Volatility, Investor Uncertainty, and Dispersion

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Abstract

This paper studies the aggregation of investor expectations of stock market return variation and its implications. We motivate theoretically that the market's expected return variance can be decomposed into the average of individuals' expected variance plus the dispersion in individuals' expected mean returns. The former can be seen as risk, while the latter is a measure of uncertainty. We illustrate this result empirically by setting up a unique survey measuring investors' expected returns and volatilities. Our finding is important to the issue of aggregating heterogeneous beliefs at the micro level in relation to pricing in financial markets. For instance, as a result it is almost per definition that individual investors are overconfident in the sense of overly narrow forecast bounds, due to neglecting individual differences of opinion about mean returns. We furthermore show that investors display a risk-return trade-off, whereas the market seems to price uncertainty.

Keywords: volatility, uncertainty, dispersion, survey data

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1 Introduction

In this paper, we focus on the aggregation of investor expectations of future stock market return variation. We motivate theoretically that aggregate market return variance can be decomposed into the average of individuals' expected variance plus the dispersion in individuals' expected mean returns. The former can be seen as risk, while the latter is a measure of uncertainty. We illustrate this result empirically by setting up a unique survey measuring investors' expected returns and volatilities. Our finding is important to the issue of aggregating heterogeneous beliefs at the micro level in relation to pricing in financial markets. For instance, as a result it is almost per definition that individual investors are overconfident in the sense of overly narrow forecast bounds, due to neglecting individual differences of opinion about mean returns. We furthermore show that investors expect a risk-return trade-off, whereas the market seems to price uncertainty.

One way to assess not directly observable return parameters such as volatility, is by estimating time-series models from data on realized returns. Many studies in this area find that GARCH-type models provide an accurate forecast for future volatility (e.g., Bollerslev [1986]; Engle [1982]; Andersen and Bollserslev [1998]). Expected return estimates, however, seem much harder to obtain (Merton [1980]). This and the existence of empirical phenomena for which standard models cannot provide a consistent rationalization have recently caused attention being dedicated to uncertainty about mean returns; see Guidolin and Rinaldi [2010] for a review on ambiguity aversion in asset pricing and portfolio choice.

In standard models, it is assumed that arbitrageurs are at the heart of asset pricing and induce non-rational investors to revise their valuations. In light of the homogeneous expectations assumption being unrealistic, less restrictive and new models have been developed for differences in beliefs . These so-called differences of opinion can result from investors interpreting the same information differently (see Harris and Raviv [1993]) and/or from investors starting with different prior beliefs (see Varian [1989]). A problem with investigating the impact of this heterogeneity on asset prices is that a benchmark cannot be identified in the case of incomplete agreement (Fama and French [2007]). To arrive at testable implications, additional assumptions can be introduced on how diverging beliefs are formed and aggregated. Empirical evidence is, however, relatively scarce and often difficult to evaluate due to dissimilar underlying models. For example, the theoretical work of Hong and Stein [2003] on asymmetric large price movements is based on differences of opinion, where fully rational investors eventually iron out mispricing. On the other hand, in the stock market model of Chen et al. [2002] arbitrageurs fail to eliminate the upward price bias and lower future returns related to differences of opinion.

An alternative way to assess differences of opinion is by exploiting data on analyst forecasts. Diether et al. [2002] find a negative cross-sectional relation between future returns and the dispersion in analysts' earnings per share forecasts, consistent with Chen et al. [2002]. Johnson [2004] offers a leverage-based explanation in support of dispersion being a proxy for idiosyncratic parameter risk. By contrast, using a factor structure, Anderson et al. [2005] establish empirically that a higher dispersion of analyst forecasts implies higher expected returns as well as higher future return volatility (see also Qu et al. [2003]). Their results suggest that the performance of asset pricing models can improve when the amount of disagreement in analysts' forecasts is taken into account.

An explicit link to uncertainty is made by Anderson et al. [2009], who emphasize the importance of aggregate measures of disagreement. Assuming that the volatility and higher-order moments of all returns are known, disagreement about mean returns is interpreted as model uncertainty. They show theoretically that uncertainty (or disagreement) about an individual stock only matters when this is correlated with (aggregate) market uncertainty. In their empirical analysis, uncertainty is quantified as the dispersion in predictions of market return forecasts constructed from the Survey of

Professional Forecasters (henceforth SPF). Their findings indicate that uncertainty among investors about the mean is important for explaining market excess returns and also matters for the cross section of stock returns.

A direct way to assess investor expectations is by using survey data on actual beliefs of financial practitioners regarding future prices and returns. A large majority of survey research centers around point forecasts of expectations in order to produce a consensus forecast of the mean return. But the individual data underlying the aggregate survey results can also be used to measure the amount of disagreement among investors. Of a more recent date are surveys that ask an investor for a confidence interval forecast to accompany her or his point forecast. DeBondt [1993] asks survey participants for expected price levels and interval estimates for which "there is only a one-in-ten chance that the actual price will turn out higher, and only a one-in-ten chance that the actual price will turn out lower" (p.358). The proportion of Standard & Poor's Index realizations outside the interval appears to be much higher than 20%. This result is indicative of overconfidence in terms of too-narrow forecast bounds (Barberis and Thaler [2003]; Alpert and Raiffa [1982]). A volatility forecast of investor expectations can be calculated from a confidence interval of return outcomes; see Keefer and Bodily [1983]. Putting this into practice for the German stock index DAX, Glaser and Weber [2005] find that volatilities predicted before September 11, 2001 underestimate the volatility of subsequently realized returns, consistent with overconfidence. However, volatility forecasts after September 11 are up to about the realized volatility. An interesting observation is that differences of opinion are also minimized after 9/11. Further research into the interrelationships between volatilities, overconfidence and uncertainty or differences of opinion among investors seems merited. Our paper takes a step in this direction.

In contrast to the use of survey data on expectations of stock prices and returns, that on expectations of inflation and gross domestic product (GDP) growth has a fairly long history in economics research (MacDonald [2000]; Pesaran and Weale [2006]). Since 1968 the SPF is conducted quarterly among forecasters on inflation and GDP growth in the U.S.. A unique feature is that SPF forecasters not only provide a point estimate but also attach probabilities to ten pre-assigned intervals for possible inflation and GDP growth outcomes. Though these probabilistic forecasts often exhibit considerable spread, Zarnowitz and Lambros [1987] and Engelberg et al. [2009] find a remarkably close relation between the associated means and the matching point estimates. The associated variance measures the individual uncertainty that the (associated) mean will be realized. Zarnowitz and Lambros [1987], Lahiri et al. [1988], and Clements [2008] find that average individual uncertainty and disagreement in point forecasts typically move together, though the former tends to be higher and less volatile than the latter. However, the link between these two measures lacks any theoretical basis and is considered to be an empirical matter (see Lahiri and Sheng [2010]; Bomberger [1996]). Giordani and Söderlind [2003] show that average individual uncertainty and disagreement in point forecasts are connected in being additive component parts of aggregate uncertainty as defined by the variance of the aggregate density forecast. They give no effect to this, however; instead, aggregate uncertainty is discarded. They do find that average individual uncertainty is the appropriate measure of collective uncertainty over future rates of inflation and GDP growth. In this paper, however, we do not follow suit.

