The revealed preference approach to collective consumption behavior: testing and sharing rule recovery*

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Abstract

We present a revealed preference methodology for empirically analyzing collective consumption behavior. First, we introduce an integer programming (IP) methodology for testing data consistency with collective consumption models that account for publicly as well as privately consumed goods. This IP methodology can include information on 'assignable quantities' for private goods. Next, we show that the IP methodology allows for recovering the personalized (Lindahl) prices for the public goods and the personalized quantities for the private

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goods. In turn, this implies recovery of the sharing rule (i.e. personalized income share levels). An empirical application demonstrates the practical usefulness of the methodology.
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1. Introduction

We present a revealed preference methodology for analyzing collective consumption behavior when demand is known for a finite set of price-income combinations. This methodology allows for empirically testing data consistency with collective consumption models and for empirically recovering the structural decision model underlying the observed collectively rational consumption behavior. Specifically, we show that such testing and recovery can make use of integer programming (IP) techniques, and we demonstrate the practical relevance through an empirical application to household consumption data. This introductory section articulates our main contributions and relates our findings to the existing literature.

Collective consumption models and the sharing rule. Collective consumption models explicitly recognize that group (e.g. household) consumption is the outcome of multi-person decision making, with each individual decision maker (e.g. household member) characterized by her or his own rational preferences. Following Chiappori (1988, 1992), they regard 'rational' group consumption as the Pareto efficient outcome of a within-group bargaining process. This collective approach contrasts with the conventional unitary approach, which models groups as if they were single decision makers. See Donni (2008) for a general discussion of collective consumption models.

The fact that the collective approach starts from individual preferences (and not group preferences) makes it particularly useful for addressing welfare-related questions that focus on the within-group distribution of the group income. In this respect, a concept that is intrinsically related to the collective approach is the so-called 'sharing rule', which describes the way in which the group income is shared between the individual members. Recovering this sharing rule can yield useful insights into the distribution of the within-group bargaining power across the individual group members; see, for example, Browning, Bourguignon, Chiappori and Lechene (1994) and Bourguignon, Browning and Chiappori (2009). In what follows, an important focus will be on sharing rule recovery.

Revealed preference conditions for collectively rational behavior. This study develops a methodology for revealed preference analysis of collective consumption models. The distinguishing feature of this revealed preference approach is that it only assumes that the value of the demand function is known for a finite number of price-income combinations. This contrasts with the more standard approach, which typically assumes a demand function of which the value is known for a continuous range of price-income combinations. See, for example, Afriat (1967), Diewert (1973) and Varian (1982) for revealed preference analysis of the unitary consumption model.

We consider collective consumption models that allow for privately consumed goods as well as publicly consumed goods. As for the privately consumed goods, we consider the possible use of 'assignable quantity' information. As a matter of fact, assignable quantity information becomes increasingly available in budget surveys (see, for example, Bonke and Browning, 2006), and is often used in empirical applications of collective models (see, for example, Browning, Bourguignon, Chiappori and Lechene, 1994). We obtain necessary and sufficient conditions for collectively rational behavior that can be formulated as 0-1 Integer Programming (IP) constraints. These conditions enable checking consistency of a given data set with particular collective consumption models. In the spirit of Varian (1982), we will refer to this as 'testing' data consistency with collective rationality.[1] Given our IP characterization, such testing boils down to verifying nonemptiness of the feasible region of specific IP problems.

At this point, it is useful to relate our analysis to the one of Cherchye, De Rock and Vermeulen (2007). Following Browning and Chiappori (1998) and Chiappori and Ekeland (2006), these authors considered a collective consumption model that does not require a prior specification of the (public or private) nature of the goods. By contrast, our focus is on models that do start from such a specification. Such a prior specification is often realistic in empirical applications of collective consumption models and is usually assumed in existing studies of these models. As for the objective

[1] Essentially, we consider 'sharp' tests: they check whether or not the data pass the revealed preference conditions exactly. If the data do not pass the conditions, then the model under study is rejected. See Varian (1990) for further discussion on the difference between revealed preference tests and traditional statistical tests.

of this study, an important advantage of assuming this prior specification is that it enables us to define data consistency tests that are both necessary and sufficient for collectively rational behavior. This implies a crucial difference with the results of Cherchye, De Rock and Vermeulen (2007), who obtain a testable necessary condition and a testable sufficient condition that in general do not coincide. In an earlier version of this study (Cherchye, De Rock and Vermeulen, 2008), we established the IP formulation of the necessary condition of Cherchye, De Rock and Vermeulen (2007). For compactness, we will abstract from a formal treatment here, but we do provide a brief discussion in Section 3.4.

Recovery of collective consumption models. Attractively, our IP characterization will naturally allow for revealed preference recovery of the decision structure that underlies observed collectively rational group behavior. Specifically, such recovery identifies personalized quantities for the private goods, personalized (Lindahl) prices for the public goods and the sharing rule (i.e. personalized income share levels) from the observed group behavior alone. It is worth indicating that this recovery typically aims at identifying the set of structural models that are consistent with a given set of observations. See Varian (1982) and, more recently, Blundell, Browning and Crawford (2008) for set identification results based on a revealed preference characterization of the unitary model.[2] A main contribution of the current study is that we develop set identification results for the collective model.

Our recovery question is essentially the revealed preference counterpart of the so-called 'identifiability' question in Chiappori and Ekeland (2009). These authors adopt a differentiable approach to tackle identifiability; this approach assumes a demand function of which the value is known for a continuous range of price-income combinations. An important difference with our results is that Chiappori and Ekeland need a so-called 'exclusive good' (i.e. a good that benefits the utility of only one group member) as a condition for unique identification (as opposed to set identification) of the household decision model; information on an exclusive good is a specific instance of what we will call assignable quantity information. By contrast, we will demonstrate that set identification is possible even in the absence of assignable quantity information. In addition, whereas the differentiable approach typically recovers

[2] Blundell, Browning and Crawford (2008) also discuss the relationship between this type of revealed preference analysis and the emerging literature on 'partial identification' (see, for example, Manski, 2003, and Chernozhukov, Hong and Tamer, 2007).

group members' income shares (i.e. the sharing rule) up to a constant under the assumption of exclusive goods, we will show the possibility to identify tight sets of income share levels even if no assignable quantity information can be used.

Structure. Section 2 gives a formal introduction to the collective consumption models on which we focus. Section 3 presents the revealed preference approach to analyzing collectively rational consumption behavior. Section 4 presents an empirical application to a sample of couples drawn from the Russia Longitudinal Monitoring Survey (RLMS). Section 5 provides a concluding discussion.

2. Rational collective consumption behavior

Throughout, we consider a group (e.g. household) that is known to consist of M group members (e.g. household members).[3] We focus on a setting with N privately consumed goods and K publicly consumed goods. The vector $\mathbf{q} \in \mathbb{R}_+^N$ represents the quantities that are privately consumed by the group and the vector $\mathbf{Q} \in \mathbb{R}_+^K$ represents the publicly consumed quantities. Next, the vector $\mathbf{q}^m \in \mathbb{R}_+^N$ represents privately consumed quantities for each member m, with $\sum_{m=1}^{M} \mathbf{q}^m = \mathbf{q}$. We assume that the preferences of every member m can be represented by a non-satiated utility function $U^m(\mathbf{q}^m, \mathbf{Q})$ that is non-decreasing in its arguments.

The empirical analyst observes T group choices characterized by prices and quantities.[4] For each observation $t \in \{1, ..., T\}$, $\mathbf{p}_t \in \mathbb{R}_{++}^N$ and $\mathbf{q}_t \in \mathbb{R}_+^N$ denote the prices and (group) quantities for the private goods, and $\mathbf{P}_t \in \mathbb{R}_{++}^K$ and $\mathbf{Q}_t \in \mathbb{R}_+^K$ denote the prices and quantities for the public goods. In general, however, the empirical analyst does not know which quantities are privately consumed by the individual group members, i.e. $\mathbf{q}_t^m$ is unobserved. If we observe how much a group member consumes of a private good, we say this good is 'assignable'; see Bourguignon, Browning and Chiappori (2009). Formally, for each member m and observation t, we define the (observed) *assignable quantities* $\mathbf{q}_t^{Am} \in \mathbb{R}_+^N$ as lower bounds for the (unobserved) quantities $\mathbf{q}_t^m$,

[3]In principle, if the empirical analyst does not know the number of group members *a priori*, then the conditions for collective rationality developed below can be tested for different values of M. As such, one can identify the number of group members (with decision power) *a posteriori*.

[4]In practice, a set of observations will consist either of multiple choice observations for the same household (e.g. our application in Section 4), or of choice observations for different households for which the empirical analyst assumes the same structural model (e.g. Blundell, Browning and Crawford, 2003 and 2008).

i.e.

$$\mathbf{q}_t^m \geq \mathbf{q}_t^{Am}.$$

Specific examples of assignable goods are so-called 'exclusive goods', i.e. goods that exclusively benefit the utility of single group members. More generally, assignable quantity information can pertain to a subset of group members and/or (private) goods. See also our empirical application in Section 4.

Summarizing, we define the set of observations

$$S = \{(\mathbf{p}_t; \mathbf{q}_t, \mathbf{q}_t^{A1}, ..., \mathbf{q}_t^{AM}, \mathbf{Q}_t), t = 1, ..., T\}.$$

In practical applications we often have $\sum_{m=1}^{M} \mathbf{q}_t^{Am} \neq \mathbf{q}_t$, i.e. some private consumption cannot be assigned prior to the analysis. It is even possible that $\mathbf{q}_t^{Am} = \mathbf{0}$, i.e. no assignable information at all is available. Therefore, we will consider *feasible personalized quantities* $\mathfrak{q}_t^m$.

Definition 1. *Let S be a set of observations. For each observation t,* feasible personalized quantities *$\mathfrak{q}_t^m \in \mathbb{R}_+^N$, $m = 1, ..., M$, satisfy $\mathfrak{q}_t^m \geq \mathbf{q}_t^{Am}$ and $\sum_{m=1}^{M} \mathfrak{q}_t^m = \mathbf{q}_t$.*

Using the concept of feasible personalized quantities, we can define the condition for a collective rationalization of a set of observations. This condition basically requires that the observed group consumption can be represented as a Pareto efficient outcome of some within-group bargaining process.

Definition 2. *Let S be a set of observations. A combination of M utility functions $U^1, ..., U^M$ provides a collective rationalization of S if for each observation t there exist feasible personalized quantities $\mathfrak{q}_t^m$ and Pareto weights $\mu_t^m > 0$, $m = 1, ..., M$, such that*

$$\sum_{m=1}^{M} \mu_t^m U^m (\mathfrak{q}_t^m, \mathbf{Q}_t) \geq \sum_{m=1}^{M} \mu_t^m U^m (\mathfrak{z}^m, \mathbf{Z})$$

for all $\mathfrak{z}^m \in \mathbb{R}_+^N$ and $\mathbf{Z} \in \mathbb{R}_+^K$ such that $\mathbf{p}_t' \left(\sum_{m=1}^{M} \mathfrak{z}^m\right) + \mathbf{P}_t'\mathbf{Z} \leq \mathbf{p}_t'\mathbf{q}_t + \mathbf{P}_t'\mathbf{Q}_t$.

Thus, a collective rationalization of the data requires that there exist feasible personalized quantities that maximize a weighted sum of the group members' utilities for the given group budget. Each Pareto weight represents the bargaining power of a member. See, for example, Browning and Chiappori (1998) for a detailed discussion.

One final remark is in order. Strictly speaking, the utility function $U^m(\mathbf{q}^m, \mathbf{Q})$ excludes consumption externalities for the privately consumed goods. Importantly, however, our setting can actually account for such externalities. Specifically, suppose a private good n is exclusively consumed by member m and characterized by externalities (e.g. male clothing or female clothing). Then, we account for externalities for member m's private consumption of good n by formally treating this good as a public good. As such, although we will not always indicate this explicitly in what follows, the quantities $\mathbf{q}^m$ must be understood as privately consumed quantities without externalities, and private quantities with externalities can be included in $\mathbf{Q}$.

