Multivariate Dynamic Probit Models: An Application to Financial Crises Mutation

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Abstract

In this paper we propose a multivariate dynamic probit model. Our model can be viewed as a non-linear VAR model for the latent variables associated with correlated binary time-series data. To estimate it, we implement an exact maximum-likelihood approach, hence providing a solution to the problem generally encountered in the formulation of multivariate probit models. Our framework allows us to study the predictive relationships among the binary processes under analysis. Finally, an empirical study of three financial crises is conducted.

J.E.L. Codes: *C35, F37*

Keywords: Non-linear VAR, Multivariate dynamic probit models, Exact maximum likelihood, Impulse-response function, Financial crises.

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1 Introduction

Since the pioneering work of Sims (1980), vector autoregressive (VAR) models have been the main tool used to analyze a set of macroeconomic time-series. This has paved the way for numerous papers proposing modifications of the standard linear VAR model. One research direction focuses on cointegrated time-series (*e.g.* Engle and Granger, 1987 ;Johansen, 1988,1991), leading to Vector Error Correction Models, while another line of research concentrates on adapting VAR specifications to the case of non-linear time-series (Gallant et al., 1993; Koop, Pesaran and Potter, 1996; Kilian and Vigfusson, 2011, among others).

Apart from models for continuous variables, discrete-choice dependent variables are commonly used, especially for forecasting. There are two main approaches. The standard approach used to forecast the probability of occurrence of such events consists of simple logit / probit models. These models have been used, for example, by Berg and Patillo, 1999; Estrella and Trubin, 2006; Fuertes and Kalotychou, 2007 to forecast recessions as well as financial (currency, banking, sovereign debt, stock market,...) crises. An alternative, more recent approach, consists of a dynamic univariate framework estimated by an exact maximum-likelihood estimation method (see Kauppi and Saikonnen, 2008; Candelon et al, 2010 or Nyberg, 2010, 2011, *inter alii*). Forecasts from these models have been shown to outperform forecasts from static models, i.e., models that rely only on the dynamics of exogenous variables. In a similar vein, Dueker (2005) estimates by simulation methods a 'mixed' Qual-VAR model including as dependent variables the latent variable underlying a binary business cycle indicator along with standard continuous indicators.

Modelling the dynamics of qualitative variables may be as important as that of continuous variables (for the improvement of Early Warning Systems, for example). Nevertheless, no multivariate extension of these dynamic binary models exists as far as we know. This omission is surprising because one would conjecture that such models could improve the accuracy of forecasts of discrete events in light of the correlation between different binary indicators in macroeconomics. To our knowledge, all existing multivariate binary models are static. Indeed, although numerous macroeconomic studies involve discrete-choice dependent variables, this particular type of non-linearity has received little attention in the literature on time-series models. The main difficulty lies in formulating a multivariate binary model and in evaluating the likelihood function. In this context, Carey, Zeger and Diggle (1993) and Glonek and McCullagh (1995) proposed generalizations of the binary logistic model to multivariate outcomes by selecting a particular parametrization for the correlations, while Ashford and Sowden (1970) and Amemiya (1972) focused on generalizing the binary probit model. Some attempts have subsequently been made to solve the computational difficulty of evaluating multivariate normal integrals. Chib and Greenberg (1998) developed a simulation-based Bayesian and non-Bayesian approach and Song and Lee (2005) relied on the Expectation-

Maximization algorithm to evaluate the likelihood function for a multivariate probit model. More recently, Huguenin, Pelgrin and Holly (2009) have shown that a multivariate probit model cannot be accurately estimated using simulation methods, as generally done in the literature. Its estimation instead requires the derivation of the exact maximum-likelihood function.

The objective of this paper is to extend the estimation methodology proposed by Huguenin, Pelgrin and Holly (2009) for non-dynamic multivariate probit models to the case of dynamic multivariate probit models. We introduce a multivariate dynamic probit model which is estimated by an exact maximum-likelihood estimation approach to produce dynamic forecasts of binary time-series correlated variables. Our model takes the form of a non-linear VAR for the latent variables associated with the binary indicators under analysis. It is easy to implement and provides a solution to the problem encountered in the estimation of multivariate probit models. For this, in a first step we extend the decomposition of higher-order integrals into integrals of lower order proposed by Huguenin, Pelgrin and Holly (2009) to the case of our multivariate dynamic model. In the second step, we evaluate the lower-order finite-range multiple integrals by using quadrature-rules over bounded intervals.

Our framework allows us to introduce dynamics in several ways. First, dynamics may be included in the form of a set of lagged binary variables. Notice the existence of threshold effects in this case, as the event under analysis occurs only if the latent variable goes beyond a certain threshold. Second, they can be introduced via the past latent variables associated with the binary indicators. This comes down to an autoregressive (AR) model, where the lagged latent variable summarizes all the past information of the system. Finally, both types of dynamics can be simultaneously considered. We thus generalize the univariate dynamic probit model developed by Kauppi and Saikkonen (2008) to a multivariate setting. We derive its exact likelihood and propose consistent and efficient estimates.

In an empirical application on three types of financial crises, namely currency, banking and sovereign debt crises, we investigate the potential spill-over from one crisis to another within a number of emerging countries. It appears that in the bivariate case mutations of a banking crisis into a currency crisis (and vice-versa) have been quite common, confirming other results in the financial crises literature (*e.g.* Glick and Hutchinson, 1999). More importantly, for the two countries (Ecuador and South Africa) which suffered from all three types of crises, the trivariate model turns out to be more parsimonious than existing models.

The rest of the paper is organized as follows. Section 2 presents the multivariate dynamic probit model. In section 3 we describe the Exact Maximum Likelihood method as well as some numerical procedures to estimate the multivariate dynamic probit model, while in section 4 the multivariate dynamic probit model is estimated for 17 emerging countries in its bivariate and trivariate form.

2 A Multivariate Dynamic Probit Model

In this section we describe the multivariate dynamic probit model and highlight its similarities with traditional VAR models. Let us denote by $y_{m,t}$, $m = \{1, 2, ..., M\}$, M binary variables taking the value one if the event under analysis occurs at time t and zero otherwise. Let $y^*_{m,t}$ be the normal latent continuous variable associated with $y_{m,t}$, and define $F_{t-1} = \sigma[(y'_s, x'_v)'|s < t, v \leq t]$ as the information set available at time t.

The corresponding multivariate probit model takes the form:

$$\begin{aligned} y^*_{m,t} &= \pi_{m,t} + \epsilon_{m,t}, \quad \forall m = \{1, 2, ..., M\} \\ y_{m,t} &= \begin{cases} 1, & \text{if } y^*_{m,t} > 0 \\ 0, & \text{otherwise} \end{cases}, \end{aligned} \tag{1}$$

where $\pi_{m,t}$ denotes the index, and the innovations $\epsilon_t = \{\epsilon_{1,t}, ..., \epsilon_{m,t}\}$ verify

$$\epsilon_t | F_{t-1} \sim IIN(0, \Omega), \tag{2}$$

such that $V(\epsilon) = I_T \otimes \Omega$, where I_T is the identity matrix of order T. Ω stands for the covariance matrix of ϵ_t, with $\Omega = (\sigma_{m,m'})$ and $\sigma_{m,m'} = \rho_{m,m'}\sigma_m\sigma_{m'}$, where $\rho_{m,m'}$ represents the correlation coefficient between the m^{th} and m'^{th} binary processes and σ_m and σ'_m are the associated standard deviations. The m events under analysis are related at time t through the associated innovations, but there is no dependence in time among these innovations (V is block-diagonal). In this model, the probability of occurrence of the m^{th} event is equal to

$$p_{m,t} = \Pr(-\epsilon_{m,t} \leq \pi_{m,t} | F_{t-1}) = \Phi(\pi_{m,t}), \ \forall m = \{1, 2, ..., M\}. \tag{3}$$

The objective of this paper is to propose a dynamic multivariate modelling of these M qualitative variables. We specify the dynamics of each stochastic process through its associated index $\pi_{m,t}$. Traditionally, the index only depends on exogenous explanatory variables $x_t = \{x_{1,t}, ..., x_{K,t}\}$, where K is the number of exogenous variables in the model. But in a dynamic model, it can also depend on the past information on the dependent variable. Formally, for a given event m, the dynamics can be introduced in two ways: either through the lagged binary variables $y_{m,t}$, or through the lagged latent variable $y^*_{m,t}$. The first equation of our multivariate dynamic probit model in (1) is hence given by:

$$y^*_{m,t} = \alpha_m + \beta_m x_{t-v} + \sum_{m'=1}^{M} \Delta_{m,m'} y_{m',t-s} + \sum_{m'=1}^{M} \Gamma_{m,m'} y^*_{m',t-s} + \varepsilon_{m,t}, \tag{4}$$

where $\alpha_m \in \mathbb{R}$ is the intercept, $\beta_m = \{\beta_{1,m}, ..., \beta_{K,m}\}$, $\beta_m \in \mathbb{R}^K$ is the vector of parameters associated with the explanatory variables x[1] and $\Delta_{m,m'} \in \mathbb{R}$ and $\Gamma_{m,m'} \in \mathbb{R}$ are the parameters of the predetermined variables giving the dynamics of the m^{th} equation of the model. $s, v > 0$ are the lags associated with the predetermined right-hand-side variables. For $s, v \geq h$ it is possible to obtain directly the h-step-ahead forecast made at time $t-h$. An iterative approach could also be envisaged. However, the computation of the forecasts becomes more burdensome since all the possible paths leading to the occurrence of the M events in h-periods' time must be taken into account (for more details on iterated forecasts in a univariate discrete-choice analysis see Kauppi and Saikkonen, 2008).

