Split Award Auctions for Supplier Retention

(Authors' names blinded for peer review)

It is common for procurement managers to frequently organize auctions among qualified suppliers to stay abreast of current supply-market pricing. The process of qualifying suppliers is expensive to the buyer; as a result, a buyer often maintains a pool of qualified suppliers, the supply base. In traditional auction models, the buyer uses the auction to find and award the business to the lowest-cost supplier in the supply base. In practice, however, sole awards can alienate losing suppliers and cause them to defect from the supply base. Therefore, to maintain the supply base — and thereby control supplier qualification costs — buyers often employ split awards. Hence, there is a trade-off between the effective purchasing cost on the one hand, and the qualification cost paid to maintain the supply base on the other. We model and investigate this trade-off and characterize (1) the optimal split award that minimizes long-run costs (purchasing and qualification) and (2) the optimal supply base size that the buyer should maintain. To characterize these results we assume that supplier's type (its relative per-unit cost) is i.i.d. across time, supplier's total cost is increasing in its type, supplier's total cost and buyer's virtual cost have single-crossing differences. We also determine that higher per-supplier qualification cost leads to a smaller supply base but does not necessarily increase the extent of multi-sourcing.

Key words: Auctions, Split awards, Multi-sourcing, Supply base.

1.   Introduction

In rapidly changing industries, frequent evolution of products and technologies makes it difficult for a procurement manager to discover the production costs of its suppliers. In such settings, buyers often deploy reverse auctions (competitive bidding) as a vehicle for pricing supply contracts (Beall et al. 2003). This is especially true in the procurement of commodity-type components in high-tech industries, where buyers regularly organize auctions to stay abreast of the market. For example, large electronics manufacturers often run quarterly auctions for procurement of commodity electronics components.

Procuring from low-cost suppliers is vital for most companies but it is equally important to procure from qualified suppliers because supply failures can be devastating for a firm. For example, petfood maker Menu Foods recalled in 2007 over 60 million packages of dog and cat food as a result of unauthorized, toxic chemical additives introduced by one of its suppliers (Myers 2007). To reduce the likelihood of such problems, buyers typically use stringent pre-qualifying procedures on

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suppliers to verify that the supplier is indeed capable of fulfilling the contract to the buyer's satisfaction. Indeed, it is a standard practice to allow only qualified suppliers to bid in auctions (Beall et al. 2003). Qualifying suppliers is a time-consuming and costly process, involving the collection of supplier information, factory audits, supplier development and evaluation. Furthermore, many elements of qualifying a supplier (e.g., product testing and face-to-face meetings with supplier engineers) typically fall on the technical workforce of the buyer firm. The buyer firm's engineers can be (and often are) reluctant to perform these tasks, viewing them as an unwelcome distraction from their primary responsibilities of engineering design. From the perspective of the procurement manager, this further introduces additional, albeit intangible, political costs in qualifying new suppliers.

To avoid the high costs associated with supplier qualification, a buyer establishes a group of suppliers, called the *supply base*, which it utilizes when awarding business. Thus the buyer has the possibility of inviting the pre-qualified suppliers from its supply base when organizing an auction, rather than spending time and money tracking down and qualifying new ones. For example, we interacted with an electronics firm in which a typical supply base for a commodity input consisted of around two to four suppliers.

However, once established, a supply base is not something that the buyer can necessarily take for granted. At the electronics manufacturer mentioned above, procurement managers noted that suppliers can disengage from the supply base. One of these procurement managers, who ran auctions for short-term contracts, pointed out that suppliers were shopping around for other customers and several were beginning to lock in contracts with these other customers. In particular, suppliers who go away from an auction empty-handed might become unavailable in the future because they have payrolls and bills to pay, and without business from the buyer must look elsewhere for other customers. The same manager recalled a supplier who used to supply the PC industry eventually, after winning little business, decided to switch gears and start focusing on customers in a different industry (defense contracting).

Thus, there exists a critical tension in the buyer's procurement strategy. On one hand, the buyer wants to procure from the cheapest supplier. This can be achieved by organizing a winner-take-all auction. On the other hand, this might alienate some suppliers from the supply base (those that did not win the auction). If the buyer wishes to maintain healthy competition in future auctions, this can saddle the buyer with a cost of qualifying new suppliers for these auctions. A trade-off hence exists between getting a low purchase price today and getting low costs of qualifying new suppliers for future auctions.

Splitting the business among multiple suppliers can help the buyer resolve this trade-off. By giving business to a supplier, the buyer increases the chance that the supplier will be available for

future competitive bidding events. It will be less likely, for example, to leave the supply base seeking greener pastures. Such supplier behavior can be captured by relating the likelihood of suppliers leaving the supply base to the amount of business awarded to them. Hence, splitting the auction volume among a few suppliers gives the buyer a lever to maintain the supply base. In fact, we spoke with some experienced procurement managers who often used split awards: they saw this approach as especially attractive in cases where keeping suppliers in the supply base was a central concern. For example, some auctions involved splits across up to four suppliers. However, since splitting the award involved purchasing from more expensive suppliers so as to retain them in the supply base, it was unclear when and how to best do this.

The objective of this paper is to provide a model capturing these relevant procurement issues, shedding light on how to optimally organize split-award auctions. For this purpose, we model a buyer that wants to minimize its long-run procurement cost, which includes the cost of purchasing in each auction, and the cost of qualifying suppliers in the supply base. To this end, our paper designs an optimal auction (mechanism) for awarding business that takes into account the future qualifying costs if suppliers defect from the supply base. Such an auction, as we show, generally results in *split awards* that multi-source among both low-cost and high-cost suppliers. We characterize how the extent of multi-sourcing depends on the cost of qualifying new suppliers and on the sensitivity of the likelihood of suppliers staying in the supply base as a function of the volume awarded. Our model is the first to analyze the use of split awards as a lever for managing supply base maintenance cost.

Choosing the supply base size is a critical decision for the buyer but it is not a priori obvious how to best make this decision. On one hand the buyer can cast a wider net by having more suppliers and hence make bidding more competitive, but on the other hand maintaining this larger pool of suppliers encourages splitting the award amongst more suppliers which effectively makes the bidding less competitive. We jointly solve the buyer's problem of deciding the number of suppliers it wants to qualify in the supply base and the amount of quantity it wants to allocate to each supplier, so as to minimize its long-run procurement cost. Our approach to solving the buyer's problem is to first design an optimal mechanism with a given supply base size, and then find the supply base size that minimizes the buyer's long-run procurement costs. We show that the buyer follows a simple and intuitive supplier qualification policy: in each period, it should qualify new suppliers into the supply base until a certain size is reached. In addition, we provide insights on how the optimal supply base size changes with the model parameters, as well as how the extent of multi-sourcing changes. Regarding the buyer's usage of multi-sourcing, even though split awards are used to mitigate the cost of qualifying new suppliers, increasing the qualification cost may actually reduce the buyer's usage of multi-sourcing: This is because the buyer may respond to

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higher qualification cost by strategically reducing the size of its supply base, thereby diminishing the opportunities to multi-source.

The outline of the paper is as follows. In §2 we review the literature. In §3 we describe the model. We then solve in §4 the buyer's mechanism design problem and find the optimal supply base size. We also discuss the sensitivity of the optimal mechanism and supply base size with respect to the model parameters and investigate the optimal mechanism and supply base composition with heterogenous supplier types. Finally we conclude the paper in §5. All proofs are included in the Appendix.

2. Literature Review

Our work is related to the auction literature, particularly works featuring multiple periods and others featuring qualification costs. It also contains some elements related to the supply risk literature.

In the auction design literature, Myerson's (1981) work was seminal in analyzing optimal auctions. We closely follow his approach in characterizing the incentives for agents (suppliers) to participate in the mechanism. However, in optimizing the principal's (buyer) objective we differ significantly from this body of work since our model analyzes optimal auctions in a multi-period setting in contrast to the single period auctions that have typically been investigated. The dynamics of multi-period auctions have seldom been explored. One exception is Klotz and Chatterjee (1995a), which consider a two-period model for defense systems procurement where suppliers face entry costs for bidding and exhibit production learning (cost reduction from one period to the next). They argue that splitting the contract award has a two-fold advantage: First, it increases market participation (in line with Klotz and Chatterjee 1995b) and second, it allows the buyer to maintain second-period cost symmetry amongst its suppliers. Elmaghraby and Oh (2004) also investigate procurement auctions in the presence of learning by doing. For a two-period model they perform a comparative analysis between sequential independent auctions in each period and an eroding price contract in which the buyer awards a multi-period contract where payment declines over time at a pre-specified rate. Both papers focus on production learning in a multi-period non-commodity procurement environment. We also provide a multi-period model, but focus instead on commodity procurement where suppliers know how to make the simple commodity (like printed circuit boards). In our model qualification cost and supply base maintenance are the key issues. Note that even for simple commodity-type items, like wheat gluten, qualification checks are critical, as evidenced by Menu Foods' experience mentioned in the Introduction.

Another paper that considers sourcing decisions in a dynamic setting is Held et al. (2008), who study a setting where awarding the business to a different sole-supplier in each period encourages

suppliers to participate in the auctions. The buyer pays the same amount regardless of who it awards the contract to, but a trade-off arises because the buyer incurs switching costs if the winning supplier is not the incumbent. In our paper the buyer's allocation decision also affects future supplier participation, but in a setting where the buyer uses split awards, the price depends on who is awarded the business, and the supplier return probabilities depend on the volume of business they won in the previous period.

The cost of qualifying suppliers has been considered in the procurement literature only for a single-period auction setting. Wan and Beil (2009), who study a buyer seeking to sole-source a new contract to the lowest-price qualified supplier, propose post-auction qualification screening to reduce the cost of qualifying losing suppliers. Unlike our paper, there is no multi-period aspect and thus no need to establish a base of qualified suppliers. In contrast to the buyer incurring the cost of including bidders (qualification cost), other papers have studied cases where instead bidders are the ones who incur cost to enter the auction. In such a setting, McAfee and McMillan (1987) find the equilibrium number of bidders who participate in the auction. Klotz and Chatterjee (1995b) and Seshadri et al. (1991) investigate the performance of procurement auction mechanisms when suppliers incur entry costs. They find that multi-sourcing outperforms single sourcing when suppliers face entry costs, while, if suppliers do not face any entry cost, then the buyer would rather single-source. In contrast, we find that multi-sourcing is optimal when the buyer incurs costs to recruit bidders, even if the suppliers do not face entry costs.

Finally, our paper is related to the broader procurement literature on multi-sourcing. Multi-sourcing has typically been analyzed in the procurement literature as a means to manage situations where any given supplier may fail to deliver the units they are assigned to produce. For example, Kleindorfer and Saad (2005), Tomlin (2006) and Federgruen and Yang (2009) discuss supply risk mitigation through multi-sourcing; Babich et al. (2007), Yang et al. (2011) and Chaturvedi and Martínez-de-Albéniz (2008) consider supply diversification to mitigate supply disruption risk, in a game-theoretic framework. In our setting, we also multi-source, but for an entirely different reason. In our paper, suppliers always fulfill their delivery obligations, but do not necessarily stay in the supply base over time. We multi-source not to ensure delivery, but rather to avoid the cost of qualifying new suppliers into the supply base.

