A Search-Theoretic Approach to Efficient Financial Intermediation*

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Abstract

This article develops a search-theoretic model of financial intermediation to study the efficiency condition of the banking sector. Competitive financial intermediation is determined by the search decisions of both households (to find adequate financial products) and banks (to attract depositors through marketing and to select borrowers through auditing). A generalization of the Hosios (1990) condition for efficiency is proposed, according to which efficient banks' bargaining power should be higher when banks are liquidity constrained to offset the (cross-market) appropriation of financial relationships' values by non-financial agents. This mechanism gives rise to a new transmission channel of interbank market failures to the real economy.

JEL Codes: C78; D83; G21

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1 Introduction

The financial crisis that began in 2007 has strengthened the role of non-financial deposits as a source of bank funding, which has become the new black.[1] Deposit funding is part of "the current “back to basics” policy" formulated by the ECB (2010)[2], and it goes back to the “back to basics” issue in finance: the efficiency of financial intermediation, which is the transformation of non-financial deposits into business loans by financial intermediaries. This paper revisits this issue. Its originality, which forms its contribution to the literature, is to assume that financial services to non-financial customers (households and entrepreneurs) are characterized by long-term relationships.[3] I provide empirical evidence that supports this assumption (see Section 2) and develop a search and matching model of financial intermediation based on this evidence. I use this model to characterize efficient financial intermediation and show its importance in understanding the transmission of interbank market failures to the production sector.

A new role for the interbank market is highlighted. The interbank market allows banks to escape from a cross-market distortion that is specific to the existence of search frictions on both sides of banks' activities, i.e., deposit and credit. Agents decide, first, to search and then, if matched, bargain interest rates. When bargaining on interest rates, a bank considers its gains and losses regardless of whether the bargain with the non-financial agent succeeds. Without an agreement, the bank would lose not only the value of one financial relationship but also that of two relationships. Indeed, without an agreement with a depositor, the bank loses this depositor as well as a creditor who can no longer

[1] This expression is borrowed from a 2012 report of the company Ernst & Young for Australian banks entitled "The rise of the deposits", in which it is stated that "Deposits are the new black, lending playing second fiddle".

[2] This episode has been documented widely, notably by the ECB (2012): "bank funding strategies needed to be adjusted quickly in order to expand the customer deposit base and reduce the share of wholesale funding." Interestingly, the ECB (2010) makes a connection between the reversal from the interbank market to the retail deposit market and the crucial role played by bank marketing in the process: "As for other sources of funding, the crisis has resulted in an increased awareness of differences between banks, with banks with established brands gaining a competitive advantage vis-à-vis their weaker competitors."

[3] These relationships are known in the literature as relationship banking, defined by Goddard et al. (2007) as follows: "One such topic is relationship banking, which can be defined simply as the provision of financial services repeatedly to the same customer". Relationship banking is not a new topic in financial intermediation. As explained below, the novelty of this paper is to consider relationship banking for both deposit and credit in a search model and to study the efficiency condition of financial intermediation.

be financed. The bank faces a liquidity problem that can be solved by a perfectly liquid interbank market. The interbank market can provide a substitute for the lost deposit to avoid the additional loss of the creditor.[4] Likewise, without an agreement with a borrower, a bank loses this borrower as well as one depositor whose funds can no longer be invested. Again, a perfectly liquid interbank market could provide a substitute for the credit to avoid the loss of the depositor. Therefore, an interbank market failure weakens the bargaining position of banks (which must maintain a strict relationship between deposits and credits) and impacts whether their provision of financial intermediation is efficient.

Efficient financial intermediation is a constrained-efficient equilibrium that is the social planner's solution to maximize steady-state welfare given search costs. From the social planner's point of view, the issue is to allocate an efficient amount of resources to search activities to ensure an efficient level of final good production. In a competitive economy, financial intermediation is determined by the search decisions of both households (to find adequate financial products) and banks (to attract depositors through marketing and to select borrowers through auditing). Even if markets are not frictionless, there is a way to reach efficiency. This method was first demonstrated by Hosios (1990) and requires equality between the agent's bargaining power and the elasticity of the matching function with respect to its search effort. In this case, the search externalities are internalized. I establish a connection between this efficiency condition and the functioning of the interbank market. The Hosios (1990) condition for efficiency is valid for a perfectly liquid interbank market. Otherwise, alternative bargaining powers are required to reach efficiency.[5] These efficient bargaining powers are different from credit and deposit markets (even if matching functions are identical), depend on the banks' liquidity constraints (on the deposit and/or credit markets), and are functions of a large set of structural parameters (not only the elasticity parameter of the matching function). The model predicts that during a crisis with interbank market failures, maintaining the efficiency of financial intermediation requires an increase in banks' bargaining power because the crisis weakens

[4] The crucial point here is that the interbank market provides immediate funds to banks, contrary to retail deposits, which are sluggish. Huang and Ratnovski (2011) note a similar difference between the interbank market and retail markets.

[5] Petrosky-Nadeau and Wasmer (2013) study the efficiency of financial intermediation with search on credit and labor markets. In their model, the Hosios (1990)'s condition is sufficient to guarantee efficiency because firms do not have the problem of liquidity considered here for banks.

the bargaining positions of banks (as explained above).

Changes in bank bargaining power are necessary to ensure efficiency, but such changes may be difficult to implement. I therefore discuss the role of sectoral and monetary policies to achieve efficiency with fixed banks' bargaining power. I also discuss the price-posting equilibrium of financial intermediation.[6] I adapt here the famous results of Shimer (1996) and Moen (1997): the competitive economy is efficient if banks post financial contracts on the market and if non-financial agents direct their search toward banks. The issue is that posted interest rates are not for one contract (for example, one loan), but they are instead for repeated contracts during the lifetime of the financial relationship, hence the strong assumption of the bank's commitment and the challenge for banks' practice, which are well known to adjust interest rates during the relationship.[7]

The remainder of the paper is structured as follows. Section 2 gives the rationale for search frictions in the banking sector. The issue of financial intermediation and its socially optimal solution are presented in Section 3. The competitive equilibrium is defined in Section 4, and its normative properties are studied in Section 5. The discussion and concluding remarks are presented in Section 6.

2 Rationale For Search Frictions

The role of financial intermediation considered in the model developed herein does not follow from structural differences between deposits and loans[8], but from the existence of search frictions in financial markets. This Section gives the rationale for search frictions in the credit and deposit markets.

[6]The search model with posted interest rates is developed in the on-line Appendix J.

[7]It is an established fact in the literature on lending relationships (see Section 2 for references).

[8]The traditional role of financial intermediation is to transform assets, which are heterogeneous with respect to size, risk, or maturity. Here, there is a one-to-one correspondence between the per-period deposit of one household and the resources needed to finance a one period firm project.

2.1 Credit Market

Applying the search model to the credit market follows the literature initiated by Diamond (1990) and developed by Den Haan et al. (2003), Wasmer and Weil (2004), and Dell'Arricia and Garibaldi (2005). Two rationales for credit search frictions have been provided. The first rationale is based on the existence of long-term relationships between lenders and borrowers, which are known as lending relationships. Berger and Udell (1998) report an average duration of lending relationship between small business firms and commercial banks of 7.77 years. This was a robust observation in financial markets at the beginning of the lending relationship literature, which has been reviewed by Berger and Udell (1995) and Elyasiani and Goldberg (2004). Both Den Haan et al. (2003) and Wasmer and Weil (2004) invoke this literature to motivate their credit market search model.[9] The second rationale for credit search frictions is provided by Dell'Ariccia and Garibaldi (2005) and Craig and Haubrich (2013). They construct databases of credit flows from banks and show that the credit market in the United States is characterized by large, cyclical flows of credit expansion and contraction that may be explained in terms of the matching friction. Based on these rationales, numerous theoretical models incorporate the credit market search model to address macroeconomic and financial issues.[10]

2.2 Deposit Market

Applying the search model to the deposit market is a contribution of this paper to the literature on frictional financial markets.[11] Search frictions have already been considered on the credit market (as explained just above), on over-the-counter financial markets, first by Duffie et al. (2005) and then by Lagos and Rocheteau (2009), among others, but not

[9] Den Haan et al. (2003) argue that "... there is a matching friction in the market to establish entrepreneur-lender relationships. This friction highlights the importance of long-term relationships". They develop this argument in the Section "Motivation for matching friction" of their paper.

[10] See, among others, Becsi et al. (2005, 2013), Chamley and Rochon (2001), Petrosky-Nadeau and Wasmer (2013), and Petrosky-Nadeau (2013).

[11] Another strand of literature considers the role of financial intermediaries in search-based models of monetary exchange *à la* Kiyotaki and Wright (1989) to explain the use of bank liabilities as a media of exchange, see He et al. (2005, 2008), and that banks improve welfare, see Berensten et al. (2007) and Gu et al. (2013).

on the deposit market.[12] The rationale for deposit market search frictions is, as for the credit market, the existence of long-term relationships between depositors and banks. I first document this fact, and I then explain it using the presence of switching costs for households and of relationship marketing by banks.

The European Commission (2009) published a survey on consumers' views regarding switching service providers to collect information about consumers' experiences switching providers and their ability to compare offers from various suppliers in several service sectors. The switching rate in the last two years is 11% for the banking industry as a whole, which is notably lower than the rates observed in other service sectors, such as car insurance (25%) or internet service (22%). The model developed in this paper does not apply to all financial services provided by banks, but to the remuneration of savings. It is therefore important to note that the switching rate for savings or investment products only remains low, approximately 13% against 9%, for current bank accounts. Furthermore, this low switching rate is not only observed in European countries. Kiser (2002) reports a mean relationship duration of 13.3 years from a survey of American consumers in Michigan.[13]

Switching costs is the most popular explanation for households' behavior on the deposit market. When a household decides to switch (or when she enters the market), she must spend time and resources to obtain information on services offered by banks; this is the search process for households on the deposit market. This search process would be costless and instantaneous without search frictions. However, the complexity of the retail financial market makes this search process costly and time-consuming. Indeed, Carlin (2009) argues that "[p]urchasing a retail financial product requires effort. Because prices in the market are complex, consumers must pay a cost (time or money) to compare prices in the market." Similarly, according to Sirri and Tufano (1998), "[e]conomists acknowledge that consumers' purchasing decisions – whether for cars or funds – are complicated by the phenomenon of costly search". Accordingly, the European Commission (2009) reports that 43% of interviewed customers anticipated or experienced difficulties switching banking services and 37% think that it is very and/or fairly difficult to compare offers in the

[12] Isoré (2012) also introduces search frictions in funding sources of banks, but with stakeholders and not with household depositors as considered here.

[13] Because the switching rate is a proxy for the (inverse of the) duration of customer relationships, it corresponds to a switching rate of approximately 7.5% per year or 15% every two years.

banking sector. Consistent with this evidence, I assume in the model developed herein that households pay search costs on the deposit market and that these search costs are interpreted as switching costs because a household must pay the costs to find another bank.[14]

Switching costs are intimately related to the practice of relationship marketing. The following observation about relationship marketing is made by Chiu et al. (2005): "marketing activities that attract, develop, maintain, and enhance customer relationships has changed the focus of a marketing orientation from attracting short-term, discrete transactional customers to retaining long-lasting, intimate customer relationships." Relationship marketing is therefore precisely devoted to increasing the cost of switching for customers. The underlying motivation of banks is to increase profits, as explained by Degryse and Ongena (2008): "Switching costs for bank customers represent an important source of rents for banks, and an important motive for the development of relationship (as opposed to transaction) banking." Sharpe (1997), Shy (2002), and Martin-Oliver et al. (2008) have established both theoretically and empirically the impact of switching costs on deposit interest rates using data for the United States, Finland, and Spain, respectively. In the model developed herein, bank search costs are interpreted as investment in relationship marketing because they are necessary to create long-term relationships with households.

3 The Issue of Financial Intermediation

This Section defines the issue of financial intermediation and presents the socially optimal solution.

3.1 Endowments and Technologies

I consider an economy with a raw good that cannot be consumed and a final (consumption) good that is produced by using the raw good as input. All agents (households, entrepreneurs, and banks) share the same linear utility function and discount factor for

[14] It is worth mentioning that if I identify both search costs and switching costs, Wilson (2012) develops a model devoted to distinguishing between search costs and switching costs.

the future, denoted $\beta \in]0,1[$ with $\beta = 1/(1+r)$, where r is the associated interest rate. There are two production technologies with different qualities. Households possess the low-quality technology that produces $\rho^h > 0$ units of final good per unit of input. Entrepreneurs possess the high-quality technology that produces $z > \rho^h$ units of final good per unit of input. Entrepreneurs have better technology, but all raw goods are initially given to households. Each household holds an asset that delivers one unit of raw good per period. The economic issue is how to avoid autarky: how do we transfer raw goods from households to entrepreneurs without a market for the raw good (e.g., without direct finance)? This is the issue of financial intermediation, solved herein in the presence of search frictions.

3.2 Search Frictions

I first characterize search frictions on the credit market. Banks invest $\kappa^c v^c$ in the search to find entrepreneurs on the credit market, where κ^c is the search cost per unit of effort and v^c is the banks' search effort (assuming a unit continuum of banks, it is equal to the search effort of the representative bank). Search is costless for entrepreneurs, and the u^c unmatched entrepreneurs search for a bank.[15] The per-period flow of new lending relationships is given by the matching function $m^c(v^c, u^c)$, which has constant returns to scale and is increasing in both arguments. The n^c matched entrepreneurs produce and remain matched with a probability $(1-\delta^c)$, where $\delta^c \in]0,1[$ is the probability of business failure.[16] The number of matched entrepreneurs evolves as follows

$$n^c_+ = (1-\delta^c)\, n^c + m^c(u^c, v^c) \tag{1}$$

where the symbol $+$ is used to denote the next-period value of state variables. The population of entrepreneurs is set to $\overline{n}^c$ and satisfies $\overline{n}^c = n^c + u^c$.