Denote by $\theta_m = (\alpha_m; \beta_m; \Delta_{m,m'}; \Gamma_{m,m'})'$ the vector of parameters for equation / event m in (4), with $\theta = (\theta_1'; \theta_2'; ...; \theta_M')'$, that will be used in the estimation of the model (see section 3).

It is clear that our model has the usual VAR-X structure, as it assumes a linear relation between the latent variables $y^*_{m,t}$ and their past. Its dynamics is then enriched by the non-linear relation between the latent variables $y^*_{m,t}$ and the observed binary ones $y_{m,t}$, which in turn depend upon $y^*_{m,t-s}$ (see the second eq. in (1)). In other words, our dynamic probit model differs from a standard VAR in two ways. First, it introduces both a linear and a non-linear dynamics. Indeed, the dynamics of the m^{th} process / event can be modeled by considering that the latent variable $y^*_{m,t}$ depends either on its lagged value via the $\Gamma_{m,m}$ coefficient, or on the past regime (0/1) through $\Delta_{m,m}$.

Second, the analysis of the interdependence, *i.e.,* predictive relationship[2] between the M qualitative variables is more complicated, as it passes through several channels.

1. Unobserved common factors can be taken into account through the contemporaneous dependence of the innovation terms ($\mathbb{E}(\epsilon_m \epsilon_{m'}) = \sigma_{mm'} \neq 0$ for $m \neq m'$).

2. For an event m, the unobservable latent variable $y^*_{m,t}$ depends on past values of other processes $y^*_{m',t-s}$ (where $m \neq m'$), themselves unobservable.

3. The latent variable may depend on past realizations of the other events, *i.e.,* $y_{m',t-s} = 1$. Formally, $y^*_{m,t}$ depends on past values of the observable variable $y_{m',t-s}$, $s > 0$, where $m \neq m'$.

4. It is possible to combine the two previous cases, assuming that $y^*_{m,t}$, depends on both the latent variable $y^*_{m',t-s}$, and past values of the observable variable, $y_{m',t-s}$, for other binary processes ($m \neq m'$).

[1]As generally assumed in discrete-choice forecasting models, the continuous variables x have predictive content v-steps-ahead (see Kauppi and Saikkonen, 2008 for a discussion on this point).

[2]In a conditional model this predictability could also be interpreted as Granger causality.

From this perspective, our model in eq.4 is a multivariate extension of the univariate dynamic probit model recently proposed by Kauppi and Saikkonen (2008).

Another novelty introduced by this non-linear VAR framework relies on the fact that our new specification enables us to compute not only marginal but also joint and conditional probabilities. The traditional marginal probabilities are associated with each binary response, $\Pr(y_m = 1|y_m^*) = \Phi(y_m^*)$, and rely on univariate discrete-choice models. In contrast, joint and conditional probabilities, *i.e.,* $\Pr(y_1 = 1, y_2 = 1, ..., y_M = 1|y^*) = \Phi_M(y^*)$, and $\Pr(y_m = 1|y_{m'}^*) = \Phi_M(y^*)/\Phi_{M-1}(y_{m'}^*)$, for $m, m' \in \{1, 2, ..., M\}$, where Φ and Φ_M represent the univariate and M-variate normal cumulative distribution functions respectively can also be obtained here.

First-order dynamics, *i.e.,* $s = 1$, are most common in empirical applications with binary-dependent indicators that stand for regime-switches, including the empirical example we present in section 4. In vector notation, the process in (4) hence becomes:

$$y_t^* = \alpha + Bx_{t-v} + \Delta y_{t-1} + \Gamma y_{t-1}^* + \epsilon_t, \tag{5}$$

where $\alpha = (\alpha_1, ..., \alpha_m)'$, B is a $M \times K$ matrix, and Δ and Γ are $M \times M$ matrices of parameters. Note that the matrices Δ and Γ summarize useful information about the dynamics of the binary processes, in particular about their persistence and predictability.

One the one hand, as previously discussed, the diagonal terms of Γ specify the persistence of each process. These parameters correspond to a first order autoregressive representation of each latent variable. An increase in the latent variable during a certain period is always transmitted to the next period, hence always increasing the probability of realization of the event (observing a value of 1). The closer these parameters are to 1, the more persistent the processes are. Notice that the diagonal elements of this matrix typically are constrained to be strictly smaller than 1 to exclude the case where the latent variable $y_{m,t}^*$ follows a random walk, which is not an interesting case in this context.

At the same time, the diagonal terms of Δ also deliver information about persistence. Indeed, they indicate to what extent the probability of the occurrence of an event (defined by the m^{th} binary indicator) depends on the regime prevailing the period before. In this situation we observe the existence of threshold effects, as the regime defined by a value of one for the binary process lasts more than one spell only if the latent variable soars sufficiently to exceed the threshold which initiated this regime in the previous period.

Altogether, we can distinguish between a linear persistence of the phenomenon, captured through the diagonal terms of Γ, and a non-linear, threshold-based one, apprehended by the diagonal terms of Δ.

On the other hand, predictability is taken into account in the off-diagonal elements of the two matrices Γ and Δ. This predictability between the M binary stochastic processes play a key role in the analysis of numerous economic events (*e.g.* financial crises, economic cycles). As in the analysis of the persistence of a binary process, both a linear and a non-linear, threshold-effect transmission can be identified. A significant off-diagonal Γ element means that no sooner the latent variable for the m binary indicator soars, then the one for another process, say m', rises. By contrast, a Δ term reveals the presence of causality only if the corresponding latent variable is high enough to impact the occurrence of another binary event.

3 Exact Maximum Likelihood Estimation

The exact maximum likelihood estimator for the multivariate dynamic probit model cannot be obtained as a simple extension of the univariate model. For this reason, the maximum likelihood method is generally considered. Holly, Huguenin and Pelgrin (2009) prove that this estimation method causes bias in the estimation of the correlation coefficients as well as in their standard deviations. Therefore, they advocate the use of exact maximum likelihood estimation. This section deals with this objective.

3.1 The Maximum Likelihood

Following Greene (2002), full information maximum-likelihood (FIML) estimates are obtained by maximizing the log-likelihood $\mathrm{LogL}(Y|Z;\theta,\Omega)$, where θ is the vector of identified parameters and Ω is the covariance matrix.[3] Under the usual regularity conditions[4] (Lesaffre and Kauffmann, 1992), the full information maximum-likelihood is given by the joint density of observed outcomes:

$$\mathrm{L}(y|z,\theta;\Omega)=\prod_{t=1}^{T} L_t(y_t|z_{t-1},\theta;\Omega), \tag{6}$$

where $y_t=(y_{1,t},...,y_{M,t})'$ and $y=[y_1,...,y_T]$. The individual likelihood $L_t(.)$ is given in Lemma 1.

Lemma 1. *The likelihood of observation t is the cumulative density function, evaluated at*

[3] Note that to identify the slope and covariance parameters, we impose that the diagonal elements of Ω to be standardized, *i.e.,* equal to one.