3.   Model Description

We model a buyer that needs to purchase a fixed, divisible quantity Q of a homogenous product. It can buy this quantity from the *pool of suppliers* (the supply base) that the buyer has at its disposal. The buyer admits only qualified suppliers (those that have survived qualification screening) into its supply base. Finding a qualified supplier involves locating a potential supplier, screening its

qualifications (e.g., through product testing, site visits, audits, etc.), and repeating the process (if needed) until finding a supplier who survives this screening. Potential suppliers can be identified via numerous sources, e.g., supplier lists, industry contacts, receiving cold calls from suppliers, etc., but the qualifications still need to be ascertained through costly screening. We assume that the buyer can qualify suppliers into its supply base from a large enough group of suppliers, that is, the buyer does not run out of suppliers to qualify. The buyer's expected cost to find a qualified supplier is given by k. Intuitively, k's size is related to the amount of qualification screening; for example, a buyer might deploy extensive and costly qualification screening when buying a critical direct input (large k), but be satisfied with lighter, less-costly screening when buying a non-critical indirect good (low k). We assume that k does not change from period to period.

In addition, the buyer does not know the cost of any of the suppliers either before or after qualifying them, because we interpret the qualification process as a way to avoid contracting with incapable or unreliable suppliers, rather than a cost discovery attempt. We let c_i denote the per unit cost of supplier i, thus for a fraction of allocation q_i made to supplier i (the actual allocation made to supplier i will be Qq_i units), the production (or opportunity) cost of supplier i will be $c_i q_i Q$. Without loss of generality we normalize the quantity such that $Q \equiv 1$ in the remainder of this paper. c_i is supplier i's private information, and the buyer is only informed of its cumulative distribution function (c.d.f.) $F(c)$ and its probability density function (p.d.f.) $f(c)$. The cost distribution is assumed to have a finite mean. In addition, we assume that $\dfrac{F(c)}{f(c)}$ is non-decreasing (that is, the distribution is *regular*), which is a common assumption in the auction literature, satisfied by many distributions, e.g., uniform, exponential, normal, gamma, etc. Moreover, also consistent with the auction design literature, we assume that the costs of the suppliers are independent and identically distributed (i.i.d.).

3.1. Single-Period Decision

First we consider a single-shot procurement event that starts with the buyer having no suppliers in its supply base. The buyer wants to minimize its *procurement cost* which includes the *cost of qualifying suppliers* and the *cost of purchasing* from the suppliers. For a given number n of suppliers, the buyer can minimize its purchasing cost by organizing a second-price auction among all these suppliers: the lowest-cost supplier will be awarded the entire allocation (that is, if supplier i is the lowest-cost supplier then $q_i = 1$) and will be paid the cost of the second lowest-cost supplier, see Myerson (1981).[1] To minimize its procurement cost the buyer needs to find the number of suppliers it should qualify into its supply base. Let $\mathbf{c} = (c_1, \ldots, c_n)$ denote the vector of unit costs of

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[1] Because the buyer needs to purchase Q units, its optimal reserve price is always the upper bound of the cost distribution, meaning if $n \geq 2$ the optimal reserve price essentially plays no active role in the auction.

n suppliers. Then the single-period expected procurement cost for the buyer would be $\mathbb{E}_{\mathbf{c}} c_{(2,n)} + kn$, where $c_{(2,n)}$ is the second-lowest cost amongst the n suppliers (the second-lowest order statistic) if $n \geq 2$, and $c_{(2,n)}$ is equal to the upper limit of the support of the cost distribution (the reserve price) if $n = 1$. We can then characterize buyer's decision on the number of suppliers it needs to qualify as

$$\min_{n \geq 1} \left\langle \mathbb{E}_{\mathbf{c}} c_{(2,n)} + kn \right\rangle. \tag{1}$$

In fact, when solving the problem above, the buyer will qualify suppliers until the marginal benefit of including an extra supplier in the auction is less than the marginal cost of qualifying that supplier. The following lemma states that the problem in Equation (1) is indeed convex in n.

LEMMA 1. *For n i.i.d. unit cost random variables with finite mean and $\dfrac{F(c)}{f(c)}$ non-decreasing, the expectation of the second order statistic is decreasing and convex in the sample size n.*

Hence the buyer can now find the optimal number of suppliers that it needs to qualify for the second-price auction. For example, when the per-unit costs of suppliers are uniformly distributed in the interval $[0,1]$, then the expected purchasing cost is $\mathbb{E}_{\mathbf{c}} c_{(2,n)} = \dfrac{2}{n+1}$ and hence the optimal number of suppliers will be either $\left\lfloor \sqrt{\dfrac{2}{k}} - 1 \right\rfloor$ or $\left\lceil \sqrt{\dfrac{2}{k}} - 1 \right\rceil$, the floor and ceiling of $\sqrt{\dfrac{2}{k}} - 1$, respectively.

3.2. Multi-period Procurement Decisions with Supplier Availability

In fast-moving industries where products and technology evolve quickly (e.g., electronics), buyers often employ short-term supply contracts which are periodically re-allocated, e.g. quarterly. We model this as a multi-period, infinite horizon setting where the buyer purchases a fixed quantity $Q = 1$ of supply in each period and does not hold inventory across periods (due to rapid product obsolescence).[2] To stay abreast of rapidly changing production technologies and the current best pricing, in such settings many buyers use auctions to re-bid the business in each period (Beall et al. 2003). In each period, we assume that supplier i's unit cost is drawn from a distribution with c.d.f. F. This assumption implies that costs are independent across periods. In fact, what really matters is that F captures suppliers' *relative* costs, hence the model easily extends to cost correlations across periods when, for example, in period t supplier i's total cost equals a publicly observed common term c_t plus the private signal on relative costs c_i governed by F. The common term c_t drops out of the analysis (because only the relative costs matter) hence our results would be

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[2] The analysis in this and the succeeding section can easily incorporate a random demand, Q, faced by the buyer, provided that suppliers remain capable of meeting the buyer's full demand and the demand is realized before the auction but after the buyer qualifies suppliers into its supply base. However, if the demand realization occurs before the buyer qualifies suppliers into its supply base, then the supply base decision would depend on the realized demand.

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robust to c_t being period-dependent and correlated across periods. Our paper's structural results even extend if suppliers' costs are correlated across suppliers within a single period.

In our model suppliers' relative costs must be independent across periods, reflecting cases where technology and production capabilities/efficiencies of suppliers change rapidly and the buyer therefore wishes to use auctions to stay abreast of the current best market pricing. This is the case in the electronics industry, for example, where the buyer uses auctions to find which suppliers currently offer the best pricing. There may be industries where product life cycles are very long, new technology does not often get introduced, and suppliers' relative costs remain very sticky across periods. Our model would not apply to such cases, because when costs are very sticky across periods, there is no need to auction in each period, and the buyer is better off just giving a long-term contract to the lowest-cost supplier in the first period (since this supplier's cost advantage will be sustained in future periods). Accordingly, there is no need for a supply base and supply base maintenance in such settings.

In this dynamic context, the buyer's procurement decision depends on how the availability of suppliers evolves over time. If the suppliers, once qualified, remain in the supply base forever then the buyer would have to qualify the suppliers only at the beginning of the horizon and can organize a winner-take-all auction in every period. The number of suppliers it qualifies at the beginning can be characterized in a similar way to the optimization problem (1) by discounting the costs associated with future auctions by a discount factor $\beta < 1$. However, there exists a risk that some suppliers might drop out of the supply base due to some *exogenous factors* like leaving the industry after being acquired, which might render them unavailable for the buyer. Therefore, there exists a *risk of supplier availability* in the buyer's supply base. We denote supplier i's availability with a random variable $A_i = [0, 1]$ which follows a Bernoulli distribution with 1 indicating that supplier i remains in the supply base for the next period and 0 indicating supplier i leaves the supply base. We can therefore characterize the risk of supplier availability by the expectation of A_i which we call *supplier i's availability*, denoted by α_i. We consider A_i to be i.i.d. across all suppliers, i.e., $\alpha_i = \alpha$.

For an i.i.d. A_i, the buyer's procurement decision does not change significantly. It can still organize a winner-take-all (second-price) auction in each period, but would now have to qualify some suppliers in each period to compensate for the suppliers that dropped out of the supply base. For an infinite horizon, we solve the buyer's problem of finding the number of suppliers that it needs to qualify in each period. Let $\mathbf{A} = (A_1, \ldots, A_n)$ denote the vector of random variables A_i for n suppliers. The buyer's decision can then be characterized through Bellman's equation as follows:

$$J(n_a) = \min_{n \geq n_a} \left\langle \mathbb{E}_{\mathbf{c}} c_{(2,n)} + k(n - n_a) + \beta \mathbb{E}_{\mathbf{A}} J\left(\sum_{i=1}^n A_i\right) \right\rangle, \tag{2}$$

where $J(n_a)$ represents the buyer's cost-to-go when it starts a period with n_a suppliers available in its supply base. The following theorem characterizes the optimal number of suppliers that the buyer should qualify into its supply base.

THEOREM 1. *For A_i i.i.d., a qualify up-to policy is optimal, i.e., there exists n^* such that, for any n_a suppliers available to the buyer, it should further qualify $\max(0, n^* - n_a)$ additional suppliers. The qualify up-to level n^* is the solution of*

$$n^* = \min(n | C_\infty(n) - C_\infty(n+1) \le 0), \tag{3}$$

where

$$C_\infty(n) = \mathbb{E}_\mathbf{c} c_{(2,n)} + k(1-\beta)n + k\beta n\bar{\alpha} \tag{4}$$

and where $\bar{\alpha} = 1 - \alpha$.

From Equation (3) we can directly infer that the optimal supply base size decreases with the qualification cost k (because $\dfrac{d(C_\infty(n) - C_\infty(n+1))}{dk} \le 0$ for all n). Indeed, for a qualification cost of 0, the buyer would want to keep an infinite supply base size and, conversely, for a very high qualification cost the buyer would want to keep a small supply base. Also from Equation (3), we can infer that the supply base size is increasing in the discount factor β, that is, the more the buyer discounts its future costs (lesser the β) the smaller the supply base it would keep. Indeed, if the buyer completely discounts its future cost ($\beta = 0$) then it would carry the smallest supply base since there is a limited advantage of carrying an extra supplier. In fact for $\beta = 0$ the buyer's problem is similar to the single-period problem in §3.1. Finally, the supply base size increases as the availability of suppliers increases (α increases). Indeed, if the suppliers are always available and never leave the supply base then the buyer would want to carry a large supply base (since it does not have to worry about any future qualification costs). Conversely for suppliers that are never available in the next period, that is $\alpha = 0$, the buyer would carry the smallest supply base. In fact with $\alpha = 0$, the problem reduces to the buyer's single-period problem discussed in §3.1.

3.3. The Impact of Allocation on Supplier Future Availability

Besides the *exogenous* factors mentioned in §3.2, it is not uncommon that suppliers who lose in the bidding process decide not to participate in future auctions. Consider the example given in the Introduction: The supplier switches to another industry after winning little business in the buyer's supply base. Essentially, the supplier, who gets less business from the buyer, is saddled with excess capacity and therefore explores options outside of the buyer's supply base. The higher the excess capacity that the supplier has, the higher the likelihood that the supplier finds it beneficial to switch from the buyer's supply base to another supply base. We present here a model that captures
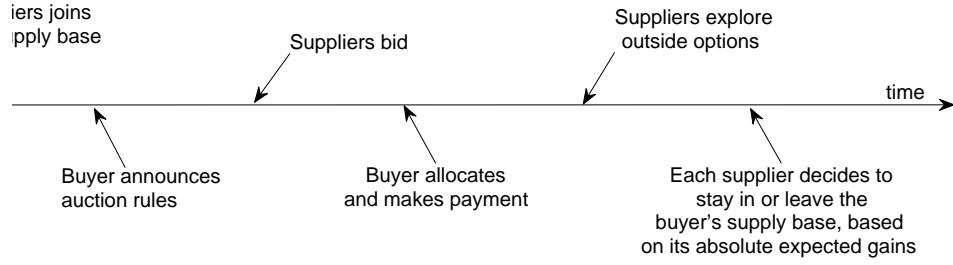
Figure 1 Timeline of events in a typical auction period.

the interaction between the buyer's present allocation to a supplier and its future availability in the buyer's supply base. This model yields a supplier total cost function and availability function.

