had experienced large losses—not least because of the restructuring—new borrowing was required. As a result, the actual debt reduction was considerably lower. Indeed, IMF (2012) predicted end-2012 debt to exceed end-2011 debt.

These developments were also reflected in the yield spreads, which fell strongly in March 2012, but started to rise again to reach a new record high by June 2012. Instead, a longer lasting reduction of yield spreads ensued at about the same time as ECB president Mario Draghi's announcement to contain yield spreads through purchases of government bonds in July. In fact, this policy ("outright monetary transactions") arguably meant to confront "unfounded fears of the reversibility of the euro".

In what follows we focus on the period 2009Q4–2012Q1. The first quarter of this period coincides with the take-off of sovereign yield spreads following the correction of the 2009 budget deficit. We limit our analysis to the period up to 2012Q1, because we are interested in studying the repercussions of an expected regime change, rather than the effect of the regime change itself, notably for the evolution of yield spreads. Moreover, in line with our modelling assumption, prior to the 2012 restructuring almost all ($> 90\%$) of Greek public debt (privately held) was issued under domestic jurisdiction, see Buiter and Rahbari (2012). We still include 2012Q1, as the restructuring of debt has taken place only at the end of that quarter.

4.2 Calibration

We use observations for the Greek economy, if available, to pin down the parameter values of the model. They are displayed in Table 1. A period in the model corresponds to one quarter. The discount factor β is set to 0.99. We assume that the coefficient of relative risk aversion, γ, takes a value of one, consistent with balanced growth. We set $\varphi = 3$, implying a Frisch elasticity of labor supply of 1/3 in line with evidence provided by Domeij and Flodén (2006). The trade price elasticity σ is set to 1.5, in line with estimates for Greece by Bennett et al. (2008), and ω to 0.2, corresponding to the 2009 export-to-GDP ratio in Greece.

Price rigidities play an important role for our quantitative results, and we perform robustness checks at the end of the next section. We assume a fairly flat Phillips curve, by setting $\xi = 0.925$. Note that such a parametrization apparently conflicts with evidence from microeconomic studies such as Nakamura and Steinsson (2008). Nonetheless, the choice of a relatively high degree of price rigidities seems appropriate in the context of our framework,

Table 1: Model calibration

	Parameter description	Value	Target / Source
β	Discount factor (steady state)	0.99	Annual interest rate 4.1%
γ	risk aversion	1	Balanced growth
φ	Inverse Frisch elasticity	3	Domeij and Flodén (2006)
σ	Trade-price elasticity	1.5	Bennett et al. (2008)
ω	Home Bias	0.2	Export-to-GDP ratio 2009
ξ	Fraction of unchanged prices	0.925	Flat Phillips curve
ϵ	Elasticity of substitution	11	Mark-up 10%
ϕ_π	Taylor-rule coefficient	0.9	PM
ψ	Tax-rule coefficient	0.009	AF
ζ	Steady-state debt-to-GDP ratio	5.13	128.3% Debt 2009Q3
δ	Haircut	0.519	51.9% Haircut 2012Q1
μ	Probability of staying in initial regime	0.78	Spread 2009Q4–2012Q1
λ	Default vs exit	0.875	CPI 2009Q4–2012Q1

as we abstract from several model features that would imply a flatter Philips curve for any given value of ξ, e.g., non-constant returns to scale in the variable factor of production or non-constant elasticities of demand.[21] Recent evidence by IMF (2013) suggests that Phillips curves indeed have been flat in the time span under consideration.

We set $\epsilon = 11$, such that the steady-state markup is equal to 10 percent. Regarding the conduct of monetary policy in case of an exit, we assume $\phi_\pi = 0.9$ such that monetary policy is passive. At the same time, we assume $\psi = 0.009$ in case fiscal policy is active, whereas we assume $\psi = 0.02$ for the regimes where fiscal policy is passive. Below, we perform a sensitivity analysis with respect to these parameter values.

We pin down a last set of parameter values by calibrating the model to match key features of the Greek economy during the period 2009Q4–2012Q1. Specifically, as spreads have been close to zero prior to 2009Q4, we assume that the Greek economy has been in steady state in 2009Q3 and set $\zeta = 5.13$ in order to match the debt-to-GDP ratio of 128.3 percent at that time.[22] For each of the 10 quarters of the period under consideration, we specify a value for the deficit shock ε_t^d so as to generate a primary budget deficit in the model which is of the same size as the one observed for Greece. Figure 4 displays the actual time series. Deficits have been large and persistent throughout the period under consideration, except for the last quarter of 2011 for which a primary surplus has been recorded.

Given this sequence of shocks, the model predicts an increase in sovereign risk which, given

[21] See Galí et al. (2007) or Eichenbaum and Fisher (2007) for further discussion of how real rigidities interact with nominal price rigidities in the context of the New Keynesian model.

