Monetary Policy Shocks, Choleski Identification, and DNK Models: An Empirical Investigation for the U.S.*

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Abstract

The identification of monetary policy shocks in structural VARs is often achieved by assuming delayed impacts on inflation and output, i.e. a Choleski economy. Alternatively, standard Dynamic New-Keynesian (DNK) models typically allow for immediate effects. We show that a DNK model estimated with U.S. data predicts a significant and persistent reaction of inflation and output to an unexpected move of the policy rate. We then use the estimated DNK as Data Generating Process (DGP) to feed Choleski-SVARs in a Monte Carlo exercise, and show that, on average, Choleski-SVARs wrongly predict muted macroeconomic responses. Intriguingly, this "in lab" evidence replicates the Choleski-SVAR impulse responses estimated with actual U.S. data in the great moderation sample.

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1 Introduction

The macroeconomic effects of a monetary policy shock are often estimated with structural vector autoregressions (SVARs). Several authors have found that, conditional on a stable macroeconomic sample like the great moderation, SVARs tend to return responses of inflation and output not significantly different from zero, or very mild at best (see, among others, Christiano, Eichenbaum, and Evans (1999), Hanson (2004), Boivin and Giannoni (2006), Mojon (2008), Castelnuovo and Surico (2009)). Figure 1 recalls this evidence. A trivariate VAR estimated with 1984:I-2008:II U.S. data suggests that, in response to a monetary policy shock identified with a standard Choleski scheme, the reaction of inflation and the output gap is basically nil.[1] Zero or weak reactions are obtained also with the Factor Augmented VAR approach, which incorporates information coming from large datasets (Boivin and Giannoni (2006), Consolo, Favero, and Paccagnini (2009)).[2] A possible interpretation of this evidence points toward the reduced influence exerted by monetary policy shocks on the economy, possibly due to financial innovations that might have enabled firms and consumers to better tackle the impact of interest rate fluctuations. Another interpretation suggests that the U.S. systematic monetary policy may have fought deviations of inflation and output from the policy targets more successfully.

This paper shows that the SVAR reactions in Figure 1 are *fully compatible* with a

[1] Giordani (2004) offers a theoretical and empirical analysis that argues that, if a measure of potential output is omitted from the SVAR, the estimated responses are doomed to be severely biased. Our vector includes a measure of the output gap constructed with the Congressional Budget Office's estimates of the U.S. potential output.

[2] Different results are typically obtained when dealing with a sample that includes the 1970s (e.g. Christiano, Eichenbaum, and Evans (2005)). Interestingly, Mojon (2008) shows that such evidence may be induced by shifts in the mean of the inflation process occurred in the 1970s and 1980s. When controlling for such shifts, the impulse responses of inflation and output to a monetary policy shock turn out to be very similar to those obtained with the great moderation sample. Moreover, SVARs estimated with samples including the 1970s often return the "price puzzle", i.e. a positive reaction of inflation to a monetary policy shock. Possibly, such reaction is an artifact driven by omitted factors (see, among others, Bernanke, Boivin, and Eliasz (2005), Forni and Gambetti (2009), and Castelnuovo and Surico (2009)).

monetary policy whose shocks actually exert a *significant* impact on macroeconomic aggregates. The story goes as follows. A popular strategy to identify a monetary policy shock is to assume a recursive (triangular, or Choleski) structure of the contemporaneous relationships of the variables modeled in the vector. This strategy is handy, in that it does not require the researcher to take a position on the identification of other shocks (see Christiano, Eichenbaum, and Evans (1999) for an extensive discussion on this issue). However, in a recent paper, Carlstrom, Fuerst, and Paustian (2009) put in evidence the distortions that may affect impulse responses generated by SVARs due to the *timing discrepancy* existing between a business cycle model admitting an *immediate* reaction of inflation and output to a monetary policy impulse vs. the Choleski-SVAR model, which instead imposes *lagged* reactions. The aim of this paper is exactly that of *assessing* the *empirical* consequences of this timing discrepancy. To hit this target, we first estimate a Dynamic New-Keynesian (DNK) model for the U.S. economy, which allows for an immediate effect of monetary policy surprises on the macroeconomic environment. Then, we employ the estimated framework as our Data Generating Process (DGP) to produce pseudo-data with which we feed Choleski-SVARs, and assess to which extent the (wrong) identifying assumption may lead to distorted impulse responses.

We find robust evidence in favor of statistically significant distortions of the SVARs' impulse responses. While the estimated DNK model predicts a *statistically significant* drop in output and inflation in response to a monetary policy shock, SVARs estimated on pseudo-data return, on average, *muted* reactions of these two variables. The estimated distortion of the responses is sizeable, with deviations with respect to the true (DNK based) responses of about 100% and 95% as for (respectively) four-quarter ahead inflation and output responses. Intriguingly, such reactions replicate the U.S. data-based SVAR evidence discussed above. Endowed with this evidence, we claim that *muted Choleski-SVAR responses* of inflation and output to a monetary policy shock

are *fully consistent* with an effective DNK policy shock. Given the popularity of the recursive identification scheme, widely applied also in the fiscal policy arena and for the scrutiny of the macro-finance interactions, this result is very powerful.

Before moving to our analysis, we make contact with some related literature. Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007) work out a condition to check, given a theoretical model, whether it is possible to recover its structural shocks with a vector autoregression. A DNK model may admit a VAR representation with infinite lags, e.g. typically, models with time delays. Ravenna (2007) discusses under which conditions a finite VAR representation exists, and shows that truncated VARs may provide misleading indications when the true DGP is an infinite order VAR. Further discussions on the distortions coming from the truncation bias, mainly on the identification of the technology shock and the dynamic reaction of hours to it, are offered by Christiano, Eichenbaum, and Vigfusson (2006) and Chari, Kehoe, and McGrattan (2008).[3] With respect to these contributions, we consider a DGP that enjoys a finite order VAR(2) representation, i.e. no truncation bias is at work, at least in population. Still, the impulse responses we estimate in our "in lab" exercise are severely distorted due to the timing discrepancy between DNK and SVARs, and turn out to be remarkably close to those we obtain with actual U.S. data.[4]

The papers closest to ours are probably Canova and Pina (2005) and Carlstrom,

[3] A somewhat related contribution is Benati and Surico (2009), who show that SVARs may display heteroskedasticity in a world in which, by construction, the DGP is homoskedastic but a policy break occurs. Benati (2010) shows that counterfactuals based on SVAR models may deliver dramatically different indications as regards the role of systematic monetary policy with respect to those obtained with a DNK model.