Assume that a supplier i commits a capacity $W_i \geq 1$ for the buyer's supply base. Suppose that, in expectation, the other industry offers the supplier the same payoff for its capacity W_i (after it has left the buyer's supply base) as the supplier would get by being in the buyer's supply base. Hence, before the auction (ex-ante) the supplier has no incentive to switch over. However, after getting q_i amount of business from the buyer's auction in a given period, the supplier would explore how profitable it is to utilize its remaining capacity $W_i - q_i$ for that period by exploring other options, e.g., it can respond to exploratory RFQs from other potential customers. In doing so it formulates an estimate of the per-unit margin $p_{out,i} - \gamma c_i$ that it expects to get in that period if it re-tools its capacity for another customer. Parameter γ moderates the impact of the supplier's cost on the margin it can make outside the supply base; for example, the dependence is nonexistent if $\gamma = 0$, and negative if $\gamma > 0$. We assume that switching over to another supply base would cost the supplier a fixed re-tooling cost of T_i. (T_i can also incorporate a profit requirement below which switching is not considered worthwhile.) Therefore, the net expected benefit that the supplier can make in that period by switching over to another supply base is given by $(p_{out,i} - \gamma c_i) \cdot (W_i - q_i) - T_i$. In Figure 1, we graphically depict the various events in a typical auction period.

We assume that ex-ante (before the buyer's auction and before the supplier participates in exploratory RFQs) the supplier does not know $p_{out,i}$, but both the buyer and the supplier share the same prior over the distribution $F_{out,i}(p)$ of $P_{out,i}$. One can then characterize the likelihood of the supplier making a net benefit by switching over to another buyer's supply base as $\mathbb{P}((P_{out,i} - \gamma c_i) \cdot (W_i - q_i) - T_i \geq 0)$. It might happen that a supplier might not get qualified by the other buyer after it has re-tooled its capacity, in which case the supplier might look for an alternate option or try to re-enter the buyer's supply base (perhaps after re re-tooling its capacity). However, for model tractability we assume that the likelihood of a supplier not getting qualified is negligibly small.

Hence, the supplier's likelihood of leaving the buyer's supply base would be the same as its likelihood of making a positive profit by switching over to another buyer's supply base, i.e., $\bar{\alpha}_i(q_i, \gamma c_i) = \mathbb{P}((P_{out,i} - \gamma c_i) \cdot (W_i - q_i) - T_i \geq 0)$. Indeed, $\bar{\alpha}_i(q_i, \gamma c_i)$ is decreasing in the amount of business q_i that the supplier gets from the buyer. Moreover, we denote by $\Omega(q_i, \gamma c_i)$ the expected value of the maximum of 0 and the additional surplus that the supplier i makes by switching over to another supply base, i.e., $\Omega(q_i, \gamma c_i) = \mathbb{E}_{P_{out,i}} \max(0, (P_{out,i} - \gamma c_i) \cdot (W_i - q_i) - T_i)$. Hence $\Omega(q_i, \gamma c_i)$ represents the value of the supplier's outside option if it gets allocated q_i amount of business by the buyer. One can therefore express $\Omega(0, \gamma c_i) - \Omega(q_i, \gamma c_i)$ as the opportunity cost incurred by the supplier in producing q_i amount of goods for the buyer. We use $S_i(q_i, c_i) = c_i \cdot q_i + \Omega_i(0, \gamma c_i) - \Omega_i(q_i, \gamma c_i)$ to denote supplier i's total cost, which is the sum of its production and its opportunity cost.

Finally, we define $B(q, c) \equiv \dfrac{\partial S(q, c)}{\partial c} \cdot \dfrac{F(c)}{f(c)} + S(q, c) + k\beta\bar{\alpha}(q, \gamma c)$. We will see later that $B(q_i, c_i)$ is the virtual cost that the buyer incurs by giving an allocation q_i to a supplier with a per-unit cost of c_i (note that this virtual cost includes the expected cost to qualify a new supplier if the supplier leaves). We make three assumptions on the cost structure of the suppliers: (a) $S(q, c)$ is increasing in c; (b) $\dfrac{\partial^2 S(q, c)}{\partial q \partial c} \geq 0$ and (c) $\dfrac{\partial^2 B(q, c)}{\partial q \partial c} \geq 0$. Assumption (a) implies that a supplier's total cost is increasing in its type. Assumptions (b) and (c) imply that supplier's total cost and the buyer's virtual cost have single crossing differences, a widely used assumption in the mechanism design literature. The single crossing conditions enables the buyer to perfectly discriminate suppliers having different types and hence avoids bunching. Moreover, assumptions (a)-(c) are satisfied when, for example, $\gamma \leq 1$ and $P_{out,i}$ is exponentially distributed or uniformly distributed.

To keep the model parsimonious, we assume that $W_i = W$, $T_i = T$ for all i and $P_{out,i} = P_{out}$ is identically distributed though not necessarily independent across suppliers (in §4.6 we relax these assumptions). This allows us to model suppliers' availability A_i as being identically distributed with the probability of staying denoted by $\alpha(q_i, \gamma c_i)$. Note that A_i need not be independent, since P_{out} can be correlated across suppliers.

Thus far we have presented a model capturing the interaction between the buyer's present allocation to a supplier and the supplier's future availability in the buyer's supply base. The model yielded supplier total cost function $S(q, c)$ and availability function $\alpha(q, \gamma c)$. We conduct our theoretical analysis using a structure that relies solely on S and α (of course satisfying assumptions (a-c)) rather than the specifics of the underlying model. In other words, the above model can be viewed as a convenient expositional tool to quickly elucidate S and α (we will also use it for generating numerical experiments in later sections). However, one may fit other models of the supplier total cost $S(q, c)$ and availability $\alpha(q, \gamma c)$ for different situations, and — as long as assumptions (a-c) hold — still use our subsequent analysis.

With suppliers' availability being dependent on the buyer's allocation, the buyer faces potential depletion of its supply base if it does not provide enough business to all the suppliers. This implies that using a winner-take-all second-price auction may no longer be a good strategy, in contrast with §§3.1-3.2. In order to reduce the future cost of qualification, the buyer might want to multi-source in this situation. Indeed, a winner-take-all auction would result in the minimum cost of purchasing. However, it would also result in a high likelihood of depletion of the supply base and therefore a high cost of qualifying suppliers for the next period. On the other hand, an equal split of business amongst the suppliers would result in a low likelihood of depletion of the supply base (and therefore low cost of qualifying new suppliers for the next period) but would result in high purchasing costs. To balance this trade-off, the buyer needs to devise an allocation rule — that does not necessarily result in single sourcing — such that buyer's present and discounted future procurement cost (qualifying and purchasing cost) is minimized. Moreover, the buyer also needs to decide the optimal number of suppliers it wants to qualify in its supply base in each period. We jointly solve the buyer's problem of designing the procurement mechanism and finding the optimal number of suppliers it needs to qualify. For this purpose, we first formulate and solve the buyer's mechanism design problem with a given supply base size. We then find the optimal supply base size that will minimize the total cost for the buyer, under the optimal mechanism.

4. Controlling Qualification Costs through Split Awards

As pointed out above, the buyer might no longer want to organize a winner-take-all second-price auction, since that would result in high future qualification costs. In order to minimize its present and future procurement costs, the buyer needs to find an appropriate rule that maps supplier bids to their allocations and payments.

4.1. Optimal Mechanism Design

For finding such an optimal rule, we use the mechanism design methodology, see Myerson (1981). Note that because costs are independent over periods, we can focus on the mechanism design problem for each period separately. We consider only truth-revealing mechanisms which by the revelation principle include an optimal mechanism, if such an optimal mechanism exists. Specifically, we consider sealed-bid mechanisms in which the suppliers bid their true marginal costs $\mathbf{c} = (c_1, \ldots, c_n)$. A mechanism is described by the rule, announced by the buyer prior to the bidding, that maps suppliers' bids to their allocations and payments. Let such a mechanism be denoted by $\mathbf{q}(\mathbf{c}), \mathbf{z}(\mathbf{c})$ where $\mathbf{q}(\mathbf{c})$ is the vector of allocations made to suppliers and $\mathbf{z}(\mathbf{c})$ is the vector of payments made to suppliers, as functions of their bids $\mathbf{c}$. Because we focus on truth-revealing mechanisms, we require that the buyer designs the allocation and payment rule such that suppliers have the incentive to reveal their true costs.

For the mechanism to be optimal, it should minimize the buyer's present and future cost. We denote this cost by $J(n_a)$, where n_a is the number of suppliers available at the beginning of the period. The buyer's optimal cost-to-go can be represented by Bellman's equation as

$$J(n_a) = \min_{n \geq n_a} \left\langle \min_{\mathbf{z},\mathbf{q}|\sum_{i=1}^{n} q_i = 1} \mathbb{E}_{\mathbf{c}} \left\{ \sum_{i=1}^{n} z_i(\mathbf{c}) + k(n - n_a) + \beta \mathbb{E}_{\mathbf{A}} J \left(\sum_{i=1}^{n} A_i(q_i(\mathbf{c}), c_i) \right) \right\} \right\rangle \tag{5}$$

s.t. $(\mathbf{z}, \mathbf{q})$ are truth-revealing.

To find the optimal solution for Equation (5), we first find the optimal mechanism with a given supply base size and then find the optimal supply base size under the optimal mechanism. Given that the buyer has n suppliers in its supply base, the buyer's mechanism design problem can be written as

$$\min_{\mathbf{z},\mathbf{q}|\sum_{i=1}^{n} q_i = 1} \mathbb{E}_{\mathbf{c}} \left\{ \sum_{i=1}^{n} z_i(\mathbf{c}) + \beta \mathbb{E}_{\mathbf{A}} J \left(\sum_{i=1}^{n} A_i(q_i(\mathbf{c}), c_i) \right) \right\} \tag{6}$$

s.t. $(\mathbf{z}, \mathbf{q})$ are truth-revealing.

We next present the conditions that the mechanism needs to satisfy in order to induce truth revelation. As in the literature (see Myerson 1981), we assume that each supplier is risk-neutral; consistent with the model introduced in §3.3 the utility U_i of supplier i can be written as

$$U_i(\mathbf{c}) = z_i(\mathbf{c}) - S_i(q_i(\mathbf{c}), c_i) \tag{7}$$

Note that U_i is formulated as supplier's relative utility, which is the difference between supplier's utility $z_i(\mathbf{c}) - c_i q_i(\mathbf{c}) + \Omega(q_i(\mathbf{c}), c_i)$ if it participates in the auction and its utility $\Omega(0, c_i)$ if it does not participate in the auction.

For the mechanism $(\mathbf{z}, \mathbf{q})$ to be truth-revealing, it should satisfy Individual Rationality (IR) and Incentive Compatibility (IC) constraints.

(IR) $U_i(\mathbf{c}) \geq 0$ for all $\mathbf{c}$,
(IC) $U_i(\mathbf{c}) \geq z_i(\hat{c}_i, \mathbf{c_{-i}}) - S_i(q_i(\mathbf{c}_{-i}, \hat{c}_i), c_i)$ for all $c_i, \hat{c}_i, \mathbf{c_{-i}}$.