[22] To be precise, Greek yield spreads in 2009Q3 have been about 1.3 percent, see Figure 1. This number is small relative to the levels observed shortly afterwards. In our model, spreads are zero in steady state only. Moreover the assumption that the economy is in steady state initially eliminates additional degrees of freedom.

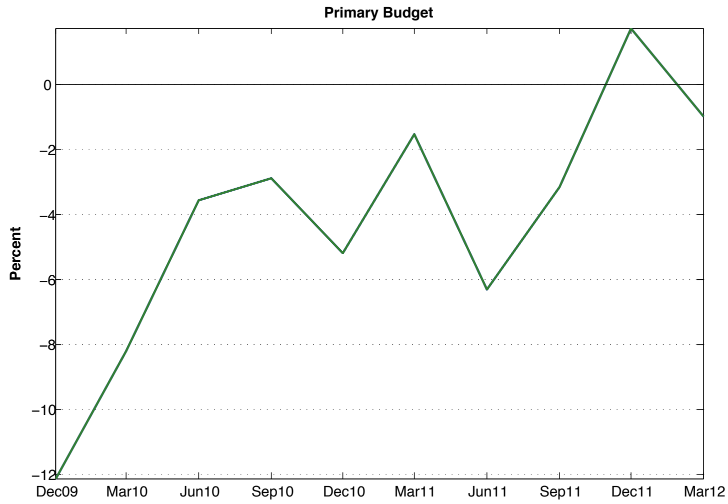

Figure 4: Primary budget balance in Greece 2009Q4–2012Q1. Source: Eurostat. Notes: quarterly observations in percent of GDP.

the other parameter values, depends on δ, μ and λ. We set $\delta = 0.519$ implying a haircut of 51.9 percent, corresponding to the actual value in 2012Q1, according to calculations by Zettelmeyer et al. (2012).[23] We identify μ and λ by targeting the increase in the risk premium and the CPI over the period under consideration. Recall that deficit shocks raise the price level only in the presence of currency risk, so targeting the change in the CPI allows us to identify λ.[24]

Our calibration yields values for $\mu = 0.78$ and $\lambda = 0.875$. These values imply a probability of exit of 2.75 percent from one quarter to the next, and of 19.25 percent of government default. The probability of leaving the euro area within one year implied by these values corresponds to 7.9 percent.[25]

4.3 Accounting for default and currency risk during the Greek crisis

We are now in the position to offer a structural account of the crisis dynamics which took off in Greece in late 2009. The panels of Figure 5 display the behavior of yield spreads, the CPI (detrended), output, and the debt-to-GDP ratio over the period 2009–2012, contrasting actual developments and those predicted by the calibrated model, driven only by deficit shocks.[26]

[23] In our model the haircut applies only to debt in excess of steady-state debt. Steady-state debt, instead, is riskless, as $\delta \leq 1$.

[24] The CPI increase is linearly detrended by two percent per annum, as there is zero trend inflation in our model.

[25] This value may appear small in light of the numbers discussed in the policy debate at the height of the crisis (see footnote 1). It is likely to be the result of our assumption that the exit probability is constant for the period under consideration. Note that this assumption does not constrain currency risk to be constant.

[26] Actual data are normalized in line with our assumption that the economy has been in steady state initially.