[4] If the parameters θ are estimated while the correlation coefficients are assumed constant, the log-likelihood function is concave. In this case the MLE exists and it is unique. Nevertheless, when θ and ρ are jointly estimated (as in our model), the likelihood function is not (strictly) log-concave as a function of ρ. Thus, the MLE exists only if the log-likelihood is not identically $-\infty$ and $E(z^T z|\rho)$ is upper semi-continuous finite and not identically 0. Furthermore, if no $\theta \neq 0$ fulfills the first order conditions for a maximum, the MLE of (θ,ρ) for the multivariate probit model exists and for each covariance matrix not on the boundary of the definition interval, the MLE is unique.

the vector w_t of a M-variate standardized normal vector with a covariance matrix $Q_t \Omega Q_t$:

$$L_t(y_t|z_{t-1}, \theta; \Omega) = \Pr(y_1 = y_{1,t}, ..., y_M = y_{M,t}) = \Phi_{M,\varepsilon_t}(w_t; Q_t \Omega Q_t), \tag{7}$$

where Q_t is a diagonal matrix whose main diagonal elements are $q_{m,t} = 2y_{m,t} - 1$ and thus depends on the realization or not of the events ($q_{m,t} = 1$ if $y_{m,t} = 1$ and $q_{m,t} = -1$ if $y_{m,t} = 0$, $\forall\ m = \{1, 2, ..., M\}$). Besides, the elements of the vector $w_t = [w_{1,t}, ..., w_{M,t}]$ are given by $w_{m,t} = q_{m,t}\pi_{m,t}$, where $\pi_{m,t}$ is the index associated with the m^{th} binary variable.

For a complete proof of Lemma 1, see Appendix 1. Thus, the FIML estimates are obtained by maximizing the log-likelihood:

$$\text{LogL}(y|z, \theta; \Omega) = \sum_{t}^{T} \text{Log}\Phi_{M,\varepsilon}(w_t; Q_t \Omega Q_t) \tag{8}$$

with respect to θ and Ω.

The main problem with FIML is that it requires the evaluation of higher-order multivariate normal integrals. Existing methods for this purpose are not sufficient to allow accurate and efficient evaluation for more than two variables (see Greene, 2002, page 714). Indeed, Greene (2002) argues that the existing quadrature methods to approximate trivariate or higher-order integrals are far from being exact. To tackle this problem in the case of a probit model, Huguenin, Pelgrin and Holly (2009) decompose the triple integral into simple and double integrals, allowing an Exact Maximum Likelihood Estimation (EML) by means of computing double integrals. They prove that the EML increases the numerical accuracy of both the slope and covariance parameters estimates, which outperform the maximum simulated likelihood method of McFadden (1989) which is generally used for the estimation of multivariate probit models. Here we extend the decomposition proposed by Huguenin, Pelgrin and Holly, (2009) to the case of our multivariate dynamic model so as to obtain a direct approximation of the trivariate normal cumulative distribution function.

The EML log-likelihood function is given by:

$$\text{LogL}(y|z, \theta; \Omega) = \sum_{t=1}^{T} \text{Log}\left[\prod_{m=1}^{M} \Phi(w_{m,t}) + G\right], \tag{9}$$

where $\Phi(w_t)$ is the univariate normal cumulative distribution function of w_t. Indeed, the log-likelihood function depends on the product of the marginal distributions (w_t) and the correction term G which captures the dependence between the m events analyzed.

The maximum likelihood estimators $\{\widehat{\theta}; \widehat{\Omega}\}_{EML}$ are the values of θ and Ω which maximize

(9):

$$\{\widehat{\theta};\widehat{\Omega}\}_{EML} = \text{Arg}\max_{\theta;\Omega}\sum_{m=1}^{M}\text{LogL}(.), \tag{10}$$

with LogL(.) given in (9).

Under the regularity conditions of Lesaffre and Kaufman (1992), the EML estimator of a multivariate probit model exists and is unique. Besides, the estimates $\{\widehat{\theta};\widehat{\Omega}\}_{EML}$ are asymptotically normally distributed and are consistent and efficient estimators of the slope and covariance parameters. It is worth noting that in a correctly specified model for which the error terms are independent across the m equations the EML function corresponds to $\sum_{t=1}^{T}\prod_{m=1}^{M}\Phi(w_{m,t})$, since the probability correction term G in eq. (9) tends toward zero.

3.2 The Empirical Procedure

Most of the empirical applications involving correlated time-series binary data (*e.g.* financial crises, economic cycles, etc.) tackle only two or three such events at a time. Accordingly, henceforth we restrict our attention to the bivariate and trivariate form of the model. This presentation simplifies the exposition of the exact maximum-likelihood estimation method and corresponds to the empirical analysis performed in section 4. Further details are provided in Appendices 1-3.

For the bivariate model, we get

$$\Phi_2(w_t;Q_t\Omega Q_t) = \Phi(w_{1,t})\Phi(w_{2,t})\frac{1}{2\pi}\int_0^{\rho_{12}}\exp\left(-\frac{1}{2}\frac{w_{1,t}^2+w_{2,t}^2-2w_{1,t}w_{2,t}}{1-\lambda_{12}^2}\right)\frac{d\lambda_{12}}{\sqrt{1-\lambda_{12}^2}} \tag{11}$$

and for the trivariate model

$$
\begin{aligned}
\Phi_3(w_t; Q_t\Omega Q_t) &= \prod_{m=1}^{3} \Phi(w_{m,t}) + G \\
&= \Phi(w_{1,t})\Phi(w_{2,t})\Phi(w_{3,t}) \\
&+ \Phi(w_{3,t}) \int_0^{\rho_{12}} \phi_2(w_{1,t}, w_{2,t}; \lambda_{12}) d\lambda_{12} \\
&+ \Phi(w_{2,t}) \int_0^{\rho_{13}} \phi_2(w_{1,t}, w_{3,t}; \lambda_{13}) d\lambda_{13} \\
&+ \Phi(w_{1,t}) \int_0^{\rho_{23}} \phi_2(w_{2,t}, w_{3,t}; \lambda_{23}) d\lambda_{23} \\
&+ \int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{\partial \phi_3(w_t; \lambda_{12}, \lambda_{13}, 0)}{\partial w_{1,t}} d\lambda_{12} d\lambda_{13} \\
&+ \int_0^{\rho_{12}} \int_0^{\rho_{23}} \frac{\partial \phi_3(\dot{w}_t; \lambda_{12}, 0, \lambda_{23})}{\partial w_{2,t}} d\lambda_{12} d\lambda_{23} \\
&+ \int_0^{\rho_{13}} \int_0^{\rho_{23}} \frac{\partial \phi_3(\ddot{w}_t; 0, \lambda_{13}, \lambda_{23})}{\partial w_{3,t}} d\lambda_{13} d\lambda_{23} \\
&+ \int_0^{\rho_{12}} \int_0^{\rho_{13}} \int_0^{\rho_{23}} \frac{\partial^3 \phi_3(\dddot{w}_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{1,t} \partial w_{2,t} \partial w_{3,t}} d\lambda_{12} d\lambda_{13} d\lambda_{23}
\end{aligned}
\tag{12}
$$

where ρ are the non-diagonal elements of the $Q_t\Omega Q_t$ matrix and λ are the non-diagonal elements of a theoretical 2×2 matrix and respectively a 3×3 matrix in which one of the correlation coefficients is null. Moreover, $\dot{w}_t$ is a vector of indices obtained by changing the order of the elements to $(w_{2,t}, w_{3,t}, w_{1,t})$. Similarly $\ddot{w}_t$ corresponds to a vector of indices of the form $(w_{3,t}, w_{1,t}, w_{2,t})$. Finally, $\dddot{w}_t$ corresponds to w_t,$\dot{w}_t$ or $\ddot{w}_t$ respectively, depending on the way the last integral is decomposed. The computation of the last term is not trivial. However, this integral can be decomposed in a non-unique way as follows:

$$
\begin{aligned}
&\int_0^{\rho_{12}}\int_0^{\rho_{13}}\int_0^{\bar{\rho}_{23}} \frac{\partial^3 \phi_3(\dddot{w}_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{1,t} \partial w_{2,t} \partial w_{3,t}} d\lambda_{12} d\lambda_{13} d\lambda_{23} \\
&\qquad = \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\ddot{w}_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{3,t}} d\lambda_{13} d\lambda_{23} - \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\ddot{w}_t; 0, \lambda_{13}, \lambda_{23})}{\partial w_{3,t}} d\lambda_{13} d\lambda_{23} \\
&\qquad = \int_0^{\rho_{12}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\dot{w}_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{2,t}} d\lambda_{12} d\bar{\rho}_{23} - \int_0^{\rho_{12}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\dot{w}_t; \lambda_{12}, 0, \lambda_{23})}{\partial w_{2,t}} d\lambda_{12} d\lambda_{23} \\
&\qquad = \int_0^{\lambda_{12}}\int_0^{\lambda_{13}} \frac{\partial \phi_3(w_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{1,t}} d\lambda_{12} d\lambda_{13} - \int_0^{\rho_{12}}\int_0^{\rho_{13}} \frac{\partial \phi_3(w_t; \lambda_{12}, \lambda_{13}, 0)}{\partial w_{1,t}} d\lambda_{12} d\lambda_{13}.
\end{aligned}
\tag{13}
$$

These finite-range multiple integrals are numerically evaluated by using a Gauss-Legendre Quadrature rule[5] over bounded intervals. In such a context, two possibilities can be considered: whether the likelihood function is directly maximized, or the first order conditions[6] are derived so as to obtain an exact score vector. As stressed by Huguenin, Pelgrin and Holly (2009), the two methods may not lead to the same results if the objective function is not sufficiently smooth. We also tackle the autocorrelation problem induced by some binary time-series variables by considering a Gallant (1987) correction for the covariance matrix of the parameters.

