

The Low Demand for Long-Term Care Insurance: Housing, Bequests, or Both?

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Abstract

It has been argued that preferences to stay in a given home and limited home-equity extraction possibilities could explain part of the low demand for long-term care insurance (LTCI). This paper constructs a life-cycle model of retired individuals with both housing and LTCI demand. I estimate different versions of this model using the Health and Retirement Study to assess the impact of housing on LTCI demand. Absent bequest motives, the model with a preference to stay in a given home matches wealth decumulation patterns, home ownership rates, and LTCI demand better than the model without it. But, it still fails to explain the slow wealth decumulation of the richest in combination with their low demand for LTCI. The specifications of the model with bequest motives are the most successful in matching all these dimensions, with or without preferences to stay in a given home. For these latter, experiments show that increasing home-equity extraction possibilities would not increase LTCI demand, casting doubt on the idea that limited home-equity extraction limits the market for LTCI.

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1 Introduction

US retirees face a high probability of needing long-term care (LTC), and in particular nursing home care, late-in-life. Hurd et al. (2014) show, using Health and Retirement Study (HRS) data, that around 40 to 45 percent of retirees aged 90 or more had a nursing-home stay in the past two years relative to the time they were interviewed. Some of these stays can be very long, and the cost of such stays is high. For instance, Brown and Finkelstein (2008) report that, in 2002, the average daily cost for a semi-private bedroom in a nursing home was \$143, which translates in an annual cost of \$52,195. For richer individuals, most of these expenditures are paid out-of-pocket which results in large out-of-pocket medical expenditures late-in-life (see De Nardi et al. (2010)).

There are two reasons why richer retirees pay so much out-of-pocket. First, Medicaid, the public program which pays for nursing home expenditures, is means-tested. As a matter of fact, richer individuals will be covered by public insurance only if they deplete their wealth first. The second reason is that relatively few retirees, even among richer ones, hold a long-term care insurance (LTCI). While for retirees with small pensions and few assets at the beginning of retirement, not holding a LTCI can easily be rationalized by Medicaid means-test and second-payer status (Brown and Finkelstein (2008)), it ends up to be much harder to rationalize it for individuals with high pensions and substantial wealth early in retirement. Indeed, richer retirees dissave at a slow rate consistent with precautionary savings due to medical expense risk (De Nardi et al. (2010)), which suggests that they should be willing to insure against this risk. Moreover, loads on LTCI are not particularly high (Brown and Finkelstein (2007)) and, arguably, not so high as to lead rich retirees to prefer to self-insure through saving rather than through LTCI insurance¹.

From a purely rational standpoint, there are few demand-side factors that might explain why richer retirees dissave little and purchase as little LTCI as observed empirically. One is the presence of strong bequest motives, in combination with the fact that bequests are luxury goods, as shown in Lockwood (2016). Another potential explanation is the fact that retirees might be unwilling to leave their homes, except if they need to go to a nursing home, while having limited access to borrowing out of housing equity (see Caplin (2002)). In this case, retirees would be “constrained” to decumulate housing equity mainly at the time they require care, so that housing would substitute for LTCI. This argument, put forward by Davidoff (2009, 2010) and based on the observation made, in particular, by Venti and Wise (2004) that retirees mainly decumulate housing equity when going to a nursing home, has (to my knowledge) not been (quantitatively) tested empirically and confronted to the bequest motive explanation. In particular, there has been no attempt to evaluate whether Davidoff’s theory could explain the extent of slow wealth decumulation in the data alongside the observed low LTCI demand. Given that housing represents a high share of retirees total net wealth (the median is about 77 percent for homeowners in my simulated sample²), and that homeownership rates decline slowly with age and remain substantial even for individuals in their 90s, it seems important to assess the potential quantitative impact of housing on LTCI demand. Even more so, as Davidoff’s argument suggests that increasing the ability of retirees to extract home equity may substantially increase the demand

¹This point is discussed in details in section 6.4 for the setting here and is also put forward in Lockwood (2016).

²The mean ratio is 78 percent. For those with net wealth larger than the value of their primary residence, these figures are 65 and 62 respectively. These numbers are computed using the Health and Retirement Study (HRS), see details below.

for LTCI. The present paper tries to fill part of the gap³.

To do so, I construct a rich life-cycle model in which single retirees make decisions about tenure choice, housing size, savings and LTCI in presence of mortality risk, LTC risk and (other) medical expense risk. The key structural parameters of the model are estimated using a method of simulated moments. I study to which extent different versions of this model, e.g. with a preference for staying in a given home (which I also call “utility cost of moving”) and/or with bequest motives, can match the empirical patterns for wealth decumulation, LTCI demand, and homeownership rates. First, it allows me to study if specific preferences for housing in combination with limited home equity extraction possibilities may alone have explained the historically low demand for LTCI alongside slow wealth decumulation and slow decline in homeownership rates with age. Second, I can compare the housing story to the bequest motive story, to see if one performs better than the other. Third, I can combine them and see how the model performs in this case. The model is particularly well suited to study whether increased possibilities for home equity extraction may lead to an increase in LTCI demand, and if so by how much. It also enables to study the effect of the exclusion of most housing wealth in the computation Medicaid’s means-test on LTCI demand (see De Nardi et al. (2012)).

First, I find that the model with housing tenure choice but without a utility cost of moving, nor bequest motives, performs poorly confirming the results of Lockwood (2016). When trying to match wealth profiles, homeownership rates and LTCI demand, this specification generates too much wealth decumulation, very rapid declines in homeownership rates with age counterfactually to the data, and LTCI demand way higher than in the data. This specification relies only on precautionary motives. To generate the slow wealth decumulation in the data, the precautionary motive must be strong, but the stronger it is the higher LTCI demand will be. This link between the motives for saving and LTCI demand explains the bad match of this specification of the model. Additionally, I show that, when not trying to match LTCI demand, the model does quite well both for wealth decumulation and the patterns of homeownership. The good match for homeownership stems partly from the precautionary saving motive in combination with rent prices which differ from maintenance costs. In this case LTCI rates are counterfactually high.

Second, the model with a utility cost of moving, but without bequest motives, performs much better relative to LTCI demand and homeownership rates. Both are close to the data. However, it generates still too much wealth decumulation relative to the data for individuals with high permanent income. The reason, as explained in section 3, stems from the fact that Davidoff’s theory (without additional elements) plays essentially when preferences to stay in a given home and limited home-equity extraction possibilities lead an individual to have, in older ages, an amount of wealth which is more than what he would have kept for pure precautionary reasons. It happens that total wealth for higher-income individuals is too high relative to housing wealth for the model to match it in combination with their observed LTCI demand.

Third, the model with bequest motives, but without a utility cost of moving, manages to match quite well all the moments. It tends to generate a bit too much wealth accumulation for higher-income individuals at older ages and slightly too little LTCI demand, but the results confirm those in Lockwood (2016) that the low demand for LTCI can be well explained by the presence of bequest motives. When combining both the utility cost of moving and bequest motives, the parameters still point towards strong bequest motives.

³Some other potential explanations are discussed below.

Finally, I study the implications of increasing the possibility of home equity extraction on LTCI demand, something which has, to my knowledge, not been assessed yet in a careful quantitative exercise. In the model with a preference to remain in a given home but no bequest motives, it generates a very large increase of LTCI demand concentrated for those with high income. Versions of the model with bequest motives (with or without a utility cost of moving) lead, however, to a very different conclusion. In this case, increased ability to extract home equity does not lead to an increase in LTCI demand and can even decrease it. This effect is most likely due to a substitution effect as increased borrowing allows better consumption smoothing, and thus improves self-insurance through savings. As these specifications most closely fit the data, this result suggests that limited home-equity extraction possibilities is unlikely to explain the low demand for LTCI, and that improving, for instance, the design of reverse mortgages may not have any effect on LTCI demand.

The rest of the paper is organized as follow. In section 2, I discuss the related literature. In section 3, I discuss Davidoff's theory in more details. In section 4, I describe the model. In section 5, I present the data set and the details of the first-stage estimation. In section 6, I show the results of the second-stage estimation and discuss the fit of different specifications of the model. In section 7, I perform different experiments and, in particular, study the effect of increased home equity extraction on LTCI demand.

2 Related Literature

This paper is, first, related to the literature trying to understand the reasons behind the low demand for LTCI in the data and, in particular, on the demand side. Brown and Finkelstein (2007) show that supply side factors are unlikely to explain the small size of the LTCI market and that demand-side factors are likely to be important. Brown and Finkelstein (2008) show that Medicaid generates an implicit tax on LTCI because of its second-payer status and means-test, which can explain the very low demand for LTCI for a significant share of the population. However, as shown in Lockwood (2016) or here, Medicaid can hardly explain (alone) the low demand for LTCI of richer individuals. Davidoff (2009, 2010) discusses the reasons for which housing may limit the demand for LTCI, a point also mentionned in the literature review by Brown and Finkelstein (2009) on the evidence about the low demand for LTCI. He shows that, if individuals have a strong preference to remain in a given home and have little access to home-equity extraction products, housing may limit the demand for LTCI. The present paper tries to assess to which extent this theory (in a more realistic model) can explain the low demand for LTCI observed empirically. Lockwood (2016) shows that bequest motives which are strong and luxury goods can explain both the low demand for LTCI and the slow dissaving rate of wealthy retirees⁴. The main difference between my paper and his stems from the inclusion of housing. In particular, this allows me to study how specific preferences relative to homeownership may affect LTCI demand, and the potential effect of increasing ability to extract home equity on LTCI demand. Mommaerts (2016) show that provision of care from family members may reduce the demand for LTCI, a channel I do not include here. Ameriks et al. (2016) use strategic surveys (as in Ameriks et al. (2011)) and show that flaws in LTCI design may explain part of the "LTCI puzzle". Kojien et al. (2016) study the optimal demand for insurance products in a life-cycle model of older households. Though housing is present in their model,

⁴In Lockwood (2012), he also studies the impact of bequest motives on annuity demand.

it is in relatively stylized way as housing and bonds are seen as perfect substitutes.

This paper is also related to the literature which tries to understand savings behaviours of retirees. Palumbo (1999) and, more recently, De Nardi et al. (2010) show that medical expenditures can help explain why retirees dissave at a slow pace in the US. Kopecky and Koreshkova (2014) study more precisely the impact of nursing home expenses in a general equilibrium model and show that their impact on savings can be large. Nakajima and Telyukova (2012), in a model in many ways close to the one here, study how housing affects the dissaving behaviour of retirees. In Nakajima and Telyukova (2016), they use a similar model to study the welfare impact of reverse mortgages and how different designs of the product may increase demand. In my model I allow for LTCI demand, an insurance channel that they do not consider. Moreover, I try to match LTCI demand which, as already mentioned in the introduction, has a large influence on identification. Given the large size of nursing home expenditures late in life (see Kopecky and Koreshkova (2014)), it is reasonable to think that introducing the possibility to purchase LTCI may have potentially large influence on the attractiveness of home-equity release products. The model here also borrows several elements from Yao and Zhang (2005). Yogo (2016) also studies the life-cycle behaviour of retirees in a model with housing. The specificities of his model is that he includes demands for risky assets and annuities, and allows for endogenous health. However, he does not include the demand for LTCI.

Ameriks et al. (2011) use strategic survey questions to identify bequest motives. De Nardi et al. (2016) include endogenous medical expenditures in a life-cycle model of retirees and match Medicaid rates. They show that Medicaid can help to identify bequest motives. I do not match Medicaid rates here, but I show that the model generates Medicaid rates close to the data and also replicates quantitatively well the fact that renters rely more on Medicaid than homeowners. De Nardi et al. (2016) use their model to study the effect of Medicaid reform on welfare. Barczyk and Kredler (2016) study a similar point but include informal care from the family as well as strategic bequest motives in a rich dynamic setting. Braun et al. (2015) study the welfare gains from means-tested social insurance programs in a general equilibrium model and find them to be large.

3 Housing as a substitute for LTCI

This section aims at giving additional intuitions on why housing may substitute for LTCI (following Davidoff (2009, 2010)), and in which cases this effect will be large or small, and also why ultimately it is a quantitative question. Consider first a life-cycle model without bequest motives, without housing and without the possibility to purchase LTCI, and in which there is a high risk of nursing home expenditures late in life. Figure 1 illustrates in a schematic way the possible outcomes in such a model. On the y -axis is total wealth and on the x -axis is age. The starting age is the age of retirement. At this age, wealth level corresponds to the one at point A. Individuals can live up to the age corresponding to point D. If the observed average wealth profile for an individual in the data corresponds to the grey curve going through A, C, and D, then estimated risk aversion will usually be higher than if the observed wealth profile were the black curve going through A, B and D.

Let's introduce housing into the picture and let's assume that individuals start as homeowners, do not

want to leave their homes except if they have to go to a nursing home, and cannot extract home equity if they stay in their homes. Housing wealth is the dotted line and total wealth is unchanged initially at A, so that, initially, the difference between the curves for total wealth and housing wealth represents “liquid wealth”. As a first approximation, there is no reason to think that putting housing into the picture should change the amount that an individual would like to save for precautionary reasons. We can thus assume that *desired* total wealth⁵ for our two fictitious types of individuals is still given by the black and grey curves.

For least risk averse individuals, we observe that total wealth will, likely, start to be higher than desired total wealth after point B. Indeed, if individuals have a really high cost to leave their homes, they will stay in it and, as withdrawing housing equity in this case is not feasible, their wealth will be higher than what they would desire to save for precautionary reasons. As a matter of fact, for these individuals housing might be a strong substitute for LTCI as they tend to be over-insured by housing quite early in retirement. The extent of over-insurance is represented by the shaded area (including the part with horizontal lines). Typically, most of the gains from LTCI (absent bequest motives) stem from being able to decumulate wealth faster, and they are small for these individuals as they already decumulate wealth fast and are constrained in how much they can decumulate. So, it is likely that this individuals will not want to purchase a LTCI if offered the possibility.

The gains to have access to a LTCI are, however, potentially still large for individuals with desired total wealth looking more like the grey curve, despite housing and borrowing constraints. Indeed, they are only over-insured by housing after point C, and the extent of over-insurance is only the shaded area with horizontal lines.