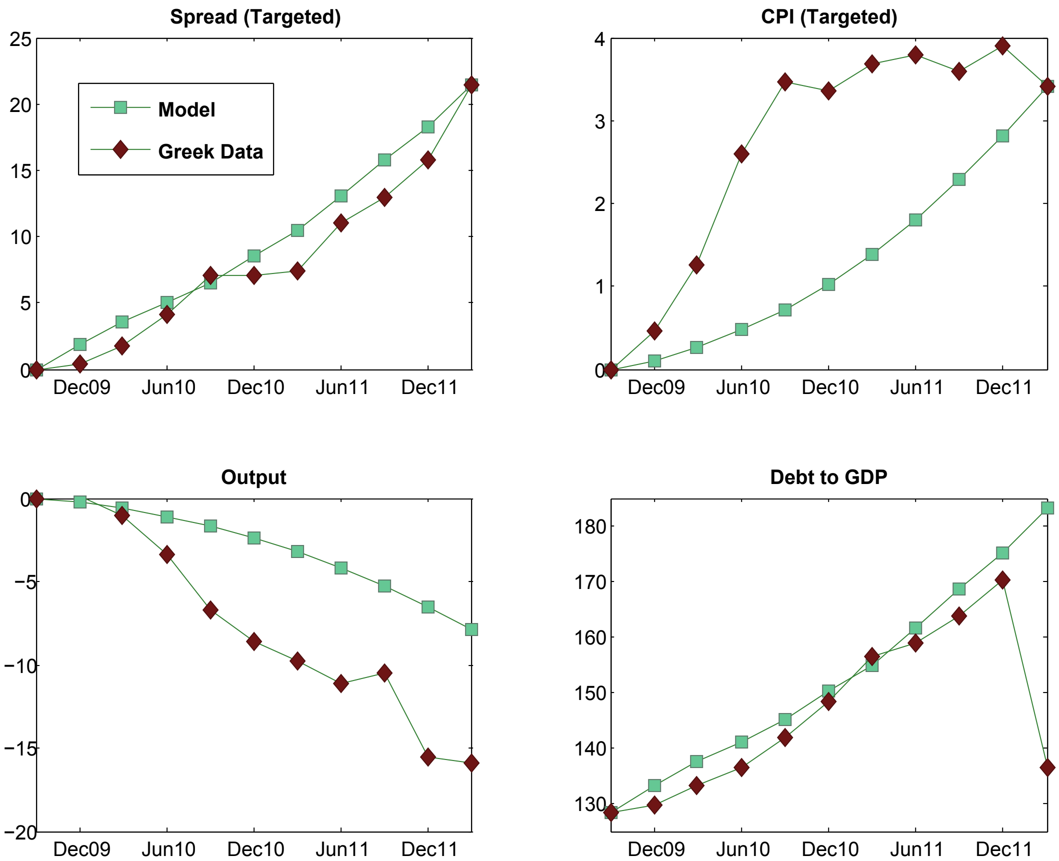

Figure 5: Evolution of key variables during crisis period. Notes: yield spread and the CPI in 2012Q1 (first row) serve as calibration target. Line with squares: model prediction; line with diamonds: data (normalized to zero in 2009Q3, except for debt); first observation: 2009Q4. Vertical axis measures percentage points for the spread, relative change in percent for the CPI and output, and levels for debt-to-GDP; data sources: Eurostat and IMF.

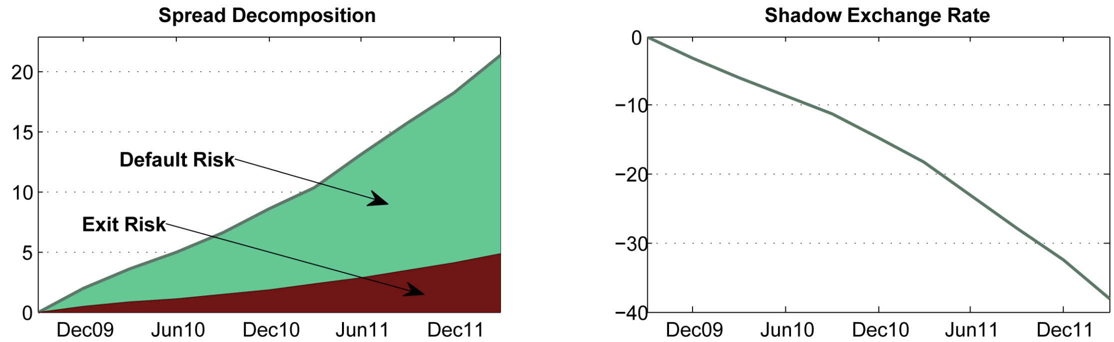

Figure 6: Decomposition of yield spread (left) and depreciation upon exit (right). Notes: currency risk measured by private sector interest rate; default risk: difference between yield spread and spread due to currency risk (all measured in percentage points); depreciation upon exit (shadow exchange rate) measured in percent.

The evolution of the yield spreads and the CPI are shown in the top panels of Figure 5. While the changes between 2009Q3 and 2012Q1 have been used as calibration targets, we note that the model prediction tracks the actual evolution of the spread rather closely. Admittedly, this is less so in case of the CPI, where the model fails to predict a first peak in late 2010.

The bottom panels show the evolution of output and the build-up of government debt. Again, we note that the model prediction for debt tracks actual developments rather closely.[27] Regarding output, however, we note that the model also accounts for half the output decline since 2009Q3. This is a noteworthy result and testifies to the importance of currency risk, because in the absence of exit expectations deficit shocks would have no bearing on real activity (see Figure 3 above). Nevertheless, default risk, by raising financing costs and thus deficits, also contributes to a deterioration of public finances which, in presence of exit expectations, further amplifies currency risk and its effect on the economy at large.

Eventually, we aim to isolate the distinct contribution of default and currency risk to the dynamics of sovereign yield spreads in Greece. Given our calibrated model this is straightforward: while the government's borrowing costs include both currency and default risk, the private sector borrowing costs only include the former. The left panel of Figure 6 shows the result. We find that currency risk accounts for slightly less than a quarter of the total spread, with the rest made up by default risk. The right panel of Figure 6 reports the shadow exchange rate, that is, the source of currency risk. At each point in time, it corresponds to the amount of depreciation were the economy to exit the currency union. It rises over time

[27] As discussed above, the restructuring of Greek debt has taken place at the end of 2012Q1, explaining the sudden decline in debt in Figure 5. By contrast, the average spread in 2012Q1 did not decline (spreads were 24.1 percent in January, 27.4 percent in February and 17.2 percent in March, see Figure 1).