4 Empirical Application

This section aims at implementing the multivariate dynamic probit methodology presented above to a system composed by three types of financial crises, *i.e.,* currency, banking and sovereign debt crises. As historical events have proven, most of the time crises do not remain restricted to a single market, but tend to spill-over into another one. We evaluate the probability of mutation of one type of crisis into another one. After a short data description, we estimate bivariate models that include only currency and banking crises, as previously done in a panel framework by Glick and Hutchison (1999). This constitutes a benchmark for the second part where the sovereign debt crises are included in the system.

[5]Details about this quadrature are available in Appendix 2.

[6]The score vector of the trivariate probit model is presented in Appendix 3.

4.1 The Database

Monthly macroeconomic indicators expressed in US dollars covering the period from January 1985 to June 2010 have been extracted for 17 emerging countries[7] from the IMF-IFS database as well as from the national banks of the countries under analysis via Datastream. [8]

The three types of crises are identified by relying on popular measures, generally considered in the literature.[9] As in Lestano and Jacobs (2004) and Candelon et al. (2012), we use a modified version of the pressure index proposed by Kaminski et al.(1998) to date currency crises. Besides, the money market pressure index proposed by Hagen and Ho (2004) is considered for the monthly identification of banking crises, while the non-parametric method based on sovereign debt spread introduced by Pescatori and Sy (2007) is used to detect debt-servicing difficulties. To this aim, the government bond returns are obtained via the JPMorgan EMDB database.

We have selected the main leading indicators used in the literature for the three types of crises that we analyze (see Candelon et al., 2012, Jacobs et al., 2003, Glick and Hutchison, 1999, Hagen and Ho, 2004, Pescatori and Sy, 2007), namely, the one-year growth rate of international reserves, the growth rate of M2 to reserves ratio, one-year growth of domestic credit over GDP ratio, one-year growth of domestic credit, one-year growth of GDP, government deficit, debt service ratio and external debt ratio.

Remarks

1. As in Kumar (2003), we dampen the magnitude of every variable using the formula: $f(x_t) = sign(x_t)\log(1 + |x_t|)$, so as to reduce the impact of extreme values.[10]

2. It should also be noted that the entire sample is used for the identification of currency and banking crises, while the identification of debt crises is realized by using data from December 1997 (See Table 1) since the CDS spread series used for the identification of sovereign debt crises are not available before 1997 in the JPMorgan EMDB database. Consequently our empirical analysis will consist of two parts, the first one analyzing the case of twin crises (currency and banking) for which the entire database can be used, while the second part focuses on the interactions between the three types of crises and is thus based on data from 1997 onwards. The data sample actually used for each of the 17 countries and the two types of analyses is available in Table 1.

[7] Argentina, Brazil, Chile, Colombia, Ecuador, Egypt, El Salvador, Indonesia, Lebanon, Malaysia, Mexico, Panama, Peru, Philippines, South Africa, Turkey and Venezuela.

[8] We choose not to include any European country, as *i*) only few of them have suffered from the three types of crises and *ii*) if this is the case it corresponds to a single episode: the recent turmoil.

[9] For a more detailed description of the three dating methods see the Candelon et al. 2011.

[10] Missing values of the series are replaced by cubic spline interpolation.

3. We only retain the countries for which the percentage of crisis periods is higher than 5% (See Table 2).[11]

4. As mentioned in section 2, there are three dynamic multivariate specifications that can be used. As shown by Candelon et al. (2010), the dynamic model including the lagged binary variable seems to be the best choice according to model selection using the Schwarz information criterion. Relying on the same univariate results, we fix the lag-number v of the continuous variables x to one. Since we cannot expect one type of crisis to affect the probability of another type of crisis immediately, in the empirical application we allow for response lags of 3, 6 and respectively 12 months for the bivariate models and of 3 or 6 months for the trivariate models[12]. Therefore, for each type of crisis we construct a lagged variable $y_{m,t-l}$ which takes the value of one if there was crisis in the past l periods or at time t, and the value of 0 otherwise:

$$y_{m,t-l} = \begin{cases} 1, & \text{if } \sum_{j=0}^{l} y_{m,t-j} > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

5. The significance of the parameters of each model is tested by using simple t-statistics based on robust estimates of standard-errors (which rely on a Gallant kernel, as in Kauppi and Saikkonen, 2008). A special attention is given to the interpretation of cross-effects which stand for the transmission channels of the shocks/crisis. Besides, the joint null of zero contemporaneous correlations between crises is tested using a log-likelihood ratio test for the trivariate models.

4.2 Bivariate Analysis

Along the lines of Kaminsky et al. (1998) it is possible to find a large number of explanatory variables that may signal the occurrence of a crisis. Nevertheless, Candelon et al. (2010) showed that a univariate dynamic probit model presents the advantage of yielding plausible results while being fairly parsimoniously parametrized. Indeed, a large part of the information is integrated either in the past state variable or in the lagged latent variable and thus, only a few explanatory variables turn out to be significant. Therefore, we consider the

[11] Argentina, Chile, Ecuador, Egypt, Indonesia, Lebanon, Mexico, South Africa and Venezuela are included in the bivariate analysis, whereas a trivariate model is specified for Ecuador and South Africa. Since the threshold has been arbitrarily set to 5%, we have also checked the borderline countries, like Colombia or Turkey in the bivariate analysis and Egypt in the trivariate analysis respectively, and similar results have been obtained.

[12] A 12 months lag is not used in the case of trivariate models since it would significantly reduce the already small number of observations we have at our disposal.

first lag of the four explanatory variables which are significant in Candelon et al. (2010), *i.e.,* one-year growth of international reserves, one-year growth of M2 to reserves for currency crises as well as one-year growth of domestic credit over GDP and one-year growth of domestic credit for banking crises, resulting in four different specifications including one explanatory variable for each type of crisis. Three different lags (3 months, 6 months and 12 months) are considered for the lagged binary variable $y_{m,t-l}$. The dynamic probit model is estimated country-by-country using the exact maximum likelihood. [13] This model is indeed a simplification as contagion (or spill-overs) from one country to another are not taken into account. A panel version of the model would lead to several problems. First, as shown by Berg et al. (2008) heterogeneity due to country specificities would have to be accounted for. Second, the estimation of a fixed effect panel would be biased without a correction on the score vector.[14] Third, in a country by country analysis contagion has to be ignored. For all these reasons, we consider this extension to be beyond the scope of this paper and leave it for future research.

Each model is estimated via maximum-likelihood, the bivariate normal cumulative distribution function being approximated using the Gauss-Legendre quadrature, as proposed by Huguenin, Pelgrin and Holly, (2009). However, the quadrature specified in Matlab by default, *i.e.,* the adaptive Simpson quadrature, has been considered as a benchmark.

Information criterion BIC is used to identify the best model for each country; the specification with the lagged binary variable turns out to be preferred. Lag lengths are determined similarly. It is worth stressing that the results are generally robust to the choice of explanatory variables and even to the choice of lags.

A summary of the results for the selected models is given in Table 3.

insert Table 3

First of all, it seems that most of the models exhibit dynamics, whatever the lag used to construct the 'past crisis' variable is. This result confirms the findings of Candelon et al. (2010) and Bussière (2007), showing that crises exhibit a regime dependence: if the country is proven to be more vulnerable than investors had initially thought, investors will start withdrawing their investments, thus increasing the probability of a new crisis. More precisely, most of the countries are found to have experienced banking and currency crises, with a significant autoregressive coefficient, *i.e.,* the crisis variable depends on its own past, *e.g.* Argentina, Egypt, Lebanon, Mexico, South Africa, Venezuela. Besides, only for a small number of cases, only one of the two types of crises is best reproduced by a dynamic model (currency crises in Chile (3 and 12 months), Mexico (6 and 12 months); banking crises in

[13] Initial conditions are introduced as given by the univariate static probit.