[4] Obviously, the timing-discrepancy issue may be by-passed with the employment of DNK models displaying lagged transmission of the policy impulses. However, several considerations are in order. First, the microfoundations of the transmission lags in the DNK are questionable. Second, as stressed by Carlstrom, Fuerst, and Paustian (2009), such DNKs have VAR exact representations often requiring infinite lags, which naturally raise a truncation bias issue that may harm the precision of the estimated SVAR impulse responses. Third, Choleski-SVAR reactions in line with those produced by DNK models with lagged transmission would not square up with our evidence presented in Figure 1 and that proposed by the contributions cited in the Introduction. Further considerations are proposed by Carlstrom, Fuerst, and Paustian (2009).

Fuerst, and Paustian (2009). Canova and Pina (2005) set up a Monte Carlo exercise in which two calibrated DGPs are employed (a limited participation model and a sticky price-sticky wage economy), and a variety of short-run "zero restrictions" VAR identification schemes are considered. They find remarkable differences between the predictions coming from the structural models and those implied by the estimated SVARs. With respect to Canova and Pina (2005), we deal with an *estimated* DGP, whose calibration is then, by construction, the best to replicate the U.S. macro dynamics of interest. Moreover, we show that the predictions of SVARs estimated on artificial data line up with the (arti)facts generated with SVARs estimated on actual U.S. data. Consequently, we offer an alternative interpretation for the "muted" SVAR's responses often found when identifying a monetary policy shock with Choleski-zero restrictions. As pointed out above, in part of our analysis we rely on some theoretical derivations put forth by Carlstrom, Fuerst, and Paustian (2009), which is essentially a theoretical investigation.

The paper develops as follows. Section 2 presents and estimates the new-Keynesian model we take as our DGP. Section 3 sets up our "in lab" experiment, with which we contrast the impulse responses generated with our DNK with those coming from the SVARs in a controlled environment. An interpretation of our results, based both on some matrix-algebra expressing the DNK-SVAR mapping as well as a battery of simulations, is provided in Section 4. Section 5 presents our robustness checks, which verify the solidity of our results to a variety of perturbations of the baseline framework. Section 6 concludes. A Technical Appendix collects some information on our Bayesian estimation of the DNK framework.

2 DNK as DGP

2.1 The standard DNK framework

We work with a standard, very popular DNK model (see e.g. King (2000), Woodford (2003a), Carlstrom, Fuerst, and Paustian (2009)). The log-linearized version of the model is the following:

$$
\begin{aligned}
(1+\beta)\pi_t &= \beta E_t \pi_{t+1} + \pi_{t-1} + \kappa y_t + \varepsilon_t^{\pi}, && (1)\\
R_t - E_t \pi_{t+1} &= \sigma(E_t y_{t+1} - y_t) + P(\rho_a - 1)a_t, && (2)\\
R_t &= \tau_R R_{t-1} + (1-\tau_R)(\tau_\pi \pi_t + \tau_y y_t) + \varepsilon_t^{R}, && (3)
\end{aligned}
$$

Eq. (1) is an expectational new-Keynesian Phillips curve (NKPC) in which π_t stands for the inflation rate, β represents the discount factor, y_t identifies the output gap, whose impact on current inflation is influenced by the slope-parameter κ, and ε_t^{π} is the "cost-push" shock. Firms set prices optimally conditional on the Calvo-lottery, and full indexation to past inflation à la Christiano, Eichenbaum, and Evans (2005) (by non-reoptimizing firms) is assumed, which implies the presence of past inflation in the supply schedule. Eq. (2) is obtained by log-linearizing households' Euler equation. Output fluctuations are driven both by expectations on future realizations of the business cycle and by the *ex-ante* real interest rate, whose impact is regulated by the degree of risk aversion σ. The convolution $P \equiv \sigma(1+\nu)(\sigma+\nu)^{-1}$ involves the inverse of the Frisch labor elasticity ν, and a_t identifies the technological shock. Eq. (3) is a standard Taylor rule postulating the systematic, inertial reaction of the policy rate to movements in inflation and the output gap. A monetary policy shock ε_t^{R} allows for a stochastic evolution of the policy rate.

The model is closed with the following stochastic processes:

$$\begin{bmatrix} \varepsilon_t^{\pi} \\ a_t \\ \varepsilon_t^{R} \end{bmatrix} = \boldsymbol{F} \begin{bmatrix} \varepsilon_{t-1}^{\pi} \\ a_{t-1} \\ \varepsilon_{t-1}^{R} \end{bmatrix} + \begin{bmatrix} u_t^{\pi} \\ u_t^{a} \\ u_t^{R} \end{bmatrix}, \boldsymbol{F} \equiv \begin{bmatrix} \rho_{\pi} & 0 & 0 \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_R \end{bmatrix}, \tag{4}$$

where the martingale differences, independent processes $\boldsymbol{u}_t$ are distributed as

$$\begin{bmatrix} u_t^{\pi} \\ u_t^{a} \\ u_t^{R} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{\pi}^2 & 0 & 0 \\ 0 & \sigma_a^2 & 0 \\ 0 & 0 & \sigma_R^2 \end{bmatrix} \right). \tag{5}$$

2.2 Model estimation

We estimate the model (1)-(5) with Bayesian methods (for a discussion of this methodology vs. alternatives, see Canova and Sala (2009)). We concentrate on U.S. data spanning the sample 1984:I-2008:II. The edges of the sample correspond to the beginning of the great moderation (McConnell and Perez-Quiros (2000)) and to the acceleration of the financial crises began with the bankruptcy of Lehman Brothers in September 2008, which triggered unconventional policy moves by the Federal Reserve Bank (Brunnermeier (2009)). We employ three observables, which we demean prior to estimation. The output gap is computed as the percentage log-deviation of the real GDP with respect to the measure of potential output provided by the Congressional Budget Office. The inflation rate is the percentage quarterly growth rate of the GDP deflator. Finally, for the short-term nominal interest rate we consider the effective federal funds rate expressed in quarterly terms (averages of monthly values, in percentage terms). The source of the data is the Federal Reserve Bank of St. Louis' website.