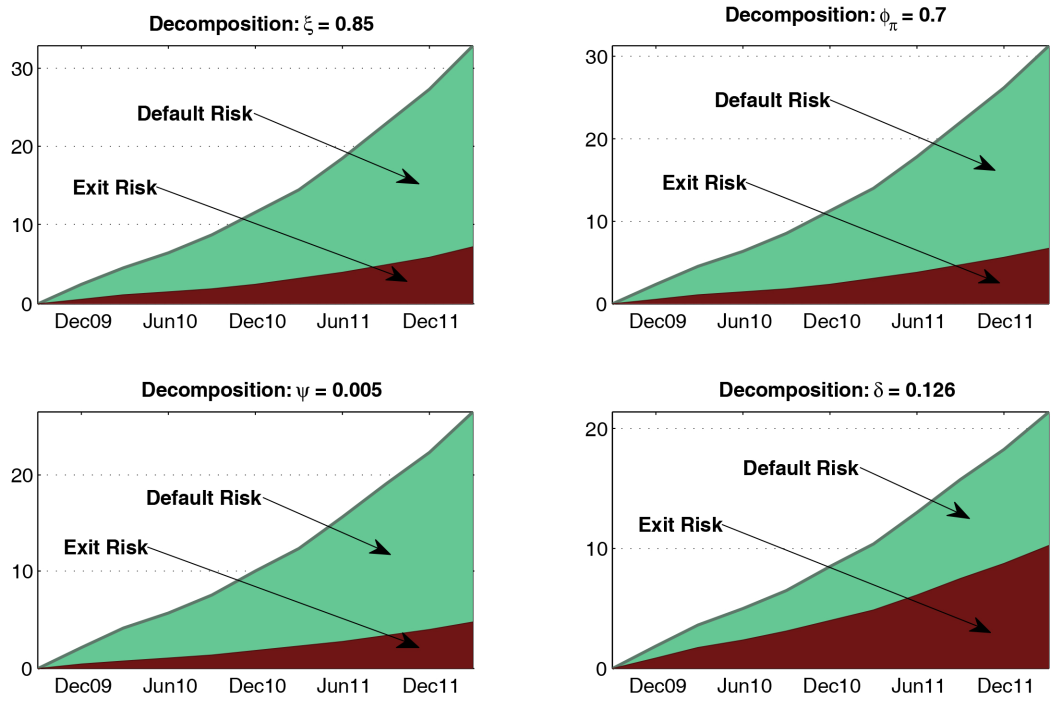

Figure 7: Robustness of the spread decomposition towards changes in $\xi, \phi_\pi, \psi, \delta$. Notes: Targets are the spread and CPI increase 2009–2012, see Figure 5.

in close sync with the evolution of debt, as inflation upon exit will be higher, the higher the debt level—in line with the fundamental insight of the fiscal theory of the price level.

Our results so far suggest a limited role for currency risk in accounting for Greek yield spreads. We will now explore to what extent these results hold up if we alter parameter values that are hard to pin down empirically and thus may appear controversial, such as the high degree of price stickiness. The degree of price stickiness governs inflation dynamics and thus the amount of currency risk necessary to generate the increase in prices observed in the data.[28] We consider a value of $\xi = 0.85$ instead of $\xi = 0.925$, effectively reducing the average price duration from 13 to about 6 quarters.

Results are shown in the upper left panel of Figure 7. Even though the model now overshoots somewhat the rise in yield spreads, we note that the relative weights of currency and default risk that make up the spread remain largely unchanged. A similar finding obtains once we

[28]In this and the following robustness checks, we perform the same targeting exercise as before, using the parameter values reported in Table 1 (except for the parameter under consideration).

lower the (counterfactual) Taylor-coefficient upon exit, ϕ_π, from 0.9 to 0.7, which implies a more accommodative monetary policy (upper right panel). We further consider a decline in ψ, the tax collection parameter, from 0.009 to 0.005, so that fiscal policy becomes "more active". Less tax collections imply that upon exit, higher levels of depreciation are necessary in order to restore equilibrium. Once again, we find that altering ψ does not significantly change the composition of default versus currency risk in yield spreads (lower left panel). Finally, we illustrate that, in principle, the model is capable of producing high risk premia driven by currency risk. In the lower right panel of Figure 7 we alter the expected fraction of default, δ, in such a way that only one quarter of the actual haircut in 2012Q1 was expected by investors. In this case, currency risk makes up about half of the rise in yields from 2009–2012.

5 Conclusion

In this paper, building on the standard New Keynesian small open economy framework, we have developed a Markov-Switching Linear Rational Expectations model of changing policy regimes. In particular, policy regimes differ in terms of government budget policies as well as in terms of the exchange rate regime. As a first result, we show that a budget policy which implies excessive deficits is not sustainable for a member of a currency union. However, such a policy regime may nevertheless be consistent with an equilibrium if market participants expect a regime change to take place at some point.