[14] See Candelon et al., (2010) for a discussion about this point.

Argentina (6 and 12 months), Ecuador, Lebanon (6 months), South Africa (12 months) and Venezuela (12 months)). Actually, in Chile a past currency crisis has only a short term positive impact on the emergence of another currency crisis, whereas a banking crisis has just a long term effect on the probability of occurrence of another banking crisis. Mexico, however, seems to be more prone to recurring currency crises than banking crises as the former type of crisis has a long-term impact on the probability of experiencing a new crisis, whereas the latter has a positive effect only in the short run. On the contrary, for Argentina, South Africa and Venezuela the impact of past banking crises on currency crises is longer (up to one year) as opposed to that of past currency crises on banking ones (up to three and six months, respectively).

Second, for the majority of the countries (Argentina, Chile, Lebanon, Mexico and Venezuela), currency and banking crises are interconnected. This link between crises can take two forms. On the one hand, a certain type of crisis increases (or diminishes) the probability of occurrence of the other type of crisis. This strong link between banking to currency crisis was emphasized by Glick and Hutchinson (1999) within a panel framework. Nevertheless, there is no reason for the transmission of shocks to be symmetric. Indeed, our country per country analysis reveals that for some countries like Argentina (3 and 6 months) a banking crisis in the past months increased the probability of a currency crisis at time t. At the same time, a banking crisis in Chile in the last 12 months reduced the probability of experiencing a currency crisis. Conversely, a currency crisis in Egypt and in Lebanon (3 months) diminished the probability of a banking crisis.

On the other hand, crisis can be contemporaneously positively correlated. This feature seems to be very stable across models (independent of the lag used). The only exceptions are Egypt and Lebanon, for which there is no instantaneous correlation in the model with 3-months lagged binary variables and Mexico, for which such a correlation appears only for the 12-months lag. To sum up, but for Egypt, all countries are characterized by a positive instantaneous correlation between currency and banking crises variables, corroborating the previous findings of Glick and Hutchinson (1999).

Third, the macroeconomic variables are rarely significant.[15] These results corroborate our previous findings (see Candelon et al. 2010): the lag variable captures most of the information summarized by the exogenous variables. Furthermore, when the coefficients of the exogenous variable are significant, they have the expected sign (an increase in the growth of international reserves diminishes the probability of a crisis in the next periods, while a surprise in the rest of indicators soars the probability of a crisis).

To summarize, these results confirm the presence of interaction between the banking and currency crises. The twin crisis phenomenon is thus confirmed empirically. Our findings are

[15] These results are available upon request.

also robust to the quadrature choice and the lags considered when constructing the dynamic binary variables.

4.3 Trivariate Analysis

But is it really enough to look at two crises only? This subsection extends the previous analysis to the trivariate case by modeling simultaneously the occurrence of currency, banking and debt crises. However, only two countries experienced these three events during a sufficiently long period. Ecuador presents for our sample an *ex-post* probability larger than 5% for each type of crisis. Such a result is not surprising if one remembers that Ecuador faced a strong financial turmoil in the late 1990, affecting first the banking sector,[16] then the Sucre[17], and the government budget. Jacone (2004) showed that institutional weaknesses, rigidities in public finances, and high financial dollarization have amplified this crisis. South Africa constitutes a borderline case as the sovereign debt crisis probability is slightly below 5%.

Each of the models is estimated for these countries using both the methodology proposed by Huguenin et al. (2009) based on the Gauss-Legendre quadrature and the direct approximation of a triple integral based on the adaptive Simpson quadrature that Matlab uses by default. Similar results are obtained for the two methods.[18] However, the latter implies a significant gain in time without any loss in accuracy proving that recently developed quadrature methods are good approximations of the normal cumulative distribution function. Besides, 6 and 12 month-lags of the crisis variable are considered.

insert Table 4

In the case of Ecuador, the results corroborate our bivariate findings: the banking crises are persistent, while currency crises are not. Nevertheless, it is clear that the bivariate model is misspecified, since it cannot capture the impact of a banking crisis on the occurrence of a currency crisis when using the 6-months lagged binary variables to account for the dynamics of these phenomena (see Table 4).

Moreover, the trivariate model turns out to be more parsimonious in terms of parameters to be estimated since the latent variable of past debt crisis has a positive effect on the probability of both a currency and a debt crises occurring. We also observe that the contemporaneous correlation matrix is diagonal, ruling out common shocks. Crises in Ecuador turn out to be exclusively driven by transmission channels, as in the late 1990, when the banking distress was diffused to the currency and the government budget.

[16] 16 out of the 40 banks existing in 1997 faced liquidity problems.

[17] The Ecuadorian currency was replaced by the U.S. dollar on March, 13, 2000.

[18] The results for Ecuador when considering a 6-months lag have been obtained with Matlab's quadrature since the model based on the Gauss-Legendre Quadrature did not converge.

In the case of South-Africa, both currency and debt crises are dynamic. There is no evidence of causality between the different types of crises, but significant contemporaneous correlation. It highlights the fact that contrary to Ecuador, South African crises did not mutate but they originated from a common shock. It is worth noting that in the sensitivity analyses performed the results are found to be robust to the choice of macroeconomic variables and the use of different lags for the past crisis variables.

4.4 Further results

To grasp better the properties of the models estimated and selected, a conditional probability analysis is provided. For sake of space, we only report the results obtained for Ecuador.[19]

First, Figure 1 reports the conditional probabilities for each type of crisis obtained from both the bi- and trivariate models considering a forecast horizon of 3 and 6 months. To allow a fair comparison, both models are estimated from the same sample, *i.e.,* from 1997 onwards. It goes without saying that the bivariate model does not provide any conditional probabilities for sovereign debt crisis.

It turns out that in this application the trivariate model yields better results than the bivariate one whatever the forecast horizon is, *i.e.,* the conditional probabilities issued from the trivariate model are higher than those obtained from the bivariate model during observed crisis periods, while they appear to be similar for calm periods. An evaluation strategy of these series of probabilities along the lines of Candelon et al. 2012 (including the percentage of correctly identified crisis and calm periods and evaluation criteria) supports this inference.[20] Besides, the conditional probabilities obtained from the trivariate model do not immediately collapse after the occurrence of the crisis, which is the case for the bivariate model. It stresses hence the vulnerability of the economy after the exit from a turmoil in particular if it affects the foreign exchange market.

Overall, the conditional probability analysis stresses the superiority of the trivariate model to scrutinize the diffusion mechanisms that occurred in Ecuador after the banking crisis in 1998. Strong interactions between the three types of crises are clearly present in particular between banking and other crises. From a more general perspective, we show that a crisis model should take into account the whole sequence of crises to be accurate.

[19] For South Africa, crisis mutation is exclusively driven by the contemporaneous correlation matrix as indicated in Table 4. Otherwise we can see that currency and sovereign debt crises are more persistent than banking ones. All figures are available from the authors upon request.

[20] These results are available upon request.

5 Conclusion

This paper proposes a multivariate dynamic probit model to produce dynamic forecasts of binary time-series correlated variables. It is easy to implement and relies on an exact maximum-likelihood estimation approach, hence providing a solution to the problem generally encountered in the estimation of multivariate probit models. For this, higher-order integrals are decomposed into lower-order finite-range multiple integrals, that are subsequently evaluated using quadrature-rules over bounded intervals. Our framework allows us to apprehend dynamics and predictability in several ways, namely through the lagged binary indicators or the lagged latent variables associated with the qualitative variables.

To illustrate this methodology we consider an application to three types of financial crises for a sample of emerging countries. We investigate the potential mutations of one crisis into another within each country and find that in the bivariate case that one crisis makes the other more likely in expectation. More importantly, for the two countries, Ecuador and South Africa, which suffered from all the three types of crises, the trivariate model turns out to be the best performing in term of conditional probabilities and comprehension of the reasons why a specific crisis mutates to another one: this can be due to either common shocks (as in South Africa) or to a strong predictive relationship (as in Ecuador).