The vector $\boldsymbol{\xi} = [\beta, \sigma, \kappa, \nu, \tau_{\pi}, \tau_y, \tau_R, \rho_a, \rho_{\pi}, \rho_R, \sigma_a, \sigma_{\pi,} \sigma_R]'$ collects the parameters characterizing the model. Given the structure we focus on, some parameters are hardly identified. Following Carlstrom, Fuerst, and Paustian (2009), we then set $\beta = 0.99$, $\kappa = 0.1275$, and $\nu = 1$, i.e. a very standard calibration. The remaining priors are collected in Table 1. Notice that such priors are fairly uninformative, above all as regards the autoregressive parameters, which are important drivers of the possible biases arising

when imposing the (wrong) Choleski-factorization to identify the monetary policy shock (Carlstrom, Fuerst, and Paustian (2009)). Some details on the Bayesian algorithm we employ are relegated in the Technical Appendix.

Our posterior estimates are reported in Table 1. Basically, all the estimated parameters assume very conventional values. One interesting result is the similarity between the estimates regarding the persistence of the technological shock ρ_a, whose posterior mean is equal to 0.89, and the degree of interest rate smoothing τ_R, whose posterior mean reads 0.84. Carlstrom, Fuerst, and Paustian (2009) put in evidence how the relative "weight" of these two persistence degrees on the dynamics of inflation and output may induce distortions in the impulse responses (we get back to this issue in the next Section). Figure 2 compares the actual series we aim at tracking with the DNK's one-step ahead predictions, which confirm the very good descriptive power of the DNK model.

3 Impulse responses: DNK vs. Choleski-SVARs

We compare the impulse responses to a monetary policy shock computed with the estimated DNK model with those stemming from Choleski-SVARs estimated with pseudo-data generated by our DNK-DGP framework. Our algorithm works as follows.

For $k = 1$ to K, we

1. sample a realization of the vector $\boldsymbol{\xi}^k$ from the estimated posterior densities;[5]

2. compute the DNK model-consistent impulse responses conditional on $\boldsymbol{\xi}^k$ to an unexpected nominal interest rate hike, and store them in the $[3xHxK]$ $\boldsymbol{DNK\_IRFs}$ matrix, which accounts for the vector of macroeconomic indicators $[\pi, a, R]'$, the

[5] We treat the structural parameters as independently distributed, i.e. we do not account for their possible cross-correlation structure.

$h = [1, ..., H]$ steps ahead of the impulse responses of interest, and the $k = [1, ..., K]$ draws of the vector of structural parameters $\boldsymbol{\xi}$;

3. compute the Choleski-SVAR-consistent impulse responses to a monetary policy shock hike with data $\boldsymbol{x}^{j}_{ps,[1:T]}$ generated with the DNK model conditional on $\boldsymbol{\xi}^{k}$ (ordering: inflation, output gap, nominal rate), and store them in the $[3xHxK]$ $\boldsymbol{SVAR\_IRFs}$ matrix.[6]

We run this algorithm by setting the number of draws $K = 10,000$, the horizon of the impulse response functions $H = 15$, and the length of the pseudo-data sample $T = 98$, i.e a sample length corresponding to that of the actual data sample (1984:I-2008:II) we employed to estimate both our DNK model and the Choleski-SVAR whose responses are plotted in Figure 1. Monetary policy shocks are normalized to induce an on-impact equilibrium reaction of the nominal rate equivalent to 25 quarterly basis points.

Figure 3 contrasts the impulse response obtained with the DNK model with those stemming from the Choleski-SVAR. This figure is telling. First, the estimated DNK predicts a *statistically significant* reaction of both inflation and the output gap (according to our 90% credible set). In particular, the unexpected interest rate hike induces an immediate recession, with the output level getting back to potential just after some quarters. Such recession leads to a persistent deflationary phase, which has its maximum expression after three quarters, but lasts more than three years. Evidently, the U.S. monetary policy's ability to influence inflation and the business cycle finds solid support out of our Bayesian estimation. In fact, a quite different picture arises when turning to our Choleski-SVARs. On average, SVARs return *muted* responses of inflation and output to a monetary policy shock, and even the 68% credible sets contain the

[6] Given that the DNK model has a finite VAR(2) representation, our SVARs are estimated with two lags. We relax this assumption in Section 4.

zero value for all the horizons of interest. In terms of message, the similarity between these SVAR responses and those reported in Figure 1 is impressive. This evidence suggests that SVAR's muted responses of inflation and output to a monetary policy shock estimated with actual U.S. data may very well be due to the Choleski-induced missidentification of the monetary policy shock, which returns zero responses when, *in fact*, monetary policy is effective. In other words, *muted SVAR responses* to a (misspecified) monetary policy shock are *fully consistent* with *significant macroeconomic reactions* to a (correctly identified) shock.

Is the distortion induced by the Choleski-decomposition quantitatively important? To answer this question we compute, per each variable j, horizon h, and draw k the percent-deviation of the SVAR response with respect to the DNK-consistent one. In particular, we compute

$$BIAS(j,h,k) = 100\left[\frac{SVAR\_IRFs[j,h,k]}{DNK\_IRFs[j,h,k]} - 1\right].$$

where $j = [\pi, a, R]$. We focus on the second and fourth quarter-ahead responses of inflation and output. We do so to assess the size of the bias in the "very short run" as well as that after one year, the latter being an horizon typically of interest for policymakers.[7] Notice that, for $h = 2$ and $h = 4$, $DNK\_IRFs(j,h,k)$ is negative as regards inflation and output. Then, for a given variable and a given horizon, a negative realization for BIAS indicates either a SVAR reaction with the correct sign but that underestimates the true (DNK) reaction, or a SVAR reaction with the wrong sign (a "price puzzle" or an "output puzzle").