3. Revealed preference analysis

We will first introduce a revealed preference condition for collectively rational consumption behavior. The condition will allow us to define the sharing rule concept that is intrinsic to the collective consumption model. As we will indicate, this characterization of collective rationality is not directly useful in practical analysis. Therefore, we also introduce an equivalent characterization in integer programming (IP) terms, which does permit practical analysis. Specifically, this IP condition allows for consistency testing and for recovering the collective decision model (including the sharing rule) that underlies observed collectively rational consumption behavior.

3.1. Revealed preference characterization

To define the revealed preference condition for a collective rationalization of the data, we first define *feasible personalized prices*. Essentially, these personalized prices capture the fraction of the aggregate prices for the publicly consumed quantities that is borne by the different group members. For each public good, the corresponding personalized prices must add up to the observed price and can thus be interpreted as Lindahl prices.

Definition 3. *Let S be a set of observations. For each observation t,* feasible personalized prices $\mathfrak{P}_t^m \in \mathbb{R}_+^K$, *$m = 1, ..., M$, satisfy $\sum_{m=1}^{M} \mathfrak{P}_t^m = \mathbf{P}_t$.*

We next specify the *Generalized Axiom of Revealed Preference* (*GARP*), which we adapt to our specific setting. Varian (1982) introduced the *GARP* condition for individually rational behavior under observed prices and quantities; i.e. he showed that it is a necessary and sufficient condition for the observed quantity choices to maximize a utility function under the given budget constraint. We focus on the same condition in terms of feasible personalized prices and quantities; the next Proposition 1 will establish that collective rationality as defined in Definition 2 requires *GARP* consistency for each individual member.

Definition 4. *Consider feasible personalized prices and quantities for a set of observations S. For $m \in \{1, ..., M\}$, the set $\{(\mathbf{p}_t, \mathfrak{P}_t^m; \mathfrak{q}_t^m, \mathbf{Q}_t); t = 1, ..., T\}$ satisfies* GARP *if there exist relations R_0^m, R^m that meet:*

(i) *if $\mathbf{p}_s'\mathfrak{q}_s^m + (\mathfrak{P}_s^m)' \mathbf{Q}_s \geq \mathbf{p}_s'\mathfrak{q}_t^m + (\mathfrak{P}_s^m)' \mathbf{Q}_t$ then $(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R_0^m\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$;*

(ii) *if $(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R_0\ (\mathfrak{q}_u^m, \mathbf{Q}_u)$, $(\mathfrak{q}_u^m, \mathbf{Q}_u)\ R_0\ (\mathfrak{q}_v^m, \mathbf{Q}_v)$, ..., $(\mathfrak{q}_z^m, \mathbf{Q}_z)\ R_0\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$ for some (possibly empty) sequence $(u, v, ..., z)$ then $(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$;*

(iii) *if $(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$ then $\mathbf{p}_t'\mathfrak{q}_t^m + (\mathfrak{P}_t^m)' \mathbf{Q}_t \leq \mathbf{p}_t'\mathfrak{q}_s^m + (\mathfrak{P}_t^m)' \mathbf{Q}_s$.*

We can now provide a characterization of collectively rational behavior. (Appendix A contains the proofs of our main results.)

Proposition 1. *Let S be a set of observations. The following conditions are equivalent:*

(i) *there exists a combination of M concave and continuous utility functions U^1, ..., U^M that provide* a collective rationalization *of S;*

(ii) *there exist feasible personalized prices and quantities such that for each member $m = 1, ..., M$, the set $\{(\mathbf{p}_t, \mathfrak{P}_t^m; \mathfrak{q}_t^m, \mathbf{Q}_t); t = 1, ..., T\}$ satisfies* GARP;

(iii) *there exist feasible personalized prices and quantities, numbers $U_j^m > 0$ and $\lambda_j^m > 0$ such that for all $s, t \in \{1, ..., T\}$: $U_s^m - U_t^m \leq \lambda_t^m\ [(\mathbf{p}_t'\mathfrak{q}_s^m + (\mathfrak{P}_t^m)' \mathbf{Q}_s) - (\mathbf{p}_t'\mathfrak{q}_t^m + (\mathfrak{P}_t^m)' \mathbf{Q}_t)]$ for each member $m = 1, ..., M$.*

Condition (ii) states that collective rationality requires individual rationality (i.e. *GARP* consistency) of each member in terms of personalized prices and quantities; and condition (iii) gives the equivalent 'Afriat inequalities' (see Varian, 1982, for extensive discussion in the context of the unitary model). In general, however, the *true* personalized prices and quantities are unobserved. Therefore, it is only required that there must exist at least one set of *feasible* personalized prices and quantities that satisfies the condition. In what follows, we will mainly focus on condition (ii).

3.2. Sharing rule

Importantly in view of our further discussion, the result in Proposition 1 also allows for the following *decentralized* interpretation of collective rationality: collective rationality at the group level requires individual rationality at the member level. Given this, collectively rational consumption behavior can also be represented as the outcome of a two-step allocation procedure: in the first step, the so-called *sharing rule* distributes the aggregate group income across the group members; in the second step, each member maximizes her/his utility subject to the resulting income share and accounting for the member's personalized prices. This second step corresponds to the *GARP* condition (ii) in Proposition 1, which effectively implies that each member behaves utility maximizing in terms of a member-specific utility function. Of course, we are not assuming that groups explicitly use the sharing rule. The two-step representation simply states that the outcome of the group allocation process can be characterized in this way.

In formal terms, we can restate the collective rationalization condition in Definition 2 as follows: a combination of utility functions U^1, ..., U^M provides a *collective rationalization* of the data if for each observation t there exist feasible personalized prices and quantities such that, for each individual member m,

$$U^m\left(\mathfrak{q}_t^m, \mathbf{Q}_t\right) \geq U^m\left(\mathfrak{z}^m, \mathbf{Z}\right)$$

for all $\mathfrak{z}^m \in \mathbb{R}_+^N$ and $\mathbf{Z} \in \mathbb{R}_+^K$ such that

$$\mathbf{p}_t'\mathfrak{z}^m + \left(\mathfrak{P}_t^m\right)'\mathbf{Z} \leq \mathbf{p}_t'\mathfrak{q}_t^m + \left(\mathfrak{P}_t^m\right)'\mathbf{Q}_t.$$

This member-specific utility maximization condition corresponds to the second step of the two-step representation introduced above. As for the first step, the income share of each member m in observation t corresponds to $\mathbf{p}_t'\mathfrak{q}_t^m + \left(\mathfrak{P}_t^m\right)'\mathbf{Q}_t$. This income share corresponds to a sharing rule underlying the observed group behavior.

In what follows, we will refer to *feasible income shares* $\mathfrak{y}_t^m = \mathbf{p}_t'\mathfrak{q}_t^m + \left(\mathfrak{P}_t^m\right)'\mathbf{Q}_t$, which are defined for every set of feasible personalized prices and quantities that yields a collective rationalization of the data.

Definition 5. *Consider feasible personalized prices and quantities for a set of obser-*

vations S such that each set $\{(\mathbf{p}_t, \mathfrak{P}_t^m; \mathfrak{q}_t^m; \mathbf{Q}_t)\,;\, t = 1, ..., T\}$, $m = 1$, ..., M, *satisfies* GARP. *For* $y_t = \mathbf{p}_t'\mathbf{q}_t + \mathbf{P}_t'\mathbf{Q}_t$ *the group income at observation t, the* feasible income share *for each member m at prices* $\mathbf{p}_t$ *and* $\mathbf{P}_t$ *is* $\mathfrak{y}_t^m = \mathbf{p}_t'\mathfrak{q}_t^m + (\mathfrak{P}_t^m)'\,\mathbf{Q}_t$.

We remark that this decentralized representation of collectively rational behavior, which follows from the Pareto efficiency assumption regarding the group bargaining process, is formally similar to the well-known decentralization result regarding collective rationality when public goods are excluded; see Chiappori (1988, 1992).[5] An important difference of the approach followed in this study is that each member's preferences may depend not only on her or his own private consumption, but also on the public consumption (implying that personalized Lindahl prices can differ from observed market prices). Intuitively, the part of each member's income share associated with the publicly consumed quantities reflects the extent to which these public quantities comply with the member's preferences for the public goods (as captured by the Lindahl prices).

The sharing rule is a core concept in this two-step representation. It can be interpreted as an indicator for the bargaining power of the individual group members: a higher relative income share of member m ($\mathfrak{y}_t^m/y_t$) is then regarded as an indication of increased bargaining power for that member; see, for example, Browning, Chiappori and Lewbel (2006). The sharing rule concept is particularly useful in applications, because it is independent of cardinal representations of preferences (in contrast to the Pareto weights in Definition 2). Given this useful interpretation, a main question in what follows concerns the recovery of feasible income shares.

We emphasize that one must be careful in drawing welfare conclusions on the basis of the income shares when public goods are involved. Because the personalized (Lindahl) prices for the public goods are endogenous, the welfare interpretation of the sharing rule is more complex with public goods than without public goods. For example, suppose that a reform increases a member's income share for given prices of the private goods, while at the same time her personalized (Lindahl) prices for the public goods also increase. Then, it is not directly clear from that member's income

[5] In the literature, a refinement of the standard sharing rule concept that accounts for public goods is the so-called 'conditional sharing rule'. This concept captures how the group shares the income for private consumption *for the given level of public consumption.* As such, in contrast to our sharing rule concept, it does not directly incorporate Lindahl prices for the public goods. See, for example, Blundell, Chiappori and Meghir (2005) for discussion.

share (calculated with Lindahl prices for the public goods) whether she effectively gains from the reform.

However, it is possible to test specific hypotheses regarding the relationship between a member's income share and her welfare/utility level. For example, suppose two observations s and t (e.g. with constant prices for the private goods, $\mathbf{p}_s = \mathbf{p}_t$) and we hypothesize that a higher income share for member m in observation s effectively corresponds to a higher utility level for that member in that observation. Formally, this hypothesis states

$$\text{if } \mathfrak{y}_s^m \geq \mathfrak{y}_t^m \text{ then } \left(\mathfrak{q}_s^m, \mathbf{Q}_s\right) R^m \left(\mathfrak{q}_t^m, \mathbf{Q}_t\right). \tag{3.1}$$

We will discuss how to test such a hypothesis by means of the IP methodology introduced next.

3.3. Integer programming characterization and recovery analysis

From the perspective of practical applications, conditions (ii) and (iii) in Proposition 1 are not directly useful, because they are expressed in terms of unobserved prices $\mathfrak{P}_t^m$ and quantities $\mathfrak{q}_t^m$. First, as for the *GARP* consistency condition (ii), we note that given prices $\mathbf{p}_t$ and quantities $\mathbf{q}_t$ and $\mathbf{Q}_t$ define infinitely many feasible specifications of these personalized prices and quantities. Any such feasible specification defines different relations R_0^m and R^m in the *GARP* consistency condition. Next, the 'Afriat inequalities' in condition (iii) are clearly nonlinear in unobserved $\lambda_t^m, \mathfrak{q}_t^m$ and $\mathfrak{P}_t^m$.

Given this, we provide an equivalent integer programming (IP) characterization of collectively rational consumption behavior.[6] Specifically, we reformulate condition (ii) in Proposition 1 in IP terms. Attractively, this IP formulation does allow for practical analysis. It enables us to use solution algorithms that are specially tailored for such problems (see, for example, Nemhauser and Wolsey, 1999). Our own application in Section 4 provides a specific illustration.

To obtain the IP formulation, we define the binary variables $x_{st}^m \in \{0, 1\}$, with $x_{st}^m = 1$ interpreted as '$(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R^m\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$' for a given specification of feasible personalized prices and quantities. We then have the following result.