As one can see, we consider the IR and IC constraints in the dominant strategy equilibrium. In other words, a supplier truthfully reveals its cost irrespective of other suppliers' costs. Alternatively one could also formulate the IR and IC constraints in the Bayesian-Nash equilibrium, that is, a supplier's truthful revelation is its best strategy only in expectation over other suppliers' costs. Indeed, compared to the Bayesian-Nash equilibrium, a dominant strategy equilibrium is a more stringent condition on the mechanism design problem. However, using the results of Mookherjee and Reichelstein (1992) (specifically Proposition 3), one can show that for our problem an optimal mechanism in the dominant strategy equilibrium gives the same expected surplus to the agents and

the principal as would an optimal mechanism in the Bayesian-Nash equilibrium.[3]Therefore, there is no loss of optimality in assuming that suppliers' truthful revelation is a dominant strategy in equilibrium. Following the approach of Dasgupta and Spulber (1989), the following lemma provides a simpler characterization of truth-revealing mechanisms.

LEMMA 2. $(\mathbf{z}, \mathbf{q})$ *represents a truth-revealing mechanism if*

1. $U_i(\mathbf{c}) = U_i(\infty, \mathbf{c_{-i}}) + \int_{t=c_i}^{\infty} \left. \frac{\partial S_i(q_i(t, \mathbf{c_{-i}}), c)}{\partial c} \right|_{c=t} dt$;

2. $q_i(c_i, \mathbf{c}_{-i})$ *is decreasing in* c_i *for all* $c_i, \mathbf{c}_{-i}$;

3. $U_i(\mathbf{c}) = z_i(\mathbf{c}) - S_i(q_i(\mathbf{c}), c_i)$;

4. $U_i(\mathbf{c_{-i}}, \infty) \geq 0$.

Employing the usual methods, it can be shown that the (IC) condition also implies point (1) of Lemma 2. We now re-formulate the buyer's problem in Equation (6) by substituting the value of z_i from point (3) and the value of U_i from point (1) of Lemma 2 and taking $U_i(\mathbf{c_{-i}}, \infty) = 0$ (which does satisfy point (4) of Lemma 2). We can then express the buyer's problem as the optimization of the objective function with respect to the allocation $\mathbf{q}$ alone, subject to point (2) in Lemma 2. Given an optimal $\mathbf{q}$, the payment scheme $\mathbf{z}$ can then be found through point (3) of Lemma 2, which guarantees that the IC and IR constraints are satisfied. Taking this approach, the buyer's mechanism design problem can be written as

$$
\min_{\mathbf{q}| \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{q_i=1}} \mathbb{E}_{\mathbf{c}} \left\{ \sum_{i=1}^{n} \left\langle \int_{t=c_i}^{\infty} \left. \frac{\partial S_i(q_i(\mathbf{c_{-i}}, t), c)}{\partial c} \right|_{c=t} dt + S_i(q_i(\mathbf{c}), c_i) \right\rangle + \beta \mathbb{E}_{\mathbf{A}} J \left(\sum_{i=1}^{n} A_i(q_i(\mathbf{c}), c_i) \right) \right\}
\tag{8a}
$$

$$
\text{s.t. } (\mathbf{q}) \text{ satisifies condition (2) of Lemma 2.}
\tag{8b}
$$

To minimize the program in (8), we first relax the associated constraint (8b) (which we verify later). We then characterize the cost-to-go function $J(.)$ by reformulating Bellman's equation in Equation (5) as

$$
J(n_a) = \min_{n \geq n_a} \left\langle \min_{\mathbf{q}| \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{q_i=1}} \mathbb{E}_{\mathbf{c}} \left\{ \begin{array}{l} \sum_{i=1}^{n} \left\langle \int_{t=c_i}^{\infty} \left. \frac{\partial S_i(q(\mathbf{c_{-i}}, t), c)}{\partial c} \right|_{c=t} dt + S_i(q_i(\mathbf{c}), c_i) \right\rangle \\ + \beta \mathbb{E}_{\mathbf{A}} J \left(\sum_{i=1}^{n} A_i(q_i(\mathbf{c}), c_i) \right) + k(n - n_a) \end{array} \right\} \right\rangle.
\tag{9}
$$

This formulation allows us to establish that a stationary policy is optimal.

---

[3] Proposition 3 of Mookherjee and Reichelstein states that an allocation rule $\mathbf{q}$ would give the same expected utility to suppliers in both Bayesian-Nash equilibrium and dominant-strategy equilibrium as long as suppliers cost function satisfies weak single crossing property and the allocation function is decreasing in supplier's type. From assumption (b) we know that suppliers' cost satisfy the weak single crossing property and we will see in Theorem 2 that the allocation rule $\mathbf{q}$ is decreasing in supplier's type.

LEMMA 3. *If the buyer starts the horizon with* 0 *suppliers then a stationary policy in which it qualifies up-to* n^* *suppliers in each period is optimal.*

Hence, assuming that the buyer starts the process with an empty supply base (0 suppliers), we can apply the stationary policy to Equation (9) and rewrite it as

$$J(0) = \frac{1}{1-\beta} \cdot \min_n \left\langle \min_{\mathbf{q}| \sum_{i=1}^{n} q_i = 1} \mathbb{E}_{\mathbf{c}} \sum_{i=1}^{n} \left\{ \int_{t=c_i}^{\infty} \left. \frac{\partial S_i(q(\mathbf{c}_{-i}, t), c)}{\partial c} \right|_{c=t} dt + S_i(q_i(\mathbf{c}), c_i) + k\beta\bar{\alpha}(q_i(\mathbf{c}), \gamma c_i) \right\} + kn(1-\beta) \right\rangle.$$

Hence the buyer's problem can be characterized as

$$\min_n \left\langle \min_{\mathbf{q}| \sum_{i=1}^{n} q_i = 1} \mathbb{E}_{\mathbf{c}} \sum_{i=1}^{n} \left\{ \int_{t=c_i}^{\infty} \left. \frac{\partial S_i(q_i(\mathbf{c}_{-i}, t), c)}{\partial c} \right|_{c=t} dt + S_i(q_i(\mathbf{c}), c_i) + k\beta\bar{\alpha}(q_i(\mathbf{c}), \gamma c_i) \right\} + kn(1-\beta) \right\rangle. \quad (10)$$

Therefore, given an n (qualify up-to level), the buyer's mechanism design problem can now be written as

$$\min_{\mathbf{q}| \sum_{i=1}^{n} q_i = 1} \mathbb{E}_{\mathbf{c}} \sum_{i=1}^{n} \left\{ \int_{t=c_i}^{\infty} \left. \frac{\partial S_i(q_i(\mathbf{c}_{-i}, t), c)}{\partial c} \right|_{c=t} dt + S_i(q_i(\mathbf{c}), c_i) + k\beta\bar{\alpha}(q_i(\mathbf{c}), \gamma c_i) \right\} \quad (11)$$

which is equivalent to

$$\min_{\mathbf{q}| \sum_{i=1}^{n} q_i = 1} \mathbb{E}_{\mathbf{c}} \sum_{i=1}^{n} \left\{ \left. \frac{\partial S_i(q_i(\mathbf{c}), c)}{\partial c} \right|_{c=c_i} \cdot \frac{F(c_i)}{f(c_i)} + S_i(q_i(\mathbf{c}), c_i) + k\beta\bar{\alpha}(q_i(\mathbf{c}), \gamma c_i) \right\} \quad (12)$$

Now, we optimize Equation (12) over each sample realization $\mathbf{c}$, which reduces the mechanism problem to

$$\min_{\mathbf{q}| \sum_{i=1}^{n} q_i = 1} \sum_{i=1}^{n} B(q_i, c_i) \quad (13)$$

Note that for problem (13) to be equivalent to (8), we will need to verify the feasibility of this mechanism, which we do below, namely, that the resulting allocations q_i satisfy the constraints (8b).

THEOREM 2. *Given a qualify up-to level* n *in an infinite horizon problem,* $(\mathbf{z}, \mathbf{q})$ *represents an optimal mechanism in dominant strategy equilibrium if*

$$\mathbf{q}(\mathbf{c}) = \operatorname*{arg\,min}_{\mathbf{q} \ s.t. \sum q_i = 1} \sum_{i=1}^{n} B(q_i, c_i) \quad (14)$$

$$z_i(\mathbf{c}) = S_i(q_i(\mathbf{c}), c_i) + \int_{t=c_i}^{\infty} \left. \frac{\partial S_i(q(\mathbf{c}_{-i}, t), c)}{\partial c} \right|_{c=t} dt. \quad (15)$$

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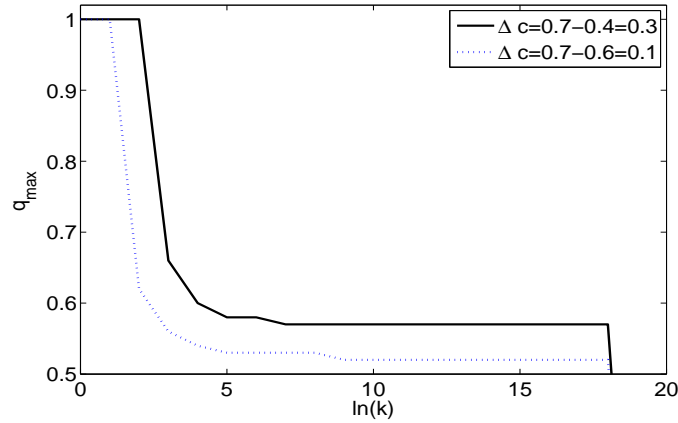Article submitted to *Management Science*; manuscript no. MS-10-01739.R1

Figure 2 **Optimal allocations with** $n = 2$, $\beta = 0.9$, $W = 1$, $T = 1$, $\gamma = 1$, P_{out} **is Exponentially distributed with mean** $\frac{1}{4}$ **and suppliers marginal cost realizations are taken from uniform distribution in the interval** $(0, 1)$.

The objective in Equation (14) is convex if $B(q, c)$ is convex in q.[4] In that case we can characterize the optimal allocations through first-order Karush-Kuhn-Tucker (KKT) conditions, that is,

$$\frac{\partial B(q_i, c_i)}{\partial q_i} = \lambda + \nu_i$$

$$\text{where } \nu_i \geq 0, q_i \geq 0, \nu_i \cdot q_i = 0 \text{ and } \lambda \text{ is such that } \sum_{i=1}^{n} q_i = 1 \text{ for } i = 1, \ldots, n \tag{16}$$

Optimal mechanism uses split awards

Note that $B(q, c)$ in the buyer's optimization program in Equation (14) is composed of two terms, namely $\frac{\partial S(q, c)}{\partial c} \cdot \frac{F(c)}{f(c)} + S(q, c)$ and $k\beta\bar{\alpha}(q, \gamma c)$. The former term is increasing in (q, c), however, the latter term, $k\beta\bar{\alpha}(q, \gamma c)$, is decreasing in q and not necessarily concave in allocation q and therefore can impose a penalty on winner-take-all allocations. Moreover, this penalty is increasing in the cost of qualifying suppliers, k. In Figure 2 we show the change in the division of allocations between the suppliers as k increases. Observe that the allocations even out (q_1 converges to 0.5) more slowly when the difference between the per-unit costs of suppliers is higher. Also, note that the buyer does not always multi-source. In fact when the relative magnitude of k is small compared to the cost of the suppliers, then the marginal savings in supply base maintenance cost achieved from multi-sourcing are less than the additional cost of purchasing from a more expensive supplier. This affect gets more pronounced as the difference between the cost of suppliers increases and therefore we see that the buyer single sources for a wider range of k as the difference in the cost of the suppliers increases.