Figure 4 displays the histograms of the distribution of the quarter-specific percentage deviations. All the objects of interest are affected by substantial distortions. The

[7] The *on-impact* reaction, which corresponds to the very first quarter in our analysis, calls for a Cholesky-induced bias *by construction*, due to the imposition of delayed effects of a monetary policy shock on inflation and output.

distributions are clearly shifted leftward with respect to the zero value, so indicating underestimation of the true effects of a monetary policy shock, or wrongly signed responses. The 68% coverage suggests that these distortions are important also once sample uncertainty is accounted for, with the possible exception of the bias affecting the four-quarter ahead output gap response. To fix ideas, Table 2 collects figures documenting these distortions. The posterior means (of the "BIAS" distributions) are all above 95%. The "uncertainty" surrounding these figures is large, but clearly support the idea of distorted SVAR responses.

4 Why do we get distorted IRFs?

4.1 Investigating the role of the timing discrepancy

Why do we get distorted IRFs from our SVARs in our "in lab" exercise? The fundamental reason is the different timing assumptions of our estimated DNK model, which allows for an *immediate* impact of the policy shock on inflation and output, vs. that of our Choleski-SVAR, which imposes a *delayed* reaction.

To show this point, we exploit Carlstrom *et al*'s (2009) theoretical results. Consider the set of unique decision rules (under equilibrium determinacy) consistent with the rational expectation assumption and the structure of the DNK model:

$$\begin{bmatrix} \pi_t \\ y_t \\ R_t \end{bmatrix} = \boldsymbol{\Gamma} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{bmatrix} + \boldsymbol{B} \begin{bmatrix} \varepsilon_t^{\pi} \\ a_t \\ \varepsilon_t^{R} \end{bmatrix}, \boldsymbol{\Gamma} \equiv \begin{bmatrix} a_1 & 0 & e_1 \\ a_2 & 0 & e_2 \\ a_3 & 0 & e_3 \end{bmatrix}, \boldsymbol{B} \equiv \begin{bmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{bmatrix} \tag{6}$$

where $\boldsymbol{\Gamma}$ and $\boldsymbol{B}$ collect convolutions of the structural parameters $\boldsymbol{\xi}$ of the DNK model.[8] Given that the third column of $\boldsymbol{B}$ does not display, in general, zeros, the monetary policy shock ε_t^R affects *all* the variables of the system *contemporaneously*.

[8] The column of zeros in $\boldsymbol{\Gamma}$ is due to the absence of lagged output in the IS equation (2). Our empirical results are robust to the introduction of past realizations of output in the aggregate demand schedule (see Section 5).

It is easy to show that the system (6) has a VAR(2) representation, which reads

$$\begin{bmatrix} \pi_t \\ y_t \\ R_t \end{bmatrix} = \boldsymbol{A}_1 \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{bmatrix} + \boldsymbol{A}_2 \begin{bmatrix} \pi_{t-2} \\ y_{t-2} \\ R_{t-2} \end{bmatrix} + \boldsymbol{B} \begin{bmatrix} u_t^{\pi} \\ u_t^{a} \\ u_t^{R} \end{bmatrix}, \tag{7}$$

where $\boldsymbol{A}_1 = \boldsymbol{\Gamma} + \boldsymbol{B}\boldsymbol{F}\boldsymbol{B}^{-1}$ and $\boldsymbol{A}_2 = -\boldsymbol{B}\boldsymbol{F}\boldsymbol{B}^{-1}\,\boldsymbol{\Gamma}$. The variance-covariance matrix of $\boldsymbol{B}\boldsymbol{u}$ is given by $\boldsymbol{B}\boldsymbol{\Omega}\boldsymbol{B}^T$, where $\boldsymbol{\Omega}$ is a diagonal $[3x3]$ matrix with the variances of the shocks positioned on the main diagonal. For the ease of exposition, and without loss of generality, we assume $\boldsymbol{\Omega} = \boldsymbol{I}_3$.

Of course, when conducting an econometric exercise, the fundamental shocks $\boldsymbol{u}_t$ are not observable, and must be inferred. To do so, the econometrician can estimate a reduced form VAR(2)

$$\begin{bmatrix} \pi_t \\ y_t \\ R_t \end{bmatrix} = \boldsymbol{A}_1 \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{bmatrix} + \boldsymbol{A}_2 \begin{bmatrix} \pi_{t-2} \\ y_{t-2} \\ R_{t-2} \end{bmatrix} + \begin{bmatrix} \zeta_t^{\pi} \\ \zeta_t^{a} \\ \zeta_t^{R} \end{bmatrix},$$

where $\boldsymbol{\zeta}_t$ is a vector of *residuals* whose variance covariance $VCV(\boldsymbol{\zeta}) = \boldsymbol{\Lambda}$ is a full (non diagonal) $[3x3]$ matrix.

To recover the unobserved structural monetary policy shock u_t^R, a researcher must impose some restrictions on e.g. the simultaneous relationships among the variables included in the vector, the long-run impact of some economic shocks, or the sign of the conditional correlations. As already stressed, a very popular choice is that of orthogonalizing the residuals by imposing a Choleski structure to the system, which assumes delayed effects of the "monetary policy shock" on the variables located before the nominal interest rate in the vector $\begin{bmatrix} \pi_t & y_t & R_t \end{bmatrix}^T$. This is done by computing the unique lower triangular matrix $\widetilde{\boldsymbol{B}}$ such that

$$\widetilde{\boldsymbol{B}}\boldsymbol{\varphi_t} = \boldsymbol{\zeta}, \text{ with } \widetilde{\boldsymbol{B}} = \begin{bmatrix} \widetilde{b}_1 & 0 & 0 \\ \widetilde{b}_2 & \widetilde{c}_2 & 0 \\ \widetilde{b}_3 & \widetilde{c}_3 & \widetilde{d}_4 \end{bmatrix}, \text{ and } \boldsymbol{\varphi_t} = \begin{bmatrix} \varphi_t^{\pi} \\ \varphi_t^{a} \\ \varphi_t^{R} \end{bmatrix}. \tag{8}$$

The Choleski "shocks" $\boldsymbol{\varphi}_t$, which are orthogonal and are assumed to have unitary variance, are then identified by computing the elements of the matrix $\widetilde{\boldsymbol{B}}$ such that

$$\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{B}}^T = \boldsymbol{\Lambda}.$$