Within our framework we study the consequences of default and currency risk. In our setup, default risk emerges because of a possible haircut on outstanding debt. Currency risk, instead, emerges because of a large scale depreciation in case the country exits the currency union. We find that the macroeconomic implications of the two sources of risk differ fundamentally. If only default risk is present, a deficit shock affects the borrowing conditions of the government, but has no further bearing on the equilibrium outcome. Instead, deficit shocks are stagflationary in the presence of currency risk.

We analyze key developments in Greece during the period 2009Q4–2012Q1 through the lens of the model. Specifically, we use the increase in yield spreads and the CPI during that period to pin down the beliefs of market participants of a default and an exit from the euro area. We find probabilities of 20 and 3 percent, respectively, for these events to take place from one quarter to the next. A decomposition of sovereign yield spreads suggests that about one quarter of the spread has been due to currency risk. Nevertheless, we stress that currency risk has a strong effect on the economy: it explains about half of the output decline during the period under consideration.

Still, our results suggest a limited role for currency risk in accounting for Greek yield spreads.

This result is particularly noteworthy in light of the rationale provided by the ECB for its promise of unlimited purchases in secondary sovereign bond market ("Outright monetary transactions" or OMT, for short), namely to restore the monetary transmission mechanism by confronting "unfounded fears of the reversibility of the euro". That said, our findings are not inconsistent with the apparent success of the ECB's OMT policy in reducing sovereign yield spreads as such. However, we leave a more detailed analysis of this policy, as well as of the developments in other European countries for future research.

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A Appendix

This section presents technical details about our solution method. Markov-Switching Linear Rational Expectations models (MS-LRE) in general are discussed in Farmer et al. (2009) and Farmer et al. (2011). We concentrate on mean-square stable minimum state variable solutions, which we obtain by applying the method of undetermined coefficients.

An MS-LRE in general has the following structure:

$$\Gamma_{\varsigma_t} x_t = E_t x_{t+1} + \Psi_{\varsigma_t} \varepsilon_t \ \ \forall \varsigma_t, \tag{A.1}$$

with x_t being a vector of endogenous random variables, ε_t being a vector of white noise structural errors, and where Γ_{ς_t} and Ψ_{ς_t} are matrices containing the model's deep parameters. They evolve over time, following a discrete time Markov Chain $\{\varsigma_t\}$, with transition matrix $P = [p_{ij}] = [Prob(\varsigma_t = j; \varsigma_{t-1} = i)]$.

A candidate solution looks as follows:

$$x_t = F_{\varsigma_t} x_{t-1} + G_{\varsigma_t} \varepsilon_t \ \ \forall \varsigma_t, \tag{A.2}$$

and it is mean-square stable (thus constitutes a rational expectations equilibrium to (A.1)) if and only if all eigenvalues of

$$(P' \otimes I_{n^2}) \text{diag}(F_{\varsigma_1} \otimes F_{\varsigma_1}, ..., F_{\varsigma_h} \otimes F_{\varsigma_h}) \tag{A.3}$$

lie within the unit circle. Here n is the number of variables considered, h denotes the number of regimes, $\otimes$ is the Kronecker-product and "diag" stacks matrices in a bigger diagonal matrix.

Specifically, as spelled out in the main text, in our model there are four distinct regimes, with transitions governed by

$$P = \begin{pmatrix} \mu & (1-\mu)\lambda & 0 & (1-\mu)(1-\lambda) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{A.4}$$

Recall that the model features two endogenous state variables ($\hat{d}_t^r$ and $p_{H,t}$) and one shock (ε_t^d). In what follows we outline the derivation of the solution (A.2) for the state variables only, so that $n = 2$.

We repeat the model equilibrium conditions for convenience:

$$y_t = E_t y_{t+1} - \frac{\varpi}{\gamma}(r_t - E_t \pi_{H,t+1}) \tag{A.5}$$
$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa \left(\varphi + \frac{\gamma}{\varpi}\right) y_t \tag{A.6}$$
$$y_t = -\frac{\varpi}{\gamma} s_t \tag{A.7}$$
$$s_t = p_{H,t} + e_t \tag{A.8}$$
$$\beta \hat{d}^r_t = \hat{d}^r_{t-1} + \zeta(\beta i_t - \pi_{H,t} - \delta_t) - \hat{t}^r_t \tag{A.9}$$
$$i_t = r_t + E_t(\delta_{t+1}) \tag{A.10}$$
$$\hat{t}^r_t = \psi_{\varsigma_t} \hat{d}^r_{t-1} - \varepsilon^d_t \tag{A.11}$$
$$\delta_t = \zeta^{-1} \delta_{\varsigma_t}\ \hat{d}^r_{t-1} \tag{A.12}$$
$$r_t = \phi_\pi \pi_{H,t} \quad \text{or} \quad e_t = 0, \tag{A.13}$$

with inflation being defined by $\pi_{H,t} = p_{H,t} - p_{H,t-1}$.