[6]Strictly speaking, the characterization of collectively rational behavior in Proposition 2 obtains a mixed integer linear programming (MILP) problem rather than a pure IP problem.

Proposition 2. *Let S be a set of observations. There exists a combination of M concave and continuous utility functions U^1, ..., U^M that provide* a collective rationalization *of S if and only if there exist $\mathfrak{P}_t^m \in \mathbb{R}_+^K$, $\mathfrak{q}_t^m \in \mathbb{R}_+^N$, $\mathfrak{y}_t^m \in \mathbb{R}_+$ and $x_{st}^m \in \{0,1\}$, $m = 1, ..., M$, that satisfy*

(i) $\sum_{m=1}^{M} \mathfrak{P}_t^m = \mathbf{P}_t$;

(ii) $\sum_{m=1}^{M} \mathfrak{q}_t^m = \mathbf{q}_t$ *and* $\mathfrak{q}_t^m \geq \mathbf{q}_t^{Am}$;

(iii) $\mathfrak{y}_t^m = \mathbf{p}_t' \mathfrak{q}_t^m + (\mathfrak{P}_t^m)' \mathbf{Q}_t$;

(iv) $\mathfrak{y}_s^m - \left(\mathbf{p}_s' \mathfrak{q}_t^m + (\mathfrak{P}_s^m)' \mathbf{Q}_t\right) < y_s x_{st}^m$;

(v) $x_{su}^m + x_{ut}^m \leq 1 + x_{st}^m$;

(vi) $\mathfrak{y}_t^m - \left(\mathbf{p}_t' \mathfrak{q}_s^m + (\mathfrak{P}_t^m)' \mathbf{Q}_s\right) \leq y_t \left(1 - x_{st}^m\right)$.

The interpretation of the different constraints is the following. Constraints (i)-(iii) directly follow from Definitions 1, 3 and 5 of feasible personalized prices, quantities and income shares. The following constraints (iv)-(vi) correspond, for each member, to the *GARP* conditions (i)-(iii) in Definition 4. Specifically, constraint (iv) implies that, if $\mathfrak{y}_s^m \geq \left(\mathbf{p}_s' \mathfrak{q}_t^m + (\mathfrak{P}_s^m)' \mathbf{Q}_t\right)$, then we must have $x_{st}^m = 1$ (which corresponds to $(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R^m\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$).[7] Next, constraint (v) imposes transitivity, i.e. $x_{su}^m = 1$ ($(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R^m\ (\mathfrak{q}_u^m, \mathbf{Q}_u)$) and $x_{ut}^m = 1$ ($(\mathfrak{q}_u^m, \mathbf{Q}_u)\ R^m\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$) imply $x_{st}^m = 1$ ($(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R^m\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$). Finally, constraint (vi) requires that, if $x_{st}^m = 1$ ($(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R^m\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$), then $\mathfrak{y}_t^m \leq \left(\mathbf{p}_t' \mathfrak{q}_s^m + (\mathfrak{P}_t^m)' \mathbf{Q}_s\right)$.

As such, Proposition 2 defines an operational necessary and sufficient test for collective rationality. If the IP constraints (i)-(vi) characterize an empty feasible region for the given data set, then a collective rationalization of the data is impossible. Conversely, if the IP constraints characterize a non-empty feasible region, then a collective rationalization of the data is certainly possible.

Proposition 2 also implies an operational way for recovering feasible personalized prices, quantities and income shares that provide a collective rationalization of the data. Specifically, for each member it identifies feasible sets of personalized prices, quantities and income shares as (non-empty) feasible sets characterized by the con-

[7] The strict inequality $\mathfrak{y}_s^m - \left(\mathbf{p}_s' \mathfrak{q}_t^m + (\mathfrak{p}_s^m)' \mathbf{Q}_t\right) < y_s x_{st}^m$ is difficult to use in IP analysis. Therefore, in practice we can replace it with $\mathfrak{y}_s^m - \left(\mathbf{p}_s' \mathfrak{q}_t^m + (\mathfrak{p}_s^m)' \mathbf{Q}_t\right) + \epsilon \leq y_s x_{st}^m$ for ϵ (> 0) arbitrarily small.

straints (i)-(vi) in Proposition 2.[8] Examples 1 and 2 in Appendix B illustrate this recovery. Importantly, they show that precise recovery (i.e. tight set identification) is possible even in the absence of assignable quantity information.

To conclude, we return to our above discussion on the welfare interpretation of the income shares. Specifically, it may be interesting to check whether a higher income share for a member effectively implies a higher welfare (or utility) level for that member. The IP formulation of the corresponding hypothesis (3.1) for member m is as follows:

$$\mathfrak{y}_s^m - \mathfrak{y}_t^m < y_s x_{st}^m. \tag{3.2}$$

The interpretation is analogous to before: if member m's income share in observation s exceeds member m's income share in member observation t ($\mathfrak{y}_s^m \geq \mathfrak{y}_t^m$), then we must have $x_{st}^m = 1$ $((\mathfrak{q}_s^m, \mathbf{Q}_s)\, R^m\, (\mathfrak{q}_t^m, \mathbf{Q}_t))$. Testing the hypothesis adds (3.2) to the constraints (i)-(vi) in Proposition 2. If the resulting IP problem defines a non-empty feasible region, then we cannot reject the hypothesis (3.1). More generally, this specific application illustrates the flexibility of the IP methodology in terms of testing alternative hypotheses about preference orderings (captured by the variables x_{st}^m) and the sharing rule (captured by the variables $\mathfrak{y}_t^m$). See Appendix B for some additional examples.

3.4. Impure goods

So far, we have focused on collective consumption models that explicitly distinguish between public and private goods. This helped to focus our discussion. Also, these models are usually considered in empirical applications (including our own application in Section 4).

Of course, in general some goods may serve different uses simultaneously. For example, expenditures on 'car use' may include a public element (e.g. car use for a family trip) and a private element (e.g. car use for work). This implies 'impure

[8] We note that our recovery results build on a mathematical symmetry between private and public consumption that was already exploited by Chiappori and Ekeland (2009). In the private goods case, the individual quantities (that add up to the household's aggregate quantities) are unobserved while every individual is faced with the same market prices. In the public goods case, personalized prices (that add up to market prices) are unobserved while all individuals consume the same quantities. As stressed by Chiappori and Ekeland (2009), this basically implies that quantities in the private goods case play the role of personalized prices in the public goods case, while quantities in the public goods case play the role of market prices in the private goods case.

goods', which are partly publicly and partly privately consumed. Clearly, if the empirical analyst observes the privately and publicly consumed components of each good, then one can use the same methodology as before. In that case, the privately consumed component and the publicly consumed component of every impure good are formally treated as different goods.

However, it is well possible that impure goods have private and public components that are not separately observed. To account for this, Browning and Chiappori (1998) introduced a collective consumption model that does not require a prior assumption regarding the public or private nature of goods. Cherchye, De Rock and Vermeulen (2007) consider the revealed preference characterization of this model. They present a testable condition that is necessary for data consistency with the model as well as a complementary sufficient condition. In general, the necessary condition and the sufficient condition do not coincide.

In an earlier version of this study (Cherchye, De Rock and Vermeulen, 2008), we established the IP formulation of the necessary condition in Cherchye, De Rock and Vermeulen (2007). This IP condition has a formally similar structure as the IP condition in Proposition 2. We showed that this IP characterization provides a useful basis for recovering the sharing rule in the case one does not use a prior specification of the (public or private) nature of the goods. A numerical example similar to Examples 1 and 2 in Appendix B demonstrated that this IP method can obtain precise recovery (i.e. tight sets of feasible income shares) even when the number of observations is small (*in casu* $T = 3$). Cherchye, De Rock, Sabbe and Vermeulen (2008) provided an empirical application of this IP methodology.

As a concluding remark, we indicate that the use of a necessary condition for collective rationality as a starting point for recovery analysis entails a subtle but important difference with the type of recovery analysis discussed before. In particular, a feasible income share for a member that satisfies the necessary condition must no longer necessarily correspond to a collective consumption model that effectively rationalizes the data. This is in sharp contrast with the recovery results based on the necessary and sufficient condition for collective rationality in Proposition 2: if a feasible income share respects this condition, then there *certainly* exists a corresponding specification of feasible personalized prices and quantities that collectively rationalizes the data.

4. Empirical application

We now apply the IP methodology to data drawn from the Russia Longitudinal Monitoring Survey (RLMS). Cherchye, De Rock and Vermeulen (2009) studied the same data set. These authors concluded consistency of these data with the collective consumption model of Browning and Chiappori (1998), which does not use a prior specification of the (public or private) nature of each good (see Section 3.4). Specifically, they tested the revealed preference conditions for this model established by Cherchye, De Rock and Vermeulen (2007).

We extend the study of Cherchye, De Rock and Vermeulen (2009) in several respects. Firstly, we will focus on various collective consumption models that do label each good as public or private *a priori*. As discussed in the previous section, we have necessary and sufficient conditions for data consistency with these models, and these conditions in turn allow for recovery (i.e. set identification) by means of the IP methodology. In particular, we will consider two limiting collective models (with, respectively, all goods public and all goods private) as well as a more realistic, intermediate model (with some goods public and other goods private). Secondly, we will illustrate the IP recovery methodology for these three models. More specifically, we will discuss recovery results for the sharing rule.[9] This part of the application will learn us more about how the set identification works in practice and how sensitive the results are to the model specification. Thirdly, our application will also consider the use of assignable quantity information for privately consumed goods. We will provide results for different degrees of assignability. However, since assignable quantity information is not available in the RLMS data, we are bound to introduce this information in some ad-hoc fashion. As such, our results here will be mainly illustrative.

At this point, it is worth emphasizing that assignable quantity information does become increasingly incorporated in household budget surveys. See, for example, Bonke and Browning (2006), who discuss a household survey with detailed information on who consumes what in the household. In this respect, we also remark that assignable quantity information is easily obtained in the context of experimental data. See, for example, Bruyneel, Cherchye and De Rock (2008), who provide an analysis

[9] For compactness, we will restrict to sharing rule recovery in what follows. Recovery of personalized prices (for publicly consumed goods) and personalized quantities (for privately consumed goods) proceeds analogously as in Examples 1 and 2 in Appendix B.

of collective consumption models on the basis of experimentally gathered data. As a matter of fact, it has been argued that the revealed preference methodology is particularly useful in combination with such experimental data. See, for example, Sippel (1997), Harbaugh, Krause and Berry (2001) and Andreoni and Miller (2002) for earlier applications that experimentally analyze individually rational behavior.

4.1. Data

Our sample consists of 148 adult couples ($M = 2$) drawn from the RLMS. No household contains other persons such as children and/or siblings, and in each household both the female member and the male member are employed. Our next analysis will consider each of the 148 households separately, which avoids (often debatable) preference homogeneity assumptions across male or female members of different households.

Our data set covers the period from 1994 to 2003. We have consumption data for each year except for the years 1997 and 1999, so that we end up with 8 ($= T$) observations (prices and quantities) per household. We consider bundles consisting of 21 ($= N + K$) nondurable goods: (1) food outside the home, (2) clothing, (3) car fuel, (4) wood fuel, (5) gas fuel, (6) luxury goods, (7) services, (8) housing rent, (9) bread, (10) potatoes, (11) vegetables, (12) fruit, (13) meat, (14) dairy products, (15) fat, (16) sugar, (17) eggs, (18) fish, (19) other food items, (20) alcohol and (21) tobacco. See Table 5.1 in Appendix C for some descriptive statistics. We refer to Cherchye, De Rock and Vermeulen (2009) for a more detailed discussion of the data.