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Appendix 1: Proof of lemma 1

By definition, the likelihood of observation t is given by:

$$\begin{aligned} L_t(y_t|z_{t-1},\theta;\Omega) &= \Pr((-q_{1,t}y^*_{1,t} \leq 0), ..., (-q_{M,t}y^*_{M,t} \leq 0)) \\ &= \Pr(-q_{1,t}\varepsilon_{1,t} \leq q_{1,t}\pi_{1,t}, ..., -q_{M,t}\pi_{M,t} \leq q_{M,t}\pi_{M,t}) \\ &= \Phi_{M,-Q_t\varepsilon_t}(w_t|0_M;\Omega) \\ &= \int_{-\infty}^{w_{M,t}} ... \int_{-\infty}^{w_{1,t}} \phi_{M,-Q_t\varepsilon_t}(Q_t\varepsilon_t,\Omega) \prod_{m=1}^{M} d\varepsilon_{m,t}. \end{aligned}$$

Since each $q_{m,t}$ takes only the values $\{-1, 1\}$, it is straightforward to show that $Q_t = Q_t^{-1}$ and $|Q_t\Omega Q_t| = |\Omega|$. Moreover, the density of an M-variate standardized normal vector $-Q_t\varepsilon_t$ with covariance matrix Ω may be re-written as the density of an M-variate standardized normal vector ε_t with variance-covariance matrix $Q_t\Omega Q_t$:

$$\begin{aligned} \phi_{M,-Q_t\varepsilon_t}(Q_t\varepsilon_t;\Omega) &= |2\pi\Omega|^{\frac{-1}{2}} \exp\left\{\frac{-1}{2}(-Q_t\varepsilon_t)'\Omega^{-1}(-Q_t\varepsilon_t)\right\} \\ &= |2\pi(Q_t\Omega Q_t)|^{\frac{-1}{2}} \exp\left\{\frac{-1}{2}\varepsilon_t'(Q_t\Omega Q_t)^{-1}\varepsilon_t\right\} \\ &= \phi_{M,\varepsilon_t}(\varepsilon_t;Q_t\Omega Q_t). \end{aligned}$$

Therefore, the likelihood of observation t is given by:

$$\begin{aligned} L_t(y_t|Z_{t-1},\theta;\Omega) &= \int_{-\infty}^{q_{M,t}\pi_{M,t}} ... \int_{-\infty}^{q_{1,t}\pi_{1,t}} \phi_{M,\varepsilon_t}(\varepsilon_t;Q_t\Omega Q_t) \prod_{m=1}^{M} d\varepsilon_{m,t} \\ &= \Phi_{M,\varepsilon_t}(Q_t\pi_t;Q_t\Omega Q_t). \end{aligned}$$

Appendix 2: The Gauss-Legendre Quadrature rule

The goal of the Gauss-Legendre Quadrature rule is to provide an approximation of the following integral:

$$\int_a^b f(x)dx. \tag{15}$$

In a first step, the bounds of the integral must be changed from $[a, b]$ to [-1,1] before applying the Gaussian Quadrature rule:

$$\int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^{1} f(z)dz, \tag{16}$$

where $z_i = \frac{b-a}{2}abs_i + \frac{b+a}{2}$ and the nodes $abs_i,\ i \in \{1, 2, ..., p\}$ are zeros of the Legendre polynomial $P_p(abs)$.

Definition 1. *Then, the standard p-point Gauss-Legendre quadrature rule over a bounded arbitrary interval [a,b] is given by the following approximation*:

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\sum_{i=1}^{p} v_i f(z_i) + R_p, \tag{17}$$

where v_i are the corresponding weights, $v_i = \frac{2}{(1-abs_i^2)\left(\frac{\partial P_p(abs)}{\partial abs}|_{abs_i}\right)^2}$, $\sum_{i=1}^{p} v_i = 2$, and R_p is the error term, $R_p = Q_p f^{(2p)}(\xi) = \frac{(b-a)^{2p+1}(p!)^4}{(2p+1)(2p!)^3} f^{2p}(\xi)$, with $\xi \in (a, b)$.

Appendix 3: The EML score vector for a trivariate dynamic probit model

For ease of notation, let us denote by $\rho_{i,j}$, $i, j = \{1, 2, 3\}$, $i \neq j$ the correlation coefficients associated to the Ω matrix. The likelihood of observation t may be written as:

$$\begin{aligned}
P_t &= \Phi_3(q_1\pi_{1,t}, q_2\pi_{2,t}, q_3\pi_{3,t}, q_1q_2\rho_{12}, q_1q_3\rho_{13}, q_2q_3\rho_{23}) \\
&= \Phi(q_1\pi_{1,t})\Phi(q_2\pi_{2,t})\Phi(q_3\pi_{3,t}) \\
&+ q_1q_2\Phi(q_3\pi_{3,t})\Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\
&+ q_1q_3\Phi(q_2\pi_{2,t})\Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\
&+ q_2q_3\Phi(q_1\pi_{1,t})\Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\
&+ q_1q_2q_3\Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
&+ q_1q_2q_3\Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
&+ q_1q_2q_3\Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{18}$$

where

$$\Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) = \int_0^{\rho_{12}} \phi_2(\pi_{1,t}, \pi_{2,t}, \lambda_{12})d\lambda_{12}$$

$$\Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) = \int_0^{\rho_{13}} \phi_2(\pi_{1,t}, \pi_{3,t}, \lambda_{13})d\lambda_{13}$$

$$\Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) = \int_0^{\rho_{23}} \phi_2(\pi_{2,t}, \pi_{3,t}, \lambda_{23})d\lambda_{23},$$

and

$$\Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) = \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{-\pi_{3,t} + \lambda_{13}\pi_{1,t} + \lambda_{23}\pi_{2,t}}{1 - \lambda_{13}^2 - \lambda_{23}^2}\phi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \lambda_{13}, \lambda_{23}, 0)d\lambda_{13}d\lambda_{23}$$

$$\Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) = \int_0^{\rho_{23}}\int_0^{\rho_{12}} \frac{-\pi_{2,t} + \lambda_{23}\pi_{3,t} + \lambda_{12}\pi_{1,t}}{1 - \lambda_{23}^2 - \lambda_{12}^2}\phi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \lambda_{23}, \lambda_{12}, 0)d\lambda_{23}d\lambda_{12}$$

$$\begin{aligned}
\Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) = &\int_0^{\rho_{12}}\int_0^{\rho_{13}} \frac{-(1-\rho_{23}^2)\pi_{1,t} + (\lambda_{12} - \lambda_{13}\rho_{23})\pi_{2,t} + (\lambda_{13} - \lambda_{12}\rho_{23})\pi_{3,t}}{1 - \lambda_{12}^2 - \lambda_{13}^2 - \rho_{23}^2 + 2\lambda_{12}\lambda_{13}\rho_{23}} \\
&\times \phi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23})d\lambda_{12}d\lambda_{23}.
\end{aligned}$$

Therefore, the first order partial derivatives can be obtained as follows:

$$\begin{aligned}\frac{\partial}{\partial \pi_1} P_t = {} & q_1 \phi(\pi_{1,t}) \Phi(q_2 \pi_{2,t}) \Phi(q_3 \pi_{3,t}) \\ & + q_1 q_2 \Phi(q_3 \pi_{3,t}) \frac{\partial}{\partial \pi_1} \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\ & + q_1 q_3 \Phi(q_2 \pi_{2,t}) \frac{\partial}{\partial \pi_1} \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\ & + q_1 q_2 q_3 \phi(\pi_{1,t}) \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial}{\partial \pi_2} P_t = {} & q_2 \phi(\pi_{2,t}) \Phi(q_1 \pi_{1,t}) \Phi(q_3 \pi_{3,t}) \\ & + q_1 q_2 \Phi(q_3 \pi_{3,t}) \frac{\partial}{\partial \pi_2} \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\ & + q_1 q_2 q_3 \phi(\pi_{2,t}) \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\ & + q_2 q_3 \Phi(q_1 \pi_{1,t}) \frac{\partial}{\partial \pi_2} \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),\end{aligned} \tag{20}$$

$$
\begin{aligned}
\frac{\partial}{\partial \pi_3} P_t = {} & q_1 \phi(\pi_{3,t}) \Phi(q_1 \pi_{1,t}) \Phi(q_2 \pi_{2,t}) \\
& + q_1 q_2 q_3 \phi(\pi_{3,t}) \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\
& + q_1 q_3 \Phi(q_2 \pi_{2,t}) \frac{\partial}{\partial \pi_3} \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\
& + q_2 q_3 \Phi(q_1 \pi_{1,t}) \frac{\partial}{\partial \pi_3} \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{12}} P_t = {} & q_1 q_2 \Phi(q_3 \pi_{3,t}) \frac{\partial}{\partial \rho_{12}} \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{12}} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{12}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{13}} P_t = {} & q_1 q_3 \Phi(q_2 \pi_{2,t}) \frac{\partial}{\partial \rho_{13}} \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{13}} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{13}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{23}} P_t = {} & q_2 q_3 \Phi(q_1 \pi_{1,t}) \frac{\partial}{\partial \rho_{23}} \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{24}
$$