Union PF–Regime 3

We start by obtaining F_{ς_3} and G_{ς_3}. Combine equations (A.5),(A.7),(A.8) to obtain the UIP-condition, combine equations (A.6),(A.7),(A.8) to obtain a second order difference equation in the producer price:

$$r_t = -E_t(\Delta e_{t+1}) \tag{A.14}$$
$$\beta E_t(p_{H,t+1}) = \underbrace{(1 + \beta + \frac{\kappa\varphi\varpi}{\gamma} + \kappa)}_{\phi_{aux}} p_{H,t} - p_{H,t-1}. \tag{A.15}$$

Union PF is absorbing, thus $E_t(\Delta e_{t+1}) = 0$ and so $r_t = 0$. Prices are solved by $p_{H,t} = \phi^{PF} p_{H,t-1}$, with $\phi^{PF} = \phi_{aux}/2\beta - \sqrt{4\phi_{aux}^2/\beta^2 - 1/\beta} < 1$, where ϕ_{aux} is specified in (A.15). As there is no default in Union PF, $i_t = r_t = 0$ (A.10), and so

$$\beta \hat{d}^r_t = (1-\psi)\hat{d}^r_{t-1} - \zeta \pi_{H,t} + \varepsilon^d_t,$$

where we suppress the regime-dependence of ψ for expositional clarity (thus $\psi_{\varsigma_3} = \psi$, and accordingly for the other regimes below). Bringing all pieces together:

$$\underbrace{\begin{bmatrix} p_{H,t} \\ \hat{d}^r_t \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} \phi^{PF} & 0 \\ \frac{\zeta(1-\phi^{PF})}{\beta} & \frac{1-\psi}{\beta} \end{bmatrix}}_{F_{\varsigma_3}} \underbrace{\begin{bmatrix} p_{H,t-1} \\ \hat{d}^r_{t-1} \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{\beta} \end{bmatrix}}_{G_{\varsigma_3}} \varepsilon^d_t.$$

Default–Regime 2

Given that regime Default is purely transitory, and hence $E_t(\delta_{t+1}) = 0$ also here (yielding again $i_t = 0$), regimes Default and Union PF differ only in how debt evolves:

$$\underbrace{\begin{bmatrix} p_{H,t} \\ \hat{d}_t^r \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} \phi^{PF} & 0 \\ \frac{\zeta(1-\phi^{PF})}{\beta} & \frac{1-\psi-\delta}{\beta} \end{bmatrix}}_{F_{\varsigma_2}} \underbrace{\begin{bmatrix} p_{H,t-1} \\ \hat{d}_{t-1}^r \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{\beta} \end{bmatrix}}_{G_{\varsigma_2}} \varepsilon_t^d.$$

Exit AF–Regime 4

In regime Exit AF, there is an independent central bank. There is no outright default. Insert the Taylor-rule into (A.5) and (A.9) to obtain a three-by-three system in $(y_t, \pi_{H,t}, \hat{d}_t^r)$:

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{\varpi}{\gamma}(\phi_\pi \pi_{H,t} - E_t \pi_{H,t+1}) \\ \pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa(\varphi + \frac{\gamma}{\varpi}) y_t \\ \beta \hat{d}_t^r &= (1-\psi)\hat{d}_{t-1}^r + \zeta(\beta\phi_\pi - 1)\pi_{H,t} + \varepsilon_t^d. \end{aligned}$$

Now guess that $\pi_{H,t} = \phi_{\pi,d}^{AF} \hat{d}_{t-1}^r + \phi_{\pi,\varepsilon}^{AF} \varepsilon_t^d$ and $y_t = \phi_{y,d}^{AF} \hat{d}_{t-1}^r + \phi_{y,\varepsilon}^{AF} \varepsilon_t^d$:

$$\pi_{H,t} = \underbrace{\frac{\phi_{\pi,d}^{AF}(1-\psi) + \phi_{y,d}^{AF}\kappa(\varphi + \frac{\gamma}{\varpi})}{1 - \phi_{\pi,d}^{AF}\zeta(\beta\phi_\pi - 1)}}_{\phi_{\pi,d}^{AF}} \hat{d}_{t-1}^r + \underbrace{\frac{\phi_{\pi,d}^{AF} + \phi_{y,\varepsilon}^{AF}\kappa(\varphi + \frac{\gamma}{\varpi})}{1 - \phi_{\pi,d}^{AF}\zeta(\beta\phi_\pi - 1)}}_{\phi_{\pi,\varepsilon}^{AF}} \varepsilon_t^d$$