As indicated above, we will focus on three different specifications of the collective consumption model. As a first exercise, we will consider two limiting specifications: the first model assumes that all goods are publicly consumed ($K = 21$; $N = 0$), and the second model imposes that all consumption is private and does not use assignable quantity information ($K = 0$; $N = 21$). However, it is clear from the above set of goods that none of these limiting cases is very realistic for the current application. Therefore, we will also consider a model with public goods as well as private goods: wood fuel, gas fuel and housing rent are public ($K = 3$), and the other goods are private ($N = 18$). Given the characteristics of the modeled goods, this choice is probably quite close to reality.

As for this model with 3 public and 18 private goods, we will provide results for different degrees of assignable quantity information. When doing so, we will start from an (admittedly extreme) base scenario that assumes $\mathbf{q}_t^{Am} = \mathbf{q}_t^m$, i.e. the within-household distribution of the privately consumed goods is fully known. In subsequent steps, we will consider less and less assignability, i.e. we account for (ever larger) deviations from the base scenario distribution (or, $\mathbf{q}_t^{Am} < \mathbf{q}_t^m$). Formally, using $\mathbf{q}_t^{Bm}$ for the private quantities of member m that correspond to the hypothesized base scenario, we define

$$\mathbf{q}_t^{Am} = \theta \mathbf{q}_t^{Bm},$$

with $0 \leq \theta \leq 1$. The parameter θ captures the extent to which we allow for deviations from the base scenario distribution, and thus accounts for imperfect assignable quantity information. For example, $\theta = 1$ implies $\mathbf{q}_t^{Am} = \mathbf{q}_t^{Bm}$, while $\theta < 1$ implies $\mathbf{q}_t^{Am} < \mathbf{q}_t^{Bm}$. Generally, lower θ values imply less stringent restrictions for the private quantities. Varying the value of θ will allow us to consider different degrees of assignability in our application.

As for the current application, because assignable quantity information is not available from the RLMS data set, our base scenario uses the observed consumption of male and female singles (or one-person households). For example, we observe that the average budget share of alcohol for male singles is (about) 5 times the corresponding budget share for female singles. Given this, in the base scenario the male consumes 5/6 of all alcohol bought by the household and the female consumes 1/6. Table 5.1 in Appendix C provides a detailed description of the base scenario distribution assumed in our analysis.

As a concluding note, we emphasize that this base scenario mainly serves illustrative purposes in the present application; it enables us to mimic practical applications with alternative degrees of assignability information. Evidently, other base scenarios can be treated in a directly analogous manner. For example, in more extensive applications it can be useful to assess the sensitivity of consistency testing and recovery results with respect to alternative, *a priori* reasonable, base scenarios. Evidently, the lower θ is, the less these results will depend on the base scenario that is assumed.

4.2. Testing

We first consider the empirical results for the two limiting collective consumption models, i.e. the case in which all goods are public and the case in which all goods are private and no assignability information is used (i.e. $\mathbf{q}_t^{Am} = \mathbf{0}$ for all m and t). Table 4.1 presents pass rates in absolute terms (Pass-number) and percentage terms (Pass-percentage). We find that the data of all households are consistent with both models. This suggests that these models effectively do provide an adequate description of the observed household behavior. However, one may also argue that the models put very little *a priori* structure on observed behavior, which makes them hardly rejectable. In addition, as indicated above, one may question the realistic nature of these 'extreme' models. Indeed, it does seem unrealistic that the 21 goods are either all public or all private. Therefore, in our main analysis we will consider the more realistic collective consumption model with some goods public and other goods private (i.e. $K = 3$ and $N = 18$).

Table 4.1: Consistency testing; limiting cases

	Pass-number	Pass-percentage
All public	148	100.00
All private (and $\mathbf{q}_t^{Am} = 0$ for all m and t)	148	100.00

As for this intermediate model, we recall that wood fuel, gas fuel and housing rent are public. For the average household in our sample, these 3 public goods account for 20.2% of the total expenditures; see Table 5.1. The remaining 18 goods are assumed to be privately consumed. For these privately consumed goods, we use the procedure described above to illustrate the use of assignable quantity information.

Table 4.2 presents pass rates for alternative values of θ. Let us first consider $\theta = 1.00$. This corresponds to the collective model where the within-household distribution for the 18 private goods is exactly equal to that in the hypothesized base scenario reported in Table 5.1. We find that this collective consumption model rationalizes the behavior of 137 households in our sample (i.e. about 93%). To some extent, this gives empirical support for the base scenario that we assume.

Next, Table 4.2 also reveals that pass rates increase if θ decreases. This is not surprising given that lower θ values comply with less assignable information for the privately consumed quantities. For one household, we need $\theta = 0.60$ for a collective rationalization, i.e. 40% of the private quantities is not assigned. For this household, the average budget share (defined over the 8 observations) of the public goods wood fuel, gas fuel and housing rent is 12.65%. Thus, our model rationalizes the behavior of this household only if less than 50% of total consumption is assigned as private consumption. The behavior of the 10 remaining households can be rationalized by a

θ equal to 0.90 (6 households) or 0.80 (4 households).

Table 4.2: Consistency testing; intermediate case

Value of θ	Pass		Power (in percentage)				
	Number	Percentage	Minimum	1st quartile	Median	3rd quartile	Maximum
1.00	137	92.57	0.00	6.05	9.00	12.05	24.20
0.90	143	96.62	0.00	2.90	4.20	5.60	10.80
0.80	147	99.32	0.00	1.08	1.80	2.80	6.80
0.60	148	100.00	0.00	0.10	0.20	0.40	2.50

To further analyze the impact of assignable quantity information, Table 4.2 provides power results for the collective rationality tests under consideration. Specifically, for each household and each θ we compute a power measure that quantifies the probability of detecting the alternative hypothesis of random behavior. Random behavior is modeled using the bootstrap method introduced by Harbaugh, Krause and Berry (2001) and Andreoni and Miller (2002): for each observation, with given prices and income, we define quantities by randomly drawing budget shares (for the 21 goods) from the set of 1184 (= 148 x 8) observed household choices.[10] Thus, our power assessment gives information on the expected distribution of violations under random choice, while incorporating information on the households' actual choices. Table 4.2 reports on the distribution of the power measure defined over the 148 households under study. These results are based on Monte Carlo-type simulations that include 1000 iterations.

We can make the following observations. First, we find that the power varies a lot across households: while it is reasonably high for some households (see in particular the maximum and 3rd quartile values for higher θ), it is also very low for other households (see the minimum and 1st quartile values). For a given household, the power of a revealed preference test of the type we consider crucially depends on the income and price variation in the data: high power is typically associated with high price variation and low income variation; see, for example, Bronars (1987) for a detailed discussion. Also, we see that power decreases (rather substantially) with

[10] See Andreoni and Harbaugh (2006) for discussions on alternative power measures that can be used in the context of data consistency tests in the revealed preference tradition.

θ. Like before, this is no surprise given that more assignable quantity information implies additional prior structure and thus more powerful tests. Generally, these results suggest that assignable quantity information can be particularly helpful to enhance the power of the collective rationality tests. Next, we recall that our analysis uses only 8 observations per household. Obviously, power can only benefit when more observations become available.

4.3. Recovery

As a preliminary exercise, we have a quick look at the recovery results for the two limiting models. Admittedly, given that the models imply so little *a priori* structure and because we only have 8 observations per household, the recovery results are rather weak. Still, even under these weak conditions we do find some sharing rule restrictions. For example, for one household observation we obtain for both limiting models that relative income shares between 40.9% and 59.1% are excluded, i.e. for the two models we have that each household member (male or female) is responsible for either at most 40.9% of the total household income or at least 59.1% of the household income. In this case, the implications of the two models are the same. Still, this is not the case in general. For example, for another household observation we find that the model with all goods public excludes relative income shares between 44.7% and 55.3%, while there is no similar sharing rule restriction for the model with all consumption private. As such, we conclude that recovery results are sensitive to the classification of commodities as public or private: different *a priori* assumptions regarding the (public or private) nature of goods can imply different sharing rule restrictions, even if no assignable quantity information is used.

Like before, our main focus is on the more realistic model with 3 public goods and 18 private goods. Given the illustrative purpose of our application, we will not provide an exhaustive account of the recovery results for all 148 households in our sample. Instead, we will concentrate on two different exercises. First, we discuss aggregate results for the different θ values considered above. Next, we particularly focus on the one household that needs $\theta = 0.60$ for a collective rationalization.[11] Because of the limited assignable quantity information (i.e. low θ value), we see this household as a natural candidate for illustrating the potential of the methodology.

[11] Recovery results for other households are available from the authors upon request.

Table 4.3 presents the aggregate sharing rule results. Specifically, it considers 4 subsets of observations: 137 households that can be rationalized for $\theta = 1.00$; 6 households that can be rationalized for $\theta = 0.90$; 4 households that can be rationalized for $\theta = 0.80$; and 1 household that can be rationalized for $\theta = 0.60$. For each subset, the table reports per observation (year) the mean and standard deviation of the lower and upper bounds on the relative income share of the female member (with the associated male share given by one minus this female share). When comparing the results for different θ values, we see that higher θ values obtain more precise recovery (i.e. tighter set identification). This suggests that higher θ values will generally lead to 'better' (i.e. more precise) recovery performance.

Table 4.3: Sharing rule recovery - female income share

	$\theta = 1.00$ (137 households)				$\theta = 0.90$ (6 households)			
Observation	Lower bound		Upper bound		Lower bound		Upper bound	
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
1	0.445	0.127	0.593	0.109	0.379	0.108	0.581	0.097
2	0.407	0.130	0.597	0.123	0.378	0.081	0.671	0.048
3	0.396	0.147	0.609	0.153	0.382	0.170	0.715	0.120
4	0.396	0.117	0.601	0.133	0.406	0.109	0.647	0.034
5	0.410	0.116	0.597	0.103	0.400	0.033	0.648	0.072
6	0.395	0.127	0.601	0.123	0.313	0.071	0.661	0.090
7	0.395	0.123	0.613	0.119	0.406	0.098	0.635	0.054
8	0.385	0.116	0.618	0.117	0.360	0.084	0.682	0.079

	$\theta = 0.80$ (4 households)				$\theta = 0.60$ (1 household)			
Observation	Lower bound		Upper bound		Lower bound		Upper bound	
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
1	0.239	0.088	0.603	0.237	0.267	-	0.667	-
2	0.372	0.033	0.628	0.085	0.658	-	0.727	-
3	0.294	0.157	0.494	0.246	0.321	-	0.721	-
4	0.187	0.193	0.644	0.325	0.130	-	0.897	-
5	0.270	0.123	0.681	0.171	0.303	-	0.703	-
6	0.351	0.057	0.646	0.075	0.204	-	0.831	-
7	0.362	0.142	0.608	0.100	0.273	-	0.296	-
8	0.314	0.046	0.676	0.107	0.297	-	0.710	-

Let us then consider the household that is rationalized for $\theta = 0.60$. The panel in

Table 4.4: Estimates of female share parameters

Sharing rule variable	Estimate	P-value
Constant	0.502	0.000
Household's real expenditure	-0.002	0.006
Relative income (male over female)	0.004	0.144
Male age - female age	-0.000	0.729
Male education level - female education level	-0.000	0.970
Number of observations	1048	
Within R-squared	0.0072	
Between R-squared	0.0160	
Overall R-squared	0.0169	

the bottom-right of Table 4.3 shows that in 2 out of the 8 observations the bounds are fairly tight: the difference between the upper and lower bounds is less than 7 percentage points in observation 2, and about 2 percentage points in observation 7. In our opinion, this is quite remarkable given $\theta = 0.60$ and an average budget share of the public goods equal to 12.65%. Also, our results reveal a rather drastic shift in the relative income share (or bargaining power) between observations 2 and 7: in observation 2 the female member was responsible for about 70% of the household income, while her income share decreases to about 30% in observation 7.