This illustrates what is done in the quantitative exercise. Typically, Davidoff’s theory may fail to be able to explain by itself the low demand for LTCI if total wealth remains much higher than housing wealth despite the possibility to purchase LTCI. In such case, other elements need to be included such as bequest motives. It is thus an empirical question, even more so as we also need to take into account, for instance, the fact that LTCI are not fair, have usually a cap on benefits, and that there are other medical expenditures. These elements are all included in the model presented in the next section.

4 Model

4.1 Preferences

Time is a year. t denotes age and ty denotes a calendar year. In the model, a single individual seeks to maximize her expected utility at each age $t = t_0, \dots, T$ where $t_0 = 67$ and $T = 104$ is the maximum age a person can reach (i.e. the probability to die from age T to $T + 1$ is zero). I consider several specifications for utility flows as the preference to stay in a given home can *a priori* take several forms. The first one I consider is:

$$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma} - \phi_s (hs_t) d_t^s \tag{1}$$

c_t is a consumption good and h_t is housing which, as detailed below, can be owned or rented. γ is the

⁵I.e. the one, she would like to have if she had the possibility to release home-equity while at home.

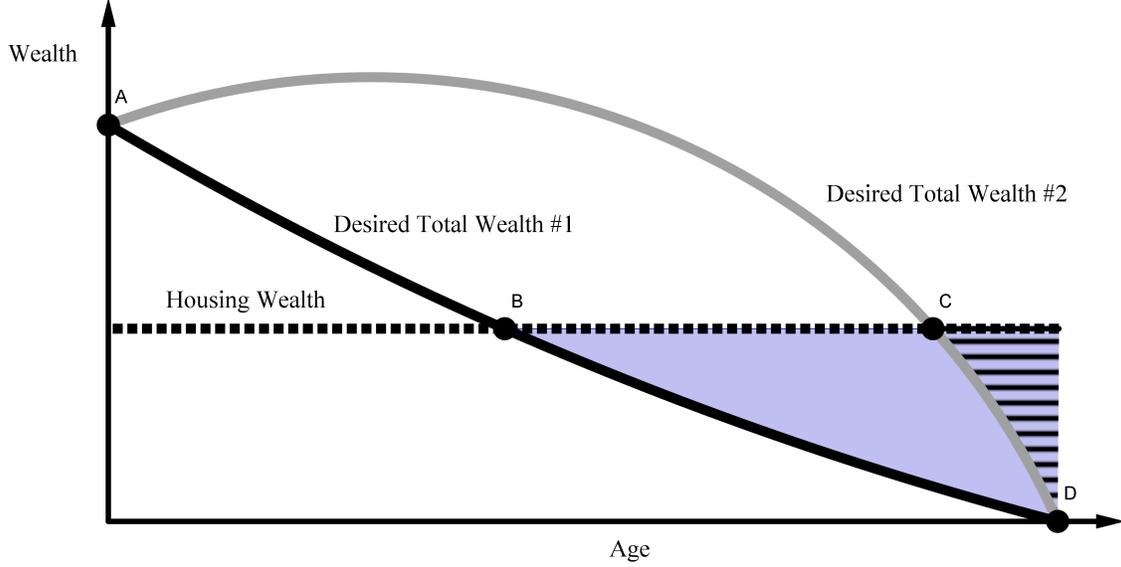


Figure 1: Wealth decumulation, housing and precautionary motives

curvature of the utility function over overall consumption⁶. d_t^s is a dummy equal to 1 if the individual was a homeowner⁷ in $t - 1$ and decides to sell her home in t , and to 0 otherwise. Thus, $\phi_s(hs_t) \geq 0$ represents a utility cost of selling a home, which I call utility cost of moving. This cost is a function of the current health state of the individual hs_t . The different health states are described below. Typically, this cost is positive except when the individual will have to move to an assisted-living facility or a nursing home. This specification is close to the one considered in Davidoff (2009), and it attempts to capture the fact that individuals who are not moving to a nursing home or assisted-living facility may be unwilling to move out of their homes. The second specification is:

$$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma} + \phi_o(t, hs_t) d_t^o \quad (2)$$

d_t^o is a dummy equal to 1 if the person decides to be a homeowner in t , and to 0 otherwise. $\phi_o(t, hs_t) \geq 0$ is an extra (additive) utility from owning a home which is allowed to depend on t and hs_t . It is important to note at this point that, in the model, existing homeowners are allowed to decide whether to move out of their home or not, or to rent. However, they are not allowed to purchase another home. Similarly, existing renters can only remain renters. This assumption is done partly for computational reasons, but also because homeownership rates decline with age in my sample and because the model, despite this assumption, can

⁶Note that, in presence of durables, γ is not the parameter of relative risk aversion, see Grossman and Laroque (1990) and Chetty and Szeidl (2007).

⁷In the model, individuals can only own one home and do not rent it to other individuals. I am interested here in the consumption aspect of housing and not in “pure” investment decisions in the housing market. A similar assumption is made in Yao and Zhang (2005) or Nakajima and Telyukova (2012, 2016).

still reproduce well median housing wealth for homeowners in combination with homeownership rates. One thing to keep in mind is that I am only considering single individuals here, for which this assumption might be less problematic than for couples. Due to this assumption, $\phi_o(t, hs_t)$ can be interpreted as some sort of habit formation regarding a specific home⁸. If ϕ_o is increasing with age for healthy retirees, it would imply that older individuals are less willing to move because they are more attached to a particular place.

The last specification is:

$$\frac{([1 + \zeta(t, hs_t) d_t^\omega] c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma} \quad (3)$$

This specification is close to (2). $\zeta(t, hs_t) \geq 0$ is an extra utility from owning the home. However, here, rather than being additive it is a complement to consumption, and thus affects the marginal utility of consumption. Two forces are worth to be noticed when this specification is adopted. If $\zeta(t, hs_t)$ is increasing in t and the individual expects to remain a homeowner, there is a rational to consume more earlier than later because, at a given level of c_t and h_t , marginal utility falls with age. But, there is also an incentive to keep enough resources in the future to be able to remain a homeowner which goes against the first force. Which dominates is unclear. Notice that both $\zeta(t, hs_t)$ and $\phi_o(t, hs_t)$ will be zero when a person has to go in an assisted-living facility or nursing home.

Individuals may also derive utility from leaving a bequest $v(Beq_t)$ if they die at age t . The specification is the same as in Lockwood (2016):

$$v(Beq_t) = \left(\frac{\phi_b}{1-\phi_b} \right)^\gamma \frac{\left(\frac{\phi_b}{1-\phi_b} c_b + Beq_t \right)^{1-\gamma}}{1-\gamma} \quad (4)$$

Beq_t is the amount of bequests left at time t , $\phi_b \in (0, 1)$ affects the strenght of the bequest motive, while $c_b \geq 0$ affects the curvature of the bequest function and to what extent bequests are luxury goods. Finally, the time preference parameter is β .

4.2 Timing

At each age t , an individual draws one out of six health states hs_t . hs_t can be: good health gh , bad health bh , receiving home care hc , in an assisted-living facility alf , in a nursing home nh , or dead. Death is naturally an absorbing state. The probability to draw a given health state depends on age t , health state hs_t , and permanent income quintile I . The health state determines the amount of LTC expenditures which are denoted, for simplicity, $M_t = M(hs_t, t, ty)$. They are a deterministic function of health state, age, and calendar year. The dependence of this latter is intended to capture the growth in real LTC and medical expenditures. This generates cohort effects.

The individual then draws a medical expenditure shock (if alive) $\varepsilon_t^m \sim \mathcal{N}\left(0, \sigma^m(I, hs_t, t, ty)^2\right)$. This shock determines the amount of medical (non-LTC) expenditures m_t which are log-normally distributed:

⁸Without this assumption, to have some sort of habit formation regarding a specific home, I would have to keep track of how long the person had been in this particular home.

$$\ln m_t = \mu^m (I, h s_t, t, ty) + \varepsilon_t^m \quad (5)$$

After these shocks are drawn, the individual makes decisions to maximize expected utility under a set of constraints described in the next subsection. In the model, the only risks are idiosyncratic and there is no aggregate risk. In particular, house prices will be deterministic.

4.3 Constraints and Medicaid

4.3.1 The no-Medicaid case

I first describe the constraints faced by an agent who is not eligible for Medicaid. Next-period liquid wealth b_t , which takes the form of a risk-free bond, is given by:

$$\begin{aligned} b_t = & (R + \mu \times 1 \{b_{t-1} < 0\}) b_{t-1} + y_t + d_{t-1}^o d_t^s p^h(ty) h_{t-1} (1 - \phi_p) \\ & + \min(B_t, M_t) - m_t - M_t - \tau_t - x_t \end{aligned} \quad (6)$$

An agent inherits liquid wealth b_{t-1} (possibly negative) from $t - 1$ multiplied by the gross real interest rate R plus an extra-interest μ if liquid wealth was negative at the end of $t - 1$. $1 \{b_{t-1} < 0\} = 1$ if $b_{t-1} < 0$ and 0 otherwise. She also receives a pension income y_t . If she was a homeowner ($d_{t-1}^o = 1$), she can decide to sell her home ($d_t^s = 1$). In this case, she receives the proceeds from selling it, which is the price of one unit of the home $p^h(ty)$ times the size of the home h_{t-1} times one minus a transaction cost. House prices are a function of calendar year ty . In the application, I also allow growth in house prices to differ for houses of different sizes. If she has a LTCI, she receives as a benefit the minimum of: the maximum amount reimbursed by the insurance policy B_t and her LTC expenditures M_t . She also has to pay for her medical bills m_t , LTC bills M_t and taxes τ_t ⁹. What remains can then be used to finance other expenditures x_t . For a person who is not in *alf* or *nh*, x_t is given by:

$$x_t = c_t + Prem_t + d_{t-1}^o (1 - d_t^s) \psi^h(ty) h_{t-1} + (d_{t-1}^o d_t^s + (1 - d_{t-1}^o)) r^h(ty) h_t \quad (7)$$

These expenditures include consumption c_t , a LTCI premium $Prem_t$ if she has a LTCI¹⁰, a maintenance cost $\psi^h(ty) h_{t-1}$ proportional to the size of the home if she decides to remain a homeowner (in that case $h_t = h_{t-1}$). If, instead, she decides to rent she has to pay the rental price in year ty $r^h(ty)$ times the size of the home h_t she decides to live in. Implicit in this constraint is the assumption that only a homeowner in $t - 1$ can be a homeowner in t , and in this case she has to stay in the same home. In particular, notice that $d_t^o = d_{t-1}^o (1 - d_t^s) + d_{t-1}^o d_t^s + (1 - d_{t-1}^o)$. The desire to remain a homeowner will stem from three elements in the model: 1) the fact that, for a house of similar size, the maintenance cost $\psi^h(ty)$ per unit of housing will be lower than the rental price $r^h(ty)$, 2) preferences and 3) house price growth. The presence of a rental premium can also be found in Campbell and Cocco (2003) or Yao and Zhang (2005), and may reflect moral hazard issues Henderson and Ioannides (1983) as well as differences in tax treatments. Given the purpose of the article, distinguishing between financial incentives for homeownership and “pure” preference is important.

⁹The computation of taxes is detailed in appendix B.

¹⁰The way LTCI works is described in details below.

When a person is in *alf* or *nh*, x_t is given by:

$$x_t = \tilde{x}_t + d_{t-1}^o (1 - d_t^s) \psi^h (ty) h_{t-1} \quad (8)$$

and

$$c_t^\omega h_t^{1-\omega} = \omega^\omega (1 - \omega)^{1-\omega} (r^h (ty))^{\omega-1} (x_{priv} + \tilde{x}_t) \quad (9)$$

An individual moving to *alf* or *nh* will not pay any premium for LTCI anymore. If she decides to remain a homeowner she will still have to pay the maintenance cost, but she will not derive utility from it (as she is not currently living in it). A renter has no reason to pay for a rent if she lives in a nursing home or assisted-living facility in this setting, so the last term in (7) disappears. When a person pays for her care in those states, the term $c_t^\omega h_t^{1-\omega}$ (in (1), (2), or (3), depending on the specification) is assumed equal to $\omega^\omega (1 - \omega)^{1-\omega} (r^h (ty))^{\omega-1} (x_{priv} + \tilde{x}_t)$. This is the result of the maximization of $c_t^\omega h_t^{1-\omega}$ under the constraint $x_{priv} + \tilde{x}_t = r^h (ty) h_t + c_t$. So, if she only pays for her care but nothing in addition ($\tilde{x}_t = 0$), she gets a utility flow equivalent to what she would have had in another health state and being a renter by spending x_{priv} on consumption c_t and housing h_t given the rental price $r^h (ty)$. $\tilde{x}_t \geq 0$ represents additional spending that she might make to improve her utility in those states.

Borrowing is limited by the following constraint:

$$b_t \geq -d_t^o p^h (ty) h_t (1 - \lambda_t) 1 \{b_{t-1} < 0\} \quad (10)$$

Typically, λ_t increases in age in the application (following Nakajima and Telyukova (2012)) reflecting limited abilities to borrow out of housing equity for elderly individuals¹¹ and scheduled repayment of existing mortgages. The term $1 \{b_{t-1} < 0\}$ implies that only those who had existing debt can have debt. This assumption leads to a better match of the percentage of homeowners in debt and is reasonable given that, in my sample, the probability to have debt today conditional on having debt yesterday is 55.9%, while this probability is only 3.3% conditional on having no debt yesterday¹². Moreover, this constraint tends to favour the utility cost of moving argument, which tends to be rejected by the data.