where

$$\frac{\partial}{\partial \pi_1}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{23}}\int_0^{\rho_{13}} \frac{\partial}{\partial \lambda_{13}}\phi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\lambda_{23},0)d\lambda_{13}\lambda_{23}$$
$$= \int_0^{\rho_{23}} \phi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\lambda_{23},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \pi_2}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{\partial}{\partial \lambda_{23}}\phi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\lambda_{23},0)d\lambda_{23}\lambda_{13}$$
$$= \int_0^{\rho_{13}} \phi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\rho_{23},0)d\lambda_{13},$$

$$\frac{\partial}{\partial \pi_3}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{13}}\int_0^{\rho_{23}} [(\pi_{3,t}-\lambda_{13}\pi_{1,t}-\lambda_{23}\pi_{2,t})^2-(1-\lambda_{13}^2-\lambda_{23}^2)]$$
$$\times \frac{1}{(1-\lambda_{13}^2-\lambda_{23}^2)^2}\phi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\lambda_{23},0)d\lambda_{13}d\lambda_{23},$$

$$\frac{\partial}{\partial \rho_{13}}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{23}} \frac{-\pi_{3,t}+\rho_{13}\pi_{3,t}+\lambda_{23}\pi_{2,t}}{1-\rho_{13}^2-\lambda_{23}^2}\phi_3(\pi_3,\pi_1,\pi_2,\rho_{13},\lambda_{23},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \rho_{23}}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{13}} \frac{-\pi_{3,t}+\lambda_{13}\pi_{3,t}+\rho_{23}\pi_{2,t}}{1-\lambda_{13}^2-\rho_{23}^2}\phi_3(\pi_3,\pi_1,\pi_2,\lambda_{13},\rho_{23},0)d\lambda_{13},$$

$$\frac{\partial}{\partial \pi_1}\Psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{23}}\int_0^{\rho_{12}} \frac{\partial}{\partial \lambda_{12}}\phi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\lambda_{23},\lambda_{12},0)d\lambda_{12}\lambda_{23}$$
$$= \int_0^{\rho_{23}} \phi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t,},\lambda_{23},\rho_{12},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \pi_2}\Psi_3(\pi_{2,t},\pi_{3,t,},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{23}}\int_0^{\rho_{12}} [(\pi_{2,t}-\lambda_{23}\pi_{3,t}-\lambda_{12}\pi_{1,t})^2-(1-\lambda_{23}^2-\lambda_{12}^2)]$$
$$\times \frac{1}{(1-\lambda_{23}^2-\lambda_{12}^2)^2}\phi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\lambda_{23},\lambda_{12},0)d\lambda_{23}d\lambda_{12},$$

$$\frac{\partial}{\partial \pi_3}\Psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{12}}\int_0^{\rho_{23}} \frac{\partial}{\partial \lambda_{23}}\phi_3(\pi_{2,t},\pi_{3,t,},\pi_{1,t},\lambda_{23},\lambda_{12},0)d\lambda_{23}\lambda_{12}$$
$$= \int_0^{\rho_{12}} \phi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\lambda_{12},0)d\lambda_{12},$$

$$\frac{\partial}{\partial \rho_{12}}\Psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{23}} \frac{-\pi_{2,t}+\lambda_{23}\pi_{3,t}+\rho_{12}\pi_{1,t}}{1-\lambda_{23}^2-\rho_{12}^2}\phi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\lambda_{23},\rho_{12},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \rho_{23}}\Psi_3(\pi_{2,t},,\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{12}} \frac{-\pi_{2,t}+\rho_{23}\pi_{3,t}+\lambda_{12}\pi_{1,t}}{1-\rho_{23}^2-\lambda_{12}^2}\phi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\lambda_{12},0)d\lambda_{12},$$

$$\begin{aligned}\frac{\partial}{\partial \pi_1}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{12}}\int_0^{\rho_{13}}\{[(1-\rho_{23}^2)\pi_{1,t}-(\lambda_{12}-\lambda_{13}\rho_{23})\pi_{2,t}-(\lambda_{13}-\lambda_{12}\lambda_{23})\pi_{3,t}]^2 \\ &-(1-\rho_{23}^2)(1-\lambda_{12}^2-\lambda 13^3-\rho_{23}^2+2\lambda_{12}\lambda_{13}\rho_{23})\}\times \\ &\frac{1}{(1-\lambda_{12}^2-\lambda_{13}^2-\rho_{23}^2+2\lambda_{12}\lambda_{13}\rho_{23})^2}\phi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\lambda_{12},\lambda_{13},\rho_{23})d\lambda_{12}d\lambda_{13},\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \pi_2}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{13}}\int_0^{\rho_{12}}\frac{\partial}{\partial\lambda_{12}}\phi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},,\lambda_{12},\lambda_{13},\rho_{23})d\lambda_{12}\lambda_{13} \\ &= \int_0^{\rho_{13}}\phi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\lambda_{13},\rho_{23})d\lambda_{13},\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \pi_3}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{12}}\int_0^{\rho_{13}}\frac{\partial}{\partial\lambda_{13}}\phi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\lambda_{12},\lambda_{13},\rho_{23})d\lambda_{13}\lambda_{12} \\ &= \int_0^{\rho_{12}}\phi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\lambda_{12},\rho_{13},\rho_{23})d\lambda_{12},\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \rho_{12}}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{13}}\frac{(1-\rho_{23}^2)\pi_{1,t}+(\rho_{12}-\lambda_{13}\rho_{23})\pi_{2,t}+(\lambda_{13}-\rho_{12}\rho_{23})\pi_{3,t}}{1-\rho_{12}^2-\lambda_{13}^2-\rho_{23}^2+2\rho_{12}\lambda_{13}\rho_{23}} \\ &\times\phi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\lambda_{13},\rho_{23}d\lambda_{13}),\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \rho_{13}}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{12}}\frac{(1-\rho_{23}^2)\pi_{1,t}+(\lambda_{12}-\rho_{13}\rho_{23})\pi_{2,t}+(\rho_{13}-\lambda_{12}\rho_{23})\pi_{3,t}}{1-\lambda_{12}^2-\rho_{13}^2-\rho_{23}^2+2\lambda_{12}\rho_{13}\rho_{23}} \\ &\times\phi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\lambda_{12},\rho_{13},\rho_{23}d\lambda_{12}),\end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) &= \int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{\partial^2}{\partial \pi_{2,t} \partial \lambda_{13}} \phi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{12} d\lambda_{13} \\
&= \int_0^{\rho_{12}} \frac{-(1-\rho_{13}^2)\pi_{2,t} + (\lambda_{12} - \rho_{13}\rho_{23})\pi_{1,t} + (\rho_{23} - \lambda_{12}\rho_{13})\pi_{3,t}}{1 - \lambda_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\lambda_{12}\rho_{13}\rho_{23}} \\
&\times \phi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \rho_{13}, \rho_{23}) d\lambda_{12} \\
&= \int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{\partial^2}{\partial \pi_{3,t} \partial \lambda_{12}} \phi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{12} d\lambda_{13} \\
&= \int_0^{\rho_{13}} \frac{-(1-\rho_{12}^2)\pi_{3,t} + (\lambda_{13} - \rho_{12}\rho_{23})\pi_{1,t} + (\rho_{23} - \rho_{12}\lambda_{13})\pi_{2,t}}{1 - \rho_{12}^2 - \lambda_{13}^2 - \rho_{23}^2 + 2\rho_{12}\lambda_{13}\rho_{23}} \\
&\times \phi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \lambda_{13}, \rho_{23}) d\lambda_{13}.
\end{aligned}
$$