Banks invest $\kappa^d v^d$ in the search to attract households to the deposit market, where κ^d is the search cost per unit of effort and v^d is the banks' search effort (assuming a unit

[15] Search is exogenous for entrepreneurs and endogenous for households and banks.

[16] After a failure, the entrepreneur builds a new business project that should be audited by banks to be financed.

continuum of banks, it is equal to the search effort of the representative bank). Unmatched households produce low-quality technology; a part, u^d, of them decide to search for a bank (and to pay the per-period cost κ^h), whereas another part, o^d, of the households prefer to remain outside the banking sector. The per-period flow of new deposit relationships is given by the matching function $m^d\left(v^d, u^d\right)$, which has constant returns to scale and is increasing in both arguments. The n^d matched households remain matched with a probability $\left(1-\delta^d\right)$, where $\delta^d \in \left]0,1\right[$ is a preference shock.[17] The number of matched households evolves as follows

$$n_+^d = \left(1-\delta^d\right) n^d + m^d\left(u^d, v^d\right) \tag{2}$$

The population of households is set to $\overline{n}^d$ and satisfies $\overline{n}^d = n^d + u^d + o^d$.

The number of productive entrepreneurs cannot exceed the number of depositors

$$n^c \leq n^d \tag{3}$$

where n^d is also the amount of deposits (each household deposits one indivisible unit of raw good) and n^c is the amount of credits (each entrepreneur borrows one indivisible unit of raw good). Finally, the matching technologies are Cobb-Douglas with the following properties

$$\begin{aligned} m^x\left(u^x, v^x\right) &= \overline{m}^x\left(v^x\right)^\varepsilon\left(u^x\right)^{1-\varepsilon} \\ q\left(\alpha^x\right) &= m^x\left(u^x, v^x\right)/v^x = \overline{m}^x\left(\alpha^x\right)^{\varepsilon-1} = p\left(\alpha^x\right)/\alpha^x \\ \partial m^x\left(u^x, v^x\right)/\partial u^x &= \left(1-\varepsilon\right) p\left(\alpha^x\right), \, \partial m^x\left(u^x, v^x\right)/\partial v^x = \varepsilon q\left(\alpha^x\right) \end{aligned} \tag{4}$$

where $\alpha^x = v^x/u^x$ is the market tightness, $q\left(\alpha^x\right)$ and $p\left(\alpha^x\right)$ are the matching probabilities, for $x = \{c, d\}$ where c stands for credit and d for deposit. Without a loss of generality, the two matching functions share the same elasticity parameter ε, but the scale parameter $\overline{m}^x$ may be different.

[17] The household decides to switch banks after a change in its demographic composition (e.g., births, divorce) or on the labor market (e.g., job loss, promotion). The household pays the search costs to find the relevant financial service given the new situation.

Lemma 1 *The market tightness variables* $\{\alpha^x\}_{x=\{c,d\}}$ *determine the degree of financial intermediation and the social welfare.*

Proof. *See Appendix A.* ■

3.3 The Socially Optimal Solution

The socially optimal equilibrium is a constrained-efficient equilibrium. The social planner chooses search efforts to maximize steady–state welfare, taking the search frictions as given. The value function associated with the problem of the social planner is

$$
\begin{aligned}
&O\left(n^c, n^d\right) = \qquad (5)\\
&\max_{u^d, \left\{n_+^x, v_+^x\right\}_{x=\{c,d\}}} \left\{n^c z + \left(\overline{n}^d - n^d - u^d\right)\rho^h + u^d\left(\rho^h - \kappa^h\right) - \kappa^d v^d - \kappa^c v^c + \beta O\left(n_+^c, n_+^d\right)\right\}\\
&-\lambda^c\left[n_+^c - \left(1-\delta^c\right)n^c - m^c\left(\overline{n}^c - n^c, v^c\right)\right]\\
&-\lambda^d\left[n_+^d - \left(1-\delta^d\right)n^d - m^d\left(u^d, v^d\right)\right]\\
&-\lambda^i\left(n^c - n^d\right)
\end{aligned}
$$

where the per-period utility flow is defined as the final goods produced by households and entrepreneurs less the search costs for households and banks. $\{\lambda^x\}_{x=\{c,d,i\}}$ are the Lagrangian multipliers associated with the constraints (1), (2), and (3). The next proposition presents the solution of (5).

Proposition 1 *The socially optimal equilibrium exists and is unique. Financial intermediation is socially optimal if the technology gap between households and entrepreneurs is sufficiently high.*

Proof. *The socially optimal allocation of resources is defined by the market tightness variables* $\{\alpha^x\}_{x=\{c,d\}}$ *that solve*

$$\alpha_o^d = \frac{\kappa^h}{\kappa^d}\frac{\varepsilon}{1-\varepsilon} \qquad (6)$$

$$\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_o^c\right)^{1-\varepsilon} + \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon} = \varepsilon\left(z-\rho^h\right) - \left(1-\varepsilon\right)\kappa^c\alpha_o^c \qquad (7)$$

The technology gap between households and entrepreneurs is $\left(z-\rho^h\right)$, *and it should be sufficiently large to ensure a positive value for* α_o^c. *See Appendix B for details.* ■

The socially optimal deposit market tightness α_o^d is given by equation (6) as a function of search costs and elasticity parameters of the matching functions.[18] To increase the amount of deposits, the social planner can increase the banks' search efforts, at the cost κ^d for the marginal productivity $m_2^d\left(u^d, v^d\right)$, or the households' participation, at the cost κ^h for the marginal productivity $m_1^d\left(u^d, v^d\right)$. Condition (6) makes equal the cost ratio, which is κ^h/κ^d, to the marginal productivity ratio, which is $m_1^d\left(u^d, v^d\right)/m_2^d\left(u^d, v^d\right) = \alpha^d\left(1-\varepsilon\right)/\varepsilon$, given the specification (4) of the matching function.

The socially optimal credit market tightness α_o^c is then given by equation (7), where α_o^d is given by (6). The LHS of (7) measures the matching costs of financial intermediation: how much it costs to collect one unit of deposit and to select one entrepreneur. The matching costs are equal to $\left(\kappa^x/\overline{m}^x\right)\left(\alpha_o^x\right)^{1-\varepsilon}$: the per-period search cost κ^x on market $x = \{c, d\}$ is divided by the probability of matching, $\overline{m}^x\left(\alpha_o^x\right)^{\varepsilon-1}$. Matching costs are discounted by $\left(r+\delta^x\right)$, the sum of the rate of time preference and of the separation probability. The RHS of (7) measures the social benefits of financial intermediation: the technology gap between households and entrepreneurs, weighted by ε, less the opportunity costs of being matched for the entrepreneur, weighted by $\left(1-\varepsilon\right)$. Outside the match, the entrepreneur would have contributed to social welfare by searching: with a marginal productivity $m_1^c\left(u^c, v^c\right)$, it would have created the value of a new match, which is equal to $\varepsilon \times \kappa^c/m_2^c\left(u^c, v^c\right)$. The last term of the RHS of (7) corresponds to the product $m_1^c\left(u^c, v^c\right) \times \varepsilon \times \kappa^c/m_2^c\left(u^c, v^c\right)$, given the specification (4) of the matching function.

4 Competitive Financial Intermediation

This Section presents the competitive equilibrium.

[18]This expression for the equilibrium tightness is common in search models with two endogenous participation rules, not one, as is usually assumed. Indeed, participation is endogenous for firms, but it is exogenous for workers in the standard labor market search model. Wasmer and Weil (2004) obtain an expression for the credit market tightness similar to (6) because they consider the endogenous participation of both entrepreneurs and bankers (but the exogenous participation of workers on the labor market). Here, participation is endogenous for banks on the deposit and credit markets, endogenous for households on the deposit market, and exogenous for entrepreneurs on the credit market.

4.1 Non-Financial Agents

Households' value functions are denoted D^y, where $y = \{h, m, u\}$ refers to the household states: non-participating, matched, and searching, respectively. They are defined by

$$D^h = \rho^h + \beta D^h \tag{8}$$

$$D^m\left(\rho^d\right) = \rho^d + \left(1 - \delta^d\right)\beta D^m\left(\rho^d\right) + \delta^d \beta D^u \tag{9}$$

$$D^u = \rho^h - \kappa^h + p\left(\alpha^d\right)\beta D^m\left(\rho^d\right) + \left[1 - p\left(\alpha^d\right)\right]\beta D^u \tag{10}$$

If the household does not participate, she produces ρ^h of final goods. If she decides to search for a bank, she still produces ρ^h but now pays κ^h as a search cost and has a probability $p\left(\alpha^d\right)$ of forming a match with a bank. When she is matched, the household receives ρ^d units of the final good as deposit interests and remains in this state with a probability $\left(1 - \delta^d\right)$. Unmatched households decide whether to search. The free entry condition on the deposit market implies $D^h = D^u$ or, equivalently,

$$\begin{aligned} \kappa^h &= p\left(\alpha^d\right)\beta\left[D^m\left(\rho^d\right) - D^u\right] \\ &= p\left(\alpha^d\right)\frac{\rho^d - \rho^h}{r + \delta^d} \end{aligned} \tag{11}$$

given (8), (9), and (10). The entry of households is such that the search cost, κ^h, is equal to search payoff: with a probability $p\left(\alpha^d\right)$, the household earns the difference between deposit interests and home production $\left(\rho^d - \rho^h\right)$ discounted by $\left(r + \delta^d\right)$.

Entrepreneurs' value functions are denoted as L^y, where $y = \{m, u\}$ refers to the entrepreneur states: matched and unmatched, respectively. They are defined by

$$L^u = p\left(\alpha^c\right)\beta L^m\left(\rho^c\right) + \left(1 - p\left(\alpha^c\right)\right)\beta L^u \tag{12}$$

$$L^m\left(\rho^c\right) = z - \rho^c + \left(1 - \delta^c\right)\beta L^m\left(\rho^c\right) + \delta^c \beta L^u \tag{13}$$

The per-period utility is zero when entrepreneurs search and $(z - \rho^c)$ when matched, where ρ^c is the amount of credit interests. The transition probabilities between states are $p\left(\alpha^c\right)$ and δ^c.

4.2 Bank Search Efforts

The representative bank maximizes the discounted sum of profits (defined as the credit interests less both the deposit interests and the search costs) subject to the constraints (1), (2), and (3). The bank value function is

$$\begin{aligned} B\left(n^c, n^d\right) &= \max_{\left\{n_+^x, v^x\right\}_{x=c,d}} \left\{\rho^c n^c - \rho^d n^d - \kappa^d v^d - \kappa^c v^c + \beta B\left(n_+^c, n_+^d\right)\right\} \\ &\quad -\lambda^c\left[n_+^c - \left(1-\delta^c\right) n^c - q\left(\alpha^c\right) v^c\right] \\ &\quad -\lambda^d\left[n_+^d - \left(1-\delta^d\right) n^d - q\left(\alpha^d\right) v^d\right] \\ &\quad -\lambda^i\left(n^c - n^d\right) \end{aligned} \tag{14}$$

The bank chooses search efforts $\{v^x\}_{x=\{c,d\}}$ given the interest rates $\{\rho^x\}_{x=\{c,d\}}$ such that

$$\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha^c\right)^{1-\varepsilon} + \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha^d\right)^{1-\varepsilon} = \rho^c - \rho^d \tag{15}$$

see Appendix C for details. The optimality condition for banks, namely (15), can be compared with its counterpart for the social planner, namely (7). The LHS terms of (7) and (15) are identical and correspond to the matching costs of financial intermediation. The RHS terms correspond to benefits of financial intermediation, which may differ. The bank's benefits are the interest margin, i.e., the difference between credit and deposit interests in (15), which may not coincide with the social benefits in (7).

4.3 Nash Bargaining

Interest rates are determined by individual Nash bargaining processes, which are repeated at each period. The interest rates satisfy

$$\left(1-\eta^d\right)\left(D^m - D^u\right) = \eta^d \Delta B^d \tag{16}$$

and

$$\left(1-\eta^c\right)\left(L^m - L^u\right) = \eta^c \Delta B^c \tag{17}$$

$(1-\eta^x)$ measures the bargaining power of banks, and η^x measures the bargaining power of non-financial customers for $x = \{c, d\}$. The surplus of non-financial customers can be directly computed using the value function definitions (9) and (10) for households and (12) and (13) for entrepreneurs. It is less direct for the bank's surplus ΔB^x.

The bank bargains with n^d depositors and n^c borrowers, with interdependencies between credits and deposits, given the constraint (3). Consider the bargaining process with the depositor. If bargaining fails, the bank is deprived of one depositor and one borrower because the bank cannot provide the raw good, which is necessary for the entrepreneur to produce.[19] The same occurs for borrowers. If bargaining fails, the bank is deprived of one borrower and one depositor because the bank cannot transform the raw good into a final good to pay the deposit interest. In fact, the depositor withdraws her unit of raw good if the utility in the unmatched state is higher than the utility in the matched state with no deposit interests, that is

$$\rho^h + \beta D^h > \left(1-\delta^d\right)\beta D^m\left(\rho^d\right) + \delta^d \beta D^u \tag{18}$$

or, equivalently, $\rho^h > \left(1-\delta^d\right)\kappa^h/p\left(\alpha^d\right)$ given the free entry condition on the deposit market (11). Hereafter, I assume that the condition (18) holds.