Survey data on future stock market returns opens up, in our view, the use of aggregate uncertainty. A principal reason for choosing a collective uncertainty measure that contains disagreement is its observed importance for explaining market excess returns. Note that this is not the case with inflation and GDP growth, since higher forecast rates of these macroeconomic variables are not associated with higher disagreement (Zarnowitz and Lambros [1987]). Thus, next to average individual variance we include disagreement for the presumptive reason that disagreement contributes to future market return volatility. Then it seems only natural to hypothesize that aggregate uncertainty measured as the variance of the aggregate density forecast represents the market's expected return variance.

The added value of this paper has both a theoretical and an empirical component. First, in the theoretical part of the paper, we show that the market's expectation of future stock market return variance can be decomposed into the average of individual variance and the dispersion in individual mean expectations. This is an application of the models first introduced in Lahiri et al. [1988] and Giordani and Söderlind [2003] for inflation expectations to a financial market setting. Second, for the accompanying empirical analysis we develop a unique survey that allows for the measurement of aggregate uncertainty. In co-operation with a large Netherlands commercial bank, we create a data set of survey expectations from a group of mass affluent retail investment clients. For more than a year and a half, we have asked participants every other week for their expectations about the future level of the AEX Index, the main stock index in the Netherlands, and their expectation of the maximum and minimum level that can be attained. By converting these extreme value levels into an expected volatility estimate using the Parkinson [1980] volatility measure, we are able to obtain individual uncertainties next to dispersion in mean returns, consistent with the theoretical decomposition.

As a first empirical test, we confront the survey-based aggregate volatility measure with two measures of future market return variance. The first one is the VAEX, being the AEX option-implied volatility index, which provides an ex-ante market expectation of future volatility. As such, this comes under the head of extracting information on investor expectations from prices of financial assets, a way so far left unmentioned. Our second measure is constructed from a GARCH model estimated on past AEX Index returns. For both measures, comparing future market return variance with the sum of average individual variance and dispersion in individual means produces empirical evidence consistent with the theoretical model. Given that the variance decomposition holds by definition, this results first and foremost confirms our choice of data and methods.

An important consequence of this result is that the variance decomposition provides a direct explanation for investor overconfidence. Recognizing that dispersion in individual means is a constituent part of future market return variance implies that investors are facing uncertainty, i.e., dispersion, as a part of risk rather than in addition to risk. As a result, risk subjectively perceived by an investor (individual variance) underestimates market risk, on average, due to fact that uncertainty is unobservable for the individual. Usually, this difference is associated with overconfidence, while our finding places it in the context of neglecting the uncertainty part of risk.

In a second empirical test, we study the expected and realized risk-return and uncertainty-return trade-offs. Since our evidence pertains to returns of the market, we can disregard idiosyncratic risk and interpret uncertainty as a form of systematic risk. This is consistent with the empirical findings of Anderson et al. [2005], who suggest that heterogeneity is a "missing factor" in empirical asset pricing models. As the next step of our analysis, we therefore investigate empirically the trade-off between risk, uncertainty, and return. Traditional risk in terms of average individual variance dominates uncertainty in explaining individuals' expected return. When considering realized market returns, however, we find an uncertainty-return trade-off. This is in agreement with the results for SPF forecasters of Anderson et al. [2009] who also find stronger evidence for an uncertainty-return trade-off[1].

The remainder of the paper is organized as follows. Section 2 theoretically shows the decomposition of expected market variance into individually expected variance and cross-sectional dispersion. Section 3 introduces the survey data and methodology. Section 4 empirically validates the decomposition model from Section 2 using the survey data. Section 5 studies the relation between investor overconfidence and cross-sectional dispersion in expected returns and Section 6 investigates the out-of-sample properties of the survey-based variance estimates. Section 7, finally, concludes.

[1]Giordani and Söderlind [2006] incorporate dispersion in their asset pricing equations derived in an Arrow-Debreu economy. Using SPF data, they find little evidence for a role of dispersion.

2 Uncertainty and Disagreement

Our goal is to examine the relation between market expected variance, individual expected return variance, and the dispersion in equity return expectations. We do so by analyzing the survey expectations from a group of investors from whom we have observations about their expected return and variance. In this section, we demonstrate theoretically how aggregate variation in return expectations of a group of individuals can be decomposed into individual expectations about returns and volatility. The following derivation is based Lahiri et al. [1988] and Giordani and Söderlind [2003], applied to a financial market setting. This is an important generalization as total sample variation has a clear meaning in financial markets, while it does not in inflation expectations.

At time t, investor i forecasts the change in the value of the stock index, r, between t and $t+1$. Assume that investor i thinks that the return r is a random variable with probability density function $f_i(r)$. Given this, the expected return according to investor i, $\bar{r}_i$, equals:

$$\bar{r}_i = \int_{-\infty}^{\infty} r f_i(r)\,\mathrm{d}r. \tag{1}$$

The average expected return simply equals the average of the expected returns over all n investors:

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} \bar{r}_i \tag{2}$$

The total sample variance over all investors, σ^2, which is the market's expected variance in our framework of financial markets, then equals:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} \int_{-\infty}^{\infty} (r-\bar{r})^2 f_i(r)\,\mathrm{d}r. \tag{3}$$

The market's expected variance, however, has got a intra-investor component and an inter-investor component. To see this, think of a market with two participants. Both have an individual expectation concerning the future price level and the variance of the return distribution. The variance expectation of the first (second) investor, however, is centered around the point forecast of the first (second) investor. Hence, the market expected variance should not only incorporate the variance estimate of the two investors, but also the spread in the point forecasts. To decompose the intra- and inter-investor components, subtract and add within Equation (3) investor i's expected return, r_i, such that we obtain

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\int_{-\infty}^{\infty}(r - \bar{r}_i + \bar{r}_i - \bar{r})^2 f_i(r)\,\mathrm{d}r. \tag{4}$$

Rewriting Equation (4) yields:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\int_{-\infty}^{\infty}\{(r - \bar{r}_i)^2 + 2(r - \bar{r}_i)(\bar{r}_i - \bar{r}) + (\bar{r}_i - \bar{r})^2\} f_i(r)\,\mathrm{d}r. \tag{5}$$

Note that the last term on the right hand side can be written as

$$\frac{1}{n}\sum_{i=1}^{n}\int_{-\infty}^{\infty}(\bar{r}_i - \bar{r})^2 f_i(r)\,\mathrm{d}r. = \frac{1}{n}\sum_{i=1}^{n}(\bar{r}_i - \bar{r})^2, \tag{6}$$

since $\int_{-\infty}^{\infty} f_i(r) = 1$.