4.3.2 The Medicaid case

Most of housing wealth is not considered in the calculation of the Medicaid asset test (De Nardi et al. (2012)). I thus define $\overline{ph}_t^{med} = \max \{0, d_{t-1}^o (1 - d_t^s) p^h (ty) h_{t-1} - \overline{ph}\}$ which is the amount of housing wealth taken into account in the computation of Medicaid. If the individual has no housing or a house worth less than \overline{ph} , she will have no housing wealth taken into account in the computation of Medicaid means-tests. If the individual has a house worth more than \overline{ph} and decides to remain in it, $p^h (ty) h_{t-1} - \overline{ph}$ will be taken into

¹¹In particular, due to the fact that many of them fail income requirements (Caplin (2002))

¹²Considering current homeowners, these figures are 65.3% and 6.2% respectively. Moreover, the amount borrowed by newly debtors is not very large with a median of around \$11,000.

account in the computation of Medicaid means-tests. Medicaid pays an amount equal to:

$$\begin{aligned}
Medicaid_t = \max & \left\{ 0, \bar{x}(hs_t) - \left[Rb_{t-1}1\{b_{t-1} \geq 0\} + y_t + d_{t-1}^o d_t^s p^h(ty)h_{t-1}(1 - \phi_p) + \overline{ph}_t^{med} \right. \right. \\
& \left. \left. + \min(B_t, M_t) - m_t - M_t - \tau_t \right] \right\}
\end{aligned} \tag{11}$$

Notice that Medicaid does not take into account existing debt in its computation ($1\{b_{t-1} \geq 0\} = 1$ if $b_{t-1} \geq 0$ and 0 otherwise). However, LTCI benefits $\min(B_t, M_t)$ are taken into account in the computation of Medicaid. This reflects the second-payer status of Medicaid and generates an implicit tax on LTCI as studied in Brown and Finkelstein (2008). Medicaid insures individuals by allowing a minimal level of expenditures in states where medical and LTC expenditures would be too high. So, b_t is now given by:

$$\begin{aligned}
b_t = Medicaid_t + (R + \mu \times 1\{b_{t-1} < 0\})b_{t-1} + y_t + d_{t-1}^o d_t^s p^h(ty)h_{t-1}(1 - \phi_p) \\
+ \min(B_t, M_t) - m_t - M_t - \tau_t - x_t
\end{aligned} \tag{12}$$

We also have:

$$c_t^\omega h_t^{1-\omega} = \omega^\omega (1 - \omega)^{1-\omega} (r^h(ty))^{\omega-1} (x_{pub} + \tilde{x}_t) \tag{13}$$

The difference between (9) and (13), is that x_{priv} has been replaced by x_{pub} . If $x_{pub} < x_{priv}$, there is public care aversion (Ameriks et al. (2011)) meaning that, all else equal, a care facility paid by Medicaid will provide lower utility than a private care facility. The other constraints from the previous subsection all apply.

The way Medicaid is formulated in (11) can lead to a strategic behaviour by homeowners. Indeed, a homeowner could receive Medicaid payments and increase her debt, so being able to have x_t in principle much higher than $\bar{x}(hs_t)$. Increasing debt, i.e. releasing home equity, can be considered as additional income which would enter the Medicaid income-test. As a matter of fact, for those who receive Medicaid I make Medicaid payment conditional on the fact that $x_t \leq \bar{x}(hs_t)$. If not Medicaid payments are set to zero. In other words, a homeowner eligible for Medicaid can give up Medicaid if he wants to (and can) consume more than $\bar{x}(hs_t)$ while remaining a homeowner¹³.

4.4 Long-Term Care Insurance

The individual is given the possibility to purchase a LTCI only at age 67 if healthy, which is the average age at which LTCI is purchased in the US (Brown and Finkelstein (2007)). This similar to what is done in Lockwood (2016). If she decides to purchase a LTCI ($LTCI_t = 1$, 0 otherwise), she has to pay a premium $Prem_t$ each year she does not receive care, if she wants to receive benefits ($\min(B_t, M_t)$ in the constraints above) when she requires care. Past 67, individuals who have a LTCI are allowed to default on their LTCI (i.e. set $LTCI_t = 0$).

¹³For interested readers, notice that the fact that Medicaid payments depend on the sale of the home prevents from getting rid of a state variable by rewriting the problem as a function of cash-on-hand.

4.5 Bequests

At the time of death, bequests are given by:

$$Beq_t = d_{t-1}^o p^h(ty) h_{t-1} (1 - \phi_p) + Rb_{t-1} \quad (14)$$

4.6 Recursive formulation

The problem is solved recursively starting from T . The value function is:

$$\begin{aligned} V_t(I, cht, hs_t, LTCI_{t-1}, b_{t-1}, d_{t-1}^o, h_{t-1}, \varepsilon_t^m) &= \max_{c_t, h_t, d_t^o, LTCI_t, Medicaid_t} u(t, c_t, h_t, d_t^o, d_t^s, hs_t, Medicaid_t) \\ &+ \beta s(I, t, hs_t) E_t [V_{t+1}(I, cht, hs_{t+1}, LTCI_t, b_t, d_t^o, h_t, \varepsilon_{t+1}^m)] \\ &+ \beta (1 - s(I, t, hs_t)) v(Beq_{t+1}) \end{aligned} \quad (15)$$

where $s(I, t, hs_t)$ is the survival probability. The expectation E_t is conditional on income quintile I , cohort cht , health state hs_t and age t . The individual solves this problem subject to the above constraints by choosing the level of consumption c_t , the size of the home h_t , whether to own or not d_t^o , whether to have a LTCI or not ($LTCI_t$), and whether to apply for Medicaid ($Medicaid_t$) if eligible. Some of these choices are restricted as mentioned above, and some variables such as b_t , Beq_{t+1} , and d_t^s are just functions of the state variables and the previous control variables. The solution method is standard and is detailed in appendix A.

5 First-stage estimation

5.1 Data, Selection, and Definitions

I use the HRS data and more specifically the RAND version of the HRS except when a specific data is not in the RAND version¹⁴. I use waves from 1998 to 2012 (included) for the first-stage estimation, hence 8 waves given that the HRS is bi-annual. For the second-stage estimation, I consider only waves until 2006 (included) given that, for computational reasons, house prices are treated as deterministic in the model. I consider only women interviewed in 1998, who are never in a couple in any of the waves considered, who never declare not to be retired and who are born before 1936 (hence 62 or older). I only select single women mainly as they are much more numerous than single men¹⁵.

All monetary values are expressed in 1998 dollars using the BEA implicit price deflator for personal consumption expenditures. I define pension income (y_t) as the sum of Social Security benefits, benefits from defined-benefits pension plans, annuities, welfare, veteran's benefits and food stamps as in De Nardi et al. (2010). For each woman in my sample, I then compute the average pension income during the observation period which defines permanent income. I then split the sample in five permanent income (or simply income) quintiles.

Appendix C provides the details of the construction of the health state variable hs_t . In short, a woman is said to be in good health gh if her self-reported health is excellent, very good, or good, and if she is not

¹⁴The RAND version of the HRS is a cleaned version of the original HRS files.

¹⁵Computational time prevents me from solving the model for both, as it would roughly double it.

classified in health states hc , alf , or nh . Similarly a woman is in bad health if her self-reported health is poor or bad, and if she is not classified in health states hc , alf , or nh . The classification in hc , alf , and nh raises some difficulties which are detailed in the appendix. In particular, one has to be careful given that some nursing home stays are short (less than a year) and that the model is annual. The *annual* transition matrices constructed using the classification chosen give outcomes in line with *monthly* ones used in previous research (Robinson (1996), Brown and Finkelstein (2008), Friedberg et al. (2014)). For instance, the unconditional mean years in nursing home for a 67 year-old that I estimate is 0.88 compared to reported unconditional means of 0.79 and 0.88 for a woman aged 65 respectively in Friedberg et al. (2014) and Brown and Finkelstein (2008). Moreover, as shown in table 8 the overall distribution of nursing home risk is close to the one found in previous research and the computation of LTCI premia match well those in Brown and Finkelstein (2007). Some of the differences may be explained by the fact that I consider single women only.

5.2 Transition matrix

To construct the transition, I use a multinomial logit in which the probability of a being in a given health state at $t + 1$ depends on a full set of dummies for health state at t , a full set of dummies for income quintile, a cubic in age t , age interacted with income quintile, and health status at t interacted with age. Given that individuals are only observed every two years in the HRS, the log-likelihood which is maximised is the one of observing a given health state at $t + 2$ given the set of the above covariates at age t . This is similar to what is done in De Nardi et al. (2016). Additional details are in appendix D, as well as comparisons with results from other transition matrices used in the literature.

Table 1 shows the results of a set of simulations to inform on longevity and nursing home risk at age 67, the beginning of the simulations in the model, for my sample of single women. The initial distribution of health states and income quintiles is based on the sample of women aged 67 to 69 in my sample. We observe the usual positive gradient between life expectancy and income quintile. While a single woman aged 67 in the first income quintile only expects to live an additional 10.7 years, this figure is 17.1 for a woman in the fifth income quintile. A higher longevity should, absent a bequest motive, lead women in higher income quintiles to dissave less all else equal as their effective patience is on average higher (the term $\beta s(I, t, hs_t)$ in (15)).

The relationship for nursing home risk implied by the estimated transition matrix is non-linear in income. Those who appear to face more risk belong to the third income quintile. Moreover, the probability to be in a nursing home more than 1 year is roughly similar for those in the first and fifth income quintile. This non-linear relationship may be explained by two opposite forces. If the distribution of health states by age were identical for each income quintile, those with higher income would be expected to face higher nursing home risk as they are expected to live longer. But, the distribution of health at a given age is usually more tilted towards worse health states for those with lower income. As a consequence, such a non-linear relationship is possible. For instance, the figures for nursing home risk are very similar for the first and fifth income quintile but, given the much lower life expectancy of those in the first income quintile, it implies that their probability to be in a nursing home while alive is much higher.

Table 1: Simulated life expectancy and nursing home risk at 67

income quintile	Life expectancy	Proba to experience nh	Proba to be in nh for more than 1 year	Proba to be in nh for more than 3 years	Proba to be in nh for more than 5 years
all	13.9	0.32	0.20	0.08	0.03
1st	10.7	0.31	0.20	0.09	0.04
2nd	12.2	0.30	0.19	0.07	0.03
3rd	14.1	0.35	0.23	0.10	0.04
4th	15.8	0.32	0.20	0.08	0.03
5th	17.1	0.32	0.19	0.07	0.03

Note: Results based on simulations using the estimated transition matrix and the distribution for income and health of individuals aged between 67 and 69 in my sample (which are then assumed to be 67 when simulating forward).

5.3 LTC expenditures

Brown and Finkelstein (2008) use for the price of a nursing home \$52,195 and for the price of an assisted-living facility \$26,280. They compute the cost of home care using Robinson’s model (Robinson (1996)), which gives skilled and unskilled hours of care by age and gender for those requiring home care. They combine it with a cost for an hour of unskilled care of \$18, a cost for an hour of skilled care of \$37, and a 35 percent coverage of home care expenditures by Medicare. I use similar figures for the cost of care. As those figures are in 2002 dollars, they are converted in 1998 dollars. Moreover, I assume that real long-term care and medical expenditures are growing at 1.5 percent per year as Brown and Finkelstein (2008) do, so I adjust the price for LTC accordingly. As a consequence, LTC risk will vary by cohort as, for instance, a 90 years-old in a nursing home will face a higher cost if she is from a cohort born late.

5.4 Medical expenditures

To estimate $\mu^m(I, hs_t, t, ty)$ and $\sigma^m(I, hs_t, t, ty)$ which determine out-of-pocket medical expenditures m_t , I do the following. First of all, I restrict the sample to those in health states gh and bh , and who do not receive Medicaid (as I want to measure out-of-pocket medical expenditures before receiving Medicaid). I will assume that the distribution of m_t for those in health states hc , alf or nh is the same as for those in bh when simulating the model.

I express out-of-pocket medical expenditures in the HRS in 1998 dollars and then divide them by $(1 + 1.5/100)^{ty-1998}$. Given the assumption of a 1.5 percent real growth rate in medical expenditures, this procedure is intended to remove year effects¹⁶. Medical expenditures in the HRS are those for the last 2 years, so I divide them by 2 and then take the log. I then regress, via OLS, the constructed variable on health state, a full set of dummies for income quintile and age. I then do a similar regression with the square of this

¹⁶To be more precise, I assume that the cost of *all* interventions or treatments increases by 1.5/100.

variable as the dependent variable. I then compute the standard deviation of medical expenditure using the fact that:

$$\sigma_t^m(I, h_{s_t}, t, 1998) = \sqrt{E[(\ln m_t)^2 | I, h_{s_t}, t, 1998] - E[\ln m_t | I, h_{s_t}, t, 1998]^2}$$

which is the formula for the variance. I then compute the mean of medical expenditures for different income quintiles and age using:

$$E[m_t | I, h_{s_t}, t, 1998] = \exp\left(E[\ln m_t | I, h_{s_t}, t, 1998] + \frac{(\sigma_t^m(I, h_{s_t}, t, 1998))^2}{2}\right)$$

I then multiply $\sigma_t^m(I, h_{s_t}, t, 1998)$ by 1.423826 following De Nardi et al. (2010) to account for the fact that the annual variance should be higher. From now on, $\sigma_t^m(I, h_{s_t}, t, 1998)$ denotes this multiplied variance. I then obtain $\mu^m(I, h_{s_t}, t, 1998)$ using:

$$\mu^m(I, h_{s_t}, t, 1998) = \log(E[m_t | I, h_{s_t}, t, 1998]) - \frac{(\sigma_t^m(I, h_{s_t}, t, 1998))^2}{2}$$

Finally, we need to include back the real growth in house prices to get $\mu^m(I, h_{s_t}, t, ty)$ and $\sigma_t^m(I, h_{s_t}, t, ty)$. Notice that under this assumption:

$$\begin{aligned} E[m_t | I, h_{s_t}, t, ty] &= 1.015^{ty-1998} E[m_t | I, h_{s_t}, t, 1998] \\ &= 1.015^{ty-1998} \exp\left(\mu^m(I, h_{s_t}, t, 1998) + \frac{(\sigma_t^m(I, h_{s_t}, t, 1998))^2}{2}\right) \\ &= \exp\left(\mu^m(I, h_{s_t}, t, 1998) + 1.015^{ty-1998} + \frac{(\sigma_t^m(I, h_{s_t}, t, 1998))^2}{2}\right) \end{aligned}$$

I thus set $\mu^m(I, h_{s_t}, t, ty) = \mu^m(I, h_{s_t}, t, 1998) + 1.015^{ty-1998}$ and $\sigma_t^m(I, h_{s_t}, t, ty) = \sigma_t^m(I, h_{s_t}, t, 1998)$. Notice that this is consistent with the assumption in footnote 16. The presence of ty , along t , in the parameters governing the distribution of m_t implies that medical expense risk differs for different cohorts.