Actually, the bank faces a liquidity problem because both credits and deposits are sluggish. If a perfectly liquid interbank market is introduced, there is no longer a double loss of financial relationships when bargaining fails because the market provides a substitute for the missing deposit or credit. To formalize this point, I introduce the function $n^y(n^x)$, which accounts for the existence of an interbank market for credit or deposit. These functions satisfy

$$n_1^y(n^x) = \begin{cases} 0 & \text{there is a substitute for } x \\ 1 & \text{no substitute} \end{cases} \tag{19}$$

for $x = \{c, d\}$, $y = \{c, d\}$ and $y \neq x$. The loss of one depositor implies the loss of one borrower if $n_1^c\left(n^d\right) = 1$ and not otherwise. Similarly, the loss of one borrower implies

[19] I assume that if the production process is interrupted, the entrepreneur should be audited once again to restart the production process.

the loss of one depositor if $n_1^d(n^c) = 1$ and not otherwise. Finally, the bank surplus is as follows

$$\begin{aligned}\Delta B^x &= \left.\frac{\partial B\left(n^c, n^d\right)}{\partial n^x}\right|_{n^y = n^y(n^x)} \qquad (20)\\ &= \rho^c - \rho^d + (1-\delta^c)\frac{\kappa^c}{q(\alpha^c)} + \left(1-\delta^d\right)\frac{\kappa^d}{q(\alpha^d)} - \left[1 - n_1^y(n^x)\right](1+r)\frac{\kappa^x}{q(\alpha^x)}\end{aligned}$$

for $x = \{c, d\}$, $y = \{c, d\}$ and $y \neq x$; see the Appendix D for details. The bank's surplus on the market x is equal to the net interest margin plus the value of financial relationships on the two markets, if these relationships are not destroyed by the probability $(1-\delta^x)$, less the discounted matching costs. If $n_1^y(n^x) = 1$, there is no substitute for the non-financial customer x and the bank cannot subtract the matching cost from its surplus. If $n_1^y(n^x) = 0$, there are substitutes and the bank's surplus is lower than if $n_1^y(n^x) = 1$. The appendix E defines the bank value functions with and without an interbank market and shows their consistency with the definition (20). The bank's liquidity constraints determine the bank's surplus and, therefore, impact the interest rates and search decisions, as shown in the next Section.

5 (In)Efficiency of Competitive Financial Intermediation

I define the competitive equilibrium and then discuss its normative properties.

5.1 Competitive Equilibrium

Definition 1 *The competitive financial intermediation with bargained interest rates is characterized by the interest rates* $\{\rho_b^x\}_{x=\{c,d\}}$ *that satisfy*

$$\rho_b^d = \rho^h - \left(1-\delta^d\right)\frac{\kappa^h}{\overline{m}^d\left(\alpha^d\right)^\varepsilon} + \left(\frac{\eta^d}{1-\eta^d}\right)(1+r)\left[\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} + n_1^c\left(n^d\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}\right] \qquad (21)$$

$$\rho_b^c = z - (r + \delta^c + p(\alpha_b^c))\left(\frac{\eta^c}{1-\eta^c}\right)\left[\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} + n_1^d(n^c)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}\right] \qquad (22)$$

where the equilibrium market tightness variables $\{\alpha_b^x\}_{x=\{c,d\}}$ *are the solution of*

$$\frac{\kappa^h}{\overline{m}^d \left(\alpha_b^d\right)^{\varepsilon}} = \left(\frac{\eta^d}{1-\eta^d}\right)\left[\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}\right] \tag{23}$$

$$\begin{aligned} & \left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} + \left(r+\delta^d\right)\left(\frac{1-\eta^c}{1-\eta^d}\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} \\ = \; & \left(1-\eta^c\right)\left(z-\rho^h\right) - \eta^c\kappa^c\alpha_b^c - \left(r+\delta^c+p\left(\alpha_b^c\right)\right)\eta^c n_1^d\left(n_b^c\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} \\ & - \left(r+\delta^d\right)\left(1-\eta^c\right)\left(\frac{\eta^d}{1-\eta^d}\right)n_1^c\left(n_b^d\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} \end{aligned} \tag{24}$$

Appendix F provides the resolution details for the Nash bargaining process, and Appendix G shows how to obtain the equilibrium conditions for market tightness.

The equilibrium deposit interest rate given by (21) is equal to (i) the value of self-production, ρ^h, (ii) less the household value of the financial relationship, $\kappa^h/p\left(\alpha^d\right)$, which is preserved by the household with the probability $\left(1-\delta^d\right)$, (iii) plus the share $\eta^d/\left(1-\eta^d\right)$ of the bank values of financial relationships, $\kappa^x/q\left(\alpha^x\right)$, discounted by $(1+r)$ for $x=\{c,d\}$. When $n_1^c\left(n^d\right)$ is equal to zero, the household receives only a share of the bank value of the financial relationship on the deposit market, as is commonly the case in models that use search frictions. Here, the novelty is that when $n_1^c\left(n^d\right)$ is equal to unity, the household also receives a share of the bank value of the financial relationship on the credit market. I refer to this phenomenon as the cross-market appropriation of financial relationship's value by non-financial agents. Because of the bank's liquidity constraint, the household succeeds in appropriating a share of the lending relationship's value, even if the household does not participate in the search externalities on the credit market. Cross-market appropriation allows households to earn higher deposit interests.

The equilibrium credit interest rate given by (22) is equal to (i) the value of business production, z, (ii) less the share $\eta^c/\left(1-\eta^c\right)$ of the bank value of financial relationships, $\kappa^x/q\left(\alpha^x\right)$, discounted by[20] $\left(r+\delta^c+p\left(\alpha_b^c\right)\right)$ for $x=\{c,d\}$. When $n_1^d\left(n^c\right)$ is equal to

[20]The discount rate can also be written as $[(1+r)+p\left(\alpha^c\right)-\left(1-\delta^c\right)]$. It is composed of the discount rate for time preference, namely $(1+r)$, and of the difference in probabilities of being matched according to the current state, namely, $p\left(\alpha^c\right)$, if unmatched, and $\left(1-\delta^c\right)$, if matched.

zero, the entrepreneur receives only a share of the bank value of the financial relationship on the credit market, as is commonly the case in models that use search frictions. As previously, cross-market appropriation occurs when $n_1^d(n^c)$ is equal to unity. Because of the bank's liquidity constraint, the entrepreneur succeeds in appropriating a share of the deposit relationship's value even if the entrepreneur does not participate in the search externalities on the deposit market. Cross-market appropriation allows entrepreneurs to pay lower credit interests.

Equations (23) and (24) show how the cross-market appropriation influences the equilibrium values of the tightness variables. When households appropriate a share of the lending relationship's value, i.e., $n_1^c\left(n_b^d\right) = 1$ in the RHS term of equation (23), they are willing to pay a higher matching cost, which corresponds to the LHS term of equation (23). For a given value of the credit market tightness α_b^c, this mechanism tends toward a fall in the deposit market tightness α_b^c. For banks, cross-market appropriation lowers the payoff of financial intermediation, as in the RHS term of equation (24) for $n_1^d(n_b^c) = n_1^c\left(n_b^d\right) = 1$; consequently, the matching costs of financial intermediation should decrease, i.e., the LHS term of equation (24). For a given value of the deposit market tightness α_b^d, this mechanism acts toward a fall in the credit market tightness α_b^c. The next Section shows how the bargaining process can offset the effects of cross-market appropriation.

5.2 Efficient Financial Intermediation

Proposition 2 *There exist values for bargaining power that make the competitive financial intermediation efficient.*

Proof. *The values of bargaining powers $\{\eta_o^x\}_{x=c,d}$ imply that $\{\alpha_b^x = \alpha_o^x\}_{x=c,d}$ where $\{\alpha_o^x\}_{x=c,d}$ solve (6) and (7) and $\{\alpha_b^x\}_{x=c,d}$ solve (23) and (24). They are*

$$\eta_o^d = (1-\varepsilon)\left[1 + \varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q\left(\alpha_o^c\right)}\right]^{-1} \leq (1-\varepsilon) \tag{25}$$

$$\eta_o^c = (1-\varepsilon) \tag{26}$$

$$\times \frac{z - \rho^h + \kappa^c \alpha_o^c - \left(r+\delta^d\right) \frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{\varepsilon + \varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q(\alpha_o^c)}}}{z - \rho^h + \kappa^c \alpha_o^c - \left(r+\delta^d\right) \frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{\varepsilon + \varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q(\alpha_o^c)}} + \left(r + \delta^c + p\left(\alpha_o^c\right)\right) n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}} \leq (1-\varepsilon)$$

where the socially optimal values for market tightness $\{\alpha_o^x\}_{x=c,d}$ do not depend on the bargaining power. See Appendix H for details. ■

Equations (25) and (26) generalize the Hosios (1990) condition for efficiency. In standard search models, the Hosios (1990) condition imposes equality between the agent's bargaining power and the elasticity of the matching function with respect to its search effort. In this case, the search externalities are internalized.

This condition is also sufficient in the search model of financial intermediation proposed in this paper without liquidity constraints: the conditions (25) and (26) reduce to

$$\eta_o^d = \eta_o^c = (1-\varepsilon) \tag{27}$$

for $n_1^c\left(n_b^d\right) = n_1^d\left(n_b^c\right) = 0$. With liquidity constraints, the Hosios (1990) condition is no longer sufficient to ensure efficiency because of cross-market appropriation. The second terms in equations (25) and (26), which multiply the elasticity coefficient $(1-\varepsilon)$, generalize the Hosios (1990) condition to correct for cross-market appropriation. The next corollary explains the correction.

Corollary 1 *Efficiency of financial intermediation requires that the banks' bargaining power increase when liquidity constraints appear to offset cross-market appropriation. Otherwise, the deposit market tightness is lower than the socially optimal level and credit rationing may occur.*

Proof. *It follows from proposition 2, see Appendix I for details.* ■

In the reference situation, there are no liquidity constraints, $n_1^c\left(n_b^d\right) = n_1^d\left(n_b^c\right) = 0$, and the Hosios (1990) condition (27) is sufficient for efficiency, $(1-\eta_o^c) = (1-\eta_o^c) = \varepsilon$. I then consider the case of a constraint for credit, and not for deposit, i.e., $n_1^d\left(n_b^c\right) = 1$

and $n_1^c\left(n_b^d\right) = 0$. Efficiency requires that $\left(1-\eta_o^d\right) = \varepsilon$ and $(1-\eta_o^c) > \varepsilon$. Banks can instantaneously find a substitute for a deposit, but not for credit. This substitute does not change the deposit market equilibrium, which is efficient under the Hosios condition (1990). It does, however, change the credit market equilibrium. According to (22), the credit interest rate should fall because of cross-market appropriation. Therefore, banks would reduce their search efforts on the credit market until the equality between matching costs and the financial intermediation payoff is restored, with the consequence of a reduction in final goods production and welfare (see Lemma 1). A solution to avoid this market failure is to increase the banks' bargaining power with entrepreneurs $(1-\eta_o^c)$ above ε, to offset the cross-market appropriation by entrepreneurs.

In the case of a constraint for deposit, and not for credit, i.e., $n_1^d(n_b^c) = 0$ and $n_1^c\left(n_b^d\right) = 1$, efficiency requires that $\left(1-\eta_o^d\right) > \varepsilon$ and $(1-\eta_o^c) = \varepsilon$. Banks can instantaneously find a substitute for credit, but not for deposit. As in the previous case, banks' bargaining power should be different than in the reference situation to preserve efficiency. According to (21), the deposit interest rate increases because of cross-market appropriation. The banks' bargaining power with households should therefore increase to $\left(1-\eta_o^d\right)$, which is above ε, to offset the cross-market appropriation by households.

If the bargaining power values are fixed to the Hosios (1990) values (27) and if there are credit/deposit constraints, inefficient financial intermediation occurs. In Appendix I, I demonstrate that the competitive deposit market tightness is equal to or below its socially optimal level. Cross-market appropriation stimulates the entry of households into the deposit market who are willing to pay higher matching costs. The competitive credit market tightness can be lower than, equal to or higher than its socially optimal level because of the existence of two effects. First, the cross-market appropriation by entrepreneurs makes the competitive credit market tightness lower because banks reduce their search efforts in response to a cut in credit interests. Second, low tightness on the deposit market reduces the matching costs of banks on this market, which are therefore willing to pay higher matching costs on the credit market. Remember that banks consider the total matching costs to be the sum of the matching costs on the deposit market and on the credit market, see the LHS of (15). For a given net interest margin, e.g., the RHS of (15), smaller matching costs on one market imply higher matching costs on the other one at equilibrium. If the first effect dominates the second, inefficient financial intermediation occurs with excessive credit rationing; otherwise, it occurs with an excessive investment of scarce resources in the financial sector. The next Section numerically illustrates these theoretical properties.

5.3 Numerical Illustration

Using a numerical example, this Section illustrates the theoretical properties of the search model of financial intermediation. The model is calibrated for the socially optimal allocation. One unit of time corresponds to one month. The discount rate is $\beta = 0.999$. The average duration of a financial relationship is set to five years both for households and entrepreneurs ($\delta^d = \delta^c = 1/(12 \times 5)$). The average search duration is shorter for households (slightly above one month) than for entrepreneurs (three months), i.e., $p\left(\alpha^d\right) = 1/1.5$ and $p\left(\alpha^c\right) = 1/3$. The corresponding rate of matching for entrepreneurs is $n^c = 0.95$ under the normalization $\overline{n}^c = 1$. The elasticity of the matching functions is set to $\varepsilon = 0.5$ and the values of the scale parameters of the matching functions are deduced to be $\overline{m}^d = 0.66$ and $\overline{m}^c = 0.30$. The productivities of households and entrepreneurs are set to $\rho^h = 4$ and $z = 5$, and the search costs are set to $\kappa^h = \kappa^d = \kappa^c = 1$. The values of the deposit and credit interest rates are deduced to be $\rho^d = 4.02$ and $\rho^c = 4.10$.