Using $\int_{-\infty}^{\infty} f_i(r) = 1$ and $\int_{-\infty}^{\infty} r f_i(r)\,\mathrm{d}r = \bar{r}_i$, the middle term on the right hand side of Equation (5) can be shown to equal zero:

$$\begin{aligned}\frac{1}{n}\sum_{i=1}^{n}\int_{-\infty}^{\infty}2(r - \bar{r}_i)(\bar{r}_i - \bar{r}) f_i(r)\,\mathrm{d}r. &= \frac{2}{n}\sum_{i=1}^{n}\int_{-\infty}^{\infty}\{r(\bar{r}_i - \bar{r}) - \bar{r}_i(\bar{r}_i - \bar{r})\} f_i(r)\,\mathrm{d}r. \qquad (7)\\ &= \frac{2}{n}\sum_{i=1}^{n}\{\bar{r}_i(\bar{r}_i - \bar{r}) - \bar{r}_i(\bar{r}_i - \bar{r})\}\\ &= 0\end{aligned}$$

Substituting the results from Equations (6) and (7) into (5) yields

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\int_{-\infty}^{\infty}(r-\bar{r}_i)^2 f_i(r)\,\mathrm{d}r + \frac{1}{n}\sum_{i=1}^{n}(\bar{r}_i-\bar{r})^2. \tag{8}$$

Note that in Equation (8) the term $\int_{-\infty}^{\infty}(r-\bar{r}_i)^2 f_i(r)\,\mathrm{d}r$ is the variance of the expectation by investor i, σ_i^2. Using this, we rewrite Equation (8) such that we obtain

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\sigma_i^2 + \frac{1}{n}\sum_{i=1}^{n}(\bar{r}_i-\bar{r})^2. \tag{9}$$

Equation (9) shows that the market's expected variance σ^2 equals the average of the individuals' expected variance over all investors, $E(\sigma_i^2)$, plus the variance of the expected return forecasts over all investors $var(\bar{r}_i)$:

$$\sigma^2 = E(\sigma_i^2) + var(\bar{r}_i). \tag{10}$$

Equation (10) shows that the market's expected variance in a sample of multiple investors equals the average expected variance plus the disagreement among the investors, as the variance of the expected return forecast over all investors, $var(r_i)$, is a measure of disagreement or dispersion. Note that dispersion is zero only if all respondents have the same forecast, i.e. $r_i = r$, implying homogeneity of expectations. In that case, aggregate variation equals the average over the expected variation of individual investors. Also note that (10) is a definition; it is not an economic model in the sense that the result is conditional on certain assumptions. In a market with more than one investor, the market's expected volatility can be decomposed into these two elements at each point in time.

We emphasize that the use of the expectation operator E and var on the right-hand-side of Equation (10) does not imply that we introduce a conditioning random variable in our statistical framework. In this respect our approach differs from that of Giordani and Söderlind [2003] where information sets are aggregated. Their model assumption of individual forecasters facing different information sets involves the problem of how to interpret the aggregate distribution. Instead, we allow investors to have different probabilities when faced with the same information as long as they are in some ways 'reasonable'. Morris [1995] argues in favor of this personalistic or subjectivist Bayesian view of probabilities (see also Savage [1954]) in, e.g., research on financial markets. According to Wallis [2005], who refers to Granger [1989] stating that "aggregating forecasts is not the same as aggregating information sets", the aggregate density forecast is a combined forecast in the tradition of the point forecast pooling literature. In addition, combining point forecasts with equal weights often outperforms more complicated weighting schemes, though an optimality result is missing. In the present paper on aggregating survey expectations on a stock index, there is no reason to depart from this practice of unweighed averaging. The aggregate distribution then captures two types of uncertainty: which forecast to trust and that forecast's uncertainty. This interpretation was discarded by Giordani and Söderlind [2003] on information-based arguments.

Intuitively, the relation between the market's expected variance, expected uncertainty, and dispersion, is best seen from the option market. In the option market, participants trade on (expected) volatility. As the option premium is conditional on the future volatility of the underlying asset, both hedging-motivated and speculation-motivated trading in options requires forming expectations about future volatility. Consider an at-the-money put option on a certain stock index. When the expected volatility of the underlying increases, both the hedger and the speculator will increase demand for the option. Increased demand will bid up the option premium until it represents the marginal investor's volatility estimate. The option premium, however, also depends on the spread in return expectations. An increase in the spread does not cause a symmetric market neutral response by optimistic and pessimistic investors due to the nonlinear pay-off structure of options. Consider again the at-the-money put option on a certain stock index. An increase in the dispersion in return expectations could cause the hedging-motivated investors with low return expectations to increase their long position in the put option. The hedgers with high return expectations, on the other hand, will compensate this increased demand by decreasing their long position, but will do this less than proportionately due to their higher (less negative) delta. The speculators with low return expectations will also increase their long-position in the put option. The positive speculators could react by decreasing their long-position or by shorting options. Writing options, however, is bounded to a certain maximum due to margin requirements. Hence, both an increase in individual's expected variance and the dispersion in return expectation can increase the option's implied volatility.

The empirical merits and consequences of the variance decomposition has, to our best knowledge, never been studied in a financial market setting. The main reason for this is, most likely, the lack of data. The next section describes the survey that we have developed in order to measure investor expectation of future volatility.

3 The Survey Data and Methods

Key to this study is to measure the expectations of investors regarding returns and volatility. To do so, we run a survey among mass affluent retail clients of the large Netherlands commercial bank ABN-Amro. The group of clients we analyze are serviced by a department on the trading floor of the bank and belong to the group of mass affluent clients, with investable capital between 100,000 and 1,000,000 USD, who trade frequently, least once per month, to take advantages of (perceived) market opportunities. The survey participants are investors who are active on the financial markets and decide on their portfolio themselves (possibly based on advice provided by the bank). Anecdotal evidence suggests that the investors are especially active in trading options, suggesting that they have a certain degree of knowledge on derivatives and have incentives to focus on market volatility.

Every other week on Friday afternoon after market close an e-mail is sent out containing the most recent closing price of the AEX Index[2] and five questions. The first three questions are (translated from Dutch):

"Today, the AEX Index closed at XXX,

- on what level will the AEX Index end on dd-mm-yyyy?
- on what level will the AEX Index end maximally on dd-mm-yyyy?
- on what level will the AEX Index end minimally on dd-mm-yyyy?"

with XXX replaced by the exact closing price of the AEX Index and with dd-mm-yyyy being a specific date of the Friday two weeks after the survey, or a Thursday in case the specific Friday is a holiday [3]. The first question refers to the expected return, the second and third questions to the expected variance. The bank uses the first question to construct a sentiment index[4] and communicates this back to their clients on the following Monday. The first survey was sent out on December 18, 2009 and at the time of writing there are 36 completed surveys[5]. Table 1 presents descriptive statistics of the survey.