5.5 Long-Term Care Insurance

I use the typical LTCI contract from Brown and Finkelstein (2008). It covers *hc*, *alf* and *nh*. There is no deductible. The maximum nominal benefit is of \$100 per day¹⁷. The average load¹⁸ computed for women was -0.06 in Brown and Finkelstein (2007). I use, however, a 18% load as in Lockwood (2016), which corresponds to the average load for men and women computed in Brown and Finkelstein (2007). The premium is constant in nominal terms. The assumed inflation rate is 3% as in Brown and Finkelstein (2008). I assume that $R = 1.02$ (see below), so I use a 2% interest rate to compute premia. Premia are computed separately for each income quintile. The parameters of the LTCI are summarized in table 2. Averaging over the different income quintiles the average premium in 1998 dollars for a 67-year old healthy female is \$2,693.

¹⁷All their figures are in 2000 dollars so I convert them in 1998 dollars in my simulations.

¹⁸The load is given by the formula $load = 1 - \frac{EPDV \text{ of benefits}}{EPDV \text{ of premiums}}$.

Table 2: LTCI parameters

Variables/ Parameters	Meaning
Load	18%
Real interest rate	2%
Inflation rate	3%
Maximum yearly nominal benefit in 1998	\$34,475

The corresponding figure in Brown and Finkelstein (2007) is \$2,073.¹⁹ The difference is mainly explained by the difference in loads. Assuming a -6% load, the computed premium is \$2,083. If I assume a 3% real interest rate as they use, instead of the 2% real interest rate that I use, the premium becomes \$1,943. LTCI premia computed using the estimated transition matrix are thus close to those reported in Brown and Finkelstein (2007)²⁰.

5.6 House prices, rents, and debt

Given the high dimensionality of the problem I have to limit myself to 8 possible values for housing size, and I have to treat house prices as deterministic. To construct housing sizes I use the distribution of house sizes for homeowners in the *simulated sample*²¹ in 1998. I use the following percentiles: 15, 30, 45, 60, 75, 90 and 95. It gives 8 bins, and possible values for h are the medians of those bins. As a matter of fact, house prices in 1998 are set to 1. To construct house prices in 2000, 2002, 2004, and 2006, I take the ratio of the house values in those years relative to the house value in 1998 for those who were homeowners in 1998 and were still homeowners in those years. For all these individuals who were in a given house value bin in 1998, I then compute the median house value increase between a give year and 1998 using these ratios²². As a consequence, the growth in house prices is allowed to differ for houses of different values²³²⁴.

After 2006, house prices are assumed to remain constant to their 2006 level. There are several reasons why I made such an assumption. First, the growth in real house prices between 1998 and 2006 was particularly large and much larger than historical averages²⁵. As a consequence, assuming that house prices would grow after 2006 would most likely lead to an “insurance-value” of housing much larger that what would be expected

¹⁹To compute this number, I interpolate linearly between the figures for ages 65 and 75 that they report. I then use the BEA implicit price deflator for personal consumption expenditures to express values in 1998 dollars.

²⁰This gives additional confidence in the construction of health states from the data.

²¹I describe its construction below.

²²Of course, when doing this all prices are expressed in 1998 dollars to capture real growth in house prices.

²³Between waves, I take the mean of the wave before and the wave after.

²⁴This leads to some heterogeneity in asset returns by income, a feature Lockwood (2016) controls for. However, I cannot include heterogeneity in returns by income for other assets given computational constraints.

²⁵To mention just one macroeconomic reference on the subject, Justiniano et al. (2013) (figure 1.2) show that house prices are roughly constant between 1970 and 1998 while there is a huge increase in house prices around the period 1998-2006, and a huge drop corresponding to the great recession.

by rational agents²⁶. Second, though the growth in house prices was much larger over that period than over the last 30 years, there is still the possibility that people thought that house prices would remain on average higher than historical standards because of a change in regime. The fact that there was not a huge movement of individuals selling their homes to become renters right prior to the crisis in order to make “real” gains by selling their homes would give some credit to this hypothesis. This is why I do not assume that house prices were expected to fall after 2006²⁷.

Given those elements, it is rather unlikely that I underestimate by much (if any) the “insurance-value” of housing. However, given the high growth in house prices during years 1998-2006 and the fact that house prices are volatile, I may be overestimating the “insurance-value” of housing²⁸. In order to account for this while not increasing the complexity of the model, I redo some of the estimations assuming a counterfactually high transaction cost of housing ϕ_p (results to be added soon). Doing so, reduces the amount that individuals are expected to get when reselling their homes, and this lowers down the “insurance-value” of housing.

To compute rent prices, I use the rent-price ratio and house prices data²⁹ from the Lincoln institute between 1998 and 2006 to construct rent prices. After 2006, I assume that house prices are equal to their 2006 level and set the rent-price ratio to its average over 1998-2006. Using this, I construct rent prices for after 2006.

Finally, I set the values for λ_t using a median regression of net debt over housing wealth³⁰, for those with net debt, on age.

5.7 Other calibrated parameters

The interest rate is set to 2% following De Nardi et al. (2010). μ , the extra interest for debtors, is set to 1.6% following Nakajima and Telyukova (2012). ϕ_p is set, in the baseline, to 6% following Yao and Zhang (2005). ψ^h (1998) is set to 1.5% based on Yao and Zhang (2005). Its value increase until 2006 and then remain constant. The increase is based on the increase of the residential implicit price deflator relative to the implicit price deflator for consumption expenditures. $\omega = 0.8$ based on Yao and Zhang (2005). For \overline{ph} , I use a value of 366,996 in 1998 dollars based on Nakajima and Telyukova (2012). I do not account for capital tax gains as, for the house bins considered and taking as base price 1998, the tax actually never applied. For similar reasons, I do not account for a tax on bequests. The computation of the income tax is described in appendix.

\underline{x} is fixed to \$6,445 in gh , bh , and hc following Brown and Finkelstein (2008). This amount is just \$355 for those in alf or nh . I set x_{priv} to \$6,090 still following Brown and Finkelstein (2008). x_{pub} is estimated from the data³¹. Finally, y_t constant over time and is set, for each income quintile, equal to its mean value

²⁶Notice that even if house prices are constant there is still a rational to own, even without a preference to stay in a given home, because the rent is usually more than the maintenance cost (see below).

²⁷It is important to keep in mind that the last wave for which I simulate the model is 2006. So, the assumptions about expectations of house prices affect the behavior of the model only through expectations. Typically, I am not simulating the model in 2008 or 2010 assuming that house prices are equal to their 2006 level, while obviously, at this time, they were not.

²⁸Typically, if individuals expect that house prices can potentially fall when they require nursing home care, it should lead housing to be a worse substitute to LTCI.

²⁹The series used are the FHFA ones (<http://www.lincolnst.edu/>).

³⁰ $\frac{\text{housing wealth} - \text{total wealth}}{\text{housing wealth}}$ with *housing wealth* as the value of the primary residence.

³¹If x_{pub} were equal to x_{priv} then the individual would get the equivalent of \$6,445 of c and h , when receiving Medicaid in

Table 3: Calibration summary

Variables/ Parameters	Meaning	Value	Source
M_t (in <i>hc</i>)	LTC expenditures when in <i>hc</i>	see text	Brown and Finkelstein (2008)
M_t (in <i>alf</i>)	LTC expenditures when in <i>alf</i>	\$23,745	Brown and Finkelstein (2008)
M_t (in <i>nh</i>)	LTC expenditures when in <i>nh</i>	\$47,160	Brown and Finkelstein (2008)
m_t	medical expenditures	see text	author's calculations
	growth rate of medical expenditures	1.5%	Brown and Finkelstein (2008)
$\underline{x}(gh, bh, hc)$	expenditures when receiving Medicaid	\$6,445	Brown and Finkelstein (2008)
$\underline{x}(alf, nh)$	expenditures when receiving Medicaid	\$355	Brown and Finkelstein (2008)
x_{priv}	equivalent expenditure when in <i>alf</i> or <i>nh</i>	\$6,090	Brown and Finkelstein (2008)
$p^h(ty)$	house prices	see text	author's calculations
$r^h(ty)$	rent prices	see text	Lincoln Institute
λ_t	borrowing limit parameter	see text	author's calculations
R	gross interest rate	1.02	De Nardi et al. (2010)
μ	extra interest for debtors	0.016	Nakajima and Telyukova (2012)
ϕ_p	transaction cost	0.06	Yao and Zhang (2005)
$\psi^h(1998)$	maintenance cost in 1998	0.015	Yao and Zhang (2005)
ω	housing share in utility	0.8	Yao and Zhang (2005)
\overline{ph}	housing exemption in Medicaid means-test	\$366,996	Nakajima and Telyukova (2012)
y_t	mean value in each income quintile	see text	author's calculations

for this particular income quintile.

6 Second-stage estimation

6.1 Sample

I simulate 3 cohorts defined according to birth years: 1929-1933 (the *young* cohort), 1919-1923 (the *middle* cohort), and 1909-1913 (the *old* cohort). For each woman in these cohorts, I replace her age in 1998 with 67, 77, and 87 respectively (the middle point in each cohort). I drop women with more than 1 million dollar in wealth and/or with a house worth more than \$400,000 in 1998³². I use waves from 1998 to 2006 (included).

For each woman, when doing the simulations, I take as initial values (i.e. in 1998) for hs_t , $LTCI_{t-1}$, b_{t-1} , d_{t-1}^o and h_{t-1} their counterpart in the data. I consider a person in the data to have a LTCI if she has a LTCI covering both home care and nursing home care. I also use the values for hs_t observed in 2000, 2002, 2004, and 2006 when doing the simulations. Between each wave, I simulate the health state based on the data generating process estimated for health state. I generate 10 fictitious set of data for each woman. The draws for health states that I use are the same for all estimations.

6.2 Moments

The moments matched³³ are:

1. median total wealth by income quintile, cohort and year;
2. the 75th percentile of total wealth by income quintile, cohort and year;
3. homeownership rate by income quintile, cohort and year;
4. median housing wealth for homeowners by income quintile, cohort and year;
5. average LTCI ownership rate by income quintile and cohort.

To construct the moments relative to LTCI, I take the mean of $LTCI_t$ for each individual across the waves that they are observed. I then match the mean of this mean by income quintile and cohort. I am mostly interested in the average LTCI ownership rate by income quintile for the *youngest cohort*. Indeed, in the model, they are the only one who can decide to have a LTCI even if they did not have one in the past. As we will see, all specifications are not as successful in matching this set of moments.

The procedure to compute the distance between the data and the model follows the procedure described in details in De Nardi et al. (2016) appendix C. I borrow elements of their exposition here to give the flavour of how the estimation is done. A reader interested in the exact computation of moment conditions (in particular of medians or other percentiles), and on the asymptotic properties of the method of simulated moments can find more details in their exposition. Notations here are voluntarily close to theirs.

alf or *nh*.

³²This removes about 2.5 percent of women from my sample.

³³I consider only moments for which they are enough observations which removes some moments for the very old.

Table 4: Summary statistics

Variable	Population	Value
Total wealth	all	129,610
	homeowners	193,443
Housing Wealth	all	56,197
	homeowners	98,715
Home ownership rate	age 67	64.1
	age 87	64.8
	age 97	48.4
LTCI ownership rate	all	4.6
	lowest income quintile	1.2
	highest income quintile	11.2

Note: Unweighted statistics. Total wealth and housing wealth in 1998 dollars. Home ownership rate and LTCI ownership rate in percentages.

In short, let Δ be the vector of values for the parameters we wish to estimate. Let χ be the values for the parameters of the model which have been determined in the first stage. The “true” values for these vectors are Δ_0 and χ_0 . Due to computational constraints, we need to treat our estimate $\hat{\chi}$ of χ , as equal to its true value χ_0 . Let $\varphi(\Delta; \chi_0)$ be a column vector storing the J moment conditions we are considering, and $\hat{\varphi}_N(\cdot)$ being its sample counterpart with N the number of independent individuals in our sample. Denoting $\hat{\mathbf{W}}_N$ a $J \times J$ weighting matrix, our estimate of Δ is given by:

$$\hat{\Delta} = \frac{N}{1 + \tau} \arg \min_{\Delta} \hat{\varphi}_N(\Delta; \chi_0)' \hat{\mathbf{W}}_N \hat{\varphi}_N(\Delta; \chi_0)$$

where τ is the ratio of the number of observations over the number of simulated observations. In practice, I use for $\hat{\mathbf{W}}_N$ the $J \times J$ matrix with off-diagonal elements equal to zero, and diagonal elements equal to the inverse of the variance for the corresponding moments in $\varphi(\cdot)$. I compute these variances from the true data.

6.3 Empirical patterns

Table 4 some summary statistics for the sample considered. We see that housing wealth³⁴ is a substantial part of total wealth, that homeownership rates are still around 50 percent for individuals in their late 90s, and that LTCI rates are low even for those in the top income quintile for which LTCI rate are 11.2 percent.

Figure 2 shows the main moments I am trying to match with the models. The upper left panel shows

³⁴Housing wealth only includes the value of the primary residence.

median wealth as a function of age and income quintile. The curves starting at age 67 are the profiles for the youngest cohort. Those starting at 77 and 87 are for the middle and older cohorts respectively. Each point of a given curve corresponds to a particular year (or wave). The first point corresponds to year 1998, the second point to year 2000... Median wealth profiles are globally monotonic in permanent income, so that, for a given cohort, the lower curve is for the bottom income quintile while the upper curve is for the top income quintile³⁵. The figure on the upper right panel reads similarly, and displays the 75th percentile of wealth as a function of income quintile and age. The figure on the bottom left panel displays homeownership rates as a function of income quintile and age. I do not show the curves for all income quintiles for the sake of readability³⁶. Finally, the figure on the lower right panel displays LTCI rates by income quintile for the youngest cohort, i.e. the one which is allowed to purchase a LTCI at age 67. I use similar figures to show the match between a specific version of the model and the data.

The figures show, in addition to the low demand for LTCI and the slow decline in homeownership rates which were directly visible from the table, that wealth decumulation is slow especially for those in the top income quintile.