This implies that the equivalence $\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{B}}^T = \boldsymbol{B}\boldsymbol{B}^T$ must hold. Solving the system, it is possible to express the elements of $\widetilde{\boldsymbol{B}}$ in terms of the objects belonging to $\boldsymbol{B}$. Then, given the restriction

$$\widetilde{\boldsymbol{B}}\boldsymbol{\varphi}_t = \boldsymbol{B}\boldsymbol{u}_t$$

imposed by (7) and (8), it is possible to express the Choleski-"shocks" $\boldsymbol{\varphi_t}$ in terms of the DNK shocks $\boldsymbol{u_t}$ and the elements belonging to the matrix $\boldsymbol{B}$. Carlstrom, Fuerst, and Paustian (2009) derive the mapping from the true DNK shocks and the Choleski-SVAR monetary policy "shock", which reads

$$\varphi_t^R = \alpha_1 u_t^\pi + \alpha_2 u_t^a + \alpha_3 u_t^R, \tag{9}$$

where the α-weights are given by the expressions

$$\begin{aligned}
\alpha_1 &= \frac{c_2 d_1 - c_1 d_2}{\Phi}, \\
\alpha_2 &= \frac{d_2 b_1 - d_1 b_2}{\Phi}, \\
\alpha_3 &= \frac{b_2 c_1 - b_1 c_2}{\Phi}, \\
\Phi &= \sqrt{(c_2 d_1 - c_1 d_2)^2 + (d_2 b_1 - d_1 b_2)^2 + (b_2 c_1 - b_1 c_2)^2}.
\end{aligned}$$

In general, the "shock" $\varphi_{\boldsymbol{t}}^R$ is a misspecified representation of the true monetary policy shock u_t^R. The standard Choleski identification scheme recovers the true policy shock only under the restriction $d_1 = d_2 = 0$, whose relevance may be appreciated

looking back at the set of decision rules (6) (see the third column vector of the matrix $\boldsymbol{B}$, i.e. the monetary policy shock impulse vector).

Unfortunately, this constraint is not consistent with standard DNK models like the one we focus on in this paper. Indeed, the calibration suggested by our estimated posterior means implies the following values for the matrices characterizing the set of decision rules (6):

$$\boldsymbol{\Gamma} \equiv \begin{bmatrix} 0.72 & 0.00 & -0.17 \\ -0.33 & 0.00 & -0.72 \\ 0.22 & 0.00 & 0.74 \end{bmatrix}, \text{ and } \boldsymbol{B} \equiv \begin{bmatrix} 1.31 & -0.04 & -0.36 \\ -1.10 & -0.17 & -1.35 \\ 0.37 & -0.03 & 0.80 \end{bmatrix}$$

Notably, $\boldsymbol{B}[1,3] = d_1 = -0.36$ and $\boldsymbol{B}[2,3] = d_2 = -1.35$. Not surprisingly, the Choleski restriction is not consistent with the DNK model we are working with. As a consequence, while $\alpha_1 \approx 0$, $\alpha_2 = -0.99$, and $\alpha_3 = 0.13$. Then, the monetary policy shock (mis)identified by the Choleski scheme is in fact a convolution of the true technology shock u_t^a, which enters the reduced form $\varphi_{\boldsymbol{t}}^R$ with a *negative* sign, and of the true monetary policy shock $u_{\boldsymbol{t}}^R$, which enters with a *positive* sign. This offers an interesting interpretation to the muted responses returned by the estimated SVARs. A negative technology shock opens a positive output gap, which exerts a positive pressure on inflation and the policy rate. At the same time, a monetary policy shock (a policy tightening) would trigger a positive reaction of the policy rate, and a negative reaction of inflation and the output gap. Then, the reduced form shock φ_t^R actually captures the joint effects of these two structural shocks, so wrongly leading to muted reactions.

The mapping going from the structural parameters $\boldsymbol{\xi}$ to the elements of the $\boldsymbol{B}$ matrix is highly non-linear, and a closed form solution to express the latter as a function of the former is not available. However, one may resort to numerical approximation to assess to what extent the calibration of the DNK model is responsible for the distortions affecting the VAR impulse responses. We then construct the densities of the α-coefficients by sampling 10,000 realizations of the structural parameters $\boldsymbol{\xi}$ from the estimated posterior

densities, and exploit the relationships discussed in this Section. Figure 5 plots these densities. Interestingly, the cost-push shock u_t^π enters the reduced form SVAR monetary policy shock with a negligible weight, close to zero. By contrast, the distribution of the weight assigned to the technology shock u_t^a is negative and "significantly" different from zero. Also the density of the loading of the u_t^R suggests values different from zero, but positive. As pointed out by Carlstrom, Fuerst, and Paustian (2009), the two effects (that of the technology shock and that of the monetary policy shock) are barely equivalent over time when the persistence of the technology shock and that of the monetary policy shock are similar, then the effect on agents' expectations is comparable. As previously stressed, this is indeed our case.

To visually appreciate to what extent the monetary policy shock is misspecified, Figure 6 contrasts the true monetary policy shock u_t^R and the reduced form φ_t^R. Evidently, the two stochastic processes display a mild comovement, with a degree of correlation as low as 0.36. Realizations displaying very different magnitudes or different signs are frequent. In the light of the differences between these two processes, the amount of distortions affecting the SVAR's impulse responses is perhaps not too surprising.

4.2 Identifying the drivers of the distortions

Which are the structural parameters mainly responsible of this econometric misspecification? Figure 7 depicts the DNK- vs.- SVAR-consistent impulse responses originated by calibrating the DNK model to our estimated posterior means.[9] The baseline scenario basically replicates the situation depicted in Figure 2. We then switch-off some selected structural parameters to isolate their participation to the IRFs. Given the emphasis placed on persistence parameters by Carlstrom, Fuerst, and Paustian (2009) in their empirical analysis, we concentrate on the interest rate smoothing coefficient as well as

[9] As regards the Choleski-SVARs, we consider population moments, which are computed by setting the sample size N=100,000.

the persistence of the cost-push, non-policy demand, and policy rate shocks.