The questionnaire is sent out to approximately 800 clients. The response started off at around 200 and has steadily decreased to a stable 80. Due to privacy issues we do not observe time-series of individuals but have a cross-section of expectations per survey date. The average value of the index was close to 340 over the sample period; the expected value of the index lies 2 points higher. Respondents therefore expect an average 2-week return of 0.68%, while the actual average return was 0.35%. The average respondent is therefore quite optimistic, although the difference is not significant. Figure 1

[2]The AEX index is the most important index at the Amsterdam Stock Exchange, consisting of 25 large cap stocks.

[3]The fourth question asks for individuals' expectations of the average expectation. The fifth question varies over the surveys and is chosen by the bank. Examples are questions about the oil price, or the level of the index at the end of the year.

[4]The sentiment index is constructed by calculating the percentage of bullish, i.e. positive, expected returns.

[5]The survey of July 30, 2010 is missing due to a technical reason causing the survey results not to be saved.

Table 1: Summary statistics

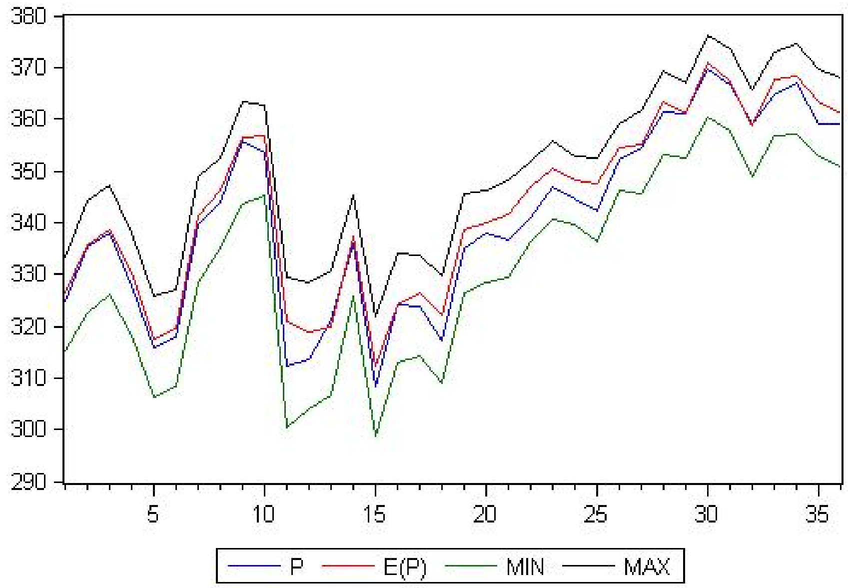
	n	(p_t)	$E(p_{t+1})$	$E(p_{min})$	$E(p_{max})$
Mean	104.889	340.19	342.46	330.92	349.51
Max	192.000	369.65	390.00	380.00	415.00
Min	68.000	308.20	275.00	240.00	282.00
St.Dev.	30.123	16.892	18.537	20.417	17.368
		(r_{t+1})	$E(r_{t+1})$	$E(r_{min})$	$E(r_{max})$
Mean		0.0035	0.0068	-0.0274	0.0277
Max		0.0687	0.1205	0.1045	0.1556
Min		-0.1161	-0.1439	-0.2369	-0.1221
St.Dev.		0.0353	0.0264	0.0316	0.0250

displays the evolution of the index and the average expectations per survey over time.

DeBondt [1993] also studied the distributional properties of expected equity returns. He surveyed the 10 and 90 percent confidence intervals and measure expected volatility based on these estimates. We apply a different methodology to measure expected volatility as we ask for the minimum and maximum instead of a confidence interval. We argue that it is relatively complicated for retail investors to understand densities and to assess probabilities and we therefore think that asking about the minimum and maximum values would yield better estimates of investor uncertainty (although we cannot verify this). Furthermore, Kahneman and Tversky [1982] show how people have biased ideas about extreme probabilities in that they have a tendency to overreact to small probabilities, and underreact to large probabilities. Another advantage of asking specifically for the maximum and minimum values is that we directly can apply the Parkinson [1980] measure that provides a volatility estimate from a high and low observation [6]. The Parkinson [1980] measure is given by:

[6] For instance Martens and van Dijk [2007] apply this measure finding that the intra-day high-low spread is a better volatility measure than one based on closing prices

Figure 1: Average Expectations over Time



$$\sigma_{i,t} = \sqrt{26}\sqrt{\frac{ln(H_{i,t}/L_{i,t})^2}{4ln(2)}}, \tag{11}$$

in which $H_{i,t}$ represents respondent i's estimate made at time t for the maximum at $t+1$ and $L_{i,t}$ a respondent i's estimate for the minimum value of the AEX at $t+1$. The $\sqrt{26}$ is used to annualize the volatility estimate[7]. The Parkinson measure assumes that log-prices prices follow a random walk for the period over which the volatility is calculated. Given that the survey asks for a minimum - maximum range at one point in time (two weeks from the survey date), the random walk assumption does not play a role.

As a measure of the market's expected variance, we use the implied volatility index VAEX. We assume that the VAEX is a good benchmark for investors as it is timely available and freely published on several websites. Furthermore, as is the case for our Parkinson measure, the implied volatility represents a forward looking measure for volatility and therefore serves as a natural comparison. The VAEX is constructed similarly as the VIX index for the S&P500, using out-of-the money index options. By construction, the VAEX represents the expected volatility for the coming month. Since we are asking survey participants for two-week returns, we implicitly assume a flat term structure of implied volatilities from two to four weeks. For robustness, we also use a GARCH-based volatility forecast as a benchmark for the market's expectation for return variance.

4 Expected Volatility

In this section, we compare market expected variance, option implied variance, with estimates of the aggregate variance, individual variance and disagreement. In other words, we test whether the theoretical decomposition derived in Section 2 holds empirically for our survey of expected uncertainty.

[7]Having 26 two-week periods in a year

Given that the decomposition holds by definition, this exercise is first and foremost a test of the appropriateness of our data and methods.

Table 2 shows the estimates for the dispersion among the respondents' return expectations ($var(\bar{r}_i)$), the average expected variance as measured by the Parkinson measure applied to each individual's maximum and minimum estimates ($E(\sigma_i^2)$), the market's expected variance (σ^2) being the sum of the former two measures, the annualized square root of the market expected variance (σ), and the annualized implied volatility estimate for the AEX Index, the VAEX, (σ_{iv}) for each individual survey. The VAEX observations are the closing values observed at the moment each survey was sent out, i.e., on Friday.