---

[4] For example, this is true when P_{out} is exponentially distributed with mean $\frac{1}{\lambda}$ and when, for $\gamma = 1$, $T \geq \frac{2k\beta}{k\beta\lambda/W - 1}$.

Allocations depend on cost spread, not just cost ordering

Also note that the optimal mechanism in Theorem 2 is designed for a dominant-strategy equilibrium. This enables the mechanism to be easily implemented. For example, one can use a clock auction. In this setting, the auction begins at a high calling price and descends according to a price clock. At each price level, suppliers decide whether to stay in the auction or permanently drop out. After all suppliers have dropped out, the buyer applies the allocation rule in Equation (14) and the corresponding payment in Equation (15) according to the drop-out bids (which play the role of $\mathbf{c}$). In a traditional winner-take-all descending clock auction (reverse English auction), it is a dominant strategy for suppliers to drop out at their true cost, unless every other supplier has dropped out, at which point the lowest-cost supplier should also drop out. In contrast, with our proposed auction rules, each supplier — including the lowest-cost supplier — has the incentive to stay in the auction exactly until its true cost is reached. This is because the quantity and profits for a supplier not only depend on its rank in the auction (being the lowest-cost supplier or not), but also on the magnitude of the cost difference with other suppliers.[5]

4.2. Optimal Supply Base Size

Having solved the mechanism design problem, we now need to find the optimal qualify up-to level n^* up to which the buyer should qualify suppliers in every period, under the optimal mechanism. For this purpose, we need to solve the buyer's objective in Equation (10). Let

$$C_{buyer}(n) = \mathbb{E}_{\mathbf{c}} \min_{\mathbf{q}} \sum_{i=1}^{n} B(q_i(\mathbf{c}), c_i) + k(1 - \beta)n \qquad (17)$$

represent the value to be minimized in n from Equation (10). From Equation (10) and the definition of $C_{buyer}(n)$ in Equation (17) we can characterize the marginal effect of the $(n+1)^{th}$ supplier as $C_{buyer}(n) - C_{buyer}(n+1)$. In the Theorem below we show that this marginal effect is decreasing in the number of suppliers n.

THEOREM 3. *The marginal value of an extra supplier $C_{buyer}(n) - C_{buyer}(n+1)$ is decreasing in n and the optimal qualify up-to level n^* can be characterized as*

$$n^* = \min\left(n | C_{buyer}(n) - C_{buyer}(n+1) \leq 0\right). \qquad (18)$$

---

[5] As one would intuitively expect, when $k = 0$, the suggested implementation, although different from the traditional reverse English auction, would result in the same outcome, namely that the lowest-cost supplier receives the entire allocation, and its payment is equal to the second-lowest cost.

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4.3. Sensitivity of the Supply Base Size

In this section we discuss the sensitivity of the optimal supply base size, as characterized in Equation (18), with respect to the model parameters. Specifically, we discuss how the optimal supply base size changes with changes in these parameters. To characterize the effect of these parameters on the supply base size, we find the change in $C_\infty(n) - C_\infty(n+1)$ as the model parameters are changed. Indeed, if the marginal savings in the buyer's cost-to-go upon adding an extra supplier are increasing with the parameters, then the optimal size of the supply base increases.

LEMMA 4. *The optimal supply base size n^* is decreasing[6] in the qualification cost k.*

Indeed, a higher k would imply a higher qualification cost for the added supplier and therefore the marginal benefit of an additional supplier will decrease as k increases. Also, as one would expect $n^*(k=0) \geq n^*(P_{out}=0) \geq n^*(P_{out}=\infty) = n^*(k=\infty) = 1$.

In Figure 3 we numerically investigate the sensitivity of the supply base as one or more of the supply base parameters are changed. We typically find that the supply base size is increasing in cost of re-tooling capacity, T, and is decreasing in the mean value of the outside option P_{out}. Indeed, with higher re-tooling cost or lower mean of P_{out}, the suppliers are less likely to leave the supply base which reduces the supply base maintenance cost and hence increases the marginal value of an extra supplier. However, note that even when suppliers do not ever leave (e.g., P_{out} is zero or T is infinity), the buyer will not wish to form an infinitely large supply base, due to the initial qualification costs of forming the supply base.

4.4. Describing the Extent of Multi-Sourcing in the Optimal Mechanism

Finally, we would like to understand how often and to what extent the buyer diversifies its purchases across the supply base. For this purpose, we consider the allocation given to the lowest-cost supplier. Let $q_{max}(c_1, \ldots, c_n) = \max_i q_i(c_1, \ldots, c_n)$ denote the allocation to the cheapest supplier (who obtains the largest allocation).[7] Define the *concentration of allocation* as $\bar{q}_{max} = \mathbb{E}_{\mathbf{c}} q_{max}(c_1, \ldots, c_n)$. Similar to §4.3, we analyze the effect of changing the model parameters on the concentration of allocation.

Changing parameters not only affects the concentration of allocation *directly* but also *indirectly* through the change in supply base size that is brought about by changing these parameters. Moreover, the direct and the indirect effects need not move the concentration of allocation in the same direction, and therefore one might expect non-monotonicity in the overall effect when an optimal supply base size is maintained.

---

[6] In this paper, we use increasing and decreasing in the weak sense.

[7] One could employ other concentration measures, such as Herfindahl-Hirschman Index (HHI), $\sum_{i=1}^n q_i^2$. Comparatively, q_{max} is easier to analyze and is similar to HHI: both vary between $\frac{1}{n}$ and 1 and $q_{max} = \frac{1}{n}$ implies perfect diversification and $q_{max} = 1$ implies sole-sourcing.
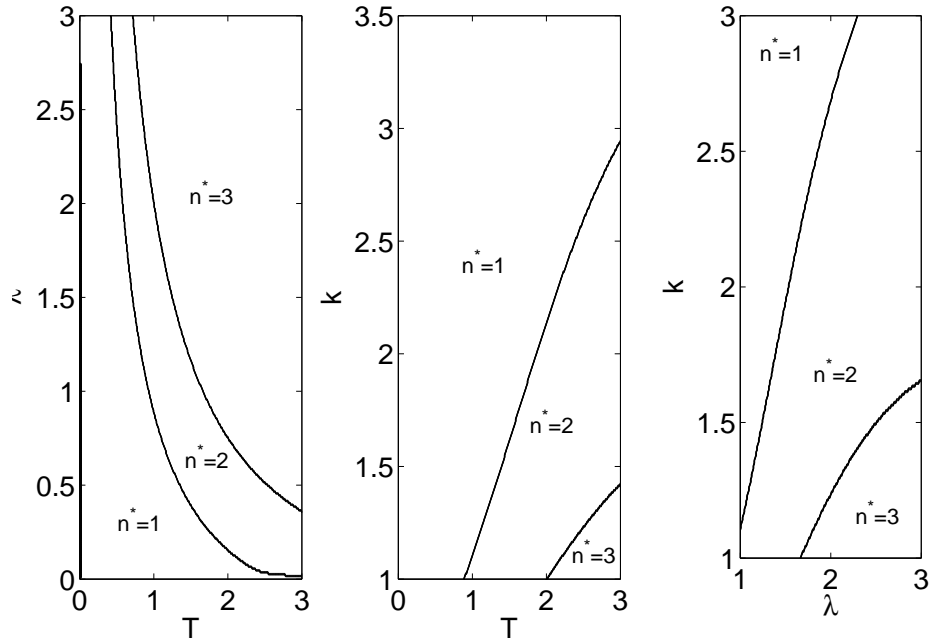
Figure 3   **Optimal supply base size plotted over the** (T, λ) **plane with** $k = 1$ **(left panel),** (T, k) **plane with** $\lambda = 1$
(middle panel), and (λ, k) **plane with** $T = 1$ **(right panel). In all panels, supplier costs are uniformly distributed**
over $[0, 1]$**,** P_{out} **is exponentially distributed with mean** $\frac{1}{\lambda}$**,** $\gamma = 1$**,** $W = 1$ **and** $\beta = 0.9$**.**

4.4.1.   Effect of Supply Base Size. First, we investigate the effect of supply base size on
the concentration of allocation.

LEMMA 5. *The concentration of allocation* $\bar{q}_{max}$ *is decreasing in the size of the supply base* n.

The intuition behind Lemma 5 is that, as the supply base size increases, the average gap between
the cost of two consecutive suppliers decreases and as a result the allocation too is less concentrated.
To summarize, all else being equal, buyers with a larger supply base will tend to move away from
winner-take-all allocations.

4.4.2.   Effect of Qualification Cost. Next, we consider the effect of changing the cost of
qualifying suppliers, k, on the concentration of allocation. We show that, as k becomes large, it
tends to a limit split award that depends on γ. Recall that γ moderates the impact of the supplier
cost on the margin it can make outside the supply base.

LEMMA 6. *For a fixed supply base size* n, *as the qualification cost* k *increases, the concentration*
$\bar{q}_{max}$ *tends to a value equal to* $\frac{1}{n}$ *when* $\gamma = 0$ *and* $\bar{\alpha}(q, 0)$ *is convex, and at least* $\frac{1}{n}$ *otherwise.*

The intuition behind Lemma 6 is that the buyer would want to spread its allocation to retain most
of its suppliers as k increases. It would do so in hopes of avoiding high costs of qualifying suppliers
in future periods. As a result, for large enough k, the buyer would split its business equally amongst
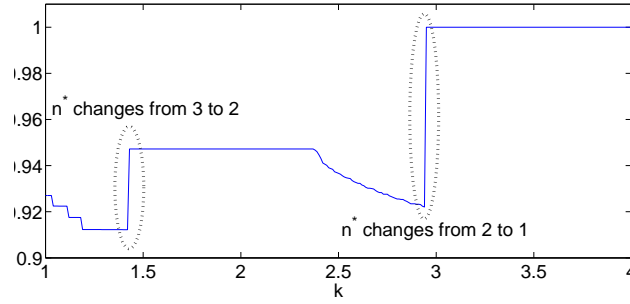
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Figure 4 **Effect of** k **on** $\bar{q}_{max}$**. Here** $\beta = 0.9$, $W = 1$, $T = 3$, $\gamma = 1$, P_{out} **is exponentially distributed with mean** 1 **and costs are uniformly distributed in** $[0, 1]$.

its suppliers. One can also show that the concentration of allocation is decreasing in k under certain conditions. [8] Interestingly, we saw in Lemma 4 that the supply base size decreases with k, which in conjunction with Lemma 5 would imply that the concentration of allocation should increase. However, the supply base size takes integer values. This implies that in between integer changes of supply base size, increasing k typically decreases the concentration of allocation; and at the value where a marginal increase in k decreases the supply base size, the concentration of allocation jumps up. We show this phenomenon for $\gamma > 0$ in Figure 4. Indeed, for large enough values of k the concentration of allocation converges to 1. To summarize, more expensive qualification pushes the buyer away from winner-take-all allocations, until qualification costs become so onerous that the buyer responds by shrinking its supply base. Hence there are two effects at play here: the direct effect typically reduces concentration as k increases, while the indirect effect reduces the supply base size which triggers an increase in the concentration of allocation. The combination of these two effects results in concentration of allocation being non-monotonic in k.