When setting $\tau_R = 0$, the effect of the monetary policy shock on inflation and output turns out to be dramatically dampened. Intuitively, this is due to the effect of interest rate smoothing on agents' expectations over the future paths of inflation and output (Woodford (2003b)). Such effect enhances the impact of a monetary policy shock on current (i.e. on impact) outcomes, so widening the gap between the DNK reactions and the "zeros" assumed by a Choleski decomposition. The second row of Figure 7 makes it clear that the absence of interest rate smoothing, more than improving the Choleski-SVAR's ability to correctly recover the true policy shock, dramatically harms monetary policy's strength. In other words, more than reducing the "artifact", it sweeps the "fact" away. Similarly to what previously found, also setting $\rho_R = 0$ leads to a weakened effect of the "true" monetary policy shock. As for the remaining persistence parameters, while imposing $\rho_\pi = 0$ seems to leave the situation unaltered, important effects come from simulating a scenario in which $\rho_a = 0$. In fact, both inflation and output SVAR reactions get the right sign and a shape very similar to the one predicted by the structural model. This finding lines up with the indications put forth by Carlstrom, Fuerst, and Paustian (2009), who find that the distortions in the reactions of inflation and output are positively correlated with the degree of persistence of the technology shock in this DNK model.

Wrapping up, the on impact distortions of the SVAR inflation and output is mainly induced by the persistence of the nominal interest rate, due to both interest rate smoothing and the persistence of the monetary policy shock. Important distortions, in terms of sign and shape, are also caused by the persistence of the technological shock. Given that all these sources of persistence find solid empirical support in the U.S. data (see e.g. Clarida, Gali, and Gertler (2000) on the interest rate smoothing, and Smets and Wouters (2007) and Justiniano and Primiceri (2008) on the technological shock), we

believe that the above documented distortions are a likely outcome as regards Choleski-SVARs estimated with U.S. data.

5 Robustness checks

We perform some checks to verify the robustness of our results.

- *Partial indexation to past inflation.* In our baseline analysis we borrow Carlstrom *et al*'s (2009) structure of the economy, which assumes full indexation to past inflation à la Christiano, Eichenbaum, and Evans (2005). In fact, the degree of indexation of the U.S. firms is not necessarily full in the sample we focus on. It is then worth checking the robustness of our findings in a model that allows for partial indexation. It is possible to show (Christiano, Eichenbaum, and Evans (2005)) that if just a fraction λ of firms reset prices as a function of past inflation, the NKPC assumes the form

$$(1+\lambda\beta)\pi_t = \beta E_t \pi_{t+1} + \lambda \pi_{t-1} + \kappa y_t + \varepsilon_t^{\pi}. \tag{10}$$

  We then estimate the model (2)-(5) and (10), and employ it as a new DGP to check the distortions arising when imposing the Choleski decomposition. The estimated degree of indexation λ (posterior mean) reads 0.04 (90% credible set: $[0.00, 0.10]$), a value in line with the estimate put forward by Cogley and Sbordone (2008) and Benati and Surico (2009).[10] The estimates of the remaining parameters of the model suggest values fairly in line with those presented in Table 2 with a few exceptions, the most remarkable being the persistence of the mark-up shock ρ_π, whose posterior mean increases to 0.99.[11] These variations notwithstanding, our

[10] The former study models a time-varying trend inflation process jointly with a consistently derived supply curve. Given the relative stability of trend inflation during the great moderation, our point estimate may be considered as comparable to Cogley and Sbordone's (2008).

[11] The Tables with the posterior densities of the models estimated in this Section are not shown for the sake of brevity, but are available upon request.

simulations still support our main result, i.e. the Choleski-induced distortions still force SVARs impulse response to be flat when, in fact, the true ones suggest a persistent deflation and a substantial recession. Figure 8 confirms the solidity of our result.

- *Hybrid IS curve.* The IS curve in our baseline model does not feature any lagged value of the output gap. In fact, habit formation in consumption offers a rationale for the introduction of lagged realizations of the output gap in the aggregate demand schedule. We then re-estimate our framework by considering the following hybrid version of the IS curve:

$$R_t - E_t\pi_{t+1} = \sigma[\phi_y E_t y_{t+1} + (1-\phi_y)y_{t-1} - y_t] + P(\rho_a - 1)a_t,$$

  where ϕ_y identifies the "degree of forward-lookingness" by the U.S. households. Our estimates indicates a posterior mean for ϕ_y equal to 0.80 and a 90% credible set of $[0.68, 0.91]$, values very close to those provided, among others, by Benati and Surico (2008) and Benati and Surico (2009). Again, Figure 9 supports the evidence in favor of the severe misspecification of the SVARs' impulse responses due to the false restrictions imposed by the Choleski identification scheme.

- *Alternative business cycle measure.* Canova (1998) shows that different filtering techniques enjoy different abilities to extrapolate business cycle frequencies out of the real U.S. series. Of course, heterogeneous business cycle representations may imply very diverse calibrations of business cycle models, so influencing the computation of the moments of interest, conditional correlations included (for some Monte Carlo exercises, see Canova and Ferroni (2009)). To check the robustness of our results, we re-estimate our benchmark model with the measure of the business cycle recently proposed by Perron and Wada (2009), which is constructed by assuming a piecewise linear trend with a break in 1973:I for the post-WWII U.S.

real GDP. Figure 9 suggests that our results are robust to this perturbation of the benchmark analysis.

- *Optimal selection of the number of lags of the SVARs.* Given that the DNK model has a finite VAR(2) representation, our SVARs are estimated with two lags. Of course, sample uncertainty may call for a different number of lags for some particular draws. We then re-run our exercise by optimally selecting, per each estimated SVAR, the number of lags according to the Schwarz criterion. Figure 10 depicts the result of this robustness check, which suggest that the impact of sample uncertainty on optimal lag-selection in this context is negligible at best.

These robustness checks suggest the solidity of our main result, i.e. the consistency between Choleski-SVARs' flat responses of inflation and output to a (misspecified) monetary policy shock and monetary policy effectiveness, is very solid.

6 Conclusions

This paper shows that flat impulse responses to a monetary policy shocks estimated with a Choleski-SVAR are fully consistent with monetary policy shocks being able to influence the macroeconomic environment. We estimate a Dynamic New-Keynesian (DNK) model with U.S. data spanning the sample 1984:I-2008:II, and verify that the DNK-impulse responses indicate a significantly negative, persistent reaction of inflation and output to a monetary policy shock. Then, we feed SVARs with pseudo-data produced with the estimated DNK model, and show that the Cholesky-SVAR "shock" offers a misspecified representation of the effects of a true monetary policy impulse. As a consequence, SVARs impulse responses are severely distorted, and suggest on average a zero-reaction to an unexpected nominal rate hike. The misspecification of the policy shock finds its foundations in the timing discrepancy existing between the structural

DNK model, which allows immediate macroeconomic reactions to a policy shock, and the Choleski-SVARs, which wrongly impose delays in the transmission mechanism. In the light of the widespread employment of the recursiveness assumption for the identification of relevant shocks in SVARs, which includes also studies on the effects of fiscal shocks, as well as investigations on the macro-finance interrelations, our results appear to be quite powerful.