Figure 2 shows the time series of expected volatilities. All numbers are annualized. For comparison reasons, the dispersion and expected volatility are the square roots of the measures shown before and therefore do not add up to the total sample volatility. Clearly, the dispersion measure is lower than the expected volatility measure, but they follow a similar pattern, consistent with Zarnowitz and Lambros [1987]. But survey-based measures also follow a similar pattern as the market's implied volatility; this finding is consistent with our model from Section 2. Also the average expected volatility of participants is typically lower than the VAEX. This becomes especially apparent between surveys 10 and 15, where volatility shoots up. The sum of the two, dispersion plus expected volatility, is, by visual inspection, the closest measure. The large increase in volatility from survey 10 is due to the start of the Greek debt crisis in May 2010. Shortly before that, at the end of February 2010, the Dutch government had resigned.

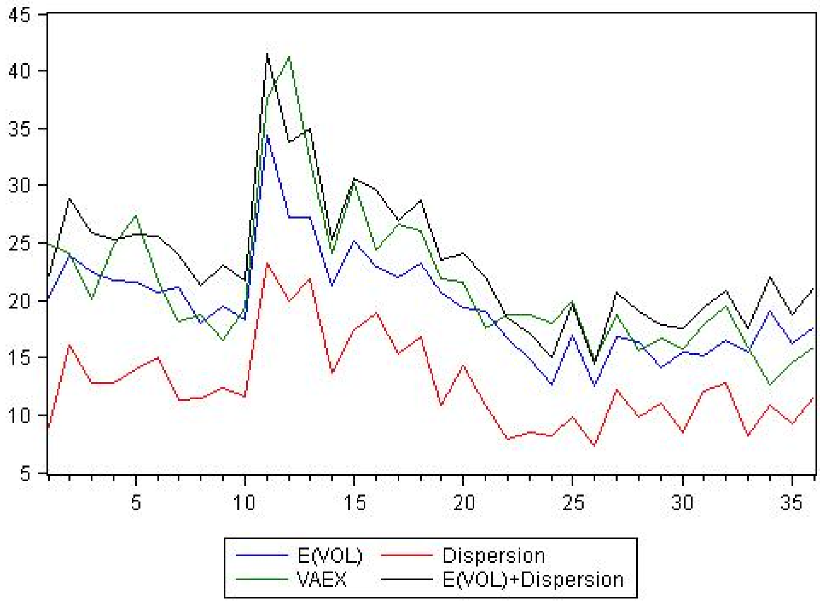
To examine the relation between market implied variance, market expected variance, individual expected variance, and disagreement, we estimate the parameters in the following regression equations.

$$\sigma_{t,iv}^2 = \alpha + \beta\sigma_t^2 + \epsilon_t \tag{12}$$

Table 2: Aggregate Variation and Volatility

survey	date	$var(\bar{r}_j)$	$E(\sigma_j^2)$	σ^2	σ	σ_{iv}
1	18-Dec-09	0.00030	0.00158	0.00188	22.122	24.859
2	31-Dec-09	0.00099	0.00222	0.00321	28.889	24.162
3	15-Jan-10	0.00064	0.00194	0.00258	25.883	20.071
4	29-Jan-10	0.00064	0.00181	0.00245	25.238	24.860
5	12-Feb-10	0.00076	0.00179	0.00255	25.751	27.410
6	26-Feb-10	0.00086	0.00165	0.00252	25.576	21.756
7	12-Mar-10	0.00050	0.00172	0.00222	24.020	18.187
8	26-Mar-10	0.00050	0.00124	0.00174	21.296	18.765
9	9-Apr-10	0.00059	0.00146	0.00205	23.070	16.505
10	23-Apr-10	0.00052	0.00130	0.00182	21.762	19.314
11	7-May-10	0.00206	0.00456	0.00662	41.496	37.676
12	21-May-10	0.00154	0.00286	0.00439	33.798	41.249
13	4-Jun-10	0.00183	0.00286	0.00469	34.923	32.348
14	18-Jun-10	0.00072	0.00174	0.00245	25.244	24.096
15	2-Jul-10	0.00117	0.00243	0.00360	30.596	30.270
16	16-Jul-10	0.00138	0.00201	0.00339	29.675	24.380
17	13-Aug-10	0.00091	0.00187	0.00278	26.899	26.650
18	27-Aug-10	0.00108	0.00208	0.00316	28.650	26.020
19	10-Sep-10	0.00046	0.00166	0.00212	23.463	21.887
20	24-Sep-10	0.00079	0.00144	0.00223	24.061	21.548
21	8-Oct-10	0.00046	0.00140	0.00185	21.956	17.619
22	22-Oct-10	0.00024	0.00107	0.00131	18.434	18.792
23	5-Nov-10	0.00027	0.00085	0.00112	17.085	18.778
24	19-Nov-10	0.00026	0.00062	0.00087	15.080	17.982
25	3-Dec-10	0.00038	0.00110	0.00148	19.613	19.903
26	17-Dec-10	0.00021	0.00060	0.00081	14.500	14.717
27	31-Dec-10	0.00057	0.00109	0.00166	20.760	18.711
28	14-Jan-11	0.00037	0.00103	0.00140	19.045	15.655
29	28-Jan-11	0.00047	0.00076	0.00124	17.922	16.600
30	11-Feb-11	0.00027	0.00092	0.00119	17.620	15.726
31	25-Feb-11	0.00057	0.00088	0.00145	19.383	17.870
32	11-Mar-11	0.00062	0.00105	0.00168	20.882	19.451
33	25-Mar-11	0.00026	0.00093	0.00118	17.546	15.926
34	8-Apr-11	0.00046	0.00140	0.00186	21.972	12.664
35	22-Apr-11	0.00033	0.00101	0.00134	18.690	14.602
36	6-May-11	0.00052	0.00120	0.00172	21.175	15.921

Figure 2: Volatility, disagreement, and uncertainty over time



$$\sigma^2_{t,iv} = \alpha + \beta E(\sigma^2_{t,i}) + \epsilon_t \tag{13}$$

$$\sigma^2_{t,iv} = \alpha + \beta var(\bar{r}_{t,i}) + \epsilon_t \tag{14}$$

$$\sigma^2_{t,iv} = \alpha + \beta E(\sigma^2_{t,i}) + \gamma var(\bar{r}_{t,i}) + \epsilon_t \tag{15}$$

The estimates for α, β, and the regression's R^2 are indicative of the in-sample power of the different measures of variation from the survey in explaining implied variance (implied volatility squared).