The Figure 1 plots the curves associated with the equations (23) and (24) for market tightness variables in Panels (a)-(b)-(c), with the equation (22) for the credit interest rate in Panels (e)-(f)-(g), and with the equation (21) for the deposit interest rate in Panels (e)-(f)-(g). To decompose the effect of liquidity constraints, three cases are shown.

1. The socially optimal allocation without liquidity constraints: $n_1^c\left(n^d\right) = n_1^d\left(n^c\right) = 0$ and $\eta^c = \eta^d = 0.5$. This corresponds to the black lines.

2. The inefficient allocation with liquidity constraints: $n_1^c\left(n^d\right) = n_1^d\left(n^c\right) = 1$ and $\eta^c = \eta^d = 0.5$. This corresponds to the red lines.

3. The socially optimal allocation with liquidity constraints: $n_1^c\left(n^d\right) = n_1^d\left(n^c\right) = 1$ and $\eta^c = 0.40$ and $\eta^d = 0.25$, according to equations (25) and (26). This corresponds to the blue lines.

Panels (a)-(d)-(g) depict the socially optimal allocation without liquidity constraints. Panels (b)-(e)-(h) show the consequences of banks' liquidity constraints for unchanged bargaining power of banks: market tightness variables fall. Inefficient financial intermediation leads to an excessive credit rationing of entrepreneurs and an amount of final goods production in the economy that is too low, see Panel (b). The Panel (e) shows the impact on the credit interest rate. When $n_1^d\,(n^c)$ switches from zero to one, it moves down the curve associated with equation (22): the credit interest rate would be lower if the credit market tightness was unchanged. However, there is an important change in the credit market tightness, see the shift to the left of the vertical line in Panel (e), which makes the equilibrium credit interest rate almost unchanged (4.11 in the inefficient case against 4.10 in the socially optimal case). The deposit interest rate is higher in the inefficient allocation than in the socially optimal allocation (4.05 against 4.02, see Panel (h)). To restore efficiency, banks bargaining power should be higher, $(1-\eta_0^c)=0.60$ and $\left(1-\eta_0^d\right)=0.75$. For these values, financial intermediation is efficient; see the blue lines in Panels (c)-(f)-(i).

6 Discussion and Concluding Remarks

I proposed in this paper a new model of financial intermediation based on search frictions. Banks should invest in search activities on the two markets, credit and deposit, to realize their financial intermediation activities. The use of the search-theoretic approach allows the definition of the constrained efficient financial intermediation and the discussion of the condition of its achievement in a competitive environment. I identified a specific source of market failures, which is the consequence of the simultaneous bargaining processes managed by banks with their customers. When a bank bargains with a depositor (or a creditor), it considers that if the bargaining process fails, a creditor will no longer be financed (or a deposit will no longer be paid). This context alters the outcome of the bargaining process, and the traditional Hosios (1990) condition for efficiency is no longer sufficient. My contribution to the search literature is therefore to generalize the Hosios (1990) condition to the case of simultaneous search on two frictional markets. In the context of the financial sector, I show that the Hosios (1990) condition corresponds to a specific environment in which banks have access to perfect substitutes for credits and deposits, for example, on the interbank market.

The search-approach of financial intermediation developed herein should be useful to address political issues, including the effectiveness of unconventional monetary policy and the importance of retail market reforms. Central banks first implemented conventional monetary policy responses to the financial crisis of 2007-2008 by lowering the nominal interest rate and restructuring banks. These responses quickly appeared to be insufficient, notably because of the zero lower bound of the nominal interest rate. Then, several central banks decided to implement unconventional monetary policy to increase liquidity and to counterbalance the liquidity crisis on the interbank market. In the search model of financial intermediation, the interest of this policy is to guarantee the efficiency of financial intermediation without implementing changes in bargaining power. This policy suppresses the deposit constraint of banks: if a bank loses one depositor, it does not necessarily lose one borrower because it has access to central bank liquidity as a source of funding. To be efficient, the unconventional monetary policy should also suppress the credit market constraint: if a bank loses one borrower, it does not necessarily lose one depositor because it has access to central bank deposits as an investment opportunity. However, the remuneration of deposits by the central bank is traditionally low, and it is at least lower than the interest yields from private non-financial borrowers. The central bank is more likely able to solve the deposit constraint than the credit constraint, but both are necessary to avoid the cross-market appropriation mechanism highlighted in this paper.

Several institutions, such as the European Commission (2013) and the U.K. Independent Commission on Banking (2001), strongly recommend reducing search and switching costs in banking retail markets, which are still high, as explained in the Section 2. These recommendations are generally motivated by the welfare losses supported by the consumers and the reduction of the market's size induced by these costs. The search model developed herein put forward the interest of this policy for stability by reducing the transmission of liquidity shocks to the production sector. However, it would be difficult to completely remove search frictions from the retail banking sector because banks adopt commercial strategies to maintain financial relationships with households. In addition, the interest of household deposits for financial stability is precisely their stability, which is closely related to the existence of financial relationships.

As a final remark, it is worth mentioning that if banks post interest rates on markets instead of bargaining over them with non-financial agents, the potential market failure identified herein would disappear. Banks would jointly determine the search efforts and the interest rates and would not be locked in the double bargaining programs described above.[21] This use of price posting strategy is specific to the search model of financial intermediation and supplements its traditional interest in (single) market search models, thus making endogenous the Hosios (1990) condition; see Shimer (1996) and Moen (1997). This price strategy, however, requires strong assumptions. The posted interests are not for one contract (the first credit to the entrepreneur, for example), but they are instead for all contracts repeated during the financial relationships. Therefore, it requires a strong commitment of banks to future interest rates. This commitment may be challenged by the individual interests of agents to renegotiate contracts, especially in times of financial crisis. Indeed, crises make it harder to commit because financial contracts are generally not state-contingent on variables such as business failure risk.

[21] See the on-line Appendix J for the resolution of the search model of financial intermediation under price positing.

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Figure

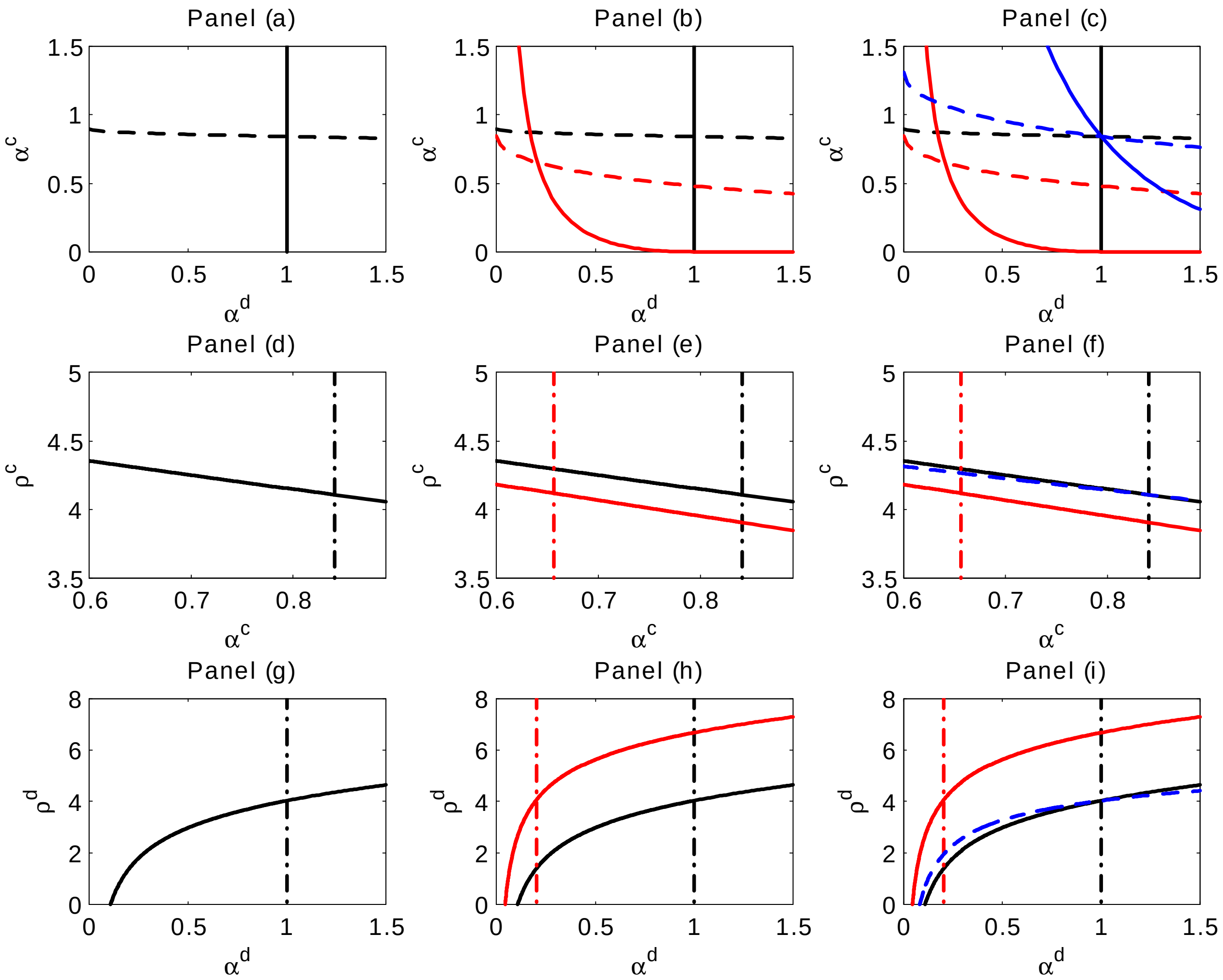

Figure 1. Equilibrium values for tightness on the credit market α^c and the deposit market α^d, panels (a)-(b)-(c), for the credit interest rate ρ^c, panels (d)-(e)-(f), and for the deposit interest rate ρ^d, panels (g)-(h)-(i). Panel (a)-(d)-(g) show the socially optimal allocation (black lines). Panel (b)-(e)-(h) show the socially optimal allocation (black lines) and the competitive equilibrium for suboptimal bargaining power (red lines). Panel (c)-(f)-(i) show the socially optimal allocation (black lines), the competitive equilibria for suboptimal (red lines) and optimal (blue lines) bargaining power.

A Proof of Lemma 1

The number of matched entrepreneurs solution of (1) is a function of the credit market tightness α^c

$$n^c(\alpha^c) = \frac{\overline{m}^c(\alpha^c)^\varepsilon}{\delta^c + \overline{m}^c(\alpha^c)^\varepsilon}\overline{n}^c \tag{A.1}$$

$n^c(\alpha^c)/\overline{n}^c$ measures the degree of financial intermediation as the rate of financing entrepreneurs. The number of searching households solution of (2) is a function of the two tightness variables $\{\alpha^c, \alpha^d\}$

$$u^d(\alpha^d, \alpha^c) = \frac{\delta^d}{\overline{m}^d(\alpha^d)^\varepsilon}n^c(\alpha^c) \tag{A.2}$$

where the constraint (3) is saturated: $n^d = n^c(\alpha^c)$. Because it is costly to collect deposits, at the equilibrium all deposits are lent. The welfare is a function of the two tightness variables $\{\alpha^c, \alpha^d\}$

$$U(\alpha^c, \alpha^d) = \frac{1}{1-\beta}\left[\begin{array}{c} n^c(\alpha^c)z + \left(\overline{n}^d - n^c(\alpha^c)\right)\rho^h \\ -\kappa^h u^d(\alpha^d, \alpha^c) - \kappa^d\alpha^d u^d(\alpha^d, \alpha^c) - \kappa^c\alpha^c(\overline{n}^c - n^c(\alpha^c)) \end{array}\right] \tag{A.3}$$

that is the final good production done by financing entrepreneurs, plus the final good production done by households, less search costs paid by searching households and banks.

B Proof of Proposition 1

The socially optimal allocation is the solution of (5), which first order conditions are

$$v^x : \kappa^x = \lambda^x\frac{\partial m^x(u^x, v^x)}{\partial v^x}, \text{ for } x = \{c, d\} \tag{B.1}$$

$$n_+^x : \lambda^x = \beta\frac{\partial O(n_+^c, n_+^d)}{\partial n_+^x}, \text{ for } x = \{c, d\} \tag{B.2}$$

$$u^d : \kappa^h = \lambda^d\frac{\partial m^d(u^d, v^d)}{\partial u^d} \tag{B.3}$$

The contributions to the value function of the marginal credit is

$$\frac{\partial O\left(n^c,n^d\right)}{\partial n^c}=z+\lambda^c\left[(1-\delta^c)-\frac{\partial m^c\left(\overline{n}^c-n^c,v^c\right)}{\partial\left(\overline{n}^c-n^c\right)}\right]-\lambda^i \tag{B.4}$$

and of the marginal deposit is

$$\frac{\partial O\left(n^c,n^d\right)}{\partial n^d}=-\rho^h+\lambda^d\left(1-\delta^d\right)+\lambda^i \tag{B.5}$$

The value of the multiplier λ^d given by (B.1) is introduced in the optimal condition (B.3) to get

$$\kappa^h=\lambda^d\frac{\partial m^d\left(u^d,v^d\right)}{\partial u^d}=\frac{\kappa^d}{\partial m^d\left(u^d,v^d\right)/\partial v^d}\frac{\partial m^d\left(u^d,v^d\right)}{\partial u^d}=\kappa^d\frac{\partial m^d\left(u^d,v^d\right)/\partial u^d}{\partial m^d\left(u^d,v^d\right)/\partial v^d} \tag{B.6}$$

For the matching function (4), it becomes

$$\alpha^d=\frac{\kappa^h}{\kappa^d}\frac{\varepsilon}{1-\varepsilon} \tag{B.7}$$

The equations (B.2) and (B.5) are used to get the value of the multiplier λ^i

$$\lambda^i=\rho^h+\left(r+\delta^d\right)\lambda^d \tag{B.8}$$

Remember that $r=1/\beta-1$. Equations (B.2) and (B.4) give

$$\lambda^c=\beta\left\{z+\lambda^c\left[(1-\delta^c)-\frac{\partial m^c\left(\overline{n}^c-n^c,v^c\right)}{\partial\left(\overline{n}^c-n^c\right)}\right]-\lambda^i\right\} \tag{B.9}$$

By using (B.8) to suppress λ^i, it becomes

$$(r+\delta^c)\lambda^c+\left(r+\delta^d\right)\lambda^d=z-\rho^h-\frac{\partial m^c\left(\overline{n}^c-n^c,v^c\right)}{\partial\left(\overline{n}^c-n^c\right)}\lambda^c \tag{B.10}$$

By using (B.1) and (4), it becomes

$$(r+\delta^c)\frac{\kappa^c}{\overline{m}^c}(\alpha^c)^{1-\varepsilon}+\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha^d\right)^{1-\varepsilon}=\varepsilon\left(z-\rho^h\right)-(1-\varepsilon)\kappa^c\alpha^c \tag{B.11}$$

The socially optimal market tightness variables $\{\alpha_o^c,\alpha_d^c\}$ solve (B.7) and (B.11).