4.5. Split Award vs. Winner-Take-All Sourcing

Finally, we compare the performance of the optimal split-award sourcing against the myopic policy of using winner-take-all sourcing in each period. For this, we calculate the percentage change in the buyer's long-term procurement cost when it moves from using the winner-take-all sourcing to using the optimal split-award mechanism and maintaining an optimal supply base size as given by Equation (18). In Figure 5 we illustrate the resulting savings in the buyer's long-term procurement cost as cost of qualifying supplier is changed. We find that benefits of using split awards as compared to winner-take-all sourcing are typically decreasing with respect to the cost of qualifying suppliers. In fact, there is clearly an advantage to multi-sourcing when k is moderate, but when $k \to 0$ there is no advantage to multi-sourcing (an infinite supply base is optimal) and when $k \to \infty$ again there

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[8] When $S(q, c) + \dfrac{\partial S(q, c)}{\partial c} \cdot \dfrac{F(c)}{f(c)}$ is concave in q, is cross derivative with respect to q and c is positive and when $B(q, c)$ is convex in q.

Figure 5 **Percentage difference in buyer's cost when $\beta = 0.9$, $T = 3$, $W = 1$, $\gamma = 1$, P_{out} is exponentially distributed with mean 1 and costs are uniformly distributed in $[0, 1]$.**

is no advantage to multi-sourcing (a single-supplier supply base is optimal and hence sole-sourcing is de facto optimal).

4.6.   Supply Base Composition with Heterogenous Supplier Types

We explore here what recommendations can be made when supplier types are heterogeneous. For this purpose, we consider r type of suppliers and use subscript $j = 1, \ldots, r$ to depict the type of the supplier. For a supplier belonging to a type-j, its per unit cost of producing good is distributed according to c.d.f. F_j. They also have different availabilities equal to $\alpha_j(q, c)$. They also have different expected value of switching over profit equal to $\Omega_j(q, c)$. Finally, they can have different qualification costs k_j too.

We are interested in understanding how the buyer should design its supply base in this case, namely the number of each type of suppliers it qualifies for the supply base. Again, we focus on the infinite horizon setting, where the optimization problem can be written, similar to Equation (9) (by relaxing condition (2) of Lemma 2), as

$$
J(n_{1,a}, \ldots, n_{r,a}) = \min_{\substack{n_1 \geq n_{1,a} \\ \vdots \\ n_r \geq n_{r,a}}} \left\{ \min_{\substack{\mathbf{q_1}, \ldots, \mathbf{q_r} \\ s.t. \\ \sum_{j=1}^{r} \sum_{i=1}^{n_j} q_{j,i} = 1}} \mathbb{E}_{\substack{\mathbf{c_1} \\ \vdots \\ \mathbf{c_r}}} \left\{ \sum_{j=1}^{r} \sum_{i=1}^{n_j} \left\langle \int_{t=c_{j,i}}^{\infty} \frac{\partial S_{j,i}(q_{j,i}(\mathbf{c_{-i}}, t), c)}{\partial c} \bigg|_{c=t} dt + \right. \\ S_{j,i}(q_{j,i}(\mathbf{c}), c_{j,i}) \right\rangle + \\ \beta \mathbb{E}_{\mathbf{A_1}, \ldots \mathbf{A_r}} J\left(\sum_{i=1}^{n_1} A_1(q_{1,i}), \ldots \sum_{i=1}^{n_r} A_r(q_{r,i}) \right) \right\} \\ + \sum_{j=1}^{r} k_j(n_j - n_{j,a}) \right\}.
$$

$$(19)$$

LEMMA 7. *If the buyer starts the horizon with 0 suppliers, then a stationary policy in which it qualifies up-to n_j^* of type-j suppliers for $j = 1 \ldots r$ is optimal.*

Therefore, similar to Equation (13), the buyer's mechanism problem can be written as

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$$\min_{\substack{\mathbf{q_1},\dots,\mathbf{q_r} \\ s.t. \sum_{j=1}^{r}\sum_{i=1}^{n_j} q_{j,i}=1}} \mathbb{E}_{\substack{\mathbf{c_1} \\ \vdots \\ \mathbf{c_r}}} \left[\sum_{j=1}^{r}\sum_{i=1}^{n_j} B_{j,i}(q_{j,i}, c_{j,i}) \right].$$

Similar to Theorem 2, it can be verified that the above allocations indeed satisfy condition (2) of Lemma 2. The optimal mechanism can then be characterized by the corollary stated below.

COROLLARY 1. *In an infinite-horizon problem* $\mathbf{z}, \mathbf{q}$ *represents an optimal mechanism in dominant strategy if and only if it satisfies*

$$\mathbf{q}(\mathbf{c}) = \min_{\substack{\mathbf{q_1},\dots,\mathbf{q_r} \\ s.t. \sum_{j=1}^{r}\sum_{i=1}^{n_j} q_{j,i}=1}} \left[\sum_{j=1}^{r}\sum_{i=1}^{n_j} B_{j,i}(q_{j,i}, c_{j,i}) \right]. \tag{20}$$

and the payment z_i *made to each supplier is given by Equation* (15).

To solve the buyer's optimal supply base size problem we follow the approach of §4.2. Denote

$$C_{buyer}(n_1,\dots,n_r) = \mathbb{E}_{\substack{\mathbf{c_1} \\ \vdots \\ \mathbf{c_r}}} \min_{\substack{\mathbf{q_1},\dots,\mathbf{q_r} \\ s.t. \sum_{j=1}^{r}\sum_{i=1}^{n_j} q_{j,i}=1}} \left[\sum_{j=1}^{r}\sum_{i=1}^{n_j} B_{j,i}(q_{j,i}, c_{j,i}) \right] + \sum_{j=1}^{r} k_j(1-\beta)n_j. \tag{21}$$

Then the buyer's optimal supply base size decision can be characterized as $\min_{n_1,\dots,n_r} C_{buyer}(n_1,\dots,n_r)$. Let $n^*_{hetero,j}$ represent the optimal number of suppliers of each type in the supply base. Also denote n^*_j as the optimal number of suppliers of each type if the supply base consisted of only type j suppliers, i.e., n^*_j can be found from the solution of Equation (18). The following corollary applies as a direct extension of Theorem 3.

COROLLARY 2. $n^*_{hetero,j} \leq n^*_j$ *for all* $j = 1\dots r$.

Figure 6 illustrates the change in the supply base composition (the change in supply base size) when $r = 2$, as the cost of qualifying a given type (type-v) of suppliers is changed. We find that the buyer maintains a homogenous supply base (consisting of a single type of suppliers) if the attributes of a particular type of suppliers move to the extreme relative to the attributes of the other type of suppliers. Specifically, we find that the proportion of a given type of suppliers in the composition of the supply base is decreasing in their qualification cost.

5. Conclusion

In this paper, we analyzed a buyer's auction design problem when it needs to organize auctions repeatedly to keep abreast of the current best supply market pricing. To have suppliers bid in these auctions the buyer incurs a cost (of qualifying suppliers) and therefore maintains a supply base to avoid qualifying new suppliers for each auction. However, suppliers tend to fall out of this pool if they are not given adequate business and therefore the buyer faces a trade-off between
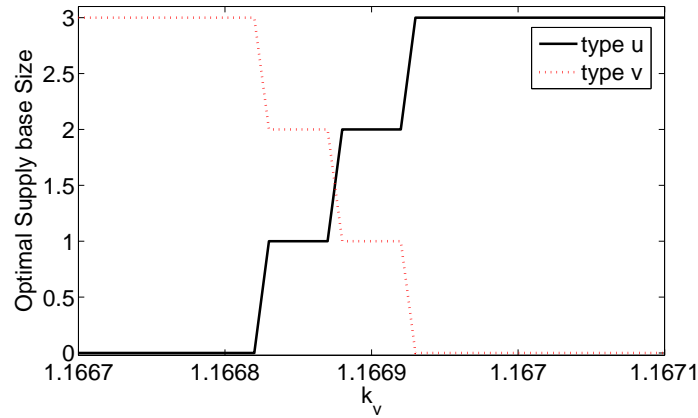
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Figure 6 **Optimal supply base size as a function of** k_v**, with** $\beta = 0.9$**,** $\gamma = 1$**,** $k_u = 1.1$**. Costs are uniformly distributed for type-**u **and type-**v **suppliers on the intervals** $[0, 1]$ **and** $[0.003, 1.003]$ **respectively.** $W = 1$ **and** $T = 3$ **for both type-**u **and type-**v **suppliers. Finally,** P_{out} **is exponentially distributed with mean** 1 **for type-**u **suppliers and with mean** $1/1.1$ **for type-**v**.**

giving adequate business to each supplier or being forced to qualify new suppliers in consequent periods. We analyze this trade-off and characterize the optimal split-award mechanism that the buyer should use in each period. We find that a buyer should typically multi-source in this scenario in order to minimize its procurement costs in the long run. We then characterize the optimal size of the supply base that the buyer should maintain over the long run.

Sensitivity analyses of the optimal supply base size with respect to qualifying cost reveals that the supply base size decreases with an increase in the cost to qualify suppliers. In addition, we evaluate how the extent of multi-sourcing under the optimal mechanism changes and we find that it increases with higher supply base size and is non-monotonic in qualification cost. Finally, we discuss the optimal mechanism and the supply base composition when suppliers are ex ante asymmetric.

This model could easily be extended to capture other aspects of the buyer's supply base maintenance costs, provided that the buyer's virtual cost function satisfies assumption (c). This could capture, for example, the buyer's costs associated with replacing poor-performing suppliers who did not get a large allocation and become inattentive to the buyer and too difficult to work with. This could also capture the buyer's costs of performing routine due diligence on suppliers who remain in the supply base; since due diligence is partly a replacement for recent familiarity with a supplier, suppliers with a smaller allocation would be less familiar and would require more re-qualification by the buyer before the next period.

Finally, it is worth mentioning some future research questions related to the model developed in this paper. An interesting extension of this work would be to analyze the optimal procurement policy when suppliers' have private information signals that are correlated across periods. The

analysis becomes quite difficult due to the complex dynamic nature of the resulting game. In fact, the problem falls in a notoriously challenging area in economics for which positive results have only been obtained in limited cases, e.g., Klotz and Chatterjee (1995a). This is certainly a challenging area for future work, but we suspect that some of the additional insights would be rather straightforward: for example, to the extent that individual suppliers' costs are more positively correlated across periods, the use of split awards would become less attractive for the buyer. Another possible extension to this work could include analyzing the optimal procurement policy when the suppliers' availability not only depends on their present allocation but also on their past allocations. In this case, one would expect that a supplier's willingness to stay in the supply base would continuously decrease over time unless it receives additional business. This would greatly complicate the buyer's optimal control problem by increasing the state space, rendering it quite analytically challenging. While this embellishment would be interesting and may include additional operational details, it will not change our paper's main idea and its main contribution, namely the use of split awards for supply base maintenance.