Equation (12) tests to what extent implied variance can be explained by the market's expected variance drawn from the surveys. Based on the analysis in the previous section, we test the hypothesis that σ^2_t is an unbiased and efficient estimator for implied variance. When this holds, we expect $\alpha = 0$ and $\beta = 1$.

Equations (13) and (14) test the explanatory power of the two components of σ^2_t, average expected uncertainty and dispersion, respectively. As these measures are only a part of the total market expectation, we expect that these measures themselves are significantly related to the implied variance but underestimate implied variance. If this is indeed true, we expect $\beta > 1$ for both measures, as implied variance is then a magnification of either average expected uncertainty or dispersion. In addition, α may deviate from zero.

The final equation, (15), tests empirically the separate contributions. If the survey-based market expected variance explains implied variance as being the sum of average expected uncertainty and disagreement, we expect $\alpha = 0$, $\beta = 1$, and $\gamma = 1$. The estimation results based on the 36 surveys are presented in Table 3.

Here we show how both parts of the variance decomposition measured

Table 3: Estimation Results Implied Volatility

	(1)	(2)	(3)	(4)
α	-0.0002	-0.0003	0.0003	-0.0001
	(0.0002)	(0.0002)	(0.0002)	(0.0002)
σ_t^2	0.9433***			
	(0.0920)			
$E(\sigma_i^2)$		1.4338***		0.8343***
		(0.1541)		(0.2602)
$var(\bar{r}_i)$			2.4363***	1.1354*
			(0.0345)	(0.5790)
R^2	0.7630	0.7351	0.7187	0.7567

Robust standard errors are in parentheses; ***, *, and * denotes significance at the 1, 5, and 10% level, respectively.

by our survey data are important in describing option implied variance. The column (1) in Table 3 shows that the estimates for α is not significantly different from zero and that the estimate for β is 0.9433 with a standard error of 0.0920. Hence, β is significantly different from zero but not significantly different from one. The R^2 is 76%. These results are exactly in line with our hypothesis that the market expected variance σ_t^2 is an unbiased and efficient estimate for the market's implied variance. Alternative proof comes from the results from Equation (15), in which implied variance is regressed on average expected volatility and disagreement. Again α is not significantly different from zero. The slope coefficient β for average expected variance is 0.8343, is significantly different from zero and is not significantly different from one. The slope coefficient γ for disagreement is 1.1354, is significantly different from zero with 90% confidence, and is not significantly different from one. The result that both slope coefficients are not significantly different from one is consistent with the hypothesis that the decomposition of the market's expected variance in terms of disagreement and average expected variance explains implied variance. From these results we conclude that the survey-based market expected variance is a precise estimator of implied variance. Apparently, the sum of average expected variance and disagreement about expected returns of a group of investors is exactly in line with implied volatility. This is suggestive of the appropriateness of our survey data.

This result implies that the respondent underestimates volatility when she or he observes only one of the two components of the market's expected variance. This is proven by the estimates of the parameters in Equations (13) and (14). Again, the α estimates are not significantly different from zero. The slope coefficient β for disagreement in Equation (13) equals 2.4363 which is significantly larger than one. Hence, the disagreement measure provides an underestimate of implied variance; on average one needs to multiply disagreement with about 2.4 to obtain an estimate for implied variance. A similar result holds for average expected uncertainty. The slope coefficient β is 1.4338 and is significantly larger than one.

The implied volatility constitutes a highly intuitive benchmark since it represents a forward looking measure of volatility, as does our survey-based measure of expected volatility. As already mentioned in the introduction, though, the volatility forecasting literature is dominated by time series techniques such as GARCH. Both implied volatility and GARCH-based volatility forecasts are shown to yield unbiased and efficient forecast of volatility; see Yu et al. [2010]. Hence, since the results in Table 3 indicate that our survey based volatility forecast describes implied volatility, it should also be directly related to GARCH-type volatility forecasts. To test this hypothesis as a robustness check, we also estimate Equations (12) to (15) using a GARCH-based volatility forecast as dependent variable[8]. The results are presented in Table 4.

The results in Table 4 are qualitatively consistent with those in Table 3. Estimation results reveal that there is information in both the expected volatility and the cross-sectional dispersion in describing the GARCH-volatility forecast. The model fit is highest for the sum of the two elements.

[8]To create a GARCH volatility forecast, we estimate a $GARCH(1,1)$ model on 10-day AEX returns using an expanding window. That is, we estimate the model from January 21, 1983 to December 18, 2009, create a volatility forecast for the next period, re-estimate the model including one additional observation, etc.

Table 4: Estimation results GARCH Volatility Forecast

	(1)	(2)	(3)	(4)
α	-0.0003	-0.0004**	0.0001	-0.0002
	(0.0002)	(0.0001)	(0.0002)	(0.0002)
σ_t^2	0.7826***			
	(0.0752)			
$E(\sigma_i^2)$		1.1854***		0.6316*
		(0.1606)		(0.3078)
$var(\bar{r}_i)$			2.0338***	1.0490*
			(0.1908)	(0.4958)
R^2	0.7400	0.7078	0.7053	0.7345

Robust standard errors are in parentheses; ***, *, and * denotes significance at the 1, 5, and 10% level, respectively.

When including both elements, in column (4), both coefficients are significant and do not deviate significantly from unity.

These results show that when we observe expectations from investors, the sum of average expected uncertainty and disagreement provides an efficient and unbiased estimate of the market's expected variance. In other words, given the fact that the variance decomposition holds by definition, the results indicate that our survey is a representative sample of the Dutch stock market.

Both the implied volatility index VAEX and our measure of sample variance are forward looking measures of market volatility. Several studies have looked into the forecasting ability of implied volatility measures; see for example Neely [2004] and Yu et al. [2010]. All studies are consistent in their conclusion that implied volatility is a strong predictor of market volatility.

Given that the in-sample results of our survey-based measures of expected volatility imply that their sum is equal to the implied volatility, a natural extension is to test whether they contain similar or different information regarding realized volatility. To do so, we regress realized market volatility in the two weeks following the survey on the volatility measures extracted from the survey. For comparability, we measure realized market volatility using the Parkinson [1980] measure. Instead of using the expected maximum and minimum levels, we now insert the realized extremes over the two weeks following each survey date. The results are presented in Table 5.

Table 5: Realized versus Expected Volatility

	(1)	(2)	(3)
C	0.0001	0.0001	0.0001
	(0.000)	(0.000)	(0.000)
$E(\sigma^2_{t,j})$	0.4323***		
	(0.070)		
$var(\bar{r}_{t,j})$		0.7272***	
		(0.098)	
σ^2_t			0.2834***
			(0.040)
R^2	0.1535	0.1462	0.159

Robust standard errors are in parentheses; ***, *, and * denotes significance at the 1, 5, and 10% level, respectively.