Equation (B.7) gives the unique and strictly positive value for α_o^d in function of the structural parameters $\left\{\kappa^h, \kappa^d, \varepsilon\right\}$, see (6). The value α_o^c solves (B.11) in function of α_o^d and other structural parameters, see also (7). Equation (B.11) is rearranged as follows

$$\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_o^c\right)^{1-\varepsilon}+\left(1-\varepsilon\right)\kappa^c\alpha_o^c=\varepsilon\left(z-\rho^h\right)-\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon} \tag{B.12}$$

The RHS term is independent of α_o^c, whereas the LHS term is strictly increasing with α_o^c and equal to 0 for $\alpha_o^c=0$. To ensure a positive value for α_o^c, the LHS term should be strictly positive, that is

$$\left(z-\rho^h\right)>\left(r+\delta^d\right)\frac{\kappa^d}{\varepsilon\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon} \tag{B.13}$$

$\left(z-\rho^h\right)$ is the gap between the household and entrepreneur technologies, $\left(r+\delta^d\right)$ the discounted rate for time preference r and for exogenous separation δ^d, and κ^d, the per-period search cost, is divided by the marginal productivity of banks' search effort on the deposit market, $\varepsilon\overline{m}^d\left(\alpha_o^d\right)^{\varepsilon-1}$. If the condition (B.13) is not satisfied, there is no financial intermediation. It is socially optimal that households use all their raw goods to produce the final good and that entrepeneurs do not produce.

C Bank's Search Efforts

The first order conditions of the program (14) are

$$v^x : \frac{\kappa^x}{q\left(\alpha^x\right)}=\lambda^x,\ \text{for } x=\{c,d\} \tag{C.1}$$

$$n_+^c : \lambda^c=\beta\frac{\partial B\left(n_+^c,n_+^d\right)}{\partial n_+^c}=\beta\left[\rho^c+\left(1-\delta^c\right)\lambda^c-\lambda^i\right] \tag{C.2}$$

$$n_+^d : \lambda^d=\beta\frac{\partial B\left(n_+^c,n_+^d\right)}{\partial n_+^d}=\beta\left[-\rho^d+\left(1-\delta^d\right)\lambda^d+\lambda^i\right] \tag{C.3}$$

Equations (C.2) and (C.3) give two expressions for λ^i

$$\lambda^i=\rho^c-\left(r+\delta^c\right)\lambda^c \tag{C.4}$$

and

$$\lambda^i = \rho^d + \left(r + \delta^d\right)\lambda^d \tag{C.5}$$

The equality between these two expressions for λ^i implies

$$\left(r + \delta^c\right)\lambda^c + \left(r + \delta^d\right)\lambda^d = \rho^c - \rho^d \tag{C.6}$$

Using the first order conditions (C.1) to get the expressions for λ^x, it becomes (15) for the matching functions (4).

D Bank's Surplus

The bank's surplus associated with the marginal credit and deposit are

$$\begin{aligned} & \left.\frac{\partial B\left(n^c, n^d\right)}{\partial n^c}\right|_{n^d=n^d(n^c)} \\ = \;& \rho^c - \rho^d n_1^d\left(n^c\right) + \lambda^c\left(1-\delta^c\right) + \lambda^d\left(1-\delta^d\right) n_1^d\left(n^c\right) - \lambda^i\left(1 - n_1^d\left(n^c\right)\right) \end{aligned} \tag{D.1}$$

and

$$\begin{aligned} & \left.\frac{\partial B\left(n^c, n^d\right)}{\partial n^d}\right|_{n^c=n^c\left(n^d\right)} \\ = \;& \rho^c n_1^c\left(n^d\right) - \rho^d + \lambda^c\left(1-\delta^c\right) n_1^c\left(n^d\right) + \lambda^d\left(1-\delta^d\right) - \lambda^i\left(n_1^c\left(n^d\right) - 1\right) \end{aligned} \tag{D.2}$$

Using the expressions for λ_i provided by (C.4) and (C.5), and the first order conditions (C.1), surplus are

$$\begin{aligned} & \left.\frac{\partial B\left(n^c, n^d\right)}{\partial n^c}\right|_{n^d=n^d(n^c)} \\ = \;& \rho^c - \rho^d + \left(1-\delta^c\right)\frac{\kappa^c}{q\left(\alpha^c\right)} + \left(1-\delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\right)} - \left[1 - n_1^d\left(n^c\right)\right]\left(1+r\right)\frac{\kappa^d}{q\left(\alpha^d\right)} \end{aligned} \tag{D.3}$$

and

$$\left.\frac{\partial B\left(n^c,n^d\right)}{\partial n^d}\right|_{n^c=n^c\left(n^d\right)} \tag{D.4}$$

$$= \rho^c - \rho^d + \left(1-\delta^c\right)\frac{\kappa^c}{q\left(\alpha^c\right)} + \left(1-\delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\right)} - \left[1 - n_1^c\left(n^d\right)\right]\left(1+r\right)\frac{\kappa^c}{q\left(\alpha^c\right)}$$

E Bank's Value Functions with and without Interbank Market

The program of the representative bank (14) is modified to introduce the interbank market. The position on the interbank market, denoted n^b, can be either positive (if it lends) or negative (if it borrows):

$$\begin{aligned} & B_{ib}\left(n^c,n^d\right) = \\ & \max_{n^b,\left\{n_+^x,v^x\right\}_{x=c,d}} \left\{\rho^c n^c + \rho^b n^b - \rho^d n^d - \kappa^d v^d - \kappa^c v^c + \beta B_{ib}\left(n_+^c, n_+^d\right)\right\} \\ & -\lambda^c\left[n_+^c - \left(1-\delta^c\right)n^c - q\left(\alpha^c\right)v^c\right] \\ & -\lambda^d\left[n_+^d - \left(1-\delta^d\right)n^d - q\left(\alpha^d\right)v^d\right] \\ & -\lambda^i\left(n^c + n^b - n^d\right) \end{aligned} \tag{E.1}$$

where ρ^b is the interbank market interest rate, which satisfies: $\lambda^i = \rho^b$. It is straightforward to check the consistency between the bank's surplus defined by (20) and those associated with (E.1). Indeed, the following relations hold

$$\Delta B_{ib}^x = \frac{\partial B_{ib}\left(n^c,n^d\right)}{\partial n^x} = \left.\frac{\partial B\left(n^c,n^d\right)}{\partial n^x}\right|_{n^y=n^y\left(n^x\right)}, \text{ for } n_1^y\left(n^x\right)=0$$

and for $x = \{c, d\}$, $y = \{c, d\}$, $y \neq x$. Without interbank market, the constraint $n^c = n^d$ is imposed in the definition of the value function (14), which becomes

$$\begin{aligned} B_{no}\left(n^c\right) &= \max_{n^c_+, \{v^x\}_{x=c,d}} \left\{\left(\rho^c - \rho^d\right) n^c - \kappa^d v^d - \kappa^c v^c + \beta B_{no}\left(n^c_+\right)\right\} \\ &\quad -\lambda^c \left[n^c_+ - \left(1-\delta^c\right) n^c - q\left(\alpha^c\right) v^c\right] \\ &\quad -\lambda^d \left[n^c_+ - \left(1-\delta^d\right) n^c - q\left(\alpha^d\right) v^d\right] \end{aligned} \tag{E.2}$$

It is straightforward to check the consistency between the bank's surplus defined by (20) and those associated with (E.2). Indeed, the following relation holds

$$\Delta B^c_{no} = \frac{\partial B_{no}\left(n^c\right)}{\partial n^c} = \left.\frac{\partial B\left(n^c, n^d\right)}{\partial n^c}\right|_{n^d = n^d(n^c)}, \text{ for } n^d_1\left(n^c\right) = 1 \tag{E.3}$$

and for $x = \{c, d\}$, $y = \{c, d\}$, $y \neq x$. Because $n^d = n^c$, the following equality holds

$$\Delta B^d_{no} = \frac{\partial B_{no}\left(n^c\right)}{\partial n^d} = \frac{\partial B_{no}\left(n^c\right)}{\partial n^c}\frac{\partial n^c}{\partial n^d} = \Delta B^c_{no} \tag{E.4}$$

The three value functions (14), (E.1), and (E.2) lead to different bank's surplus, but to equivalent optimality conditions with regard to the search efforts $n^c_+, \{v^x\}_{x=c,d}$ (for given values of interest rates).

F Nash Bargaining

For the value functions (8)-(9)-(10), and using the optimality condition on the deposit market (11), the household's surplus is

$$\begin{aligned} D^m - D^u &= \rho^d - \rho^h + \left(1-\delta^d\right)\beta\left[D^m\left(\rho^d\right) - D^u\right] \\ &= \rho^d - \rho^h + \left(1-\delta^d\right)\frac{\kappa^h}{p\left(\alpha^d\right)} \end{aligned} \tag{F.1}$$

For the value functions (12)-(13), and using the Nash solution (17), the entrepreneur's surplus is

$$\begin{aligned} L^m - L^u &= z - \rho^c + (1 - \delta^c - p(\alpha^c))\,\beta\,(L^m - L^u) \\ &= z - \rho^c + (1 - \delta^c - p(\alpha^c))\,\beta \frac{\eta^c}{1-\eta^c} \Delta B^c \end{aligned} \tag{F.2}$$

The surplus (D.4) and (F.1) are introduced in the Nash solution (16) to get

$$\begin{aligned} &\rho^d - \rho^h + \left(1 - \delta^d\right) \frac{\kappa^h}{p(\alpha^d)} \\ = &\frac{\eta^d}{1-\eta^d} \left\{ \rho^c - \rho^d + (1-\delta^c) \frac{\kappa^c}{q(\alpha^c)} + \left(1-\delta^d\right) \frac{\kappa^d}{q(\alpha^d)} - \left[1 - n_1^c\left(n^d\right)\right](1+r) \frac{\kappa^c}{q(\alpha^c)} \right\} \end{aligned} \tag{F.3}$$

Using the optimality condition (15), it becomes

$$\begin{aligned} &\rho^d - \rho^h + \left(1 - \delta^d\right) \frac{\kappa^h}{p(\alpha^d)} \\ = &\frac{\eta^d}{1-\eta^d} \left\{ (1+r) \frac{\kappa^c}{q(\alpha^c)} + (1+r) \frac{\kappa^d}{q(\alpha^d)} - \left[1 - n_1^c\left(n^d\right)\right](1+r) \frac{\kappa^c}{q(\alpha^c)} \right\} \end{aligned} \tag{F.4}$$

and

$$\rho^d = \rho^h - \left(1-\delta^d\right) \frac{\kappa^h}{p(\alpha^d)} + (1+r)\left(\frac{\eta^d}{1-\eta^d}\right)\left[\frac{\kappa^d}{q(\alpha^d)} + n_1^c\left(n^d\right)\frac{\kappa^c}{q(\alpha^c)}\right] \tag{F.5}$$

The surplus (F.2) is introduced into (17) to get

$$\rho^c = z - (r + \delta^c + p(\alpha^c)) \frac{\eta^c}{1-\eta^c} \beta \Delta B^c \tag{F.6}$$

For the bank's surplus (D.3), it becomes

$$\begin{aligned} \rho^c &= z - (r + \delta^c + p(\alpha^c)) \frac{\eta^c}{1-\eta^c} \beta \left[\rho^c - \rho^d + (1-\delta^c) \frac{\kappa^c}{q(\alpha^c)} + \left(1-\delta^d\right) \frac{\kappa^d}{q(\alpha^d)} \right] \\ &\quad + (r + \delta^c + p(\alpha^c)) \frac{\eta^c}{1-\eta^c} \left[1 - n_1^d(n^c)\right] \frac{\kappa^d}{q(\alpha^d)} \end{aligned} \tag{F.7}$$

Using the optimality condition (15), it becomes

$$\rho^c = z - (r + \delta^c + p(\alpha^c)) \left(\frac{\eta^c}{1-\eta^c}\right) \left[\frac{\kappa^c}{q(\alpha^c)} + n_1^d(n^c)\frac{\kappa^d}{q(\alpha^d)}\right] \tag{F.8}$$

or, equivalently

$$\rho^c = z - [1 + r + p(\alpha^c) - (1-\delta^c)] \left(\frac{\eta^c}{1-\eta^c}\right) \left[\frac{\kappa^c}{q(\alpha^c)} + n_1^d(n^c)\frac{\kappa^d}{q(\alpha^d)}\right] \tag{F.9}$$