Which are the implications of our study? To be clear, our results do *not* call for a rejection of the SVAR approach. Vector autoregressions are clearly useful to establish stylized facts when different, competing models are a-priori equally sensible. As Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007) put it (page 1025), "Despite pitfalls, it is easy to sympathize with the enterprise of identifying economic shocks from VAR innovations if one is not dogmatic in favor of a particular fully specified model." The identification of such shocks, however, requires care, in the light of the drawbacks that may emerge when assuming the wrong economic structure. Two possibly complementary ways to tackle this issue are available. First, as regards model calibration, one can follow Sims (1989) and Cogley and Nason (1995), who impose the same restrictions on SVARs estimated with actual data and those estimated with pseudo-data generated with a business cycle model, so rendering the comparison of the two structures' dynamics more consistent from a logical standpoint (for a recent application of this strategy, see Blanchard and Riggi (2009)). Second, a "sign restriction" approach, which constraints some dynamics of the SVAR to line up with common indications coming from a variety of models of interest or conventional wisdom, can be exploited to sharpen macroeconomic shocks' identification. Indeed, the differences between conditional correlations arising under the Choleski vs. sign restriction identification schemes may be dramatic (see, among others, Canova (2007), Chapter 4, and Castelnuovo and Surico (2009)). A non-exhaustive list of recent applications with sign

restrictions include Canova and de Nicoló (2002), Franchi (2004), Peersman (2005), Canova and Pina (2005), Uhlig (2005), and Benati and Surico (2008). Canova and Paustian (2010) offers a thoughtful analysis on the use of sign restrictions to conduct business cycle's model validation.

7 Estimation Procedure

To perform our Bayesian estimations we employed DYNARE (release 4.0), a set of algorithms developed by Michel Juillard and collaborators. DYNARE is freely available at the following URL: http://www.dynare.org/.

The simulation of the target distribution is basically based on two steps.

- First, we initialized the variance-covariance matrix of the proposal distribution and employed a standard random-walk Metropolis-Hastings for the first $t \leq t_0 = 20,000$ draws. To do so, we computed the posterior mode by the "csminwel" algorithm developed by Chris Sims. The inverse of the Hessian of the target distribution evaluated at the posterior mode was used to define the variance-covariance matrix C_0 of the proposal distribution. The initial VCV matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional variance of the state variables. We used the steady-state of the model to initialize the state vector in the Kalman filter.

- Second, we implemented the "adaptive Metropolis" (AM) algorithm developed by Haario, Saksman, and Tamminen (2001) to simulate the target distribution. Haario, Saksman, and Tamminen (2001) show that their AM algorithm is more efficient that the standard Metropolis-Hastings algorithm. In a nutshell, such algorithm employs the history of the states (draws) so to "tune" the proposal distribution suitably. In particular, the previous draws are employed to regulate

the VCV of the proposal density. We then exploited the history of the states sampled up to $t > t_0$ to continuously update the VCV matrix C_t of the proposal distribution. While not being a Markovian process, the AM algorithm is shown to possess the correct ergodic properties. For technicalities, see Haario, Saksman, and Tamminen (2001).

We simulated two chains of 1,000,000 draws each, and discarded the first 90% as burn-in. To scale the variance-covariance matrix of the chain, we used a factor so to achieve an acceptance rate belonging to the [23%,40%] range. The stationarity of the chains was assessed via the convergence checks proposed by Brooks and Gelman (1998). The region of acceptable parameter realizations was truncated so to obtain equilibrium uniqueness under rational expectations.

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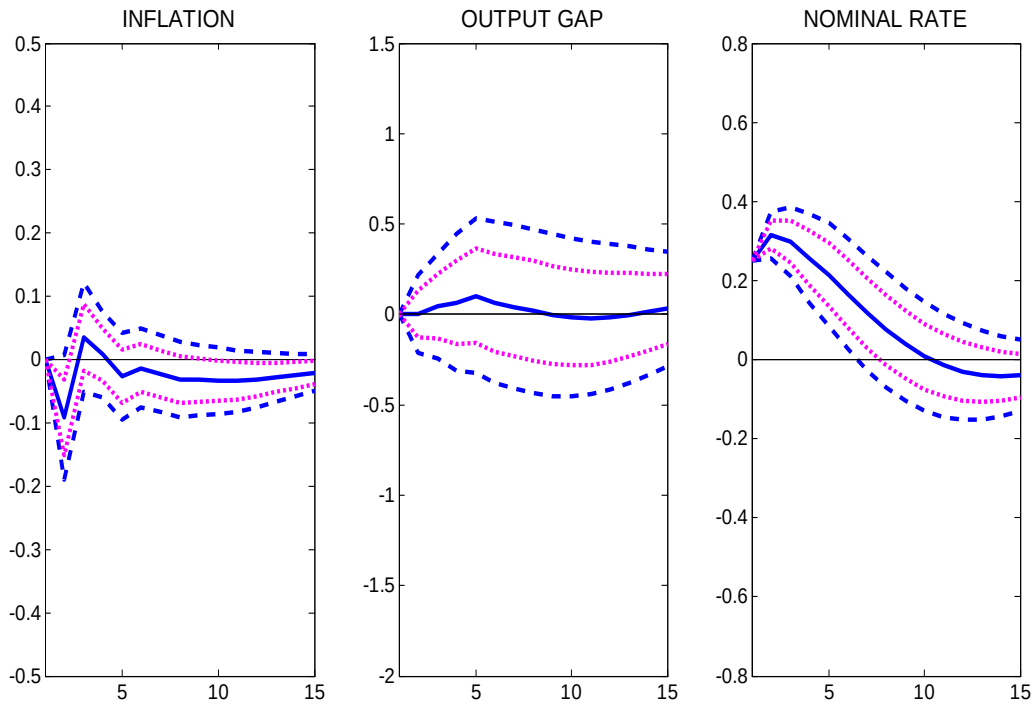

Figure 1: **SVAR impulse response functions to a monetary policy shock.** Sample: 1984:I-2008:II. Variables: Quarterly GDP inflation, CBO output gap, quarterly federal funds rate - source: FREDII. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: quarterly inflation, output gap, quarterly federal funds rate). Solid blue line: Mean response; Dashed blue lines: 90% confidence bands; Magenta dotted lines: 68% confidence bands (analytically computed). VAR estimated with a constant, a linear trend, and three lags.