As a final exercise, we relate the observed variation in the sharing rule to specific household characteristics. In fact, the literature on collective consumption models has paid considerable attention to relating the within-household sharing of resources to so-called distribution factors such as the spouses' relative incomes; see, for example, Browning, Bourguignon, Chiappori and Lechene (1994) and Bourguignon, Browning and Chiappori (2009). We address the same issue by starting from the recovery results summarized in Table 4.3. Specifically, for each household observation we calculate the average of the recovered lower and upper bounds on the female income share. Next, we regress this average on the household's real expenditure, the ratio of male income over female income, age difference (male minus female) and difference in education level (male minus female); Browning, Bourguignon, Chiappori and Lechene (1994) considered the same variables in their study.[12] Evidently, because we have only 8

[12] In a certain sense, our analysis is complementary to the one of Lacroix and Radtchenko (2008). Focusing on data from the same RLMS surrey and adopting a collective labor supply model, these

observations per household, we could not conduct a separate regression analysis for each different household. Instead, we have put together all households to obtain one panel data set.

Table 4.4 gives the regression results for the random effects estimator.[13] We see that the household's real expenditure has a significant impact on the sharing rule when using a five percent significance level; other variables turn out to be non-significant. We can thus conclude that higher real expenditure is associated with a decrease of the share going to the female household member, *ceteris paribus*. Interestingly, this outcome for Russian couples contrasts with the one of Browning, Bourguignon, Chiappori and Lechene (1994); for Canadian couples, these authors found a positive relationship between the female income share and real expenditures.

5. Summary and concluding discussion

We have presented an integer programming (IP) methodology for revealed preference analysis of collective consumption models. This methodology applies to collective models that account for publicly as well as privately consumed goods and that incorporate the possibility of assignable quantity information. The methodology allows for testing data consistency with specific collective consumption models. In addition, it enables us to recover (i.e. set identify) feasible personalized prices (for publicly consumed goods), feasible personalized quantities (for privately consumed goods) and feasible income shares (i.e. the sharing rule) that are consistent with observed collectively rational behavior. The methodology applies to any number of group members and any number of observed group consumption choices (i.e. quantities and prices).

In principle, precise recovery (i.e. tight set identification) can be obtained even if no assignable quantity information is available. Our empirical application to RLMS data suggests that additional assignable quantity information generally entails a more powerful empirical analysis (i.e. more stringent data consistency conditions and tighter set identification). Also, we can expect data sets with more observations

authors analyze the impact on the sharing rule of the dramatic economic changes in the Russian economy in the period under consideration.

[13] A Hausman test could not reject the null hypothesis that the household specific fixed effect and the explanatory variables are uncorrelated (P-value of regressions with only time-varying regressors equals 0.54). We therefore restrict attention to the results obtained by the random effects estimator, because it is more efficient than the fixed effects estimator.

to entail a more powerful analysis. In this respect, we note that data sets with more observations and more assignable quantity information become increasingly available. Thus, we can expect our methodology to be particularly useful in combination with such data sets.

As for future applications of the methodology, we indicate that recovery of the feasible personalized prices, personalized quantities and income shares in turn allows for empirically addressing welfare-related issues that are specific to the collective consumption model. As indicated in the introduction, the collective approach is particularly useful for investigating questions that pertain to the within-group distribution of the group income (i.e. the sharing rule). The methodology presented in this study paves the way for empirically addressing these issues by following a revealed preference approach.

In this respect, we distinguish at least two different types of welfare-related applications of collective consumption models that build on sharing rule recovery. A first type of applications fits in the 'targeting view' of Blundell, Chiappori and Meghir (2005), which takes as a starting point that the effectiveness of a specific benefit or tax also depends on the particular group (e.g. household) member to whom it has been targeted. These authors argue that a unitary set-up, which implicitly assumes income pooling at the aggregate group level, fails to adequately deal with such targeting considerations. A second type of applications analyzes welfare at the individual group member level rather than at the aggregate group level. For example, Browning, Chiappori and Lewbel (2006) and Lewbel and Pendakur (2008) suggest a collective approach for comparing the cost-of-living of individuals living alone with the one of the same individuals living in a multi-member household.

Appendix A: Proofs

A.1 Proof of Proposition 1

Varian (1982) has proven equivalence between conditions (ii) and (iii), so we can restrict to proving equivalence between conditions (i) and (iii).

1. Necessity. Under condition (i), we have that each $(\mathfrak{q}_t^m, \mathbf{Q}_t)$ solves the problem

$$\max_{(\mathfrak{z}^m, \mathbf{Z})} \sum_{m=1}^{M} \mu_t^m U^m \left(\mathfrak{z}^m, \mathbf{Z}\right) \text{ s.t. } \mathbf{p}_t' \left(\sum\nolimits_{m=1}^{M} \mathfrak{z}^m\right) + \mathbf{P}_t'\mathbf{Z} \leq \mathbf{p}_t'\mathfrak{q}_t + \mathbf{P}_t'\mathbf{Q}_t.$$

Given concavity, the functions U^m are subdifferentiable, which carries over to their weighted sum $\sum\limits_{m=1}^{M} \mu_t^m U^m$.[14] An optimal solution to the above maximization problem must therefore satisfy (for η_t the Lagrange multiplier associated with the budget constraint)

$$\mu_t^m U_{\mathfrak{q}_t^m}^m \leq \eta_t \mathbf{p}_t \text{ and } \sum\nolimits_{m=1}^{M} \mu_t^m U_{\mathbf{Q}_t}^m \leq \eta_t \mathbf{P}_t,$$

for $U_{\mathfrak{q}_t^m}^m$ a subgradient of the function U^m defined for the vector $\mathfrak{q}^m$ and evaluated at $\mathfrak{q}_t^m$, and $U_{\mathbf{Q}_t}^m$ a subgradient defined for $\mathbf{Q}$ and evaluated at $\mathbf{Q}_t$. Let $\mathfrak{P}_t^m = \frac{\mu_t^m U_{\mathbf{Q}_t}^m}{\eta_t}$ for $m = 1, ..., M-1$, $\mathfrak{P}_t^M = \mathbf{P}_t - \sum\nolimits_{m=1}^{M-1} \mathfrak{P}_t^{M-1}$ and $\lambda_t^m = \frac{\eta_t}{\mu_t^m}$. We thus get for each m

$$U_{\mathfrak{q}_t^m}^m \leq \lambda_t^m \mathbf{p}_t \text{ and } U_{\mathbf{Q}_t}^m \leq \lambda_t^m \mathfrak{P}_t^m. \tag{5.1}$$

Next, concavity of the functions U^m implies for each m

$$U^m\left(\mathfrak{q}_s^m, \mathbf{Q}_s\right) - U^m\left(\mathfrak{q}_t^m, \mathbf{Q}_t\right) \leq U_{\mathfrak{q}_t^m}^m\left(\mathfrak{q}_s^m - \mathfrak{q}_t^m\right) + U_{\mathbf{Q}_t}^m\left(\mathbf{Q}_s - \mathbf{Q}_t\right). \tag{5.2}$$

Substituting (5.1) in (5.2) and setting $U_k^m = U^m\left(\mathfrak{q}_k^m, \mathbf{Q}_k\right)$ $(k = s, t)$ obtains condition (iii) of the proposition.

2. Sufficiency. Under condition (iii), for any $(\mathfrak{z}^m, \mathbf{Z})$ such that $\mathbf{p}_t'\left(\sum\nolimits_{m=1}^{M} \mathfrak{z}^m\right) + \mathbf{P}_t'\mathbf{Z} \leq \mathbf{p}_t'\mathfrak{q}_t + \mathbf{P}_t'\mathbf{Q}_t$ we can define for all m

$$U^m\left(\mathfrak{z}^m, \mathbf{Z}\right) = \min_{s \in \{1,...,T\}} \left[U_s^m + \lambda_s^m [\left(\mathbf{p}_s'\mathfrak{z}^m + \left(\mathfrak{P}_s^m\right)'\mathbf{Z}\right) - \left(\mathbf{p}_s'\mathfrak{q}_s^m + \left(\mathfrak{P}_s^m\right)'\mathbf{Q}_s\right)]\right]. \tag{5.3}$$

Varian (1982) proves that $U^m\left(\mathfrak{q}_t^m, \mathbf{Q}_t\right) = U_t^m$. Next, given $\mu_t^m \in \mathbb{R}_{++}$, we have

[14] To be precise, $-U^m$ $(m = 1, ..., M)$ is convex and therefore subdifferentiable. This, of course, does not affect our argument.

that

$$\sum_{m=1}^{M} \mu_t^m U^m \left(\mathfrak{z}^m, \mathbf{Z}\right) \leq \sum_{m=1}^{M} \mu_t^m \left[U_t^m + \lambda_t^m [\left(\mathbf{p}_t' \mathfrak{z}^m + \left(\mathfrak{P}_t^m\right)' \mathbf{Z}\right) - \left(\mathbf{p}_t' \mathfrak{q}_t^m + \left(\mathfrak{P}_t^m\right)' \mathbf{Q}_t\right)]\right].$$

Without losing generality, we concentrate on $\mu_t^m = (1/\lambda_t^m)$, which obtains

$$\sum_{m=1}^{M} \mu_t^m U^m \left(\mathfrak{z}^m, \mathbf{Z}\right) \leq \sum_{m=1}^{M} \mu_t^m U_t^m + \left(\mathbf{p}_t' \left(\sum\nolimits_{m=1}^{M} \mathfrak{z}^m\right) + \mathbf{P}_t' \mathbf{Z}\right) - \left(\mathbf{p}_t' \mathbf{q}_t + \mathbf{P}_t' \mathbf{Q}_t\right).$$

Since $\mathbf{p}_t' \left(\sum_{m=1}^{M} \mathfrak{z}^m\right) + \mathbf{P}_t' \mathbf{Z} \leq \mathbf{p}_t' \mathbf{q}_t + \mathbf{P}_t' \mathbf{Q}_t$, we thus have

$$\sum_{m=1}^{M} \mu_t^m U^m \left(\mathfrak{z}^m, \mathbf{Z}\right) \leq \sum_{m=1}^{M} \mu_t^m U_t^m = \sum_{m=1}^{M} \mu_t^m U^m \left(\mathfrak{q}_t^m, \mathbf{Q}_t\right),$$

which proves that $(\mathfrak{q}_t^m, \mathbf{Q}_t)$ maximizes $\sum_{m=1}^{M} \mu_t^m U^m \left(\mathfrak{z}^m, \mathbf{Z}\right)$ subject to $\mathbf{p}_t' \left(\sum_{m=1}^{M} \mathfrak{z}^m\right) + \mathbf{P}_t' \mathbf{Z} \leq \mathbf{p}_t' \mathbf{q}_t + \mathbf{P}_t' \mathbf{Q}_t$. We conclude that the functions U^m in (5.3) provide a collective rationalization of the set S. These functions have the properties listed in condition (i) of the proposition (compare with Varian, 1982).