G Equilibrium

The competitive market tightness variables are $\{\alpha_b^x, \rho_b^x\}_{x=c,d}$. Introducing the expression of ρ_b^d given by (F.5) into (11) gives

$$\frac{\kappa^h}{p\left(\alpha_b^d\right)} = \left(\frac{\eta^d}{1-\eta^d}\right) \left[\frac{\kappa^d}{q\left(\alpha_b^d\right)} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{q\left(\alpha_b^c\right)}\right] \tag{G.1}$$

which corresponds to (23) for the specification (4) of the matching functions. For the expressions of ρ_b^d given by (F.5) and of ρ_b^c given by (F.9), the bank's net interest margin is

$$\begin{aligned} \rho_b^c - \rho_b^d \quad = \quad & z - \rho^h - (r + \delta^c + p(\alpha_b^c)) \left(\frac{\eta^c}{1-\eta^c}\right) \left[\frac{\kappa^c}{q(\alpha_b^c)} + n_1^d(n_b^c)\frac{\kappa^d}{q\left(\alpha_b^d\right)}\right] \\ & + \left(1-\delta^d\right)\frac{\kappa^h}{p\left(\alpha_b^d\right)} - (1+r)\left(\frac{\eta^d}{1-\eta^d}\right) \left[\frac{\kappa^d}{q\left(\alpha_b^d\right)} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_b^c)}\right] \end{aligned} \tag{G.2}$$

Using (G.1), (G.2) becomes

$$\begin{aligned} \rho_b^c - \rho_b^d \quad = \quad & z - \rho^h - (r + \delta^c + p(\alpha_b^c)) \left(\frac{\eta^c}{1-\eta^c}\right) \left[\frac{\kappa^c}{q(\alpha_b^c)} + n_1^d(n_b^c)\frac{\kappa^d}{q\left(\alpha_b^d\right)}\right] \\ & - \left(r+\delta^d\right)\left(\frac{\eta^d}{1-\eta^d}\right) \left[\frac{\kappa^d}{q\left(\alpha_b^d\right)} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_b^c)}\right] \end{aligned} \tag{G.3}$$

The expression of the net interest margin given by (G.3) is therefore introduced into (15) to get

$$
\begin{aligned}
& (r+\delta^c)\frac{\kappa^c}{q(\alpha_b^c)}+\left(r+\delta^d\right)\left(\frac{1-\eta^c}{1-\eta^d}\right)\frac{\kappa^d}{q\left(\alpha_b^d\right)} \qquad \text{(G.4)}\\
= \; & (1-\eta^c)\left(z-\rho^h\right)-\eta^c\kappa^c\alpha_b^c-(r+\delta^c+p(\alpha_b^c))\,\eta^c n_1^d(n_b^c)\frac{\kappa^d}{q\left(\alpha_b^d\right)}\\
& -\left(r+\delta^d\right)(1-\eta^c)\left(\frac{\eta^d}{1-\eta^d}\right)n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_b^c)}
\end{aligned}
$$

which corresponds to (24) for the specification (4) of the matching functions.

H Proof of Proposition 2

The competitive equilibrium is constrained-efficient for specific values for the bargaining powers. To obtain these values, the optimality condition (G.1) is expressed as follows

$$
\eta^d=\frac{\kappa^h}{\alpha_b^d\kappa^d}\left[1+n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_b^d\right)}{q(\alpha_b^c)}+\frac{\kappa^h}{\alpha_b^d\kappa^d}\right]^{-1} \qquad \text{(H.1)}
$$

Assuming $\alpha_b^x=\alpha_o^x$ and using the value for α_o^d given by (6), it gives (25). Then, the optimality condition (G.4) is expressed as follows

$$
\begin{aligned}
(r+\delta^c)\frac{\kappa^c}{q(\alpha_b^c)}+\left(r+\delta^d\right)\frac{\kappa^d}{q\left(\alpha_b^d\right)} \; = \; & (1-\eta^c)\left(z-\rho^h\right) \qquad \text{(H.2)}\\
& -\eta^c\kappa^c\alpha_b^c-\left(r+\delta^d\right)\left(\frac{\eta^d-\eta^c}{1-\eta^d}\right)\frac{\kappa^d}{q\left(\alpha_b^d\right)}\\
& -(r+\delta^c+p(\alpha_b^c))\,\eta^c n_1^d(n_b^c)\frac{\kappa^d}{q\left(\alpha_b^d\right)}\\
& -\left(r+\delta^d\right)(1-\eta^c)\left(\frac{\eta^d}{1-\eta^d}\right)n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_b^c)}
\end{aligned}
$$

where the LHS term is identical to that of (7) for $\alpha_b^x=\alpha_o^x$. Because (H.2) is linear with respect to η^c, it is possible to rearrange the terms to get the expression of η_o^c such that the RHS term of (H.2) is equal to the LHS term of (7). It is

$$\eta_o^c = (1-\varepsilon) \times \frac{z-\rho^h+\kappa^c\alpha_o^c-\left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left(\frac{\eta_o^d}{1-\varepsilon}\right)\left[\frac{\kappa^d}{q\left(\alpha_o^d\right)}+n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}\right]}{z-\rho^h+\kappa^c\alpha_o^c-\left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\kappa^d}{q\left(\alpha_o^d\right)}+\eta_o^d n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}\right]+\left(r+\delta^c+p\left(\alpha_o^c\right)\right)n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}} \tag{H.3}$$

I then introduce the expression of η_o^d given by (25) into (H.3) to get

$$\eta_o^c = (1-\varepsilon) \times \frac{z-\rho^h+\kappa^c\alpha_o^c-\left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)}+n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{1+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_b^d\right)}{q\left(\alpha_b^c\right)}}\right]}{z-\rho^h+\kappa^c\alpha_o^c-\left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\kappa^d}{q\left(\alpha_o^d\right)}+\frac{(1-\varepsilon)n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{1+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_b^d\right)}{q\left(\alpha_b^c\right)}}\right]+\left(r+\delta^c+p\left(\alpha_o^c\right)\right)n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}} \tag{H.4}$$

and after simplification

$$\eta_o^c = (1-\varepsilon) \times \frac{z-\rho^h+\kappa^c\alpha_o^c-\left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)}+n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{1+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q(\alpha_o^c)}}\right]}{z-\rho^h+\kappa^c\alpha_o^c-\left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)}+n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{1+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q(\alpha_o^c)}}\right]+\left(r+\delta^c+p\left(\alpha_o^c\right)\right)n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}} \tag{H.5}$$

and finally the solution (26), using once again the expression of η_o^d given by (25).

I Proof of Corollary 1

The reference situation is $n_1^c\left(n_b^d\right) = n_1^d\left(n_b^c\right) = 0$, banks do not have liquidity constraints. In this case, the equations (25) and (26) reduce to

$$\eta_o^x = 1-\varepsilon, \text{ for } x = \{c, d\}. \tag{I.1}$$

which is known as the Hosios (1990) condition for efficiency.

Credit constraint (and not deposit constraint) means that $n_1^d\left(n_b^c\right)=1$ and $n_1^c\left(n_b^d\right)=0$. The socially optimal values of bargaining power are $\eta_o^d=(1-\varepsilon)$, according to (25), and

$$\eta_o^c=(1-\varepsilon)\frac{z-\rho^h+\kappa^c\alpha_o^c-\left(r+\delta^d\right)\frac{\kappa^d}{\varepsilon q\left(\alpha_o^d\right)}}{z-\rho^h+\kappa^c\alpha_o^c-\left(r+\delta^d\right)\frac{\kappa^d}{\varepsilon q\left(\alpha_o^d\right)}+\left(r+\delta^c+p\left(\alpha_o^c\right)\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}}<(1-\varepsilon) \qquad (I.2)$$

according to (26). If η^c remains equal to $(1-\varepsilon)$, the deposit market tightness remains at its socially optimal value, $\alpha_b^d=\alpha_o^d$, but it is not the case for the credit market tightness α_b^c that solves

$$\begin{aligned}\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}+(1-\varepsilon)\kappa^c\alpha_b^c &= \varepsilon\left(z-\rho^h\right)-\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon} \\ &\quad -\left(r+\delta^c+\overline{m}^c\left(\alpha_b^c\right)^{\varepsilon}\right)(1-\varepsilon)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon}\end{aligned} \qquad (I.3)$$

where α_o^c solves (7), which can be rewritten as follows

$$\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_o^c\right)^{1-\varepsilon}+(1-\varepsilon)\kappa^c\alpha_o^c=\varepsilon\left(z-\rho^h\right)-\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon} \qquad (I.4)$$

The RHS terms of (I.3) and (I.4) are identical and increasing with the credit market tightness. Because, the LHS term of (I.3) is strictly lower than the LHS term of (I.4), the competitive credit market tightness is below its optimal value, $\alpha_b^c<\alpha_o^c$, and the rate of financing entrepreneurs is too low. The competitive equilibrium is still unique, if it exists, because the RHS term of (I.3) is increasing with α_b^c whereas the LHS term of (I.3) is decreasing (and the deposit market tightness α_o^d is constant).

Deposit constraint (and not credit constraint) means that $n_1^d\left(n_b^c\right)=0$ and $n_1^c\left(n_b^d\right)=1$. The socially optimal values of bargaining power are $\eta_o^c=(1-\varepsilon)$, according to (26), and

$$\eta_o^d=(1-\varepsilon)\left[1+\varepsilon\frac{\kappa^c}{\kappa^d}\frac{\overline{m}^d\left(\alpha_o^d\right)^{1-\varepsilon}}{\overline{m}^c\left(\alpha_o^c\right)^{1-\varepsilon}}\right]^{-1}<(1-\varepsilon) \qquad (I.5)$$

according to (25). If η^d remains equal to $(1-\varepsilon)$, the optimality condition (23) becomes

$$\frac{\kappa^h}{\overline{m}^d\left(\alpha_b^d\right)^{\varepsilon}} = \left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} + \frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}\right] \tag{I.6}$$

for the specification (4). Given the constraint $(\kappa^c/\overline{m}^c)\left(\alpha_b^c\right)^{1-\varepsilon} > 0$, the optimality condition (I.6) implies an upper-limit on α_b^d, which is the socially optimal value α_o^d

$$\alpha_b^d < \frac{\kappa^h}{\kappa^d}\frac{\varepsilon}{1-\varepsilon} = \alpha_o^d \tag{I.7}$$

Using (6) to get $\kappa^h = \alpha_o^d \kappa^d (1-\varepsilon)/\varepsilon$, (I.6) becomes

$$\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}\frac{\alpha_o^d}{\alpha_b^d} = \frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} + \frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} \tag{I.8}$$

The relative gap between optimal and competitive tightness variables is deduced for (I.8) as

$$\frac{\alpha_o^d - \alpha_b^d}{\alpha_b^d} = \frac{(\kappa^c/\overline{m}^c)\left(\alpha_b^c\right)^{1-\varepsilon}}{\left(\kappa^d/\overline{m}^d\right)\left(\alpha_b^d\right)^{1-\varepsilon}} > 0 \tag{I.9}$$

For $\eta^c = \eta^d = (1-\varepsilon)$, the optimality condition (24) becomes

$$\begin{aligned} & (r+\delta^c)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} + (1-\varepsilon)\kappa^c\alpha_b^c + \left(r+\delta^d\right)(1-\varepsilon)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} \\ = \; & \varepsilon\left(z-\rho^h\right) - \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} \end{aligned} \tag{I.10}$$

Using (I.4), (I.10) is rearranged as

$$\begin{aligned} & (r+\delta^c)\frac{\kappa^c}{\overline{m}^c}\left[\left(\alpha_b^c\right)^{1-\varepsilon} - \left(\alpha_o^c\right)^{1-\varepsilon}\right] + (1-\varepsilon)\kappa^c\left(\alpha_b^c - \alpha_o^c\right) \\ = \; & \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left[\left(\alpha_o^d\right)^{1-\varepsilon} - \left(\alpha_b^d\right)^{1-\varepsilon}\right] - \left(r+\delta^d\right)(1-\varepsilon)\frac{\kappa^c}{q\left(\alpha_b^c\right)} \end{aligned} \tag{I.11}$$

Excessive credit rationing occurs if $\alpha_b^c < \alpha_o^c$. Because the RHS term of (I.11) is strictly increasing with the ratio α_b^c/α_o^c and equal to zero for $\alpha_b^c = \alpha_o^c$, the case $\alpha_b^c < \alpha_o^c$ corresponds to a negative value for the LHS term of (I.11), that is

$$\frac{\kappa^d}{\overline{m}^d}\left[\left(\alpha_o^d\right)^{1-\varepsilon} - \left(\alpha_b^d\right)^{1-\varepsilon}\right] < (1-\varepsilon)\frac{\kappa^c}{q\left(\alpha_b^c\right)} \tag{I.12}$$

Excessive credit rationing occurs for a small gap between optimal and competitive tightness variables.