The results indicate that all volatility forecast measures have strong explanatory power for realized volatility. Consistent with the previous results, both survey measures have positive and significant explanatory power for realized volatility. The coefficient, though, is significantly smaller than unity; this conclusion holds for individual uncertainty $E(\sigma^2_{t,j})$ in column (1) and dispersion $var(\bar{r}_{t,j})$ in column (2), and the sum of the two elements in column (3). The explanatory power of $E(\sigma^2_{t,j})$ is somewhat higher than that of $var(\bar{r}_{t,j})$. Interestingly, the sum of $var(\bar{r}_{t,j})$ and $E(\sigma^2_{t,j})$ has a somewhat higher R^2 than its individual components. Hence, consistent with the in-sample results, both elements contain unique information in explaining realized volatility. These results are consistent with Anderson et al. [2005] and Anderson et al. [2009] in the sense that they find that dispersion in analyst forecasts is positively related to return volatility. Inconsistent with Anderson et al. [2009] is the fact that we also find a positive association between individual expected variance and market volatility.

This section has shown that the theoretical variance decomposition put forward in Section 2 actually holds for our survey data. This implies that our survey is indeed a representation of the average investor in the AEX index. The next two sections further investigate what the decomposition implies for investor behavior, in Section 5, and asset pricing, in Section 6.

5 Individual overconfidence

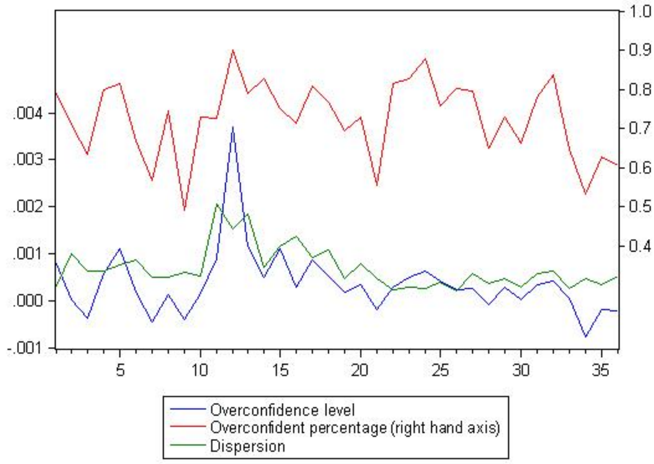
Overconfidence implies that investors place too much weight on their private information. As a result, their confidence interval regarding future prices becomes too narrow: risk is underestimated. Our results have direct implications for the notion of investor overconfidence. In previous work, we show that over 72.3% of the investors in the survey are overconfident in that their individual volatility estimate is too narrow; see Huisman et al. [2012]. The current paper gives an explanation for this finding. Given that the market's expectation for future volatility also depends on the cross-sectional dispersion, each individual will tend to understate market volatility. This might be an important explanation for the result found in the literature that investors are overconfident, because if one observes only one of both components such as done in many studies, one clearly underestimates volatility in case there is disagreement among investors. For instance, several surveys ask respondents about their expected returns, but not about their expected volatilities. Uncertainty in the market is then seen as the disagreement among the respondents, but Equation (10) shows that this is only one part of the total variation.

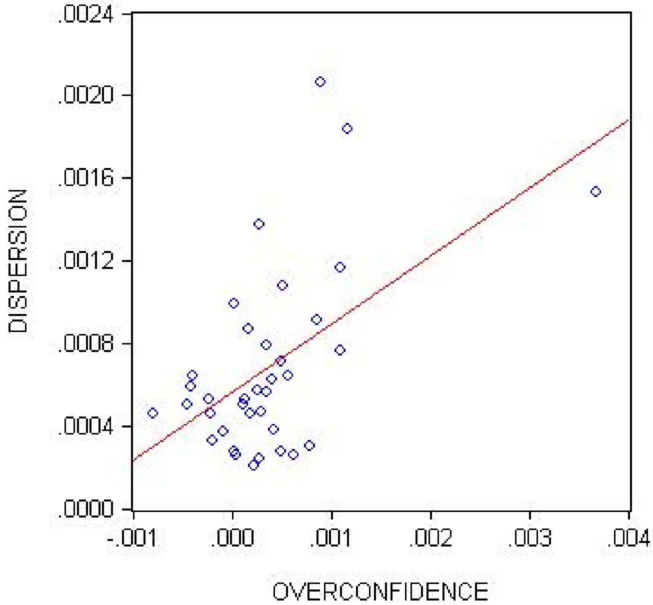
To study the relation between overconfidence and dispersion in more detail, we calculate each investor's overconfidence by means of the difference between the VAEX and the expected volatility. Figure 3 displays the percentage of overconfident respondents per survey, the average overconfidence per survey, and the dispersion in return beliefs.

The line on top of the upper graph in Figure 3 represents the percentage of overconfident responses. On average, 75% of the responses are overconfident, which is significantly above 50%. This percentage, however, changes over time. The lower two lines represent the average level of overconfidence and the dispersion. Both are of the same magnitude, and follow the same pattern. The lower graph in Figure 3 represents the scatter plot of the level of overconfidence and dispersion. The positive relation is illustrated by the regression line; the constant term does not significantly differ from zero. The slope coefficient is 0.88 and highly significant; this number is also not significantly different from one. In other words, we find a one-to-one relation between the level of overconfidence and dispersion.

It is important to observe here that our result does not imply that investors are not overconfident. By the logic of the model, we can explain overconfidence on the market level by means of dispersion. When investors have heterogeneous expectations about returns, investors almost per definition underestimate future market volatility because they do not take dispersion into account. However, it can still be the case that investment decisions are too risky relative to a participant's risk appetite. Increasing individual's variance expectations would only result in an increase in $E(\sigma^2)$, and a proportionate increase in the market's expected variance σ^2. The overconfidence would remain. If participants were aware of the relation between dispersion and overconfidence, the only way to decrease the level of overconfidence is to decrease the cross-sectional dispersion in return expectations. In a world of homogeneous expectations, the market volatility does equal the average individual expected uncertainty. As soon as expectations about expected returns diverge, market volatility exceeds average expected uncertainty. Under strict investor rationality, there is still room for dispersion because investors need only be unconditionally homogeneous. Hence, as long as dispersion between rational investors is completely random, the rational expectations hypothesis still holds. This randomness, however, does directly feed into market volatility causing the rational traders to underestimate market volatility as well.