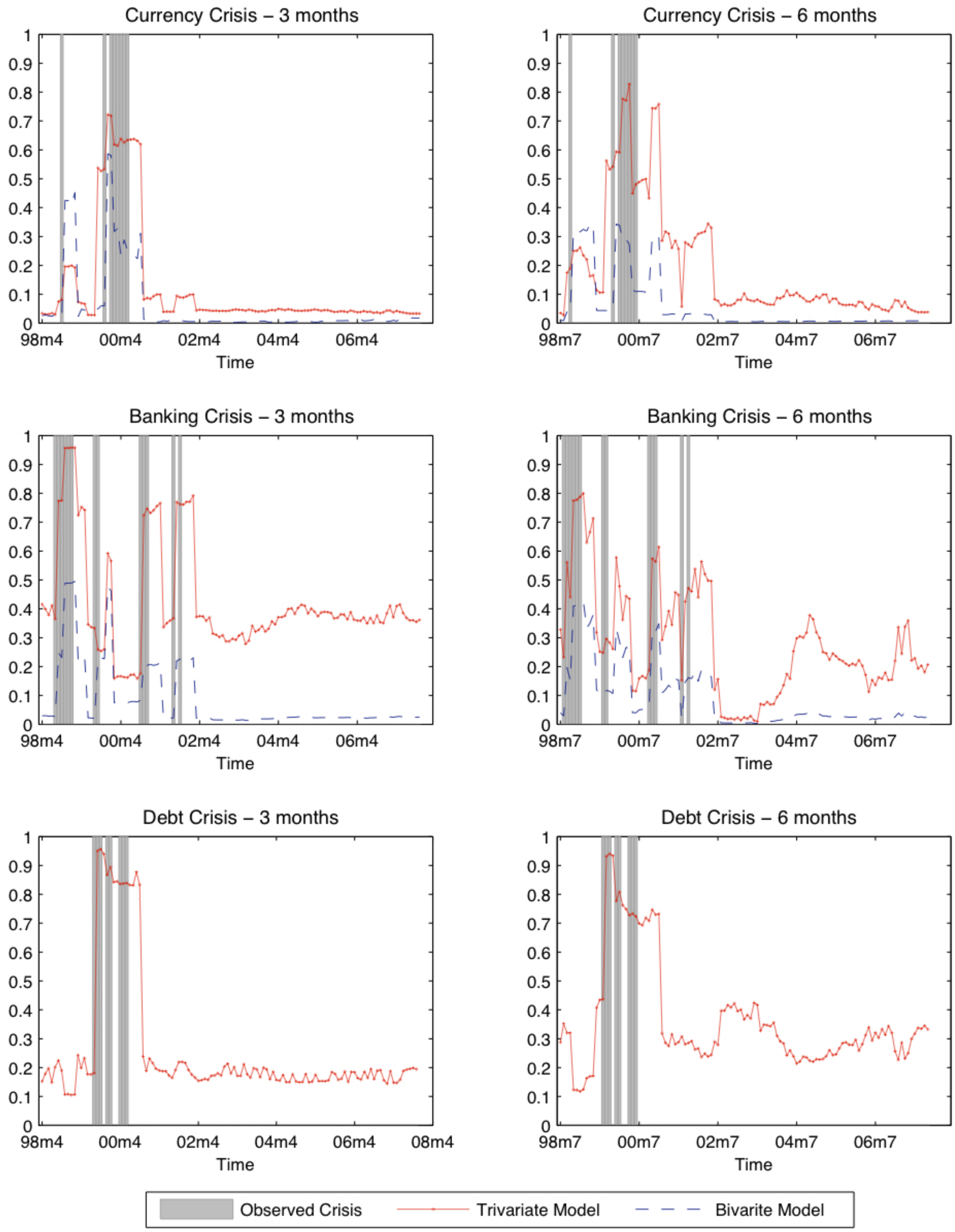

Figure 1: Conditional crisis probabilities - Ecuador

Note: Probabilities at time t are calculated including observed information prior 3 or 6 months.

Table 1: Database

Country	Bivariate model	Trivariate model
Argentina	February 1988 - May 2010	December 1997 - May 2010
Brazil	September 1990 - May 2010	December 1997 - May 2010
Chile	January 1989 - May 2009	May 1999 - May 2010
Colombia	February 1986 - August 2009	December 1997 - August 2009
Ecuador	January 1994 - November 2007	December 1997 - November 2007
Egypt	February 1986 - June 2009	July 2001 - June 2009
El Salvador	January 1991 - November 2008	April 2002 - November 2008
Indonesia	January 1989 - August 2009	May 2004 - August 2009
Lebanon	January 1989 - April 2010	April 1998 - April 2010
Malaysia	January 1988 - March 2010	December 1997 - March 2010
Mexico	January 1988 - May 2010	December 1997 - May 2010
Peru	January 1990 - May 2010	December 1997 - May 2010
Philippines	January 1995 - February 2008	December 1997 - February 2008
South Africa	January 1988 - August 2009	December 1997 - August 2009
Turkey	January 1988 - May 2010	December 1997 - May 2010
Venezuela	February 1986 - November 2009	December 1997 - November 2009

Note: Data sample.

Table 2: Percentage of crisis periods

	Bivariate model		Trivariate model		
	Currency crisis	Banking crisis	Currency crisis	Banking crisis	Debt crisis
Argentina	**5.13**	**8.90**	4.00	**6.67**	**10.0**
Brazil	3.77	**7.19**	0.00	3.33	2.67
Chile	**6.07**	**10.0**	**5.79**	**5.79**	3.31
Colombia	4.95	**9.90**	**9.22**	**12.8**	0.00
Ecuador	**5.73**	**9.93**	**6.67**	**10.8**	**6.67**
Egypt	**6.76**	**9.96**	4.17	**7.30**	**7.30**
El Salvador	3.65	**9.85**	0.00	0.00	2.50
Indonesia	5.30	**9.90**	0.00	**14.0**	**6.25**
Lebanon	**9.62**	**9.96**	1.38	**8.97**	2.76
Malaysia	3.10	**10.0**	4.05	**6.08**	4.73
Mexico	**6.50**	**9.93**	0.00	**9.33**	0.00
Panama	0.00	**9.89**	0.00	**6.38**	0.00
Peru	4.45	**8.22**	0.00	**10.7**	0.00
Phillipines	4.90	**9.80**	**5.69**	**6.50**	3.25
South Africa	**6.71**	**9.89**	**7.09**	**7.80**	4.26
Turkey	4.80	**8.56**	4.00	**6.67**	0.00
Venezuela	**7.33**	**10.1**	4.17	**7.64**	2.78

Note: The entries represent the proportion of crises period over the whole sample. It is indicated in bold as it exceeds 5%.

Table 3: Bivariate Analysis

Country		3 months		6 months		12 months	
		Δ	Ω	Δ	Ω	Δ	Ω
Argentina	currency banking	$\begin{bmatrix} + & + \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & + \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$
Chile	currency banking	$\begin{bmatrix} + & . \\ . & . \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & . \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & - \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$
Ecuador	currency banking	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$
Egypt	currency banking	$\begin{bmatrix} + & . \\ - & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ - & + \end{bmatrix}$	$\begin{bmatrix} 1 & - \\ - & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & - \\ - & 1 \end{bmatrix}$
Lebanon	currency banking	$\begin{bmatrix} + & . \\ - & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$
Mexico	currency banking	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & . \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & . \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$
South Africa	currency banking	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$
Venezuela	currency banking	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$

Note: Three different lags of the dependent variable are used, namely 3, 6 and 12 months. 'Δ' stands for the parameters of the lagged crisis variables, while Ω represents the covariance matrix. A'+'/'-' sign means that the coefficient is significant and positive/ negative, while a '.' indicates its non-significance. For example, in the case of Argentina, 3 months, all the parameters are positive and significative except for the impact of a currency crisis on the probability of occurrence of banking crises. Similarly, the correlation coefficient between currency and banking crises is significative.

Table 4: Trivariate Analysis

		3 months		6 months	
Country		Δ	Ω	Δ	Ω
Ecuador	currency banking sovereign	$\begin{bmatrix} . & . & + \\ . & + & . \\ . & . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . & . \\ . & 1 & . \\ . & . & 1 \end{bmatrix}$	$\begin{bmatrix} . & + & + \\ . & + & . \\ . & . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . & . \\ . & 1 & . \\ . & . & 1 \end{bmatrix}$
South Africa	currency banking sovereign	$\begin{bmatrix} + & . & . \\ . & . & . \\ . & . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . & + \\ . & 1 & . \\ + & . & 1 \end{bmatrix}$	$\begin{bmatrix} + & . & . \\ . & . & . \\ . & . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . & + \\ . & 1 & . \\ + & . & 1 \end{bmatrix}$

Note: Two different lags of the dependent variable are used, namely 3 and 6 months. 'Δ' stands for the parameters of the lagged crisis variables, while Ω represents the variance-covariance matrix. A'+'/'-' sign means that the coefficient is significant and positive/ negative, while a '.' indicates its non-significance. For example, in the case of Ecuador, 3 months, sovereign debt crises have a positive and significative impact on the probability of occurrence of currency crises.