Deposit and credit constraints mean that $n_1^d(n_b^c) = 1$ and $n_1^c\left(n_b^d\right) = 1$. Efficiency requires $\eta_o^x < \varepsilon$ for $x = \{c, d\}$. For $\eta^c = \eta^d = (1-\varepsilon)$, the optimality condition (23) becomes (I.6) and the relation (I.9) holds, with $\alpha_o^d > \alpha_b^d$. The optimality condition (24) becomes

$$\begin{aligned} & (r+\delta^c)\frac{\kappa^c}{\overline{m}^c}(\alpha_b^c)^{1-\varepsilon} + (1-\varepsilon)\kappa^c\alpha_b^c + \left(r+\delta^d\right)(1-\varepsilon)\frac{\kappa^c}{\overline{m}^c}(\alpha_b^c)^{1-\varepsilon} \\ = \; & \varepsilon\left(z-\rho^h\right) - \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} - \left(r+\delta^c+\overline{m}^c(\alpha_b^c)^{\varepsilon}\right)(1-\varepsilon)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} \end{aligned} \tag{I.13}$$

Using (I.4), (I.13) is rearranged as

$$\begin{aligned} & (r+\delta^c)\frac{\kappa^c}{\overline{m}^c}\left[(\alpha_b^c)^{1-\varepsilon} - (\alpha_o^c)^{1-\varepsilon}\right] + (1-\varepsilon)\kappa^c(\alpha_b^c - \alpha_o^c) \\ = \; & \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left[\left(\alpha_o^d\right)^{1-\varepsilon} - \left(\alpha_b^d\right)^{1-\varepsilon}\right] - \left(r+\delta^d\right)(1-\varepsilon)\frac{\kappa^c}{\overline{m}^c}(\alpha_b^c)^{1-\varepsilon} \\ & - \left(r+\delta^c+\overline{m}^c(\alpha_b^c)^{\varepsilon}\right)(1-\varepsilon)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} \end{aligned} \tag{I.14}$$

Excessive credit rationing occurs if $\alpha_b^c < \alpha_o^c$. Because the RHS term of (I.14) is strictly increasing with the ratio α_b^c/α_o^c and equal to zero for $\alpha_b^c = \alpha_o^c$, the case $\alpha_b^c < \alpha_o^c$ corresponds to a negative value for the LHS term of (I.14), that is

$$\frac{\kappa^d}{\overline{m}^d}\left[\left(\alpha_o^d\right)^{1-\varepsilon} - \left(\alpha_b^d\right)^{1-\varepsilon}\right] < (1-\varepsilon)\left[\frac{\kappa^c}{\overline{m}^c}(\alpha_b^c)^{1-\varepsilon} + \frac{r+\delta^c+\overline{m}^c(\alpha_b^c)^{\varepsilon}}{r+\delta^d}\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}\right]$$

Excessive credit rationing occurs for a small gap between optimal and competitive tightness variables.

J Competitive Financial Intermediation with Posted Interest Rates (On-line Appendix)

The resolution of the search model of financial intermediation with price posting is inspired by that of Kaas and Kircher (2011) developed for the labor market with large firms.

J.1 Households

The value function associated with the non-participating state is unchanged and given by (8). The value function associated with the searching state is

$$D^u = \rho^h - \kappa^h + p\left(\alpha^d\left(\rho_+^d\right)\right)\beta D^m\left(\rho_+^d\right) + \left[1 - p\left(\alpha^d\left(\rho_+^d\right)\right)\right]\beta D^u \tag{J.1}$$

where $D^m\left(\rho_+^d\right)$ is the value function associated with the matched state (9) for the posted interest rate ρ_+^d. The free entry condition on the deposit market implies

$$D^h = D^u \tag{J.2}$$

or equivalently

$$\kappa^h = p\left(\alpha^d\left(\rho_+^d\right)\right)\beta\left[D^m\left(\rho_+^d\right) - D^u\right] \tag{J.3}$$

given (8) and (J.1). Using (J.1), (9), and (J.2), the matching surplus is

$$D^m\left(\rho_+^d\right) - D^u = \frac{\rho_+^d - \rho^h}{1 - \left(1 - \delta^d\right)\beta} \tag{J.4}$$

The free entry condition (J.3) becomes

$$\frac{\kappa^h}{p\left(\alpha^d\left(\rho^d\right)\right)} = \frac{\rho_+^d - \rho^h}{r + \delta^d} \tag{J.5}$$

for the surplus (J.4).

Assuming that two interest rates are posted on the deposit market $\left\{\rho_+^d, \overline{\rho}_+^d\right\}$, $\overline{\rho}_+^d$ being the equilibrium rate and ρ_+^d the rate posted by a bank that deviates from the equilibrium value. Households can search for a bank that offers the equilibrium rate or for the deviating bank. At the equilibrium, the search payoffs should be equal or, equivalently, the surplus (J.1) is the same for ρ_+^d and $\overline{\rho}_+^d$

$$\begin{aligned} & \rho^h - \kappa^h + p\left(\alpha^d\left(\rho_+^d\right)\right)\beta D^m\left(\rho_+^d\right) + \left[1 - p\left(\alpha^d\left(\rho_+^d\right)\right)\right]\beta D^u \\ = \; & \rho^h - \kappa^h + p\left(\alpha^d\left(\overline{\rho}_+^d\right)\right)\beta D^m\left(\overline{\rho}_+^d\right) + \left[1 - p\left(\alpha^d\left(\overline{\rho}_+^d\right)\right)\right]\beta D^u \end{aligned} \tag{J.6}$$

Simplifications give

$$p\left(\alpha^d\left(\rho_+^d\right)\right)\beta\left[D^m\left(\rho_+^d\right) - D^u\right] = p\left(\alpha^d\left(\overline{\rho}_+^d\right)\right)\beta\left[D^m\left(\overline{\rho}_+^d\right) - D^u\right] \tag{J.7}$$

The elasticity of the matching probability of banks with respect to posted interest rate is

$$\begin{aligned} \frac{\partial q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d}\frac{\rho_+^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)} &= \frac{\partial q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}\frac{p\left(\alpha^d\left(\rho_+^d\right)\right)}{q\left(\alpha^d\left(\rho_+^d\right)\right)} \\ &\quad \times \frac{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d}\frac{\rho_+^d}{p\left(\alpha^d\left(\rho_+^d\right)\right)} \end{aligned} \tag{J.8}$$

The first term in bracket of the LHS term of (J.8) is determined by the specification of the matching technology. For the specification (4), it is equal to

$$\frac{\partial q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}\frac{p\left(\alpha^d\left(\rho_+^d\right)\right)}{q\left(\alpha^d\left(\rho_+^d\right)\right)} = \frac{\frac{\partial q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial a^d}\frac{a^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)}}{\frac{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial a^d}\frac{a^d}{p\left(\alpha^d\left(\rho_+^d\right)\right)}} = -\left(\frac{1-\varepsilon}{\varepsilon}\right) < 0 \tag{J.9}$$

The second term in bracket of the LHS term of (J.8) is deduced as follows. First, the matching probability associated with the interest rate ρ_+^d consistent with (J.7) is

$$p\left(\alpha^d\left(\rho_+^d\right)\right) = p\left(\alpha^d\left(\overline{\rho}_+^d\right)\right)\frac{D^m\left(\overline{\rho}_+^d\right) - D^u}{D^m\left(\rho_+^d\right) - D^u} \tag{J.10}$$

which elasticity is

$$\frac{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d}\frac{\rho_+^d}{p\left(\alpha^d\left(\rho_+^d\right)\right)} = \frac{\rho_+^d}{D^m\left(\rho_+^d\right) - D^u}\left(-\frac{\partial D^m\left(\rho_+^d\right)}{\partial \rho_+^d}\right) \tag{J.11}$$

The partial derivative of the value function associated with the state matched, defined by

(9), is

$$\frac{\partial D^m\left(\rho_+^d\right)}{\partial \rho_+^d} = 1 + \left(1-\delta^d\right)\beta \frac{\partial D^m\left(\rho_+^d\right)}{\partial \rho_+^d} = \frac{1}{1-\left(1-\delta^d\right)\beta} \tag{J.12}$$

Elasticity (J.11) becomes with (J.4) and (J.12)

$$\frac{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d} \frac{\rho_+^d}{p\left(\alpha^d\left(\rho_+^d\right)\right)} = -\frac{\rho_+^d}{\rho_+^d - \rho^h} \tag{J.13}$$

Given (J.9) and (J.13), elasticity (J.8) is therefore equal to

$$\frac{\partial q\left(\alpha_+^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d} \frac{\rho_+^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)} = \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{\rho_+^d}{\rho_+^d - \rho^h}\right) > 0 \tag{J.14}$$

This expression will be necessary to determine the bank's pricing strategy. The sign of the partial derivative is positive: if the deviating bank increases its posted interest rates, more households search toward this bank. Therefore, the probability to find new depositors for this bank is higher even if its search effort (v^d) is constant. It can be used to get the following expression of posted deposit interest rate

$$\rho_+^d = \rho^h + \frac{q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial q\left(\alpha_+^d\left(\rho_+^d\right)\right)/\partial \rho_+^d}\left(\frac{1-\varepsilon}{\varepsilon}\right) \tag{J.15}$$

J.2 Entrepreneurs

The value function associated with the matched state is defined by (13). The value function associated with the searching state is

$$L^u = p\left(\alpha^c\left(\rho_+^c\right)\right)\beta L^m\left(\rho_+^c\right) + \left(1 - p\left(\alpha^c\left(\rho_+^c\right)\right)\right)\beta L^u \tag{J.16}$$

Assuming that two interest rates are posted on the market $\left\{\rho_+^c, \overline{\rho}_+^c\right\}$, $\overline{\rho}_+^c$ being the equilibrium rate and ρ_+^c the rate posted by a bank that deviates from the equilibrium value. Being matched yields a surplus equal to

$$\begin{aligned} L^m\left(\rho_+^c\right) - L^u &= z - \rho_+^c + \left[1-\delta^c - p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\beta\left[L^m\left(\rho_+^c\right) - L^u\right] \\ &= \frac{z-\rho_+^c}{1-\left[1-\delta^c - p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\beta} \end{aligned} \tag{J.17}$$

Entrepreneurs can search for a bank that offers the equilibrium rate or for the deviating bank. At the equilibrium, the search payoffs should be equal or, equivalently, the surplus (J.17) is the same for ρ_+^c and $\overline{\rho}_+^c$

$$\begin{aligned} & p\left(\alpha^c\left(\rho_+^c\right)\right)\beta L^m\left(\rho_+^c\right)+\left(1-p\left(\alpha^c\left(\rho_+^c\right)\right)\right)\beta L_+^u && \text{(J.18)} \\ = \; & p\left(\alpha^c\left(\overline{\rho}_+^c\right)\right)\beta L^m\left(\overline{\rho}_+^c\right)+\left(1-p\left(\alpha^c\left(\overline{\rho}_+^c\right)\right)\right)\beta L_+^u && \text{(J.19)} \end{aligned}$$

Simplifications give

$$p\left(\alpha^c\left(\rho_+^c\right)\right)=p\left(\alpha^c\left(\overline{\rho}_+^c\right)\right)\frac{L^m\left(\overline{\rho}_+^c\right)-L^u}{L^m\left(\rho_+^c\right)-L^u} \tag{J.20}$$

The elasticity of the matching probability of banks with respect to posted interest rate is

$$\begin{aligned} \frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{q\left(\alpha^c\left(\rho_+^c\right)\right)} = & \frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}\frac{p\left(\alpha^c\left(\rho_+^c\right)\right)}{q\left(\alpha^c\left(\rho_+^c\right)\right)} && \text{(J.21)} \\ & \times\frac{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{p\left(\alpha^c\left(\rho_+^c\right)\right)} \end{aligned}$$

The first term in bracket of the LHS term of (J.21) is determined by the specification of the matching technology. For the specification (4), it is equal to

$$\frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}\frac{p\left(\alpha^c\left(\rho_+^c\right)\right)}{q\left(\alpha^c\left(\rho_+^c\right)\right)}=\frac{\frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \alpha^c}\frac{p\left(\alpha^c\left(\rho_+^c\right)\right)}{\alpha^c}}{\frac{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \alpha^c}\frac{q\left(\alpha^c\left(\rho_+^c\right)\right)}{\alpha^c}}=-\left(\frac{1-\varepsilon}{\varepsilon}\right) \tag{J.22}$$

The second term in bracket of the LHS term of (J.21) is deduced from the no-arbitrage condition (J.20) as follows

$$\frac{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{p\left(\alpha^c\left(\rho_+^c\right)\right)}=\frac{\rho_+^c}{L^m\left(\rho_+^c\right)-L^u}\left(\frac{-\partial L^m\left(\rho_+^c\right)}{\partial \rho_+^c}\right) \tag{J.23}$$

The partial derivative of the value function associated with the matched state, defined by (13), is

$$\frac{\partial L^m\left(\rho_+^c\right)}{\partial \rho_+^c}=-1+\left(1-\delta^c\right)\beta\frac{\partial L^m\left(\rho_+^c\right)}{\partial \rho_+^c}=\frac{-1}{1-\left(1-\delta^c\right)\beta} \tag{J.24}$$

With (J.17) and (J.24), the elasticity (J.23) becomes

$$\frac{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{p\left(\alpha^c\left(\rho_+^c\right)\right)}=-\frac{\rho_+^c}{z-\rho^c}\frac{1-\left[1-\delta^c-p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\beta}{1-\left(1-\delta^c\right)\beta} \tag{J.25}$$

Given (J.22) and (J.25), the elasticity (J.21) is therefore equal to

$$\frac{\partial q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)}{\partial \rho_{+}^{c}} \frac{\rho_{+}^{c}}{q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)}=-\left(\frac{1-\varepsilon}{\varepsilon}\right) \frac{1-\left[1-\delta^{c}-p\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)\right] \beta}{1-\left(1-\delta^{c}\right) \beta}\left(\frac{\rho_{+}^{c}}{z-\rho_{+}^{c}}\right)<0 \tag{J.26}$$

This expression will be necessary to determine the bank's pricing strategy. The sign of the partial derivative is negative: if a bank decreases its posted interest rates, more entrepreneurs search toward this deviating bank. Therefore, the probability to find new borrowers for this bank is higher even if its search effort (v^c) is constant. This last expression can be used to get the following expression of posted credit interest rate

$$\rho_{+}^{c}=z+\frac{q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)}{\partial q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right) / \partial \rho_{+}^{c}}\left(\frac{1-\varepsilon}{\varepsilon}\right) \frac{1-\left[1-\delta^{c}-p\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)\right] \beta}{1-\left(1-\delta^{c}\right) \beta} \tag{J.27}$$