Acknowledgements

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Appendix

Proof of Lemma 1: Let $c_{j,n}$ denote the jth order statistic (of unit cost) among n samples. Then

$$\mathbb{E}_{\mathbf{c}}c_{2,n} = \int_{c=0}^{\infty}\mathbb{P}(c_{2,n} \geq c)dc = \int_{c=0}^{\infty}\mathbb{P}(c_{1,n} \geq c)dc + \int_{c=0}^{\infty}\mathbb{P}(c_{2,n} \geq c \geq c_{1,n})dc$$
$$= \int_{c=0}^{\infty}(1-F(c))^n dc + \int_{c=0}^{\infty}nF(c)(1-F(c))^{n-1}dc.$$

Performing integration by parts on the second integral term yields

$$\mathbb{E}_{\mathbf{c}}c_{2,n} = \int_{c=0}^{\infty}(1-F(c))^n dc + \int_{c=0}^{\infty}\frac{d}{dc}\left(\frac{F(c)}{f(c)}\right)(1-F(c))^n dc$$
$$= \int_{c=0}^{\infty}e^{n\ln(1-F(c))}dc + \int_{c=0}^{\infty}\frac{d}{dc}\left(\frac{F(c)}{f(c)}\right)e^{n\ln(1-F(c))}dc.$$

For $\frac{F(c)}{f(c)}$ non-decreasing in c, the above expression is indeed decreasing and convex in n. ∎

Proof of Theorem 1: For a T-period horizon, denote by $n_{a,t}$ (the state variable) the number of suppliers available at the beginning of period t and denote by n_t (the decision variable) the supply base size after the buyer has qualified the additional $n_t - n_{a,t}$ suppliers. Also, denote by $J_t(n_{a,t})$ the cost-to-go at the beginning of period t. Then, the DP algorithm can be written as

$$J_t(n_{a,t}) = \min_{n_t \geq n_{a,t}}\left\langle \mathbb{E}_{\mathbf{c}}c_{(2,n_t)} + k(n_t - n_{a,t}) + \mathbb{E}_{\mathbf{A}}J_{t+1}\left(\sum_{i=1}^{n_t}A_i\right)\right\rangle,$$

$$J_T(n_{a,T}) = \min_{n_T \geq n_{a,T}}\left\langle \mathbb{E}_{\mathbf{c}}c_{(2,n_T)} + k(n_T - n_{a,T})\right\rangle.$$

From Shaked and Shanthikumar (1988) we know that $\mathbb{E}_{\mathbf{A}}J_{t+1}\left(\sum_{i=1}^{n}A_i\right)$ is convex in n if $J_{t+1}(n)$ is convex in n and A_i for $i = 1, 2 \ldots n$ are independent non-negative random variables (one can find the argument by following Example 4.3, Proposition 3.7 and Definition 2.6 of their paper). Also from Lemma 1, we know that $\mathbb{E}_{\mathbf{c}}c_{(2,n)}$ is convex in n. Hence a unique n_T^* exists such that the buyer qualifies $\max(0, n_T^* - n_{a,T})$ suppliers in the last period. Thus, $J_T(n_{a,T})$ is convex in $n_{a,T}$ and therefore by induction it follows that a qualify up-to policy is optimal for each period.

For an infinite horizon we can express the optimal cost-to-go $J(n_a)$ in any period as $J(n_a) = \lim_{T\to\infty}J_t(n_a)$ for any t. Therefore $J(n_a)$ is convex in n_a and a unique n^* exists such that the buyer

qualifies $\max(0, n^* - n_a)$ suppliers. From the Bellman's Equation in Equation (2) the optimal qualify up-to level can then be found as $n^* = \arg\min_n \left\langle \frac{\mathbb{E}_{\mathbf{c}} c_{(2,n)} + k\beta \sum_{i=1}^{n} \bar{\alpha}}{1 - \beta} + kn \right\rangle$. This is equivalent to $n^* = \arg\min_n \left\langle \mathbb{E}_{\mathbf{c}} c_{(2,n)} + k\beta n\bar{\alpha} + kn(1 - \beta) \right\rangle$. ∎

Proof of Lemma 2: Let $\Xi_i(\hat{c}_i, c_i)$ represent the utility of supplier i that has a marginal cost c_i but reports $\hat{c}_i$, that is, $\Xi_i(\hat{c}_i, c_i) = z_i(\mathbf{c}_{-i}, \hat{c}_i) - S_i(q(\mathbf{c}_{-i}, \hat{c}_i), c_i)$. Using conditions (3) we know that $U_i(\mathbf{c}_{-i}, \hat{c}_i) = z_i(\mathbf{c}_{-i}, \hat{c}_i) - S_i(q(\mathbf{c}_{-i}, \hat{c}_i), \hat{c}_i)$. Hence from condition (1) we get $z_i(\mathbf{c}_{-i}, \hat{c}_i) = U_i(\mathbf{c}_{-i}, \infty) + \int_{t=\hat{c}_i}^{\infty} \left. \frac{\partial S_i(q(\mathbf{c}_{-i}, t), c)}{\partial c} \right|_{c=t} dt + S_i(q(\mathbf{c}_{-i}, \hat{c}_i), \hat{c}_i)$. Putting this value back in the expression of $\Xi_i(\hat{c}_i, c_i)$, we get $\Xi_i(\hat{c}_i, c_i) = U_i(\mathbf{c}_{-i}, \infty) + \int_{t=\hat{c}_i}^{\infty} \left. \frac{\partial S_i(q(\mathbf{c}_{-i}, t), c)}{\partial c} \right|_{c=t} dt + S_i(q(\mathbf{c}_{-i}, \hat{c}_i), \hat{c}_i) - S_i(q(\mathbf{c}_{-i}, \hat{c}_i), c_i)$. This can also be written as $\Xi_i(\hat{c}_i, c_i) = U_i(\mathbf{c}_{-i}, c_i) - \int_{t=c_i}^{\hat{c}_i} \left. \frac{\partial S_i(q(\mathbf{c}_{-i}, t), c)}{\partial c} \right|_{c=t} dt + \int_{t=c_i}^{\hat{c}_i} \left. \frac{\partial S_i(q(\mathbf{c}_{-i}, \hat{c}_i), c)}{\partial c} \right|_{c=t} dt$. Finally, from condition (2) and the assumption $\frac{\partial^2 S(q, c)}{\partial q \partial c} \geq 0$ we get that $\int_{t=c_i}^{\hat{c}_i} \left. \frac{\partial S_i(q_i(\mathbf{c}_{-i}, t), c)}{\partial c} \right|_{c=t} dw \geq \int_{t=c_i}^{\hat{c}_i} \left. \frac{\partial S_i(q_i(\mathbf{c}_{-i}, \hat{c}_i), c)}{\partial c} \right|_{c=t} dw$ for all $\mathbf{c}_{-i}, \hat{c}_i, c_i$ and hence $\Xi_i(\hat{c}_i, c_i) \leq U_i(\mathbf{c}_{-i}, c_i)$, which is precisely the (IC) condition. Conditions (1) and (4) imply the (IR) condition. ∎

Proof of Lemma 3: Define
$$S(n) = \min_{\mathbf{q}| \sum_{i=1}^{n} \mathbf{q_i} = 1} \mathbb{E}_{\mathbf{c}} \left\{ \sum_{i=1}^{n} \left\langle \int_{t=c_i}^{\infty} \left. \frac{\partial S_i(q(\mathbf{c}_{-i}, t), c)}{\partial c} \right|_{c=t} dt + S_i(q_i(\mathbf{c}), c_i) \right\rangle + \beta \mathbb{E}_{\mathbf{A}} J\left(\sum_{i=1}^{n} A_i(q_i(\mathbf{c}), c_i) \right) \right\}$$
The Bellman's equation in (9) can then be expressed as $J(n_a) = \min_{n \geq n_a}(S(n) + k(n - n_a))$. For any c_{n+1} and any $q_1, \ldots, q_n$ and $q_{n+1} = 0$ we get $J\left(\sum_{i=1}^{n} A_i(q_i, c_i) \right) \geq J\left(\sum_{i=1}^{n+1} A_i(q_i, c_i) \right)$ (since $\alpha(0, c_i) \geq 0$) and $\int_{t=c_{n+1}}^{\infty} \left. \frac{\partial C_{n+1}(0, c)}{\partial c} \right|_{c=t} dt + C_{n+1}(0, c_{n+1}) = 0$. Hence by optimality of $\mathbf{q}$, we get $S(n+1) \leq S(n)$. Since $S(n)$ is strictly non-negative, it implies that there must exist a finite n^* such that a policy of maintaining the supply base size at $\max(n^*, n_a)$ is optimal. In fact, starting the horizon with 0 suppliers, the buyer would initially qualify n^* suppliers, such that, $n^* = \arg\min_{n \geq 0}(S(n) + kn)$ and would qualify-upto n^* in all future periods. ∎

Proof of Theorem 2: For $\frac{\partial^2 B(q, c)}{\partial q \partial c} \geq 0$, point (2) of Lemma 2 gets satisfied if $q(\mathbf{c}_{-i}, c_i)$ is decreasing in c_i, which we show next. For $c_i^1 \geq c_i^0$ and any two sample paths of costs $(c_i^0, \mathbf{c}_{-i})$ and $(c_i^1, \mathbf{c}_{-i})$, let $q_1^0, \ldots, q_n^0$ and $q_1^1, \ldots, q_n^1$ denote the respective allocations obtained from the minimization program in Equation (14). Then by optimality of these allocations we get

$$\sum_{\substack{j \in n \\ j \neq i}} B(q_j^0, c_j) + B(q_i^0, c_i^0) \leq \sum_{\substack{j \in n \\ j \neq i}} B(q_j^1, c_j) + B(q_i^1, c_i^0) \tag{22}$$

and similarly

$$\sum_{\substack{j \in n \\ j \neq i}} B(q_j^1, c_j) + B(q_i^1, c_i^1) \leq \sum_{\substack{j \in n \\ j \neq i}} B(q_j^0, c_j) + B(q_i^0, c_i^1) \tag{23}$$

Adding Equations (22) and (23) gives us $B(q_i^0, c_i^0) + B(q_i^1, c_i^1) \leq B(q_i^1, c_i^0) + B(q_i^0, c_i^1)$. Which implies that $B(q_i^1, c_i^1) - B(q_i^1, c_i^0) \leq B(q_i^0, c_i^1) - B(q_i^0, c_i^0)$. For $\dfrac{\partial^2 B(q, c)}{\partial q \partial c} \geq 0$ and $c_i^1 \geq c_i^0$, this implies that $q_i^1 \leq q_i^0$. $\blacksquare$

Proof of Theorem 3: Define $\Xi_{buyer}(c_1, \ldots, c_n) = \min_{\mathbf{q}} \sum_{i=1}^{n} B(q_i(\mathbf{c}), c_i)$. We can then write $\Xi_{buyer}(c_1, \ldots, c_n, \infty) = \Xi_{buyer}(c_1, \ldots, c_n)$ (because $B(0, \infty) = 0$). Therefore

$$\Xi_{buyer}(c_1, \ldots, c_n, c_{n+1}) = - \int_{w=c_{n+1}}^{\infty} \frac{d\Xi_{buyer}(c_1, \ldots, c_n, w)}{dw} dw + \Xi_{buyer}(c_1, \ldots, c_n).$$

Using the envelope theorem, we get $\dfrac{d\Xi_{buyer}(c_1, \ldots, c_n, w)}{dw} = \dfrac{\partial B(q_{n+1}(\mathbf{c}, w), c)}{\partial c} \Big|_{c=w}$ and therefore

$$\Xi_{buyer}(c_1, \ldots, c_n, c_{n+1}) - \Xi_{buyer}(c_1, \ldots, c_n) = - \int_{w=c_{n+1}}^{\infty} \frac{\partial B(q_{n+1}(\mathbf{c}, w), c)}{\partial c} \Big|_{c=w}.$$

From Equation (17) we know that $C_{buyer}(n) = \mathbb{E}_{\mathbf{c_n}} \Xi(c_1, \ldots, c_n) + k(1 - \beta)n$, where $\mathbf{c_n}$ denotes an n dimensional vector of virtual costs. Therefore

$$C_{buyer}(n+1) - C_{buyer}(n) = - \mathbb{E}_{\mathbf{c_{n+1}}} \int_{w=c_{n+1}}^{\infty} \frac{\partial B(q_{n+1}(\mathbf{c_n}, w), c)}{\partial c} \Big|_{c=w} + k(1 - \beta).$$