6.4 Results for a model without utility cost of moving and without bequest motives

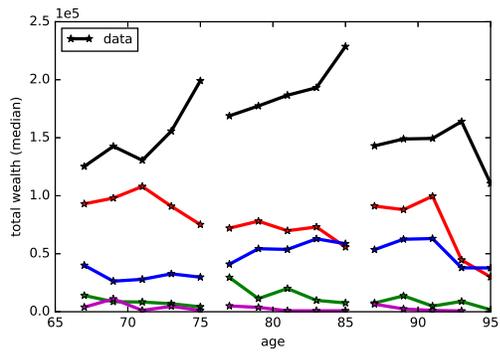
I first study the model presented in section 4 when there is no utility cost of moving (i.e. in which current utility is just $(c_t^\omega h_t^{1-\omega})^{1-\gamma} / (1-\gamma)$ and thus does not depend on d_t^s or d_t^o) and no bequest motive. Column (1) of table 5 displays the estimated parameters for this model when the moments matched are all those of section 6.2.

Figure 3 displays the match between the model and the data. The moments generated by the model are represented by dotted curves, while those of the data are represented by the plain ones. This figure reads similarly to figure 2. We can see that this version of the model does poorly along many dimensions. First, wealth decumulation tends to be too large for those in the upper income quintile. Second, homeownership rates fall with age at much faster pace than in the data and this is true across the income distribution. Finally, LTCI ownership rates are counterfactually high. For those in the upper income quintile, it is around 60 percent while it is only around 10 percent in the data. These large misses are reflected in the particularly high χ^2 statistic of the over-identification test.

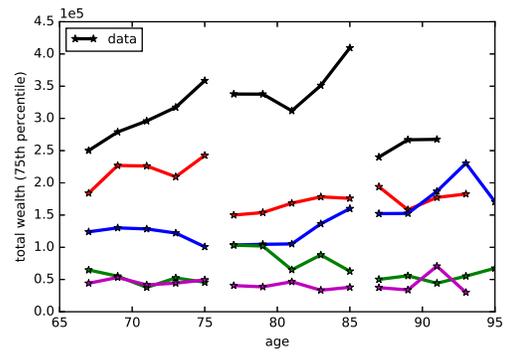
To study further why this specification performs poorly, I reestimate the model removing LTCI rates in the set of moment conditions. The estimated parameters are displayed in column (2) of table 5. We can observe that the values for β and γ are higher, while the value for x_{pub} is lower. All these elements tend to lead to higher saving rates. A higher γ and lower x_{pub} are expected to increase precautionary motives, while a higher β makes individuals more patient and thus tends to tilt consumption towards older ages. Figure 4 displays the results for this specification. Wealth and homeownership rates profiles are now close to those in the data. However, this comes at the cost of counterfactually even higher LTCI rates (not matched). This

³⁵For wealth and homeownership rate profiles, the color code is as follow: bottom income quintile (purple), second income quintile (green), third income quintile (blue), fourth income quintile (red) and upper income quintile (black).

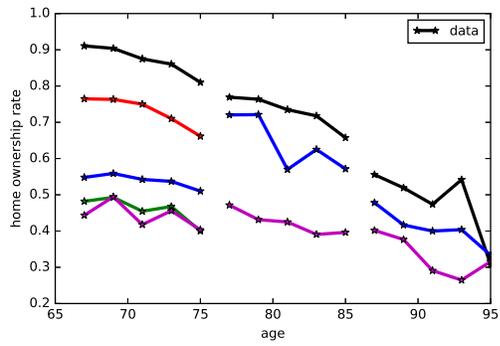
³⁶Notice that the corresponding moments are nevertheless matched in the estimation.



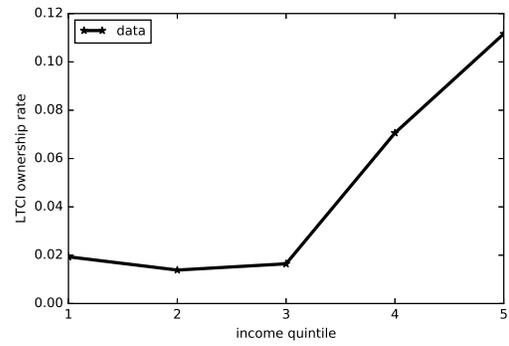
(a) Total wealth (median)



(b) Total wealth (75th percentile)



(c) Homeownership rate



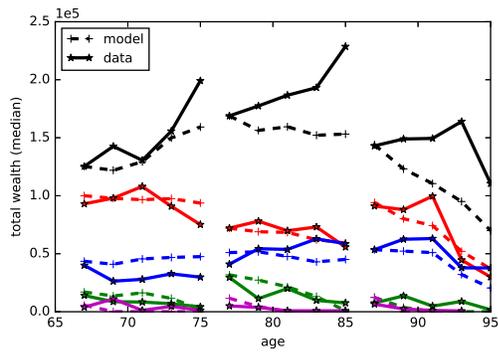
(d) LTCI ownership rate

Figure 2: Moments from the data

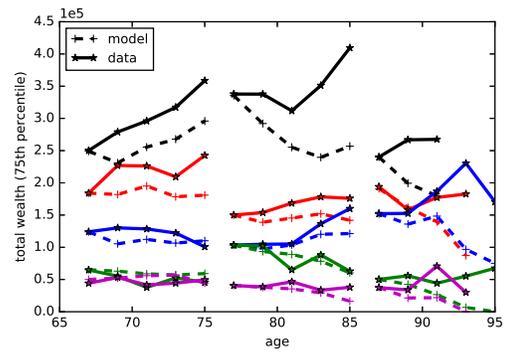
Table 5: Estimated structural parameters without utility cost of moving and without bequest motive

	(1)	(2)	(3)
β	1.00 (0.01)	1.10 (0.02)	1.09 (0.03)
γ	3.48 (0.02)	4.23 (0.30)	4.30 (0.31)
x_{pub}	7,764 [†] (283)	4,609 [†] (456)	4,560 [†] (54)
Utility	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$
LTCI rate as moment	Yes	No	No
Allowed to purchase LTCI at 67	Yes	Yes	No
χ^2 statistic	3,177	956	937
Degrees of freedom	304	279	279

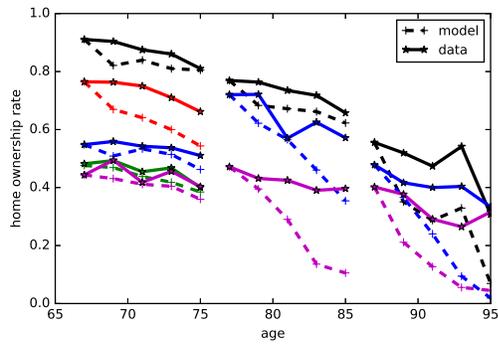
Note: Standard errors are in parentheses. NA stands for Non Applicable. The model is solved by scaling all dollar values in 1,000 of dollars in order to avoid numerical imprecisions which may arise when doing divisions by large numbers. It can occur with a CRRA utility function with $\gamma > 1$, as in this case we are indeed doing a division as $1 - \gamma < 0$. Due to this, the values with a [†] here are in fact the values in the code times 1,000.



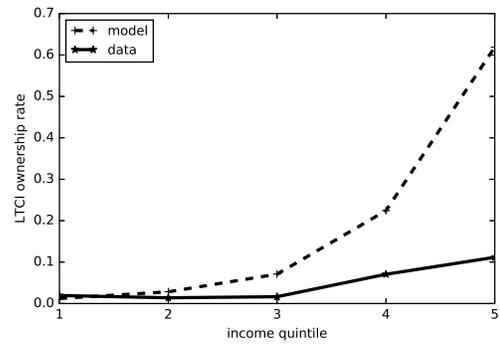
(a) Total wealth (median)



(b) Total wealth (75th percentile)



(c) Homeownership rate



(d) LTCI ownership rate

Figure 3: Model without bequest motive and without utility cost of moving

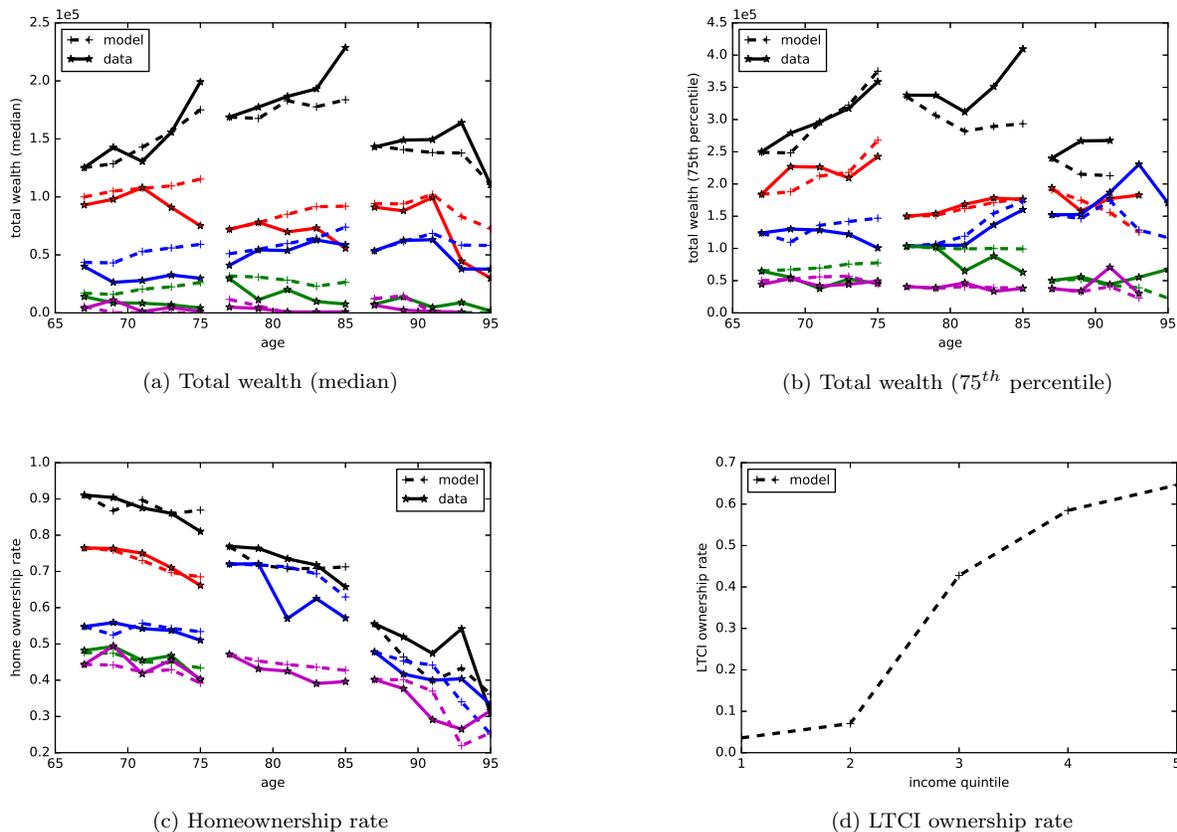


Figure 4: Model without bequest motive and without utility cost of moving - LTCI rate not matched

is particularly true for individuals in the third and fourth income quintiles which in the data have LTCI ownership rates of 1.6 and 7.1 percent in the data, while here they are above 40 and 50 percent respectively.

These results confirm the point made in Lockwood (2016) that the life-cycle model with only precautionary saving can match well wealth profiles, but cannot match both wealth profiles and LTCI demand. Indeed, to generate the slow observed dissaving in the data, precautionary saving must be high. But high precautionary motives lead to a high demand for insurance despite current loads and the presence of Medicaid.

A second point worth noticing is that precautionary motives alone are able to explain high homeownership rates late in life. Typically, in this case, individuals want to keep large amounts of overall wealth (which can be made of housing or other assets) until late in life. Housing provides a return for two reasons: the growth rate in house prices and the fact that the maintenance cost is lower than the rental cost. Even if the first reason is absent, individuals with high precautionary motives may still remain homeowners until very late. In Achou (2015), I show that, in a model in which house prices do not grow, this channel can be sufficient to lead to a slow decline in homeownership rates. As a matter of fact, it is not clear that the strenght of bequest motives can be well-identified if no additional moments are included beside wealth and homeownership rates.

In column (3) of table 5, I estimate a version of the model where LTCI rates are not part of the moment

conditions and where LTCI purchase is not allowed at 67. Parameter estimates as well as wealth and homeownership profiles (not displayed) are very similar to those of the previous estimation (the one in column (2)). It shows that the previous remarks are not dependent on allowing for the purchase LTCI.

6.5 Results for a model with utility cost of moving and/or bequest motives

I now turn to the model in which there is a utility cost of moving (i.e. utility can be given by equation (1), (2), or (3)) and/or a bequest motive. Column (1) (resp. (2) and (3)) of table 6 displays the estimated parameters when utility is given by (1) (resp. (2), and (3)) and when there is no bequest motive. Column (4) shows the results when there is no utility cost of moving (i.e. in which current utility is just $(c_t^\omega h_t^{1-\omega})^{1-\gamma} / (1-\gamma)$ and thus does not depend on d_t^s or d_t^o), but in which a bequest motive is present. Finally, column (5) shows the results when utility is given by (1) and when there is a bequest motive. The moments matched are all those mentioned in section 6.2.

When applicable $\phi_s(hs_t)$ is equal to a constant if hs_t is equal to gh , bh or hc and to 0 otherwise. Similarly, $\phi_o(t, hs_t) = \phi_o^1 + \phi_o^2(t - 67)$ and $\zeta(t, hs_t) = \zeta_o^1 + \zeta_o^2(t - 67)$ if hs_t is equal to gh , bh or hc and to 0 otherwise. It implies that the utility cost of moving is only present when an individual is not in an assisted-living facility or nursing home.

The outcome of specifications (1) to (3)³⁷ are very similar to one another. As a consequence, I only display them, in figure 5, for specification (1). While the model still generates too much wealth decumulation for those in the top income quintile, it does much better than the model with no utility cost of moving for homeownership rates and LTCI demand. For LTCI, the pattern is particularly striking. While in figure 3 around 60 percent of individuals in the top income quintile were purchasing a LTCI, the figure is now only about 16 percent and much closer to its data counterpart of about 11 percent. This observation tends to show that Davidoff's channel can have substantial quantitative implications for LTCI demand.