J.3 Banks

The state variable ϱ_{+}^{x} for the market $x=\{c, d\}$ measures the amount of interests received or paid by the bank, which evolves as follows

$$\varrho_{+}^{x}=\left(1-\delta^{x}\right) \varrho^{x}+q\left(\rho_{+}^{x}\right) v^{x} \rho_{+}^{x}, \text { for } x=\{d, c\} \tag{J.28}$$

This variable is a state variable because the bank cannot revise interest rates posted in the past. The representative bank maximizes

$$\begin{aligned}
& P\left(n^{c}, n^{d}, \varrho^{c}, \varrho^{d}\right) \\
= & \max_{\left\{\varrho_{+}^{x}, n_{+}^{x}, v^{x}, \rho_{+}^{x}\right\}_{x=\{d, c\}}}\left\{\varrho^{c}-\varrho^{d}-\kappa^{d} v^{d}-\kappa^{c} v^{c}+\beta P\left(n_{+}^{c}, n_{+}^{d}, \varrho_{+}^{c}, \varrho_{+}^{d}\right)\right\} \\
& -\lambda^{c}\left[n_{+}^{c}-\left(1-\delta^{c}\right) n^{c}-q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right) v^{c}\right] \\
& -\lambda^{d}\left[n_{+}^{d}-\left(1-\delta^{d}\right) n^{d}-q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right) v^{d}\right] \\
& -\lambda^{i}\left(n^{c}-n^{d}\right) \\
& -\mu^{d}\left[\varrho_{+}^{d}-\left(1-\delta^{d}\right) \varrho^{d}-q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right) v^{d} \rho_{+}^{d}\right] \\
& -\mu^{c}\left[\varrho_{+}^{c}-\left(1-\delta^{c}\right) \varrho^{c}-q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right) v^{c} \rho_{+}^{c}\right]
\end{aligned} \tag{J.29}$$

The first order conditions of the program (J.29) are

$$n^c_+ : \lambda^c = \beta \frac{\partial P\left(n^c_+, n^d_+, \varrho^c_+, \varrho^d_+\right)}{\partial n^c_+} = \beta \left[\left(1-\delta^c\right)\lambda^c - \lambda^i\right] \tag{J.30}$$

$$n^d_+ : \lambda^d = \beta \frac{\partial P\left(n^c_+, n^d_+, \varrho^c_+, \varrho^d_+\right)}{\partial n^d_+} = \beta \left[\left(1-\delta^d\right)\lambda^d + \lambda^i\right] \tag{J.31}$$

$$\varrho^c_+ : \mu^c = \beta \frac{\partial P\left(n^c_+, n^d_+, \varrho^c_+, \varrho^d_+\right)}{\partial \varrho^c_+} = \beta \left[1 + \left(1-\delta^c\right)\mu^c_+\right] \tag{J.32}$$

$$\varrho^d_+ : \mu^d = \beta \frac{\partial P\left(n^c_+, n^d_+, \varrho^c_+, \varrho^d_+\right)}{\partial \varrho^d_+} = \beta \left[-1 + \left(1-\delta^d\right)\mu^d_+\right] \tag{J.33}$$

$$v^x : -\kappa^x + \lambda^x q\left(\alpha^x\left(\rho^x_+\right)\right) + \mu^x q\left(\alpha^x\left(\rho^x_+\right)\right)\rho^x_+ = 0,\ x = \{c, d\} \tag{J.34}$$

$$\rho^x_+ : \lambda^x q_1\left(\alpha^x\left(\rho^x_+\right)\right)v^x + \mu^x\left[\frac{\partial q\left(\alpha^x\left(\rho^x_+\right)\right)}{\partial \rho^x_+}\rho^x_+ + q\left(\alpha^x\left(\rho^x_+\right)\right)\right]v^x = 0,\ x = \{c, d\} \tag{J.35}$$

The price posting strategy is determined by (J.35), the first order condition of program associated with the posted interest rate ρ^x_+ on the market $x = \{d, c\}$ The first term account for the impact of ρ^x_+ on the creation of new financial relationship, which values are λ^x, according to $q_1\left(\alpha^x\left(\rho^x_+\right)\right)v^x$, namely the reaction of the matching probability times the search effort, v^x. The sign of $q_1\left(\alpha^x\left(\rho^x_+\right)\right)$ depends on the market x. The second term account for the impact of ρ^x_+ on the variation of the value function induced by the new amount of interests, which is equal to μ^x the derivative of the value function with respect to ϱ^d_+. Varying ρ^x_+ impacts directly the amount of interests for the flow of new customers, $q(\cdot)v^x$, and indirectly because of the variation in this flow, $q_1(\cdot)\rho^x_+ v^x$. The equilibrium value of the two multipliers are $\mu^c = 1/\left(r+\delta^c\right)$ and $\mu^d = -1/\left(r+\delta^d\right)$. The sign of μ^c is positive and that of μ^d negative, because credit interests increase the value function whereas deposit interests lower it. Variation in interests are discounted at the rate $\left(r+\delta^x\right)$ that takes into account the preference for the present and duration of the commitment for posted interest rates.

Lagrangian multiplier values $\{\mu^x\}_{x=c,d}$ are deduced from (J.32) and (J.33) as

$$\mu^c = \frac{1}{r+\delta^c}, \text{ and } \mu^d = -\frac{1}{r+\delta^d} \tag{J.36}$$

Lagrangian multiplier values $\{\lambda^x\}_{x=c,d}$ are deduced from (J.34) and (J.36)

$$\lambda^c = \frac{\kappa^c}{q\left(\alpha^c\left(\rho^c\right)\right)} - \frac{\rho_+^c}{r+\delta^c} \tag{J.37}$$

$$\lambda^d = \frac{\kappa^d}{q\left(\alpha^d\left(\rho^d\right)\right)} + \frac{\rho_+^d}{r+\delta^d} \tag{J.38}$$

Equation (J.30) and (J.31) give two expressions for λ^i

$$\lambda^i = \left(r+\delta^d\right)\lambda^d, \text{ and } \lambda^i = -\left(r+\delta^c\right)\lambda^c \tag{J.39}$$

The equality between these two expressions for λ^i given by (J.39) yields

$$\left(r+\delta^d\right)\lambda^d + \left(r+\delta^c\right)\lambda^c = 0 \tag{J.40}$$

and, for the values of λ^x given by (J.37) and (J.38), (J.40) becomes

$$\left(r+\delta^c\right)\frac{\kappa^c}{q\left(\alpha^c\left(\rho^c\right)\right)} + \left(r+\delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\left(\rho^d\right)\right)} = \rho_+^c - \rho_+^d \tag{J.41}$$

The interpretation of the RHS term of (J.41) is similar to that of (7) for the social planner, except that the representative bank considers that average search costs on market x, namely $\kappa^x / q\left(\alpha^x\left(\rho^x\right)\right)$, because it takes as given the market tightness contrary to the social planner. The LHS of (J.41) is the private return for financial intermediation, also known as the net interest margin.

Posted interest rate strategies satisfy (J.35), or equivalently

$$\rho_+^x : \frac{\lambda^x}{\mu^x} + \rho_+^x = -\frac{q\left(\alpha^x\left(\rho_+^x\right)\right)}{\partial q\left(\alpha^x\left(\rho_+^x\right)\right)/\partial\left(\rho_+^x\right)}, \; x = \{c,d\} \tag{J.42}$$

Using (J.34), (J.42) becomes

$$\rho_+^x : \frac{1}{\mu^x}\frac{\kappa^x}{q\left(\alpha^x\left(\rho_+^x\right)\right)} = -\frac{q\left(\alpha^x\left(\rho_+^x\right)\right)}{\partial q\left(\alpha^x\left(\rho_+^x\right)\right)/\partial\left(\rho_+^x\right)}, \; x = \{c,d\} \tag{J.43}$$

and, with (J.36), (J.43) is finally

$$\left(r+\delta^{c}\right)\frac{\kappa^{c}}{q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)}=-\frac{q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)}{\partial q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)/\partial\rho_{+}^{c}} \tag{J.44}$$

and

$$\left(r+\delta^{d}\right)\frac{\kappa^{d}}{q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)}=\frac{q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)}{\partial q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)/\partial\rho_{+}^{d}} \tag{J.45}$$

J.4 Equilibrium

Financial intermediation is constraint efficient when non-financial agents direct search and banks posted interest rates. The competitive economy is characterized by $\left\{\alpha_p^x,\rho_p^x\right\}_{x=c,d}$ with $\alpha_p^x=\alpha_o^x$ for $x=c,d$. Equations (J.27) and (J.44) give the equilibrium credit interest rate

$$\rho_p^c=z-\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_p^c\right)^{1-\varepsilon}+\kappa^c\alpha_p^c\right] \tag{J.46}$$

Equations (J.15) and (J.45) give the equilibrium deposit interest rate

$$\rho_p^d=\rho^h+\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_p^d\right)^{1-\varepsilon} \tag{J.47}$$

For the interest rate (J.47), the condition (J.5) determines the deposit market tightness

$$\alpha_p^d=\frac{\kappa^h}{\kappa^d}\left(\frac{\varepsilon}{1-\varepsilon}\right) \tag{J.48}$$

which is identical to the social planner solution, see (6). The net interest margin associated with (J.46) and (J.47) is

$$\begin{aligned}\rho_p^c-\rho_p^d \quad = \quad & z-\rho^h-\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_p^c\right)^{1-\varepsilon}+\kappa^c\alpha_p^c\right] \\ & -\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_p^d\right)^{1-\varepsilon}\end{aligned} \tag{J.49}$$

This expression is introduced into the condition (J.41) to get the credit market tightness

$$\begin{aligned} & \left(r+\delta^{d}\right)\frac{\kappa^{d}}{q\left(\alpha_{p}^{d}\right)}+\left(r+\delta^{c}\right)\frac{\kappa^{c}}{q\left(\alpha_{p}^{c}\right)} \\ = \quad & z-\rho^{h}-\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r+\delta^{c}\right)\frac{\kappa^{c}}{\overline{m}^{c}}\left(\alpha_{p}^{c}\right)^{1-\varepsilon}+\kappa^{c}\alpha_{p}^{c}\right]-\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r+\delta^{d}\right)\frac{\kappa^{d}}{\overline{m}^{d}}\left(\alpha_{p}^{d}\right)^{1-\varepsilon} \end{aligned} \tag{J.50}$$

or equivalently

$$\left(r+\delta^{c}\right)\frac{\kappa^{c}}{\overline{m}^{c}}\left(\alpha_{p}^{c}\right)^{1-\varepsilon}+\left(r+\delta^{d}\right)\frac{\kappa^{d}}{\overline{m}^{d}}\left(\alpha_{p}^{d}\right)^{1-\varepsilon}=\varepsilon\left(z-\rho^{h}\right)-\left(1-\varepsilon\right)\kappa^{c}\alpha_{p}^{c} \tag{J.51}$$

which is identical to the social planner solution, see (7).

K Notations (Not intended for publication)

- General notations

y^x is the parameter or variable y on the x market where $x = \{c, d\}$ denotes the credit market (c) and the deposit market (d)

y_+ is the tomorrow value of the state variable y

- Population

$\overline{n}^d > 0$: total population of households

$u^d > 0$: households that search for a bank on the deposit market (unmatched)

$n^d > 0$: households matched with a bank

$o^d > 0$: unmatched households that do not search (no-participating)

$\overline{n}^c > 0$: total population of entrepreneurs

$u^c > 0$: entrepreneurs that search for a bank on the credit market

$n^c > 0$: entrepreneurs matched with a bank

$[0, 1]$: size of the continuum of banks

- Costs

$\kappa^x > 0$: bank's search cost on the market $x = \{c, d\}$

$\kappa^h > 0$: household's search cost on the deposit market

- Production technologies

$\rho^h \geq 0$: domestic production of final good by one household

$z > 0$: production of final good by one entrepreneur

- Interest rates

$\beta = 1/(1+r) \in]0,1[$: the discount factor (identical for households, entrepreneurs, and banks)

$r = (1/\beta - 1) > 0$: interest rate to discount for time-preference

$\rho^x > 0$: interest rates on market $x = \{c, d\}$

$\rho^x_+ > 0$: posted interest rates on market $x = \{c, d\}$ for future periods

$\varrho^x > 0$: predetermined interests on market $x = \{c, d\}$ when interest rates are posted

- Additional parameters and variables

$\delta^x \in]0,1[$: exogenous bank exit rates of the non-financial agents on market $x = \{c, d\}$

$\eta^x \in]0,1[$: Nash bargaining power of non-financial agents on market $x = \{c, d\}$

λ^x : Lagrangian multipliers associated with the dynamic constraint on n^x for $x = \{c, d\}$

μ^x : Lagrangian multipliers associated with the specific dynamic constraint on ϱ^x for $x = \{c, d\}$

λ^i : Lagrangian multiplier associated with the bank's balance sheet constraint $n^c \leq n^d$

- Value functions

D^s : value function for households associated with the $s = \{h, u, m\}$ states, where h is for outside the banking sector, u for unmatched and searching for a bank, and m for matched with a bank

L^s : value function for entrepreneurs associated with the $s = \{u, m\}$ states, where u is for unmatched and searching for a bank, and m for matched with a bank

O : value function for the social planner

P : value function for the bank when interest rates are posted

B : value function for the bank when interest rates are bargained

ΔB^x : bank's surplus in the Nash bargaining program on market $x = \{c, d\}$

- The matching process.

$\overline{m}^x > 0$: scale parameter of the matching technology on market $x = \{c, d\}$

$\varepsilon \in]0, 1[$: elasticity parameter of the matching functions

$\alpha^x = v^x / u^x$: market tightness of the x market for $x = \{c, d\}$

$p(\alpha^x)$: matching probability of non-financial agents on the x market

$q(\alpha^x)$: matching probability of search effort for banks on the x market