For symmetric suppliers (having the same cost distribution), the above equation can equivalently be written as $C_{buyer}(n+1) - C_{buyer}(n) = - \mathbb{E}_{\mathbf{c_{n+1}}} \int_{w=c_1}^{\infty} \dfrac{\partial B(q_1(w, \mathbf{c_n}), c)}{\partial c} \Big|_{c=w} + k(1 - \beta)$. Similarly,

$$C_{buyer}(n+2) - C_{buyer}(n+1) = - \mathbb{E}_{\mathbf{c_{n+2}}} \int_{w=c_1}^{\infty} \frac{\partial B(q_1(w, \mathbf{c_{n+1}}), c)}{\partial c} \Big|_{c=w} + k(1 - \beta).$$

For $\dfrac{\partial^2 B(q, c)}{\partial q \partial c} \geq 0$, we get $q_1(w, \mathbf{c_n}) = q_1(w, \mathbf{c_n}, \infty) \geq q_1(w, \mathbf{c_n}, c_{n+1})$ for all $w, \mathbf{c_n}, c_{n+1}$. Hence $\dfrac{\partial B(q_1(w, \mathbf{c_n}), c)}{\partial c} \geq \dfrac{\partial B(q_1(w, \mathbf{c_{n+1}}), c)}{\partial c}$ for all $w, \mathbf{c_n}, c_{n+1}, c$. Therefore $C_{buyer}(n+2) - C_{buyer}(n+1) \geq C_{buyer}(n+1) - C_{buyer}(n)$. and hence $C_{buyer}(n) - C_{buyer}(n+1)$ is decreasing in n. $\blacksquare$

Proof of Lemma 4: Differentiating $C_{buyer}(n)$ w.r.t. k (and applying the envelope theorem) we get

$$\frac{dC_{buyer}(n)}{dk} = \mathbb{E}_{\mathbf{c_n}} \sum_{i=1}^{n} \bar{\alpha}(q_i(\mathbf{c_n}), c_i) + (1 - \beta)n.$$

where $q_i(\mathbf{c_n})$ represents the optimal allocation to supplier i when there are n suppliers in the supply base. Similarly

$$\frac{dC_{buyer}(n+1)}{dk} = \mathbb{E}_{\mathbf{c_{n+1}}} \sum_{i=1}^{n+1} \bar{\alpha}(\hat{q}_i(\mathbf{c_{n+1}}), c_i) + (1 - \beta)(n + 1).$$

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where $\hat{q}_i(\mathbf{c_{n+1}})$ represents the optimal allocation to supplier i when there are $n+1$ suppliers in the supply base. Taking the difference between the above two expressions we get

$$\frac{d(C_{buyer}(n) - C_{buyer}(n+1))}{dk} = \mathbb{E}_{\mathbf{c_{n+1}}} \left\langle \sum_{i=1}^{n} \left(\bar{\alpha}(q_i(\mathbf{c_n}), c_i) - \bar{\alpha}(\hat{q}_i(\mathbf{c_{n+1}}), c_i) \right) - \bar{\alpha}(\hat{q}_{n+1}(\mathbf{c_{n+1}}), c_{n+1}) \right\rangle - (1-\beta).$$

For any sample path, that is, for any $c_1, \ldots, c_{n+1}$ we get, from the proof of Theorem 3, $q_i \geq \hat{q}_i$, for all $i = 1, \ldots, n$. Since $\bar{\alpha}(q, c)$ is decreasing in q therefore $\frac{d(C_{buyer}(n) - C_{buyer}(n+1))}{dk} \leq 0$. ∎

Proof of Lemma 5: We order the marginal cost such that $c_1 \leq c_2 \leq \ldots \leq c_n$. Also we define the maximum allocation χ_{max} as a function of the differences between consecutive order statistics of marginal costs, $\mathbf{\Delta c} = \Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1} = c_2 - c_1, c_3 - c_2, \ldots, c_n - c_{n-1}$, that is $\chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu) = q_{max}(c_1, c_2, \ldots, c_n)$, where ν is the cost of a supplier (could be any supplier) such that one can infer $c_1, \ldots, c_n$ from $\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu$ (note that ν controls for change in allocation when the costs of each supplier is shifted). Indeed, ν is distributed according to the cost of the suppliers, i.e., with c.d.f. f and p.d.f. F.

The remainder of the proof is organized as follows. We first show that the probability distribution function of any Δc_i can be characterized by $h(w)$. We next show that the survival function of any Δc_i is decreasing in the sample size n (i.e. the ith order statistic with sample size n stochastically dominates in the first order the ith order statistic with sample size $n+1$). Finally we use this result to show that the expectation of χ_{max} is decreasing in n.

The probability density function of the difference between two consecutive order statistics $m+1$ and m of virtual cost, for sample size n, can be written as (see David and Nagaraja 2003)

$$h_{m,n}(w) = \int_{x=0}^{\infty} F^{m-1}(x)[1 - F(x+w)]^{n-m-1} f(x) f(x+w) dx.$$

Indeed, with n suppliers, we can express $\bar{q}_{max}(n) = \mathbb{E}_{\mathbf{\Delta c}, \nu} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu)$ and for $n+1$ suppliers $\bar{q}_{max}(n+1) = \mathbb{E}_{\mathbf{\Delta c}, \nu} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \Delta c_n, \nu)$.

Since adding an extra supplier will not increase the allocation of the cheapest supplier supplier (follows directly from the proof of Theorem 3). Thus, for any sample path $\Delta c_1, \ldots, \Delta c_{n-1}, \nu$, representing $\mathbb{E}_{\Delta c_{i,n}}$ as the expectation taken over the distribution of Δc_i when n suppliers are present, we can write

$$\chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu) \geq \mathbb{E}_{\Delta c_{n,n+1}} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \Delta c_n, \nu), \tag{24}$$

However, the density function of the difference between consecutive order statistics will be different for n and for $n+1$ suppliers, i.e., $h_{m,n}(w)$ will be different from $h_{m,n+1}(w)$. Let $\bar{H}_{m,n}(z)$ denote

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the survival function of the difference between consecutive order statistics of marginal cost for n suppliers. Therefore

$$
\bar{H}_{m,n}(z) = \int_{w \overline{\approx} z}^{\infty} h_{m,n}(w)dw = \int_{x=0}^{\infty} \int_{w \overline{=} z}^{\infty} F^{m-1}(x)[1-F(x+w)]^{n-m-1} f(x)f(x+w)dwdx
$$
$$
= \int_{x=0}^{\infty} F^{m-1}(x) \frac{[1-F(x+z)]^{\bar{n}-m}}{n-m} f(x)dx.
$$

Indeed, $\dfrac{[1-F(x+z)]^{n-m}}{n-m}$ is decreasing in n and therefore $\bar{H}_{m,n}(z) \geq \bar{H}_{m,n+1}(z)$ for all $1 \leq m \leq n-1$.

From Shaked and Shanthikumar (1988), we know that for any two random variables X and Y (with c.d.f. F and G respectively) and a function $\phi(z)$ that is increasing in z the following is true: $\mathbb{E}_X \phi(x) \geq \mathbb{E}_Y \phi(y)$ if $\bar{F}(x) \geq \bar{G}(x)$.

From the feasibility of the mechanism we know that the allocation can only decrease. Therefore any increase in Δc_i with all the other Δc_j, for $j \neq i$, and ν held constant would imply that q_k would not increase for all $k \geq i+1$ and q_l would not decrease for all $l \leq i$, in order to maintain the constraint $\sum q_i = 1$. Hence, $\chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1})$ is non-decreasing in Δc_i, for all $1 \leq i \leq n-1$ when for all $j \neq i$, Δc_j are held constant. Therefore,

$$
\mathbb{E}_{\Delta c_{i,n}} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu) \geq \mathbb{E}_{\Delta c_{i,n+1}} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu). \tag{25}
$$

Moreover, for any sample path $\Delta c_1, \ldots, \Delta c_{n-1}$ increasing c_1 implies shifting all the costs $c_1, \ldots, c_n$ to the right. Since ν is distributed according to f, hence adding a supplier does not change its distribution.

Indeed, using Equation (24) alongwith Equation (25), it follows that for any sample path $\Delta c_1, \ldots, \Delta c_{n-2}$

$$
\mathbb{E}_{\Delta c_{n-1,n},\nu} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu) \geq \mathbb{E}_{\Delta c_{n-1,n+1},\nu} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu)
$$
$$
\geq \mathbb{E}_{\Delta c_{n-1,n+1},\nu} \left(\mathbb{E}_{\Delta c_{n,n+1}} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \Delta c_n, \nu) \right)
$$

Because this is true for every sample path $\Delta c_1, \ldots, \Delta c_{n-2}$,

$$
\mathbb{E}_{\Delta \mathbf{c},\nu} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \nu) \geq \mathbb{E}_{\Delta \mathbf{c},\nu} \left(\mathbb{E}_{\Delta c_{n,n+1}} \chi_{max}(\Delta c_1, \Delta c_2, \ldots, \Delta c_{n-1}, \Delta c_n, \nu) \right),
$$

and hence $\bar{q}_{max}(n+1) \leq \bar{q}_{max}(n)$. ∎

Proof of Lemma 6: Let $\phi(q,c) = S(q,c) + \frac{\partial S(q,c)}{\partial c} \cdot F(c)/f(c)$. For any k, let the optimal allocation be $q_1, \ldots, q_n$. Optimality of allocation implies that for any $q_i > q_j$ a transfer of an amount $\epsilon > 0$ from q_i to q_j would result in a non-optimal solution (a higher cost), i.e.

$$
\sum_{l=1,l\neq i,l\neq j}^{n} B(q_l,c_l) + \phi(q_i,c_i) + \phi(q_j,c_j) + k\beta\bar{\alpha}(q_i) + k\beta\bar{\alpha}(q_j) \leq
$$
$$
\sum_{l=1,l\neq i,l\neq j}^{n} B(q_l,c_l) + \phi(q_i-\epsilon,c_i) + \phi(q_j+\epsilon,c_j) + k\beta\bar{\alpha}(q_i-\epsilon) + k\beta\bar{\alpha}(q_j+\epsilon)
$$

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Which implies that

$$\phi(q_i, c_i) + \phi(q_j, c_j) - \phi(q_i - \epsilon, c_i) - \phi(q_j + \epsilon, c_j) \leq k\beta(\bar{\alpha}(q_i - \epsilon) + \bar{\alpha}(q_j + \epsilon) - \bar{\alpha}(q_i) - \bar{\alpha}(q_j))$$

However, for $\bar{\alpha}$ convex the right hand side of the above inequality is less than 0 and for $k \to \infty$ the above inequality can not be true if $q_i > q_j$. In fact, for $k \to \infty$, the above inequality holds only if $q_i = q_j$. Applying this argument for all the pairs of allocation results in $q_i = \frac{1}{n}$ for all i as $k \to \infty$. ∎

Proof of Lemma 7: Similar to the proof of Lemma 3, finite optimal supply base levels $n_1^*(0) \ldots n_r^*(0)$ exist for each type of suppliers that minimize the cost-to-go at the beginning of the horizon (when the buyer has an empty supply base). Hence, starting any consequent period with $n_{j,a} \leq n_j^*(0)$ for $j = 1 \ldots r$ suppliers, a stationary policy of qualifying up-to $n_j^*(0)$ for $j = 1 \ldots r$ is optimal. ∎