To further see this, I solved the model keeping the parameters equal to those in column (1) of table 6 instead for ϕ_s which was set to 0³⁸. Homeownership rates fell with age at a much faster rate than in the data. For instance, for all income quintiles, the homeownership rate was about 0 at 95 compared to around 30 percent in the data. Conversely, LTCI rates increased substantially at more than 20 and 40 percent for the fourth and fifth income quintile respectively. For the other specifications, the results were both qualitatively and quantitatively very close when setting the parameters affecting the utility cost of moving to 0.

About parameter estimates, we can see that both β and γ for specifications (1) to (3) in table 6 are lower than those of specification (1) in table 5. This is because the model with a utility cost moving needs lower patience and lower prudence to generate low dissaving rates and a slow decline in homeownership rates with age. To give an interpretation of the size of the utility cost of moving in specification (1) in table 6, I solve for x^{low} that satisfies:

$$\left| \left(\omega^\omega (1-\omega)^{1-\omega} (\gamma^h)^{\omega-1} \right)^{1-\gamma} \left[\frac{(x^{high})^{1-\gamma}}{1-\gamma} - \frac{(x^{low})^{1-\gamma}}{1-\gamma} \right] \right| = \phi_s \quad (16)$$

³⁷From now on, specifications correspond to the columns in table 6 excepted stated otherwise.

³⁸Figures for these exercises are in the online appendix.

Table 6: Estimated structural parameters with utility cost of moving and/or bequest motive

	Specifications				
	(1)	(2)	(3)	(4)	(5)
β	0.93 (0.01)	0.96 (0.01)	0.91 (0.01)	0.95 (0.01)	0.97 (0.01)
γ	2.92 (0.05)	1.86 (0.06)	2.03 (0.06)	2.72 (0.06)	2.02 (0.27)
x_{pub}	5,862 [†] (1,259)	5,349 [†] (662)	4,546 [†] (70)	4,424 [†] (311)	4,011 [†] (631)
ϕ_b	NA	NA	NA	0.95 (0.01)	0.94 (0.01)
c_b	NA	NA	NA	10,404 [†] (692)	10,367 [†] (780)
Utility	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$ $-\phi_s (hs_t) d_t^s$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$ $+\phi_o (t, hs_t) d_t^o$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$ $\times (1 + \zeta (t, hs_t) d_t^o)^{1-\gamma}$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$ $-\phi_s (hs_t) d_t^s$
Bequest	No	No	No	Yes	Yes
$\phi_s (hs_t = gh, bh, hc)$ (0 otherwise)	$3.90e^{-2}$ ($1.40e^{-2}$)	NA	NA	NA	$2.60e^{-3}$ (0.14)
$\phi_o (t, hs_t = gh, bh, hc)$ $= \phi_o^1 + \phi_o^2 (t - 67)$ (0 otherwise)	NA	ϕ_o^1 $2.11e^{-2}$ ($0.29e^{-2}$) ϕ_o^2 $2.25e^{-2}$ ($0.36e^{-2}$)	NA	NA	NA
$\zeta (t, hs_t = gh, bh, hc)$ $= \zeta_o^1 + \zeta_o^2 (t - 67)$ (0 otherwise)	NA	NA	ζ_o^1 $2.50e^{-1}$ ($1.11e^{-2}$) ζ_o^2 $2.12e^{-2}$ ($2.43e^{-3}$)	NA	NA
χ^2 statistic	1,264	1,540	1,576	1,009	983
Degrees of freedom	303	302	302	302	301

Note: Standard errors are in parentheses. NA stands for Non Applicable.

The model is solved by scaling all dollar values in 1,000 of dollars in order to avoid numerical imprecisions which may arise when doing divisions by large numbers. It can occur with a CRRA utility function with $\gamma > 1$, as in this case we are indeed doing a division as $1 - \gamma < 0$. Due to this, the values with a [†] here are in fact the values in the code times 1,000.

e^{-i} stands for $\times 10^{-i}$

Table 7: Medicaid rates and debt rates

		Specifications					
		Data	(1)	(2)	(3)	(4)	(5)
<i>Medicaid rates</i>	overall	22.1	20.7	20.9	20.8	20.2	20.6
	1 st income quintile	56.3	77.8	78.3	78.3	77.5	78.9
	2 nd income quintile	36.6	19.6	19.6	19.4	18.8	19.1
	3 rd income quintile	12.5	9.4	9.6	9.5	8.8	9.0
	4 th income quintile	5.9	4.7	4.9	4.7	3.9	4.0
	5 th income quintile	0.8	0.6	0.6	0.6	0.3	0.3
	owners	12.1	11.8	11.5	11.9	10.9	11.9
renters	34.2	32.6	32.9	31.2	31.5	31.6	
<i>Debt rates</i>	overall	8.2	12.3	12.0	11.8	10.5	10.9
	1 st income quintile	7.8	14.2	14.8	14.9	14.2	15.3
	2 nd income quintile	9.7	13.4	12.7	12.6	12.5	13.1
	3 rd income quintile	7.4	10.7	10.0	9.8	10.1	10.3
	4 th income quintile	7.9	12.3	11.9	11.0	10.8	10.8
	5 th income quintile	8.3	11.3	11.1	10.9	6.2	6.2

Notes: all values in percentages.

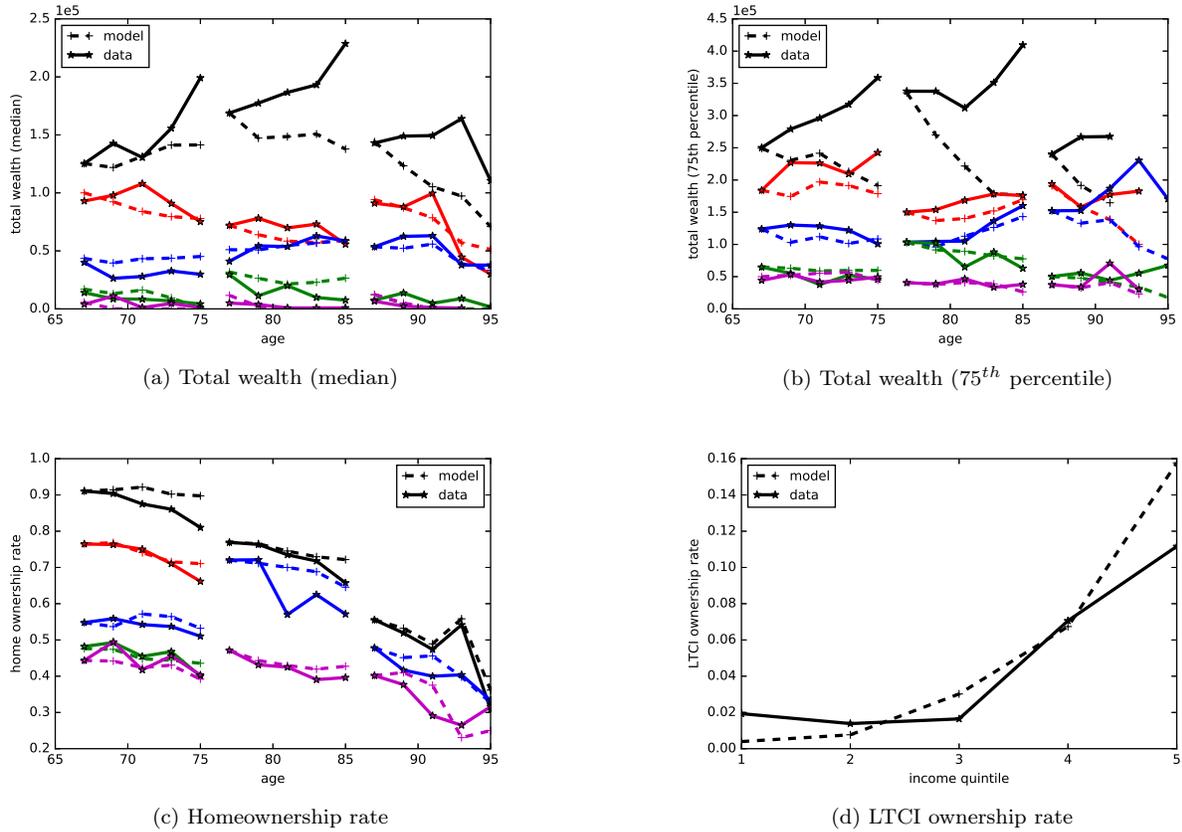


Figure 5: Model without bequest motive and utility cost moving as in equation (1)

The left-hand side corresponds to the absolute value of the difference in utility between spending x^{high} and x^{low} when an individual rents housing at r^h and allocates his spending optimally between housing and consumption. I take $r^h = 4.5/100$ and $x^{high} = \$30,000$. The corresponding x^{low} is $\$3,409$. Hence, the cost of moving is somehow equivalent to the cost of spending $\$3,409$ instead of $\$30,000$ for a year which is arguably large.

Table 7 displays the Medicaid rates and debt rates for the different specifications of the model in table 6. The overall Medicaid rate is close to the one in the data in all specifications despite the fact that it is not matched. The fact that homeowners rely much less on Medicaid than renters (despite Medicaid housing exemption) is also well reproduced. The model matches less well the percentage in Medicaid for individuals in lower income quintiles. For richer individuals, the match is quite good. The model usually overpredicts debt rates compared to the data. Though, the order of magnitude is quite similar. The fact that the implied debt rates are not low relative to the data suggest that the way the borrowing constraint is modelled (see equation (10)) does not remove a substantial insurance channel used by households in the data. I discuss the question of borrowing further below.

In column (4) of table 6, I introduce bequest motives and remove the utility cost of moving. As show by

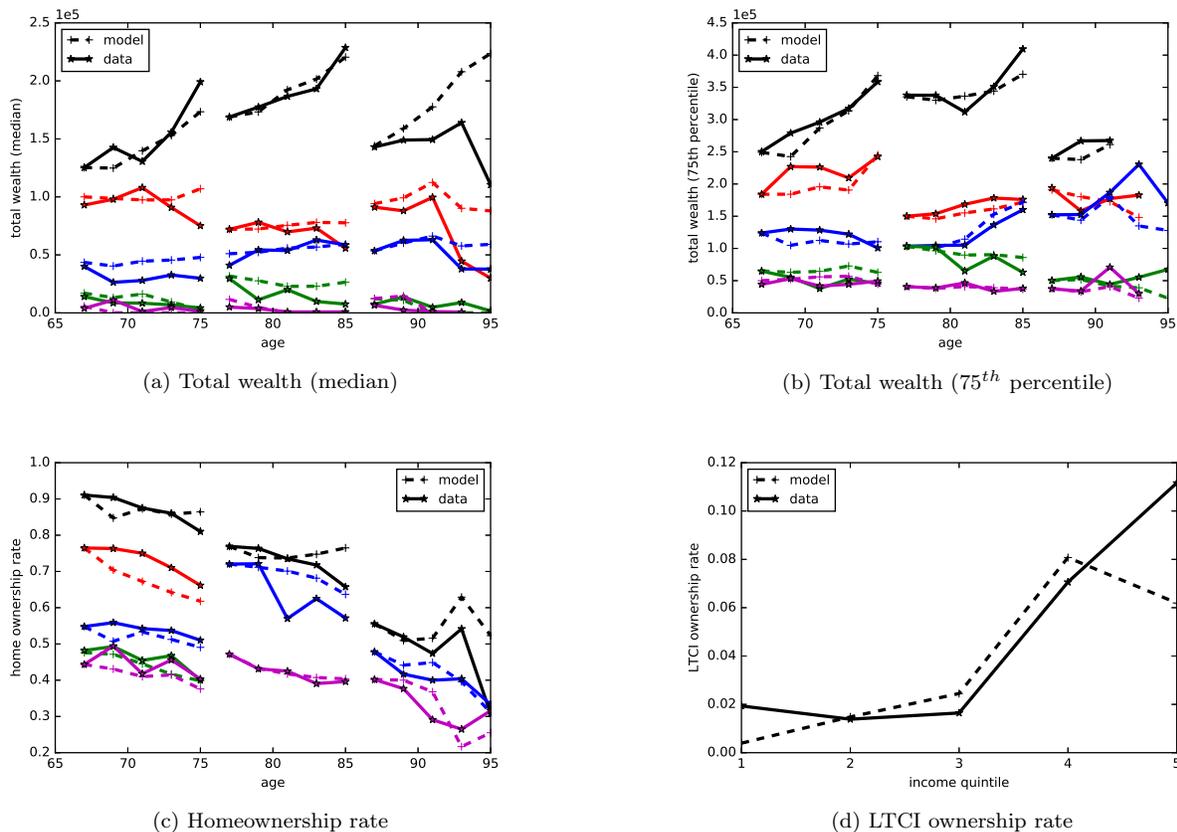


Figure 6: Model with bequest motive and without utility cost moving

the χ^2 statistic, this specification performs better than specifications (1), (2) or (3). The bequest motive is strong with a value for ϕ_b of 0.95. The outcome of the model are displayed in figure 6. The model is able to replicate well the slow decumulation of wealth even at the top, the high homeownership rates until late in life, and the low LTCI demand even for those in the top income quintile. It even generates over-accumulation of wealth late in life and a bit too little LTCI demand. But overall, it does arguably better than the model with only a utility cost of moving. Column (5) show the estimates of the model with bequest motives when utility is given by equation (1), i.e. the utility cost of moving is also present. The estimation results are very similar to those in column (4) and the utility cost of moving is no more well-identified and much smaller than in column (1). The outcome of the model in such a case is very close to the one in figure 6.

7 Experiments

In this section, I study the effect of the housing exemption in Medicaid means-test calculation on LTCI demand, as it may lead individuals to remain homeowners for a longer time and thus have some effect on LTCI demand. I then study, the effect of loosening borrowing constraints on LTCI demand.