A.2 Proof of Proposition 2

1. Necessity. Suppose there exist feasible personalized prices and quantities such that for each member $m = 1, ..., M$, the set $\{(\mathbf{p}_t, \mathfrak{P}_t^m; \mathfrak{q}_t^m, \mathbf{Q}_t)\,;\ t = 1, ..., T\}$ satisfies *GARP*. Then the corresponding specification of $\mathfrak{P}_t^m$, $\mathfrak{q}_t^m$, $\mathfrak{y}_t^m$ and x_{st}^m satisfies constraints (i)-(vi) in Proposition 2. First, constraints (i)-(iii) are satisfied because the feasible personalized prices, quantities and income shares are consistent with Definitions 1, 3 and 5. Next, to see consistency with constraints (iv)-(vi), consider any sequence $(u,\ v,\ ...,\ z)$ such that $\mathbf{p}_s' \mathfrak{q}_s^m + \left(\mathfrak{P}_s^m\right)' \mathbf{Q}_s \geq \mathbf{p}_s' \mathfrak{q}_u^m + \left(\mathfrak{P}_s^m\right)' \mathbf{Q}_u$, $\mathbf{p}_u' \mathfrak{q}_u^m + \left(\mathfrak{P}_u^m\right)' \mathbf{Q}_u \geq \mathbf{p}_u' \mathfrak{q}_v^m + \left(\mathfrak{P}_u^m\right)' \mathbf{Q}_v$, ..., $\mathbf{p}_z' \mathfrak{q}_z^m + \left(\mathfrak{P}_z^m\right)' \mathbf{Q}_z \geq \mathbf{p}_z' \mathfrak{q}_t^m + \left(\mathfrak{P}_z^m\right)' \mathbf{Q}_t$. (Trivially, another sequence does not impose restrictions on x_{st}^m.) Constraint (iv) then implies $x_{su}^m = x_{uv}^m = ... = x_{zt}^m = 1$, and constraint (v) consequently obtains $x_{st}^m = 1$. Constraint (vi) is then automatically satisfied because the set $\{(\mathbf{p}_t, \mathfrak{P}_t^m; \mathfrak{q}_t^m; \mathbf{Q}_t)\,;\ t = 1, ..., T\}$ satisfies *GARP*, and thus $\mathbf{p}_t' \mathfrak{q}_t^m + \left(\mathfrak{P}_t^m\right)' \mathbf{Q}_t \leq \mathbf{p}_t' \mathfrak{q}_s^m + \left(\mathfrak{P}_t^m\right)' \mathbf{Q}_s$ when-ever $\mathbf{p}_s'\mathfrak{q}_s^m + (\mathfrak{P}_s^m)'\mathbf{Q}_s \geq \mathbf{p}_s'\mathfrak{q}_u^m + (\mathfrak{P}_s^m)'\mathbf{Q}_u$, $\mathbf{p}_u'\mathfrak{q}_u^m + (\mathfrak{P}_u^m)'\mathbf{Q}_u \geq \mathbf{p}_u'\mathfrak{q}_v^m + (\mathfrak{P}_u^m)'\mathbf{Q}_v$, ..., $\mathbf{p}_z'\mathfrak{q}_z^m + (\mathfrak{P}_z^m)'\mathbf{Q}_z \geq \mathbf{p}_z'\mathfrak{q}_t^m + (\mathfrak{P}_z^m)'\mathbf{Q}_t$ (which corresponds to $(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R^m\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$).

2. Sufficiency. If there exist $\mathfrak{P}_t^m$, $\mathfrak{q}_t^m$, $\mathfrak{y}_t^m$ and x_{st}^m that satisfy constraints (i)-(vi) in Proposition 2, then there exist feasible personalized prices and quantities such that for each member $m = 1, ..., M$, the set $\{(\mathbf{p}_t, \mathfrak{P}_t^m; \mathfrak{q}_t^m, \mathbf{Q}_t); t = 1, ..., T\}$ satisfies *GARP*. We prove *ad absurdum*. Suppose that for any specification of the feasible personalized prices and quantities that satisfy constraints (i)-(vi) in Proposition 2 we have a sequence $(u, v, ..., z)$ such that, for some m, $(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R_0^m\ (\mathfrak{q}_u^m, \mathbf{Q}_u)$, $(\mathfrak{q}_u^m, \mathbf{Q}_u)\ R_0^m\ (\mathfrak{q}_v^m, \mathbf{Q}_v)$, ..., $(\mathfrak{q}_z^m, \mathbf{Q}_z)\ R_0^m\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$ and $\mathbf{p}_t'\mathfrak{q}_t^m + (\mathfrak{P}_t^m)'\mathbf{Q}_t > \mathbf{p}_t'\mathfrak{q}_s^m + (\mathfrak{P}_t^m)'\mathbf{Q}_s$. By construction, $(\mathfrak{q}_s^m, \mathbf{Q}_s)\ R_0^m\ (\mathfrak{q}_u^m, \mathbf{Q}_u)$, $(\mathfrak{q}_u^m, \mathbf{Q}_u)\ R_0^m\ (\mathfrak{q}_v^m, \mathbf{Q}_v)$, ..., $(\mathfrak{q}_z^m, \mathbf{Q}_z)\ R_0^m\ (\mathfrak{q}_t^m, \mathbf{Q}_t)$ implies $\mathbf{p}_s'\mathfrak{q}_s^m + (\mathfrak{P}_s^m)'\mathbf{Q}_s \geq \mathbf{p}_s'\mathfrak{q}_u^m + (\mathfrak{P}_s^m)'\mathbf{Q}_u$, $\mathbf{p}_u'\mathfrak{q}_u^m + (\mathfrak{P}_u^m)'\mathbf{Q}_u \geq \mathbf{p}_u'\mathfrak{q}_v^m + (\mathfrak{P}_u^m)'\mathbf{Q}_v$, ..., $\mathbf{p}_z'\mathfrak{q}_z^m + (\mathfrak{P}_z^m)'\mathbf{Q}_z \geq \mathbf{p}_z'\mathfrak{q}_t^m + (\mathfrak{P}_z^m)'\mathbf{Q}_t$. In terms of the constraints (i)-(vi), this means that there always exists a sequence $(u, v, ..., z)$ such that, on the one hand, $x_{su}^m = x_{uv}^m = ... = x_{zt}^m = 1$ (because of constraint (iv)) and thus $x_{st}^m = 1$ (because of constraint (v)) while, on the other hand, $\mathbf{p}_t'\mathfrak{q}_t^m + (\mathfrak{P}_t^m)'\mathbf{Q}_t > \mathbf{p}_t'\mathfrak{q}_s^m + (\mathfrak{P}_t^m)'\mathbf{Q}_s$ and thus constraint (vi) is violated. This contradiction completes the proof.

Appendix B: Numerical examples

In this appendix, we illustrate the IP methodology by means of two fully worked examples. Interestingly, these examples demonstrate that the methodology can obtain precise recovery (i.e. tight sets of feasible income shares, personalized prices and personalized quantities) even when the number of observations is small (*in casu* $T = 3$) and no assignable quantity information can be used (i.e. $\mathbf{q}_t^{Am} = \mathbf{0}$ for all m and t).[15] These tight sets can be identified because there is a large variation in the observed prices and aggregate quantities. In general, for a given price-quantity variation, we can -of course- expect the sets to become tighter when more information can be used (e.g. because T gets larger or $\mathbf{q}_t^{Am} > \mathbf{0}$ for some m and t). Such additional information can also include specific hypotheses about the decision structure underlying observed group behavior (*in casu* the sharing rule, feasible personalized prices and the feasible

[15] This may be important for observational data on collective structures that do not contain any assignable quantity information. For example, data on extended families may lack exclusive goods for some household members.

personalized quantities). In fact, as also shown in the examples, our approach allows for testing such assumptions.

We will focus on two limiting specifications of the collective model in Definition 2. These models impose specific *a priori* structure on the group behavior: (1) the model in which all goods are public, and (2) the model in which all goods are private. Evidently, examples with some goods private and other goods public can be constructed in a directly analogous manner.

B.1 All consumption public

Example 1 considers the case in which all goods are publicly consumed and, thus, there is no private consumption ($N = 0$). In terms of the general condition for collective rationality in Definition 2, this means that we consider member-specific utility functions $U^m(\mathbf{q}^m, \mathbf{Q}) = V^m(\mathbf{Q})$. It is worth emphasizing that this setting is more general than may seem at first sight. *Stricto sensu*, the mere implication is that the (observed) aggregate quantities (fully) enter all utility functions. As discussed at the end of Section 2, this allows for private consumption with externalities of a particular good n by member m as long as that good n is exclusively consumed by that member m.

Example 1. *Consider a two-member household ($M = 2$) that consumes three goods ($K = 3$). Suppose three observations with quantities and prices (for $0 < \epsilon < 1$)*[16]

$$\begin{aligned}
\mathbf{Q}_1 &= (1,0,0)', \mathbf{P}_1 = (1+\epsilon, 1, \epsilon/2)', \\
\mathbf{Q}_2 &= (0,1,0)', \mathbf{P}_2 = (1, 1+\epsilon, \epsilon/2)', \\
\mathbf{Q}_3 &= (0,0,1)', \mathbf{P}_3 = (0.5+\epsilon/2, 0.5+\epsilon/2, 1)'.
\end{aligned}$$

As a preliminary step, we note that these prices and quantities imply

$$\begin{aligned}
y_1 &= 1+\epsilon,\ \mathbf{P}_1'\mathbf{Q}_2 = 1,\ \mathbf{P}_1'\mathbf{Q}_3 = \epsilon/2, \\
y_2 &= 1+\epsilon,\ \mathbf{P}_2'\mathbf{Q}_1 = 1,\ \mathbf{P}_2'\mathbf{Q}_3 = \epsilon/2, \\
y_3 &= 1,\ \mathbf{P}_3'\mathbf{Q}_1 = 0.5+\epsilon/2,\ \mathbf{P}_3'\mathbf{Q}_2 = 0.5+\epsilon/2.
\end{aligned}$$

[16] We emphasize that we use zero quantities only for mathematical elegance. Of course, this use of zero quantities does not affect the core of our arguments in this and the following example.

Step 1. *We first consider the restrictions on the binary variables x^1_{st} and x^2_{st} ($s, t \in \{1, 2, 3\}$, $s \neq t$) for the current data. As a first result, we must have $x^1_{st} = 1$ or $x^2_{st} = 1$ for any s and t. Specifically, constraint (iv) in Proposition 2 implies*

$$\mathfrak{y}^1_s - \left(\mathfrak{P}^1_s\right)' \mathbf{Q}_t < y_s x^1_{st} \text{ and } \mathfrak{y}^2_s - \left(\mathfrak{P}^2_s\right)' \mathbf{Q}_t < y_s x^2_{st}.$$

Combining these two constraints, and using that $\mathbf{P}_s = \mathfrak{P}^1_s + \mathfrak{P}^2_s$ and $y_s = \mathfrak{y}^1_s + \mathfrak{y}^2_s$, yields

$$y_s - \mathbf{P}'_s \mathbf{Q}_t < y_s \left(x^1_{st} + x^2_{st}\right);$$

and thus, because $y_s > \mathbf{P}'_s \mathbf{Q}_t$, we necessarily have $x^1_{st} = 1$ or $x^2_{st} = 1$ for any s and t.

As a second result, we obtain that $x^m_{st} = 1$ implies $x^l_{ts} = 1$ ($m, l \in \{1, 2\}$, $m \neq l$) for any s and t. Specifically, for $x^m_{st} = 1$ constraint (vi) in Proposition 2 entails

$$\mathfrak{y}^m_t - \left(\mathfrak{P}^m_t\right)' \mathbf{Q}_s \leq 0 \; (= y_t \left(1 - x^m_{st}\right)).$$

Using $\mathbf{P}_t = \mathfrak{P}^1_t + \mathfrak{P}^2_t$, $y_t = \mathfrak{y}^1_t + \mathfrak{y}^2_t$ and $y_t > \mathbf{P}'_t \mathbf{Q}_s$, this obtains

$$\mathfrak{y}^l_t > \left(\mathfrak{P}^l_t\right)' \mathbf{Q}_s, \text{ and thus } x^l_{ts} = 1 \text{ because of constraint (iv).}$$

As a third result, we cannot have $x^1_{st} = 1$ and $x^2_{st} = 1$ for any s and t. If $x^1_{st} = 1$ and $x^2_{st} = 1$, then constraint (vi) in Proposition 2 requires

$$\mathfrak{y}^1_t - \left(\mathfrak{P}^1_t\right)' \mathbf{Q}_s \leq 0 \text{ and } \mathfrak{y}^2_t - \left(\mathfrak{P}^2_t\right)' \mathbf{Q}_s \leq 0.$$

In turn, using $\mathbf{P}_t = \mathfrak{P}^1_t + \mathfrak{P}^2_t$ and $y_t = \mathfrak{y}^1_t + \mathfrak{y}^2_t$, this yields

$$y_t - \mathbf{P}'_t \mathbf{Q}_s \leq 0,$$

which is excluded because $y_t > \mathbf{P}'_t \mathbf{Q}_s$.