Figure 3: Overconfidence and Dispersion





6 Risk, Uncertainty, and Returns

The risk-return trade-off constitutes one of the most fundamental relationships in finance, as theoretically described in Merton [1973]. More recently, though, as reviewed in Guidolin and Rinaldi [2010], uncertainty has been shown to influence both investor decision making and subsequent asset pricing. An uncertain situation, then, is defined as Knightian uncertainty, following Knight [1921], where the probability distribution of the situation is unknown. Theoretically, uncertainty in individual decision making is often dealt with by means of the maximin utility principle, as axiomatized by Gilboa and Schmeidler [1989], meaning that agents evaluate choices by maximizing the utility of the worst outcome. Empirically, though, it is not so evident how to operationalize the uncertainty construct, or, more specifically, how to distinguish uncertainty from risk. Anderson et al. [2009] argue, building on Merton [1980], that rather precise estimation of volatility can be achieved while estimation of the drift remains problematic. In their view, therefore, uncertainty is related to uncertainty in first moments while higher moments are completely known. In their empirical application, Anderson et al. [2009] correspondingly proxy risk by means of an ARCH-type model and uncertainty by the dispersion in analyst forecasts[9]. The empirical results indicate that return is mainly related to uncertainty, and not to risk.

The advantage of our survey data is that we directly observe both perceived risk and uncertainty of investors in the interpretation of Anderson et al. [2009]. Individual uncertainty, $E(\sigma^2_{t,j})$, represents the risk component from Anderson et al. [2009] as it measures an individual's assessment of the second moment. Dispersion, $var(\bar{r}_{t,j})$, on the other hand, measures the uncertainty surrounding the first moment, thus the uncertainty component from Anderson et al. [2009]. In addition, because we survey individuals' expected

[9]To be more specific, Anderson et al. [2009] impute analysts' real return forecasts from company profit forecasts by means of the Gordon Growth model.

return, we can study the relation between both expected return and risk & uncertainty and realized return and risk & uncertainty. Table 6 presents the estimation results for the relation between average individual expected return, $\bar{r}$, expected risk, $E(\sigma^2_{t,j})$, and uncertainty $var(\bar{r}_i)$.

Table 6: Expected Return and Risk & Uncertainty

	(1)	(2)	(3)
α	0.004	0.003	0.006***
	(0.002)	(0.002)	(0.002)
σ^2_t	1.575*		
	(0.879)		
$E(\sigma^2_{t,j})$		3.002**	
		(1.326)	
$var(\bar{r}_i)$			2.273
			(2.414)
R^2	0.059	0.105	-0.003

Robust standard errors are in parentheses; ***, *, and * denotes significance at the 1, 5, and 10% level, respectively.

The results in Table 6 reveal that investors individually display a risk-return trade-off, but no uncertainty-return trade-off. Column (1) shows that expected returns are positively and significantly related to the aggregate uncertainty σ^2_t. Columns (2) and (3), however, indicate that this effect is fully driven by the risk component, and not by the uncertainty component. The relation between expected return and uncertainty in Column (3) does not come out significantly, while the relation between expected return and risk, in Column (2), does. Note that the estimate of α in column (2) is insignificant while it is significant in column (3); this is consistent with the reference model.

Subsequent question is whether investor expectations also materialize, hence whether realized market returns are also driven by risk rather than uncertainty. Table 7 presents the estimation results for the relation between realized market return r, expected risk, $E(\sigma^2_{t,j})$, and uncertainty $var(\bar{r}_i)$.

Table 7: Realized Return and Risk & Uncertainty

	(1)	(2)	(3)
C	-0.008	-0.007	-0.008
	(0.008)	(0.008)	(0.007)
σ^2_t	5.089**		
	(2.539)		
$E(\sigma^2_{t,j})$		6.567	
		(3.916)	
$var(\bar{r}_{t,j})$			16.578***
			(6.556)
R^2	0.030	0.020	0.045

Robust standard errors are in parentheses; ***, *, and *
denotes significance at the 1, 5, and 10% level, respectively.

The results in Table 7 show that realized market returns are related to uncertainty, but not to risk. This is consistent with the findings of Anderson et al. [2009] but not with our results for the expected returns in Table 6. Market returns are again significantly related to aggregate uncertainty, σ^2_t in Column (1). When breaking down the aggregate uncertainty in individual uncertainty $E(\sigma^2_{t,i})$, and dispersion, $var(\bar{r}_i)$, however, returns are found to be significantly related to dispersion (Column (3)), and not to individual risk (Column (2)).

Intuitively, the contrasting results from Tables 6 and 7 can be explained as follows. Individual investors are consistent in that they combine a higher return expectation with a higher variance expectation. Individuals only observe individual expected variance, and not the dispersion in expected returns. This already became apparent in the previous section, where the consistent negative bias in volatility forecasts (overconfidence) was shown to be significantly related to dispersion. Hence, individuals cannot take the dispersion, or uncertainty, into account when forming return expectations. Given that individual expected variance is incorporated in individual return estimates, we can expect that this information is already priced in the current market price because it is common knowledge. Under reasonable market efficiency, individual expected variance will not be significantly related to the returns in the two weeks after the expectations are formed. The uncertainty part, however, is unknown or unobserved to investors. In the two weeks following the expectation formation, investors learn about the uncertainty part of market risk. Because uncertainty is an integral part of total uncertainty, this will be gradually priced as the full extend becomes known. Therefore, the uncertainty is significantly related to the realized returns in the two weeks following expectation formation.

Alternatively, we can explain the results from Table 7 from a more behavioral perspective. We can argue that realized returns are significantly related to dispersion because dispersion represents overconfidence, as shown in the previous section. As investors are overconfident, they will have the tendency to trade more and take on more risk (see Barber and Odean [2000], Barber and Odean [2001]). Because overconfidence is systemic in our case, it will drive the market price up accordingly; see DeLong et al. [1990]. This also explains the contrast between the results of Anderson et al. [2005], who find a positive relationship between dispersion and return, and the results of Chen et al. [2002] and Diether et al. [2002], who find a negative relationship between dispersion and future return. Overconfidence, measured by dispersion, causes prices to rise above fair value. As a result, there has to be mean reversion in future periods.

7 Conclusion

In this paper, we study how individual expectations about the volatility of equity market returns aggregates to the market's expectation from both a theoretical and empirical perspective. It is show theoretically that the market's expected volatility consists of the sum of the average individual expected variance plus the cross-sectional dispersion in expected mean returns. Using a unique survey data set of expectations of mass affluent retail clients of a

Netherlands commercial bank, we examine the relationship laid bare by the theoretical model between implied volatility (an indicator for expected market volatility), individual expected uncertainty and cross-sectional disagreement about expected returns. Our result show that the sum of individual expected uncertainty and cross-sectional disagreement almost perfectly explains the implied volatility index. This result implies that it is almost per definition the case that individuals are overconfident, since focusing on investors' expected volatility only does not take the cross-sectional dispersion into account. Hence, individual estimates of market volatility will tend to be too low compared to market volatility, except for the case that investors are homogeneous in which there is no disagreement about expected return. Finally, the empirical results indicate that investors display a risk-return trade-off, while the market seems to price dispersion.

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