7.1 Effect of the housing exemption in Medicaid means-test calculation

In this experiment I set \overline{ph} to 0, so that housing is fully included in Medicaid means-test. The results for this experiment can be found in the online appendix. In the end, including all housing in the computation of Medicaid has relatively moderate effects on LTCI demand. For specifications (1), (2) and (3), the increase is concentrated at the top income quintile. The LTCI rates for this income quintile are, for these specification, respectively around 20, 17, and 24 percent against around respectively 16, 14, and 16 percent in the estimation. The effect is moderate mainly because the negative impact of setting \overline{ph} to 0 on the homeownership rate is concentrated among lower income quintiles. For specification (4), there is no increase in LTCI demand for the top income quintile. Only a moderate increase (around 3 percentage points) is observed for those in the third income quintile. For specification (5), there is no increase in LTCI demand for the top income quintile.

7.2 Increasing home-equity extraction possibilities

Davidoff (2009, 2010) argued that increasing home-equity extraction may increase the demand for LTCI, because of the potential negative impact of housing equity on LTCI demand. In order to assess this question, I study what happens to LTCI demand when λ_t is set to 0.2 at all ages³⁹. In the estimation, its value was, for instance, of 0.71 at 67 and 0.86 at 97. Moreover, I allow all homeowners to borrow, not only those who had debt previously. That is, the constraints is no more given by (10) but by:

$$b_t \geq -d_t^o p^h (ty) h_t (1 - \lambda_t) \quad (17)$$

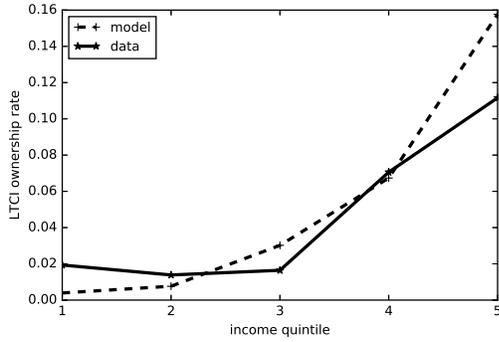
hence, the term $1 \{b_{t-1} < 0\}$ does not appear anymore. In figure 7, I show the result of this experiment for specifications (1), (4) and (5) in table 6 respectively. The results for (2) and (3) are close to those for (1). Panels on the left show LTCI demand for the youngest cohort in the model (vs the data) with the original borrowing constraint. Panels on the right show what happens in the experiment.

The experiment for specification (1) confirm Davidoff's point that, in a model without bequest motives, with a utility cost of moving and limited housing-equity extraction possibilities, improving borrowing possibilities (through for instance reverse mortgages) would lead to higher LTCI demand. This increase is, however, concentrated at the top with LTCI rates going from about 16 to 50 percent. However, as discussed in 6.5, this specification does significantly worse than specifications (4) or (5).

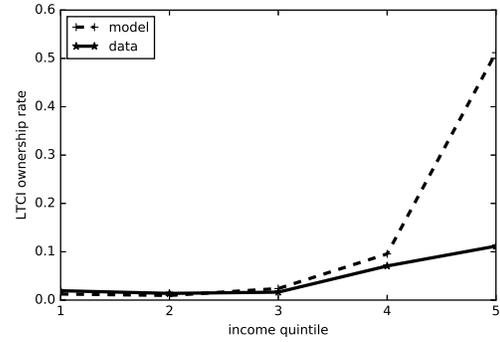
For these specifications, the result of the experiment is strikingly different. Indeed, not only LTCI rates do not increase but even tend to decrease. This likely stems from the fact that increasing borrowing limits improves consumption-smoothing when self-insuring through savings only. This additional insurance plays negatively for LTCI demand, while, at the same time, bequest motives remove most of the gains from decumulating wealth faster, which would be allowed by purchasing a LTCI.

It may still be argued that specifications (4) and (5) might underweight the utility cost of moving, and thus underweight the effect of increased home-equity extraction possibilities on LTCI demand. Typically, the utility cost of moving is 0 in specification (4) and is low (but not well-identified) in (5). As a consequence, I performed an additional estimation of specification (5) but this time fixing the parameters for γ , β , x_{pub} and

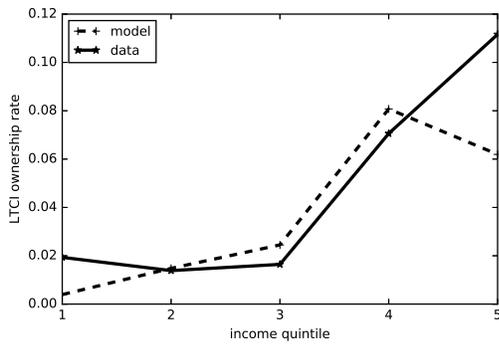
³⁹NB: \overline{ph} is set to its initial value here.



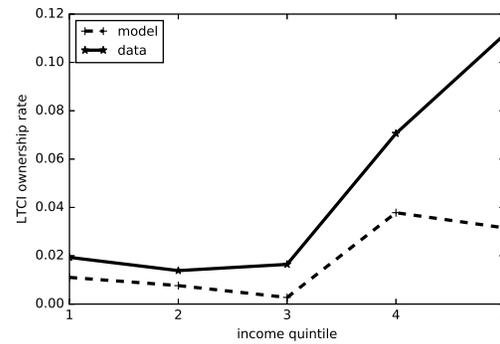
(a) LTCI ownership rate - specification (1)



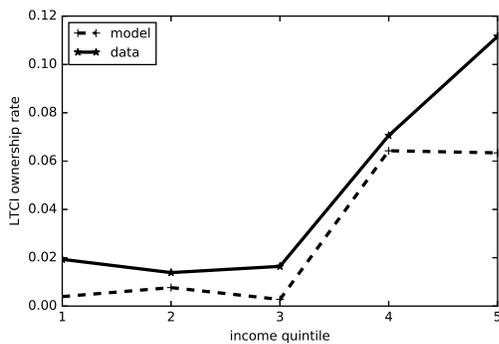
(b) LTCI ownership rate - specification (1) with increased borrowing



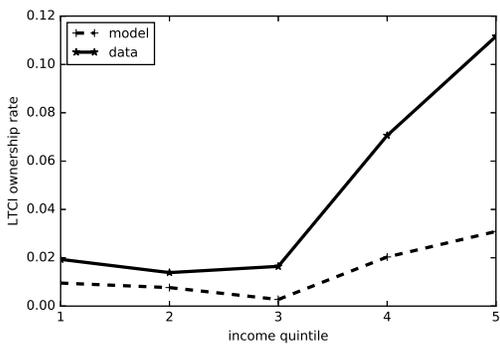
(c) LTCI ownership rate - specification (4)



(d) LTCI ownership rate - specification (4) with increased borrowing



(e) LTCI ownership rate - specification (5)



(f) LTCI ownership rate - specification (5) with increased borrowing

Figure 7: Effect of home equity extraction on LTCI demand

ϕ_s to their values in (1). The values for ϕ_b and c_b were then set in order to minimize the distance between the model and the data. In the end, this estimation somehow consists in giving a high weight to the utility cost of moving story to explain the observed patterns in the data, and letting the residual be explained by bequest motives. The obtained values for ϕ_b and c_b are respectively 0.97 (s.e. 0.003) and 15,141 (s.e. 602)⁴⁰. In this case, the experiment gives very similar results and leads to an even stronger decline in LTCI demand. LTCI demand goes from around 14 percent before the experiment to 2 percent after. As a consequence, even when the utility cost of moving is high, bequest motives wipe out all the effect of increased home-equity extraction possibilities on LTCI demand.

8 Robustness

TBA

9 Conclusion

In this paper, I show that, despite its size in households' portfolios and its theoretical appeal, housing is unlikely to play a major role in the explanation of the low demand for LTCI even when home-equity extraction possibilities are limited. The model with a utility cost of moving (but no bequest motives) and limited ability to withdraw housing equity can better match LTCI demand alongside wealth decumulation and homeownership rates than the model without it. However, this model performs significantly worse than the model with bequest motives, be there a utility cost of moving or not, in particular for upper-income individuals.

I use the model to perform several experiments. In particular, I study how increasing the ability to extract home equity affects the demand for LTCI. In the specifications of the model which perform best (i.e. those including a bequest motive), allowing individuals to extract home-equity while still in their homes does not increase *at all* the demand for LTCI. Hence, it does not appear that housing equity, and constraints on its adjustment, has a significant impact on LTCI demand.

The results in this paper bring support to those from related recent papers. In particular, they are line with Lockwood (2016). Indeed, they demonstrate that limited possibilities for home equity extraction and preferences to stay in one home are not a fully satisfactory alternative to bequest motives in order to explain the slow dissaving rate and low LTCI demand of retirees in upper-income quintiles. Even when I impose a large utility cost of moving and tight limits on home equity extraction, the bequest motive which matches best the data still points to an almost-absent effect of home-equity on LTCI demand. Thus, it appears that bequest motives are so strong as to make LTCI unattractive for richer individuals even if they have access to home-equity release products such as reverse mortgages.

The results here also complement those in Nakajima and Telyukova (2016). They use a model close to the one here and find fairly small welfare gains of introducing reverse mortgages. Their model does not include LTCI demand however and, thus, they do not use it to infer bequest motives or a utility cost of moving. As

⁴⁰Standard errors were computed treating the values of γ , β , x_{pub} and ϕ_s as measured without error.

shown above, matching LTCI demand can significantly help identify these two potential forces. Despite this, the fact that upper-income individuals do not purchase LTCI in my model, even when allowed to borrow more, because of strong bequest motives, parallels their result that bequest motives are a major factor in explaining the low take-up rate of reverse mortgages.

Overall, these results suggest that demand for financial products targeted towards the elderly - i.e. reverse mortgages, LTCI, and annuities (see Lockwood, 2012) - may bring significantly less welfare gains than originally thought.

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A Computational details

The computational method is standard. I discretize the grid for b_t using 180 points with more density at low values. The maximum point on the grid is \$800,000. The grid for b_t differs between renters and homeowners with houses of different sizes (it also changes with year), because the lowest possible value for b_t is set by the borrowing constraint $b_t \geq -d_t^o p^h(ty) h_t (1 - \lambda_t) 1 \{b_{t-1} < 0\}$. The grid for consumption has 180 points as well and goes from \$300 and \$250,000. I discretize medical expenditures m_t using gaussian quadrature with 4 nodes. In order to save some computational time, I regroup together the two lowest values for m_t as they are very close and small. I sum their weights and take their means as a node. This implies that I have in the end only 3 nodes but still can capture a large part of tail risk as the highest node for m_t has a high value. I solve the model backwards separately for each income and cohort to find the value function $V(\cdot)$. I then use this value function to simulate the model forward⁴¹. For values of a given variable which lie within the grids I consider I use linear interpolation. I use extrapolation for values which fall outside the grids.

To find the maximum I cannot use fast algorithms as the value function has many kinks, and am constrained to use grid search. For renters, I can limit the range of values to look for on the consumption grid by using the (intratemporal) first order condition between c and h .

The codes for the model, and some for the first stage estimation, are in Python 3. I use the Anaconda distribution which can be downloaded freely from <https://www.continuum.io/downloads>. Python is an object-oriented interpreted programming language very powerful and easy to use. The Anaconda distribution includes the most popular libraries for numerical work or data analysis (such as Numpy, Scipy, Matplotlib or Pandas to name a few). While Python has many advantages, it has the drawback to be interpreted and not compiled, hence slow for intensive for loops which need to be used when solving the model.

Fortunately, Python offers ways to circumvent this issue. As a matter of fact, the most computationally-intensive part of the program (which is to solve for the model for a given set of parameters) is coded using Cython (<http://cython.org/>). Cython allows to create (easily) functions which are compiled in C. Seljebotn (2009) has been a very useful reference to allow me to gain most from the speed increase allowed by Cython. A large part of the speed gain has to do with types declaration and removal of unnecessary checks once the code has been debugged.

One iteration of the model (including the computation of the distance statistics for which I use Pandas) takes around 12 minutes when parallelizing on a Lenovo P300 work station running on Windows 10 with an Intel Core I7-4790 processor and 32Go of RAMs. Most of the estimations were performed on the supercomputers Mammouth série II and Mammout parallèle II from the University of Sherbrooke. In the replication material, I give some hint on how the Cython function can be compiled on Windows or Linux.

The procedure to find the set of structural parameters which minimizes the distance statistic is standard. I first supply an initial guess for the vector of structural parameters to be estimated and I then run a Nelder-Mead simplex algorithm⁴² starting from this initial guess. I do this for several initial guesses⁴³. Among the

⁴¹Given that the decision to sell a home or to default on a LTCI may lead to jumps in the decision rules, this procedure is much more straightforward than solving for policy rules and then using them for simulations.

⁴²The Nelder-Mead algorithm used is the one from the Scipy library: <https://docs.scipy.org/doc/scipy-0.18.1/reference/optimize.minimize-neldermead.html>.

⁴³While it is easy to give plausible initial values for some of “usual” parameters such as β , γ or c , it is more difficult for some others such as the utility cost of moving. For these parameters, I usually start with a guess which I improve before running a

results from these different simplex algorithms, I take the one which gives the smallest distance between the data and the model. Looking for the minimum takes several days.

B Income and Taxes

The definition of pension income defined is the same manner as in De Nardi et al. (2010) It is the sum of Social Security benefits, benefits from defined-benefits pension plans, annuities, welfare, veteran’s benefits and food stamps. For each individual, I take the average pension income over the different observations and defined it as permanent income. I then construct 5 permanent income groups (i.e. 5 income quintiles) and take the average permanent income in each group. This latter is used as the pension income for each group.

Income taxes are computed taking taxable income as the sum of defined-benefits pension plans, annuities and asset income ($(R - 1)b_{t-1}$ if $b_{t-1} \geq 0$ in the model). The sum of defined-benefits pension plans and annuities for each of the previously defined income group is computed using the same procedure as for pension income. The parameters for the tax are taken from appendix A of the supplement to French and Jones (2011).