As a fourth result, we cannot have (i) $x^m_{21} = 1$ and $x^l_{31} = 1$ or (ii) $x^m_{12} = 1$ and $x^l_{32} = 1$ ($m \neq l$). For example, consider $x^m_{21} = 1$ and $x^l_{31} = 1$. (The argument for $x^m_{12} = 1$ and $x^l_{32} = 1$ is directly analogous.) In that case, constraint (vi) in Proposition

2 requires

$$y_1 - (\mathfrak{P}_1^m)' \mathbf{Q}_2 - \left(\mathfrak{P}_1^l\right)' \mathbf{Q}_3 \leq 0 \ (= \ y_1 \left(2 - x_{21}^m - x_{31}^l\right)),$$

which is excluded because $y_1 > \mathbf{P}_1' (\mathbf{Q}_2 + \mathbf{Q}_3)$ *and, by construction,* $\mathbf{P}_1' (\mathbf{Q}_2 + \mathbf{Q}_3) \geq (\mathfrak{P}_1^m)' \mathbf{Q}_2 + \left(\mathfrak{P}_1^l\right)' \mathbf{Q}_3$.

Given these four results, we necessarily obtain $x_{13}^m = x_{12}^m = x_{32}^m = 1$ *and* $x_{23}^l = x_{21}^l = x_{31}^l = 1$. *It is easily verified that this specification satisfies the necessary and sufficient condition in Proposition 2, i.e. the corresponding feasible region defined by constraints (i)-(vi) is non-empty.*

Step 2. *Next, we consider recovery of the sharing rule. Using constraint (vi) in Proposition 2 (together with* $\mathbf{P}_3'\mathbf{Q}_1 \geq (\mathfrak{P}_3^m)' \mathbf{Q}_1$ *and* $\mathbf{P}_3'\mathbf{Q}_2 \geq \left(\mathfrak{P}_3^l\right)' \mathbf{Q}_2$, *which hold by construction), we obtain*

$$\begin{aligned} x_{13}^m &= 1 \Rightarrow \mathfrak{y}_3^m \leq \mathbf{P}_3'\mathbf{Q}_1 = 0.5 + \epsilon/2 \Rightarrow \mathfrak{y}_3^l = y_3 - \mathfrak{y}_3^m \geq 0.5 - \epsilon/2, \\ x_{23}^l &= 1 \Rightarrow \mathfrak{y}_3^l \leq \mathbf{P}_3'\mathbf{Q}_2 = 0.5 + \epsilon/2 \Rightarrow \mathfrak{y}_3^m = y_3 - \mathfrak{y}_3^l \geq 0.5 - \epsilon/2; \end{aligned}$$

or, when ϵ *becomes arbitrarily small we obtain very tight sets (around 0.5) of feasible income shares* $\mathfrak{y}_3^1$ *and* $\mathfrak{y}_3^2$.

Similarly, we get

$$\begin{aligned} x_{32}^m &= 1 \Rightarrow \mathfrak{y}_2^m \leq \mathbf{P}_2'\mathbf{Q}_3 = \epsilon/2 \Rightarrow \mathfrak{y}_2^l = y_2 - \mathfrak{y}_2^m \geq 1 - \epsilon/2, \\ x_{31}^l &= 1 \Rightarrow \mathfrak{y}_1^l \leq \mathbf{P}_1'\mathbf{Q}_3 = \epsilon/2 \Rightarrow \mathfrak{y}_1^m = y_1 - \mathfrak{y}_1^l \geq 1 - \epsilon/2; \end{aligned}$$

which again obtains tight sets of $\mathfrak{y}_t^m$ *and* $\mathfrak{y}_t^l$ $(t = 1, 2)$ *when* ϵ *gets small. For example,* ϵ *arbitrarily close to zero yields* $\mathfrak{y}_1^m \approx 1$, $\mathfrak{y}_1^l \approx 0$ *and* $\mathfrak{y}_2^m \approx 0$, $\mathfrak{y}_2^l \approx 1$.

Two remarks are in order. First, this result can be interpreted in terms of 'bargaining power' of the individual members, for which the sharing rule can be interpreted as an indicator. Specifically, consider ϵ *arbitrarily small. In that case, member* m *can be conceived as the (quasi) 'dictator' in situation 1 (i.e. member* m *is solely responsible for the full household budget or* $\mathfrak{y}_1^m \approx y_1$*) while the other member* l *is the 'dictator' in situation 2 (*$\mathfrak{y}_2^l \approx y_2$*); in situation 3, finally, the aggregate income is split equally over the two members (*$\mathfrak{y}_3^1 \approx \mathfrak{y}_3^2 \approx 0.5y_3$*).*

Second, the proposed method allows for imposing a whole series of additional restrictions on the sharing rule (or, alternatively, for testing specific hypotheses about *the sharing rule). For instance, suppose that in our current example we impose (or assume) that the feasible income share of the husband (member 1) is higher than that of the wife (member 2) in situation 1, i.e.* $\mathfrak{y}_1^1 \geq \mathfrak{y}_1^2$*. This immediately obtains* $1-\epsilon/2 \leq \mathfrak{y}_1^1 \leq 1$, $0 \leq \mathfrak{y}_1^2 \leq \epsilon/2$ *and* $0 \leq \mathfrak{y}_2^1 \leq \epsilon/2$, $1-\epsilon/2 \leq \mathfrak{y}_2^2 \leq 1$*; and, thus, for* ϵ *arbitrarily small the mere restriction* $\mathfrak{y}_1^1 \geq \mathfrak{y}_1^2$ *implies that the husband is the 'dictator' in situation 1* ($\mathfrak{y}_1^1 \approx y_1$) *and the wife is the 'dictator' in situation 2* ($\mathfrak{y}_2^2 \approx y_2$)*. Alternatively, one can put upper and lower bounds (or test corresponding assumptions) on the relative income share of some member* m *in situation* t*, i.e.* $\underline{y}_t^m \leq \mathfrak{y}_t^m/y_t \leq \overline{y}_t^m$ *for* $\underline{y}_t^m, \overline{y}_t^m \in [0,1]$*. For instance, our result implies that any lower bound* $\underline{y}_t^m > \epsilon/2$ *for some* m *and all* t *will be rejected for this specific data structure. Finally, additional sharing rule restrictions can impose a specific relationship between feasible income shares of the same member* m *in different situations (e.g. time periods). For instance, suppose that we assume in the current example that the feasible income share of the husband must be higher in situation 1 than in situation 2, i.e.* $\mathfrak{y}_1^1 \geq \mathfrak{y}_2^1$*; this directly obtains* $1-\epsilon/2 \leq \mathfrak{y}_1^1 \leq 1$, $0 \leq \mathfrak{y}_2^1 \leq \epsilon/2$ *and* $0 \leq \mathfrak{y}_1^2 \leq \epsilon/2$, $1-\epsilon/2 \leq \mathfrak{y}_2^2 \leq 1$.

Step 3. *Let us then consider recovery of the feasible personalized (Lindahl) prices. As a starting point, we use our conclusion for the feasible income shares, which can be summarized as*

$$\begin{aligned}
1-\epsilon/2 &\leq \mathfrak{y}_1^m \leq 1+\epsilon \text{ and } 0 \leq \mathfrak{y}_1^l \leq \epsilon/2, \\
0 &\leq \mathfrak{y}_2^m \leq \epsilon/2 \text{ and } 1-\epsilon/2 \leq \mathfrak{y}_2^l \leq 1+\epsilon, \\
0.5-\epsilon/2 &\leq \mathfrak{y}_3^1 \leq 0.5+\epsilon/2 \text{ and } 0.5-\epsilon/2 \leq \mathfrak{y}_3^2 \leq 0.5+\epsilon/2.
\end{aligned}$$

For the given data structure, this implies (for $(\mathfrak{P}_t^m)_k$ *the* k*-th entry of* $\mathfrak{P}_t^m$*)*

$$\begin{aligned}
1-\epsilon/2 &\leq (\mathfrak{P}_1^m)_1 \leq 1+\epsilon \text{ and } 0 \leq \left(\mathfrak{P}_1^l\right)_1 \leq \epsilon/2, \\
0 &\leq (\mathfrak{P}_2^m)_2 \leq \epsilon/2 \text{ and } 1-\epsilon/2 \leq \left(\mathfrak{P}_2^l\right)_2 \leq 1+\epsilon, \\
0.5-\epsilon/2 &\leq \left(\mathfrak{P}_3^1\right)_3 \leq 0.5+\epsilon/2 \text{ and } 0.5-\epsilon/2 \leq \left(\mathfrak{P}_3^2\right)_3 \leq 0.5+\epsilon/2.
\end{aligned}$$

We thus get very tight sets of feasible values for $(\mathfrak{P}_t^m)_t$ *and* $\left(\mathfrak{P}_t^l\right)_t$ *when* ϵ *gets arbitrarily small. To illustrate the impact of additional structure, suppose that* $(\mathfrak{P}_1^1)_1 > (\mathfrak{P}_1^2)_1$*, i.e. the husband contributes more to the first good in situation 1.*

For ϵ arbitrarily small, this mere restriction implies that the husband 'pays' (quasi) everything of the first good in situation 1 ($(\mathfrak{P}_1^1)_1 \approx (\mathbf{P}_1)_1$), while the wife pays everything of the second good in situation 2 ($(\mathfrak{P}_2^2)_2 \approx (\mathbf{P}_2)_2$); finally, in situation 3 the expenditure for the third good is equally split ($(\mathfrak{P}_3^1)_3 \approx (\mathfrak{P}_3^2)_3 \approx 0.5\,(\mathbf{P}_3)_3$).

B.2 All consumption private

Example 2 considers the case that excludes public consumption ($K = 0$). In other words, all consumption is private. This model is commonly referred to as the 'egoistic' model in the literature on collective consumption behavior. In terms of the general condition for collective rationality in Definition 2, this means that we consider member-specific utility functions $U^m(\mathbf{q}^m, \mathbf{Q}) = V^m(\mathbf{q}^m)$. It is worth noting that this case actually also encompasses a wider class of member-specific utilities that model 'altruism' in a specific way: it also includes so-called 'caring preferences', which correspond to utility functions $U^m(\mathbf{q}^m, \mathbf{Q}) = W^m\left(V^1(\mathbf{q}^1), ..., V^M\left(\mathbf{q}^M\right)\right)$ that depend not only on member m's own 'egoistic' utility but also on the other member l's utility defined in terms of $\mathbf{q}^l$. Chiappori (1992) argues that every Pareto efficient outcome in terms of caring preferences (W^m) is also Pareto efficient in terms of egoistic preferences (V^m). In other words, under Pareto efficiency the empirical implications of caring preferences are indistinguishable from those of egoistic preferences.

Example 2. *We recapture the situation of Example 1, with corresponding observed prices and quantities. In this case all goods are privately consumed so that $N = 3$ and*

$$\begin{aligned}
\mathbf{q}_1 &= (1,0,0)', \mathbf{p}_1 = (1+\epsilon, 1, \epsilon/2)', \\
\mathbf{q}_2 &= (0,1,0)', \mathbf{p}_2 = (1, 1+\epsilon, \epsilon/2)', \\
\mathbf{q}_3 &= (0,0,1)', \mathbf{p}_3 = (0.5+\epsilon/2, 0.5+\epsilon/2, 1)'.
\end{aligned}$$

We note that this example does not include assignable quantity information.