$$y_t = \underbrace{\frac{\phi_{y,d}^{AF}(\frac{1-\psi}{\beta} + \frac{\phi_{\pi,d}^{AF}\zeta}{\beta}(\beta\phi_\pi - 1)) - \frac{\phi_{\pi,d}^{AF}\varpi}{\gamma\beta}(\beta\phi_\pi - 1)}{1 + \frac{\varpi}{\gamma}\frac{\kappa}{\beta}(\varphi + \frac{\gamma}{\varpi})}}_{\phi_{y,d}^{AF}} \hat{d}_{t-1}^r$$

$$+ \underbrace{\frac{\phi_{y,d}^{AF}(\zeta\phi_\pi\phi_{\pi,\varepsilon}^{AF} - \frac{\zeta}{\beta}\phi_{\pi,\varepsilon}^{AF} + \frac{1}{\beta}) - \frac{\varpi}{\gamma}(\phi_\pi - \frac{1}{\beta})\phi_{\pi,\varepsilon}^{AF}}{1 + \frac{\varpi}{\gamma}\frac{\kappa}{\beta}(\varphi + \frac{\gamma}{\varpi})}}_{\phi_{y,\varepsilon}^{AF}} \varepsilon_t^d$$

Verify the guess first for $\phi_{\pi,d}^{AF}$ and $\phi_{y,d}^{AF}$ to obtain a quadratic equation in $\phi_{\pi,d}^{AF}$. The root which implies stable dynamics is given by $\phi_{\pi,d}^{AF} = -p/2 + \sqrt{p^2/4 - q}$, where

$$\begin{aligned} p &= -\left(\frac{1}{\beta}(\beta - 1 + 2\psi) + \frac{\varpi\kappa}{\gamma\beta}(\varphi + \frac{\gamma}{\varpi})\right) / \frac{\zeta(\beta\phi_\pi - 1)}{\beta} \\ q &= \left(\frac{\psi}{\beta}(\beta - 1 + \psi) + \frac{\varpi\kappa}{\gamma\beta}(\varphi + \frac{\gamma}{\varpi})(\beta\phi_\pi - 1 + \psi)\right) / \frac{\zeta^2(\beta\phi_\pi - 1)^2}{\beta}. \end{aligned}$$

The three other coefficients are linear in $\phi^{AF}_{\pi,d}$. We obtain:

$$\underbrace{\begin{bmatrix} p_{H,t} \\ \hat{d}^r_t \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} 1 & \phi^{AF}_{\pi,d} \\ 0 & \frac{1-\psi+\zeta(\beta\phi_\pi-1)\phi^{AF}_{\pi,d}}{\beta} \end{bmatrix}}_{F_{\varsigma_4}} \underbrace{\begin{bmatrix} p_{H,t-1} \\ \hat{d}^r_{t-1} \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} \phi^{AF}_{\pi,\varepsilon} \\ \frac{\zeta(\beta\phi_\pi-1)\phi^{AF}_{\pi,\varepsilon}+1}{\beta} \end{bmatrix}}_{G_{\varsigma_4}} \varepsilon^d_t.$$

Union AF–Regime 1

Given the closed-form expressions of the solutions for all target regimes, we now solve for regime Union AF. As in Union PF above, the equilibrium is characterised by the second order difference equation in prices (A.15). Split up $E_t(p_{H,t+1})$ into conditional expectations and evaluate each of them separately:

$$E_t(p_{H,t+1}|\text{Default}) = \phi^{PF} p_{H,t} \tag{A.16}$$

$$E_t(p_{H,t+1}|\text{Exit AF}) = p_{H,t} + \phi^{AF}_{\pi,d}\hat{d}^r_t \tag{A.17}$$

$$E_t(p_{H,t+1}|\text{Union AF}) = ?. \tag{A.18}$$

The third conditional expectation depends on the solution of regime Union AF which we have not yet worked out. To obtain an expression for bond yields, use the law of iterated expectations and combine (A.10) and (A.12):

$$i_t = -(1-\mu)(1-\lambda)E_t(e_{t+1}|\text{Exit AF}) + (1-\mu)\lambda\zeta^{-1}\delta\hat{d}^r_t. \tag{A.19}$$

Replace $E_t(e_{t+1}|\text{Exit AF})$ by combining (A.7) and (A.8):

$$i_t = (1-\mu)(1-\lambda)\left(E_t(p_{H,t+1}|\text{Exit AF}) + \frac{\gamma}{\varpi}\phi^{AF}_{y,d}\hat{d}^r_t\right) + (1-\mu)\lambda\zeta^{-1}\delta\hat{d}^r_t. \tag{A.20}$$