C Health states

I consider 6 possible health states hs , which include death. Apart from death, the first health state is the good-health (or healthy) state gh . It corresponds to individuals declaring as self-reported health of excellent, very good, or good, and which do not belong to any other health states defined below. The second health state is the bad-health state bh . This corresponds to individuals declaring as self-reported health of fair or poor, and which do not belong to any other health states defined below. The third health state is home care hc . As noted in Friedberg et al. (2014), the HRS has only limited information on home care. Using whether or not the person had used some home care in the previous period lead to a percentage of home care users much higher than in the NCLTS data (see Friedberg et al. (2014) table 4B) which were used by Robinson (1996). In the end, I decided to consider a person as receiving home care if the person declared having received home care in the HRS data and if a measure of mobility called *mobila* in the RAND version of the HRS was equal to 5 on a scale of 5. In the HRS, individuals are asked whether or not they had difficulties (i) walking several blocks, (ii) walking one block, (iii) walking across the room, (iv) climbing several flight of stairs and (v) climbing one flight of stairs. *Mobila* is equal to the number of difficulties people declare to have. So if an individual answers no for each potential difficulty (i) to (v) *mobila* will be equal to 0, while, if an individual answers yes for each one, *mobila* will be equal to 5. Using this definition for home health care gives me home care rates globally in line with the NCLTS data. For instance, only about 2 to 3% of women aged between 60 and 70 receive home health care using my measure, while this rate goes to around 6% at age 90. This is of a similar order of magnitude that what can be found in Friedberg et al. (2014) (table 4).

simplex algorithm. For instance for $\phi_s (hs_t = gh, bh, hc)$, I start by a value based on the utility function: $\left| \frac{c_l^{1-\gamma}}{1-\gamma} - \frac{c_h^{1-\gamma}}{1-\gamma} \right|$ where c_l and c_h are respectively a low and a high value (say \$5,000 and \$30,000). This initial guess roughly says that selling a home while not in *alf* or *nh* is equivalent to the loss of utility that one would have going from c_h to c_l for a year. I then increase the value of this guess, if it improves the fit of the model I continue to to increase it further. If it does not, I decrease it and if the fit improves I continue further. I then start the simplex algorithm at the end of this procedure.

The fourth health state is assisted-living facility (*alf*). To determine if a person lives in an assisted-living facility, I use a similar procedure to the one in Friedberg et al. (2014) for the HRS data.

Most importantly for the purpose of the paper is the definition of the fifth health state, nursing home care (*nh*). Indeed, nursing home care constitute most of out-of-pocket health expenditures (and thus of long-term care expenditures) late-in-life and generate substantial tail risk. One difficulty is that, using an annual model, it is easy to overstate expenditures in that state. This comes from the fact that many long-term care spells are less than a year. If one considers that a person who declares to live in a nursing home is in the *nh* state, which in the model lasts at least a year, then nursing-home risk will be overestimated. At some point, I was using this measure for nursing home care and considering that people who where in the home care state where those who had received some home care in the HRS. This gave me average years spent in nursing home care much higher than those reported by Brown and Finkelstein (2008) or Friedberg et al. (2014) who use monthly transitions, and hence premiums on long-term care insurance much higher than those reported in Brown and Finkelstein (2007). They were about twice as high as those reported by Brown and Finkelstein (2007).

To avoid this over-statement issue, and given that for our purpose what we are mainly interested in is large risk (so not about short nursing home stays), I first compute the number of days (or nights) a person was in a nursing home in the past two year if the person answered the core interview. I add to those days the number of days in nursing home from the exit interview of the next wave, if any. This is done to capture end-of-life long stays. If the computed number of days is equal to or greater than 300, the person is considered to be in a nursing home.

In the next section, I describe the construction of the transition matrices. I show, in particular, that the statistics relative to nursing home risk generated using this matrices are close to those in Brown and Finkelstein (2008) and Friedberg et al. (2014). Moreover, generated premiums on LTCI happen to be very close to those reported in Brown and Finkelstein (2007), which give further confidence in the definitions above.

D Transition matrices and long-term care statistics

To construct my transition matrices I use a procedure similar to the one in De Nardi et al. (2016). The data are bi-annual so we only observe transitions from current health state to the one two years from now. Denoting $p_{ijt} = P(hs_{t+1} = j | hs_t = i)$ where t is a year, we have:

$$P(hs_{t+2} = j | hs_t = i) = \sum_k P(hs_{t+1} = j | hs_t = k) P(hs_{t+2} = j | hs_{t+1} = k) = \sum_k p_{ikt} p_{kjt+1}$$

If we assume that $p_{ijt} = \gamma_{ijt} / \sum_k \gamma_{ikt}$ with $\gamma_{i0t} = 1$ and $\gamma_{ikt} = \exp(x\beta_k)$, then we can estimate yearly transition probabilities by maximum likelihood. I take as covariates: a complete set of dummies for current health status and income, a cubic in age, age interacted with health status and income interacted with health status⁴⁴.

⁴⁴To find the vector of parameters that maximize the log likelihood, I first run a multinomial logit regression for two-years

Table 8: Comparison of nursing home risk

	Robinson	Friedberg et al.	Estimated transition matrix
Percent Using	0.44	0.58	0.32
Unconditionnal mean years in <i>nh</i>	0.88	0.79	0.88
Conditionnal mean years in <i>nh</i>	2.00	1.37	2.77
Probability of using for more than a 1 year	0.18	0.21	0.20
Probability of using for more than 3 years	0.10	0.09	0.08
Probability of using for more than 5 years	0.05	0.04	0.03

Robinson column: Figures based on table 1 from Brown and Finkelstein (2008) which displays figures for nursing home risk for women aged 65.

Friedberg et al. column: Figures based on Friedberg et al. (2014) table 8 HRS column for women aged 65.

Estimated transition matrix: Results based on simulations using the estimated transition matrix and the distribution for income and health of individuals aged between 67 and 69 in my sample (which are then assumed to be 67 when simulating forward).

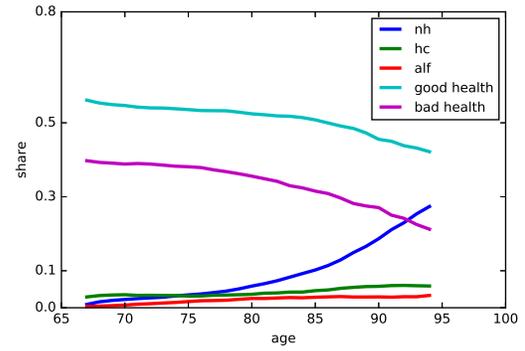
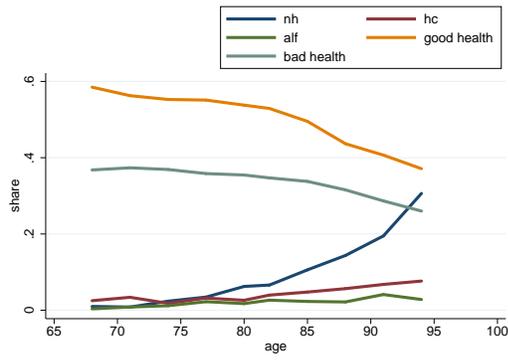
To assess the quality of the transition matrix, I take the distribution of women aged 67 to 69 in my sample and simulate health histories⁴⁵. Table 8 shows the statistics for nursing home use and compares them with the figures for women from the Robinson model and the updated version of the Robinson model in Friedberg et al. (2014)⁴⁶. The probability of having been in a nursing home is smaller in my simulations which is normal given that I do not consider short stays. However, the unconditionnal mean years in nursing home I compute is close to those computed with other transition matrices. Hence, nursing home risk does not appear to be overestimated in that dimension. Moreover, the distribution of nursing home length is very close to the one in Friedberg et al. (2014). Overall, the transition matrix seems to compare well to existing and popular (monthly) transition matrices with respect to nursing home care. For home care, the average number of years in care is lower at 0.56 than the one reported in Brown and Finkelstein (2008) (0.80) but remains of the same order of magnitude. Moreover, as discussed in the main text, computed premia for LTCI are very similar to those in Brown and Finkelstein (2008) when using the same parameters as theirs, which gives additional credibility to the transition matrix estimated here.

Figure 8 displays, on the left panel, the proportion of individuals in each health state by age in the data, and, on the right one, those simulated using the transition matrix. The figures are very close to each other and, in particular, we see that the transition matrix generates the steep increase of the proportion of individuals in nursing homes as they age. Figure 9 shows the proportion in nursing home by age for the first and fifth income quintile from the data (left) and simulated (right). Early in retirement almost no rich individual is in a nursing home while the proportion of poorer individuals in nursing home is much higher. This difference disappears around age 90, something that is well captured by the transition matrix.

transition in Stata. I then use the estimated parameters as the initial vector in a simplex algorithm to find the parameters of yearly transition. The code (`trans_mat_log.py`) to run this simplex algorithm is available in the replication codes folder. The log-likelihood that I obtain with this code appears very close to the one found by Stata for the two-years transition.

⁴⁵I assume that these women are all 67 when performing those simulations.

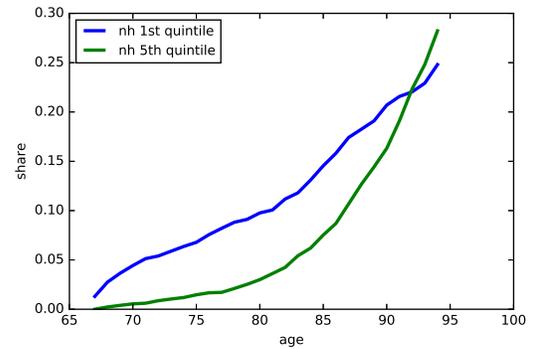
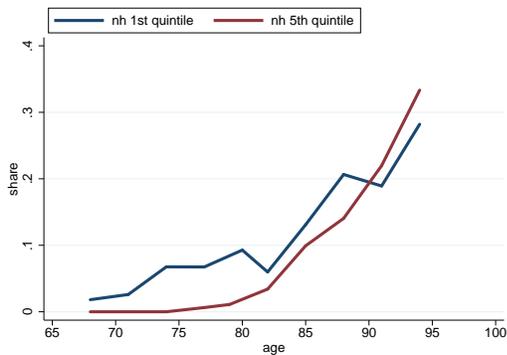
⁴⁶The figures are taken from Brown and Finkelstein (2008) and Friedberg et al. (2014). Notice that the simulations in those papers are for women starting at age 65, while for my simulations are for single women starting at age 67.



(a) Data

(b) Simulated

Figure 8: Percentages in health state by age



(a) Data

(b) Simulated

Figure 9: Nursing home rates by age for individuals in bottom and top income quintiles

E Medical expenditures

Table 9 shows the result from the OLS regression of the log of out-of-pocket medical expenditures and its square described in section 9. We see that these variables are increasing in bad health, age and income quintile which seems natural.

Though out-of-pocket “pure” medical expenditures (m_t) include only a non-persistent term, it can be noticed that *overall* medical expenditures ($m_t + M_t$) will be persistent due to the fact that nh in particular is a quite persistent state. De Nardi et al. (2010) are right to insist on the importance of using an AR(1) term in their specification of overall out-of-pocket medical expenditure as they have only two health states (good and bad). Here the health measure is more detailed as there are five possible health states while alive. Through a more detailed health measure, I am likely to capture most of the AR(1) component that they use. In particular, an OLS regression of medical expenditure (in level) on a full set of dummies for my health state measure⁴⁷ and a rich set of controls has an adjusted R^2 of 0.196, while using the measure in De Nardi et al. (2010) and the same set of controls the R^2 is only 0.038. This can be explained by the fact, put forward by DFJ2010, that most of out-of-pocket medical expenditures in old age are due to nursing home which they can’t capture perfectly with their health measure.

Given that, conditional on the health states I consider, the variance of out-of-pocket medical expenditure appears much lower than theirs, and that m_t does not include LTC expenditures, having a non-persistent error term does not appear to be a strong assumption. Moreover De Nardi et al. (2010) find that saving behaviours of the elderly are much more driven by the mean of medical expenditures conditional on health state than by the conditional variance.

F Robustness

TBA

⁴⁷Hence, in this case I consider all individuals not only those in gh or bh .

Table 9: Out-of-pocket medical expenditure regression (OLS)

	(1)	(2)
	$\log(m)$	$(\log(m))^2$
<i>bh</i>	0.501*** (0.0298)	6.501*** (0.378)
2nd inc. quintile	0.193** (0.0610)	2.181** (0.773)
3rd inc. quintile	0.159** (0.0574)	1.765* (0.726)
4th inc. quintile	0.206*** (0.0561)	2.374*** (0.711)
5th inc. quintile	0.291*** (0.0553)	3.300*** (0.700)
age	0.0131*** (0.00182)	0.174*** (0.0230)
Constant	5.053*** (0.153)	25.46*** (1.932)
Observations	9505	9505
Adjusted R^2	0.0351	0.0367

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 10: Estimated structural parameters with utility cost of moving and/or bequest motive when $\phi_p = 20/100$

	Specifications				
	(1)	(2)	(3)	(4)	(5)
β	0.94	0.94	0.92	TBA	TBA
γ	2.85	1.96	1.94		
x_{pub}	7,699 [†]	5,198 [†]	5,773 [†]	†	†
ϕ_b	NA	NA	NA		
c_b	NA	NA	NA	†	†
Utility	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$ $-\phi_s (hs_t) d_t^s$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$ $+\phi_o (t, hs_t) d_t^o$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$ $\times (1 + \zeta (t, hs_t) d_t^o)^{1-\gamma}$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$	$\frac{(c_t^\omega h_t^{1-\omega})^{1-\gamma}}{1-\gamma}$ $-\phi_s (hs_t) d_t^s$
Bequest	No	No	No	Yes	Yes
$\phi_s (hs_t = gh, bh, hc)$ (0 otherwise)	$4.19e^{-2}$	NA	NA	NA	
$\phi_o (t, hs_t = gh, bh, hc)$ $= \phi_o^1 + \phi_o^2 (t - 67)$ (0 otherwise)	NA	ϕ_o^1 $1.83e^{-2}$ ϕ_o^2 $2.16e^{-2}$	NA	NA	NA
$\zeta (t, hs_t = gh, bh, hc)$ $= \zeta_o^1 + \zeta_o^2 (t - 67)$ (0 otherwise)	NA	NA	ζ_o^1 $2.83e^{-1}$ ζ_o^2 $2.13e^{-2}$	NA	NA

Note: Standard errors are in parentheses. NA stands for Non Applicable. The model is solved by scaling all dollar values in 1,000 of dollars in order to avoid numerical imprecisions which may arise when doing divisions by large numbers. The values with a [†] here are in fact the values in the code times 1,000.