As for the feasible income shares, an analogous reasoning as in Steps 1 and 2 of

Example 1 yields the conclusion (for $m \neq l$)

$$\begin{aligned} 1-\epsilon/2 &\leq \mathfrak{y}_1^m \leq 1+\epsilon \text{ and } 0 \leq \mathfrak{y}_1^l \leq \epsilon/2, \\ 0 &\leq \mathfrak{y}_2^m \leq \epsilon/2 \text{ and } 1-\epsilon/2 \leq \mathfrak{y}_2^l \leq 1+\epsilon, \\ 0.5-\epsilon/2 &\leq \mathfrak{y}_3^1 \leq 0.5+\epsilon/2 \text{ and } 0.5-\epsilon/2 \leq \mathfrak{y}_3^2 \leq 0.5+\epsilon/2. \end{aligned}$$

Focusing on the feasible personalized quantities, this implies (using constraint (iv), for $(\mathfrak{q}_t^m)_n$ the n-th entry of $\mathfrak{q}_t^m$)

$$\begin{aligned} 1-\epsilon/2 &\leq (1+\epsilon)(\mathfrak{q}_1^m)_1 \leq 1+\epsilon \text{ and } 0 \leq (1+\epsilon)\left(\mathfrak{q}_1^l\right)_1 \leq \epsilon/2, \\ 0 &\leq (1+\epsilon)(\mathfrak{q}_2^m)_2 \leq \epsilon/2 \text{ and } 1-\epsilon/2 \leq (1+\epsilon)\left(\mathfrak{q}_2^l\right)_2 \leq 1+\epsilon, \\ 0.5-\epsilon/2 &\leq \left(\mathfrak{q}_3^1\right)_3 \leq 0.5+\epsilon/2 \text{ and } 0.5-\epsilon/2 \leq \left(\mathfrak{q}_3^2\right)_3 \leq 0.5+\epsilon/2. \end{aligned}$$

We thus obtain

$$\begin{aligned} (1-\epsilon/2)/(1+\epsilon) &\leq (\mathfrak{q}_1^m)_1 \leq 1/(1+\epsilon) \text{ and } 0 \leq \left(\mathfrak{q}_1^l\right)_1 \leq \epsilon/\left(2(1+\epsilon)\right), \\ 0 &\leq (\mathfrak{q}_2^m)_2 \leq \epsilon/\left(2(1+\epsilon)\right) \text{ and } (1-\epsilon/2)/(1+\epsilon) \leq \left(\mathfrak{q}_2^l\right)_2 \leq 1/(1+\epsilon), \\ 0.5-\epsilon/2 &\leq \left(\mathfrak{q}_3^1\right)_3 \leq 0.5+\epsilon/2 \text{ and } 0.5-\epsilon/2 \leq \left(\mathfrak{q}_3^2\right)_3 \leq 0.5+\epsilon/2. \end{aligned}$$

This yields very tight sets of feasible values for $(\mathfrak{q}_t^m)_t$ and $\left(\mathfrak{q}_t^l\right)_t$ when ϵ gets arbitrarily small. To illustrate the impact of additional structure, suppose that $(\mathfrak{q}_1^1)_1 > (\mathfrak{q}_1^2)_1$, i.e. the husband consumes more of the first good in situation 1. For ϵ arbitrarily small, this sole restriction immediately obtains that the husband consumes (quasi) everything of the first good in situation 1 ($(\mathfrak{q}_1^1)_1 \approx (\mathbf{q}_1)_1$), while the wife consumes everything of the second good in situation 2 ($(\mathfrak{q}_2^2)_2 \approx (\mathbf{q}_2)_2$); finally, in situation 3 the third good is equally split ($(\mathfrak{q}_3^1)_3 \approx (\mathfrak{q}_3^2)_3 \approx 0.5\,(\mathbf{q}_3)_3$).

Appendix C: Data

The construction of the base scenarios s_n^1 and s_n^2. As explained in the main text, in our application we construct the base scenario by using the data that are available for (male and female) singles (or one-person households): we consider average budget shares for male and female singles as obtained from the same RLMS

Table 5.1: Descriptive statistics

Goods	Mean budget shares for couples	(std. dev.)	s_n^1	s_n^2
Food outside the home	0.029	(0.107)	0.638	0.362
Clothing	0.073	(0.158)	0.275	0.725
Car fuel	0.054	(0.123)	0.856	0.144
Wood fuel	0.034	(0.134)	(public)	
Gas fuel	0.022	(0.072)	(public)	
Luxury goods	0.018	(0.097)	0.675	0.325
Services	0.191	(0.222)	0.436	0.564
Housing rent	0.146	(0.170)	(public)	
Bread	0.103	(0.141)	0.483	0.517
Potatoes	0.010	(0.054)	0.393	0.607
Vegetables	0.018	(0.055)	0.312	0.688
Fruit	0.013	(0.029)	0.406	0.594
Meat	0.093	(0.119)	0.544	0.456
Dairy products	0.047	(0.063)	0.371	0.629
Fat	0.025	(0.049)	0.396	0.604
Sugar	0.047	(0.092)	0.473	0.527
Eggs	0.011	(0.022)	0.532	0.468
Fish	0.016	(0.039)	0.405	0.595
Other food	0.017	(0.041)	0.510	0.490
Alcohol	0.014	(0.041)	0.837	0.163
Tobacco	0.016	(0.058)	0.992	0.008
Household characteristics	**Mean**	**(st.dev.)**		
Age male	60.8	(12.20)		
Age female	61.7	(10.28)		
Education level male	7.66	(2.43)		
Education level female	7.71	(2.46)		
Real income male (Dec. 2003 RUB)	10,765	(14,953)		
Real income female (Dec. 2003 RUB)	8,393	(11,179)		
Real expenditures (Dec. 2003 RUB)	21,773	(33,544)		

data set. Specifically, let member 1 be the male in the household and member 2 the female, and let w_n^1 and w_n^2 represent the (mean) budget share of the private good n for a single male and a single female, respectively. Then, we define the male quantities $\mathbf{q}_t^{B1}$ and the female quantities $\mathbf{q}_t^{B2}$ in the base scenario as follows (with $(\mathbf{q}_t)_n$ the n-th entry of $\mathbf{q}_t$):

$$\left(\mathbf{q}_t^{B1}\right)_n = s_n^1 \ (\mathbf{q}_t)_n \text{ and } \left(\mathbf{q}_t^{B2}\right)_n = s_n^2 \ (\mathbf{q}_t)_n$$

with

$$s_n^1 = \frac{w_n^1}{w_n^1 + w_n^2} \text{ and } s_n^2 = \frac{w_n^2}{w_n^1 + w_n^2}.$$

Intuitively, the value of s_n^1 (s_n^2) indicates the relative importance of good n in the consumption pattern of the male (female). For example, suppose $w_n^1/w_n^2 = \alpha_n$, so that α_n reflects the relative importance of good n for the male as compared to the female. In that case, we have $\left(\mathbf{q}_t^{B1}\right)_n / \left(\mathbf{q}_t^{B2}\right)_n = s_n^1/s_n^2 = \alpha_n$.

References

[1] Afriat, S. (1967), "The construction of utility functions from expenditure data", *International Economic Review*, 8, 67-77.

[2] Andreoni, J. and J. Miller (2002), "Giving According to GARP: An experimental test of the consistency of preferences for altruism", *Econometrica*, 70, 737-753.

[3] Andreoni, J. and W.T. Harbaugh (2006), "Power indices for revealed preference tests", University of Wisconsin-Madison Department of Economics Working Paper, 2005-10, version March 2006.

[4] Blundell, R., M. Browning and I. Crawford (2003), "Nonparametric Engel Curves and. Revealed Preference", *Econometrica*, 71, 205-240.

[5] Blundell, R., M. Browning and I. Crawford (2008), "Best nonparametric bounds on demand responses", *Econometrica*, 76, 1227-1262.

[6] Blundell, R., P.-A. Chiappori and C. Meghir (2005), "Collective labor supply with children", *Journal of Political Economy*, 113, 1277-1306.

[7] Bonke, J. and M. Browning (2006), “Allocation within the household: direct survey evidence”, Working paper 286, Department of Economics, University of Oxford.

[8] Bourguignon, F., Browning, M and P.-A. Chiappori (2009), “Efficient intra-household allocations and distribution factors: implications and identification”, *Review of Economic Studies*, 76, 503-528.

[9] Bronars, S. (1987), “The power of nonparametric tests of preference maximization”, *Econometrica*, 55, 693-698.

[10] Browning, M, F. Bourguignon, P.-A. Chiappori and V. Lechene (1994), “Income and Outcomes: A Structural Model of Intrahousehold Allocations”, *Journal of Political Economy*, 102, 1067-1096.

[11] Browning, M. and P.-A. Chiappori (1998), “Efficient intra-household allocations: a general characterization and empirical tests”, *Econometrica*, 66, 1241-1278.

[12] Browning, M., P.-A. Chiappori and A. Lewbel (2006), “Estimating consumption economies of scale, adult equivalence scales, and household bargaining power”, Boston College Working Paper WP588, Department of Economics, Boston College, version 2006.

[13] Bruyneel, S., L. Cherchye and B. De Rock (2008), “How to model collective decisions? Evidence from a laboratory experiment”, ECORE discussion paper 2007/116, ECORE, Université Libre de Bruxelles, version 2008.

[14] Cherchye, L., B. De Rock, J. Sabbe and F. Vermeulen (2008), “Nonparametric tests of collectively rational consumption behavior: an integer programming procedure”, *Journal of Econometrics*, 147, 258-265.

[15] Cherchye, L., B. De Rock and F. Vermeulen (2007), “The collective model of household consumption: a nonparametric characterization”, *Econometrica*, 75, 553-574.

[16] Cherchye, L., B. De Rock and F. Vermeulen (2008), “The revealed preference approach to collective consumption behavior: testing, recovery and welfare analysis”, *CentER Discussion Paper* No. 2007-73, University of Tilburg, version 2008.

[17] Cherchye, L., B. De Rock, and F. Vermeulen (2009), "Opening the black box of intra-household decision-making: theory and non-parametric empirical tests of general collective consumption models", *Journal of Political Economy*, 117, 1074-1104.

[18] Chernozhukov, V., H. Hong and E. Tamer (2007), "Estimation and confidence regions for parameter sets in econometric models", *Econometrica*, 75, 1243 -1284.

[19] Chiappori, P.-A. (1988), "Rational household labor supply", *Econometrica*, 56, 63-89.

[20] Chiappori, P.-A. (1992), "Collective labor supply and welfare", *Journal of Political Economy*, 100, 437-467.

[21] Chiappori, P.-A. and I. Ekeland (2006), "The micro economics of group behavior: general characterization", *Journal of Economic Theory*, 130, 1-26.

[22] Chiappori, P.-A. and I. Ekeland (2009): "The micro economics of efficient group behavior: identification", *Econometrica*, 77, 763-799.

[23] Diewert, E. (1973), "Afriat and revealed preference theory", *Review of Economic Studies*, 40, 419-425.

[24] Donni O. (2008), "Household behavior and family economics", in W.-B. Zhang (ed.), Mathematical Models in Economics, in *The Encyclopaedia of Life Support System (EOLSS)*, Developed under the Auspices of the UNESCO, Oxford: Eolss Publishers.

[25] Harbaugh, W.T., K. Krause and T. Berry (2001), "GARP for Kids: On the development of rational choice behavior", *American Economic Review*, 91, 1539-1545.

[26] Lacroix, G. and N. Radtchenko (2008), "The Changing Intra-Household Resource Allocation in Russia", IZA Discussion Papers 3733, Institute for the Study of Labor (IZA).

[27] Lewbel, A. and K. Pendakur (2008), "Estimation of Collective Household Models with Engel Curves", *Journal of Econometrics*, 147, 350-358.

[28] Manski, C. (2003), *Partial identification of probability distributions*, New York: Springer Verlag.

[29] Nemhauser, G. and L. Wolsey (1999), *Integer and combinatorial optimization*, John Wiley, New York.

[30] Sippel, R. (1997), "An experiment on the pure theory of consumer behaviour", *Economic Journal*, 107, 1431-44.

[31] Varian, H. (1982), "The nonparametric approach to demand analysis", *Econometrica*, 50, 945-972.

[32] Varian, H. (1990), "Goodness-of-fit in optimizing models", *Journal of Econometrics,* 46, 125–140.