Now insert (A.17) into (A.20) and set $\hat{d}^r_t = \beta^{-1}\left((1-\psi)\hat{d}^r_{t-1} + \zeta(\beta i_t - (p_{H,t} - p_{H,t-1})) + \varepsilon^d_t\right)$ to obtain an expression for the yield i_t purely as a function of today's producer price and the relevant state variables $(p_{H,t-1}, \hat{d}^r_{t-1}, \varepsilon^d_t)$:

$$i_t = \vartheta_1 p_{H,t} + \vartheta_2 p_{H,t-1} + \vartheta_3 \hat{d}^r_{t-1} + \vartheta_4 \varepsilon^d_t, \tag{A.21}$$

with $\vartheta_1, ..., \vartheta_4$ being coefficient functions of the structural parameters. Plugging back (A.21) into (A.17) yields a similar expression for $E_t(p_{H,t+1}|\text{Exit AF})$:

$$E_t(p_{H,t+1}|\text{Exit AF}) = \eta_1 p_{H,t} + \eta_2 p_{H,t-1} + \eta_3 \hat{d}^r_{t-1} + \eta_4 \varepsilon^d_t, \tag{A.22}$$

with, again, $\eta_1, ..., \eta_4$ being coefficient functions of the structural parameters.

We are now in the position to apply the guess-and-verify method. Guess that, while in regime Union AF, producer prices evolve as $p_{H,t} = \phi_p^{UAF} p_{H,t-1} + \phi_d^{UAF} \hat{d}^r_{t-1} + \phi_\varepsilon^{UAF} \varepsilon_t^d$ and solve (A.18):

$$E_t(p_{H,t+1}|\text{Union AF}) = \phi_p^{UAF} p_{H,t} + \frac{\phi_d^{UAF}}{\beta}\left((1-\psi)\hat{d}^r_{t-1} + \zeta(\beta i_t - (p_{H,t} - p_{H,t-1})) + \varepsilon_t^d\right),$$

the third conditional expectation needed to evaluate the full of $E_t(p_{H,t+1})$. Finally, replace i_t by (A.21) and rearrange (A.15) to verify the guess:

$$\begin{aligned} p_{H,t} = & \underbrace{\frac{-(\mu\phi_d^{UAF}\zeta(\beta\vartheta_2+1) + (1-\mu)(1-\lambda)\beta\eta_2 + 1)}{\mu(\beta\phi_p^{UAF} + \phi_d^{UAF}\zeta(\beta\vartheta_1 - 1)) + (1-\mu)(\beta\lambda\phi^{PF} + \beta(1-\lambda)\eta_1 - \phi_{aux})}}_{\phi_p^{UAF}} p_{H,t-1} \\ & + \underbrace{\frac{-(\mu\phi_d^{UAF}(1-\psi+\zeta\beta\vartheta_3) + \beta(1-\mu)(1-\lambda)\eta_3)}{\mu(\beta\phi_p^{UAF} + \phi_d^{UAF}\zeta(\beta\vartheta_1 - 1)) + (1-\mu)(\beta\lambda\phi^{PF} + \beta(1-\lambda)\eta_1 - \phi_{aux})}}_{\phi_d^{UAF}} \hat{d}^r_{t-1} \\ & + \underbrace{\frac{-(\mu\phi_d^{UAF}(\beta\zeta\vartheta_4+1) + (1-\mu)(1-\lambda)\beta\eta_4)}{\mu(\beta\phi_p^{UAF} + \phi_d^{UAF}\zeta(\beta\vartheta_1 - 1)) + (1-\mu)(\beta\lambda\phi^{PF} + \beta(1-\lambda)\eta_1 - \phi_{aux})}}_{\phi_\varepsilon^{UAF}} \varepsilon_t^d \end{aligned}$$

Verify the guess first for ϕ_p^{UAF} and ϕ_d^{UAF} to obtain a cubic polynomial in ϕ_d^{UAF}. The polynomial has three real roots, all of which imply explosive paths for the state variables while in Union AF. However, for the calibrated model we verify that at most one of these solution candidates satisfies mean-square stability (the root lying in [0,0.5]). The coefficients ϕ_p^{UAF} and ϕ_ε^{UAF} are linear functions of ϕ_d^{UAF}.

We thus obtain:

$$\underbrace{\begin{bmatrix} p_{H,t} \\ \hat{d}^r_t \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} \phi_p^{UAF} & \phi_d^{UAF} \\ \frac{\zeta(\beta(\vartheta_1\phi_p^{UAF}+\vartheta_2)-(\phi_p^{UAF}-1))}{\beta} & \frac{1-\psi+\zeta(\beta(\vartheta_1\phi_d^{UAF}+\vartheta_3)-\phi_d^{UAF})}{\beta} \end{bmatrix}}_{F_{\varsigma_1}} \underbrace{\begin{bmatrix} p_{H,t-1} \\ \hat{d}^r_{t-1} \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} \phi_\varepsilon^{UAF} \\ \frac{\zeta(\beta(\vartheta_1\phi_\varepsilon^{UAF}+\vartheta_4)-\phi_\varepsilon^{UAF})+1}{\beta} \end{bmatrix}}_{G_{\varsigma_1}} \varepsilon_t^d.$$