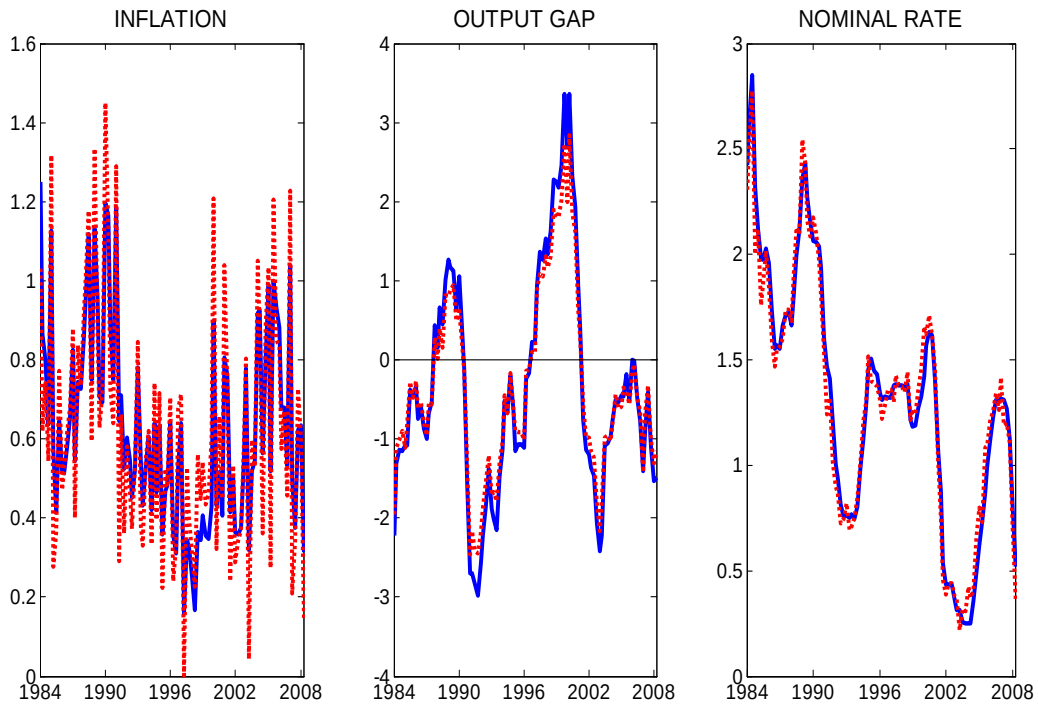

Figure 2: **Actual series vs. DNK's one-step ahead forecasts.** Solid blue line: Actual series; Dotted red lines: DNK's predictions.

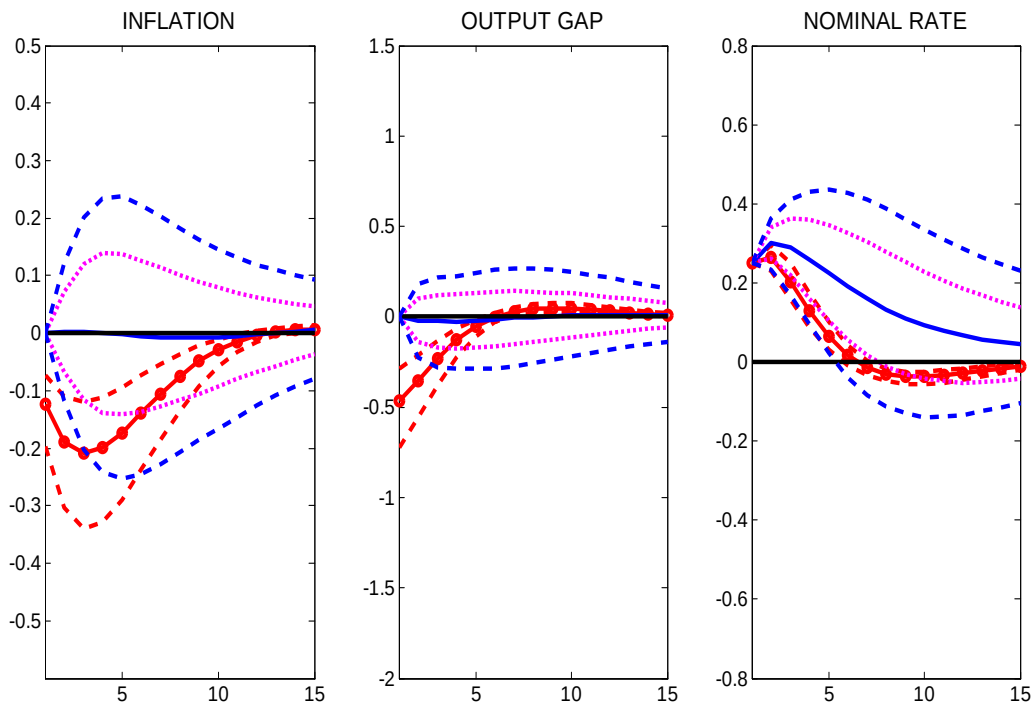

Figure 3: **DNK and VAR impulse response functions to a monetary policy shock.** Circled red line: DNK Bayesian mean impulse response; Dashed red lines: 90% credible sets. Solid blue line: VAR mean impulse response; Dashed blue lines: 90% confidence bands; Magenta dotted lines: 68% confidence bands. Moments computed over the impulse response function distributions simulated by drawing 10,000 realizations of the vector of parameters of the DNK model, which is also used to generate the pseudo-data to feed the SVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: Inflation, output gap, nominal rate). VAR estimated with two lags.

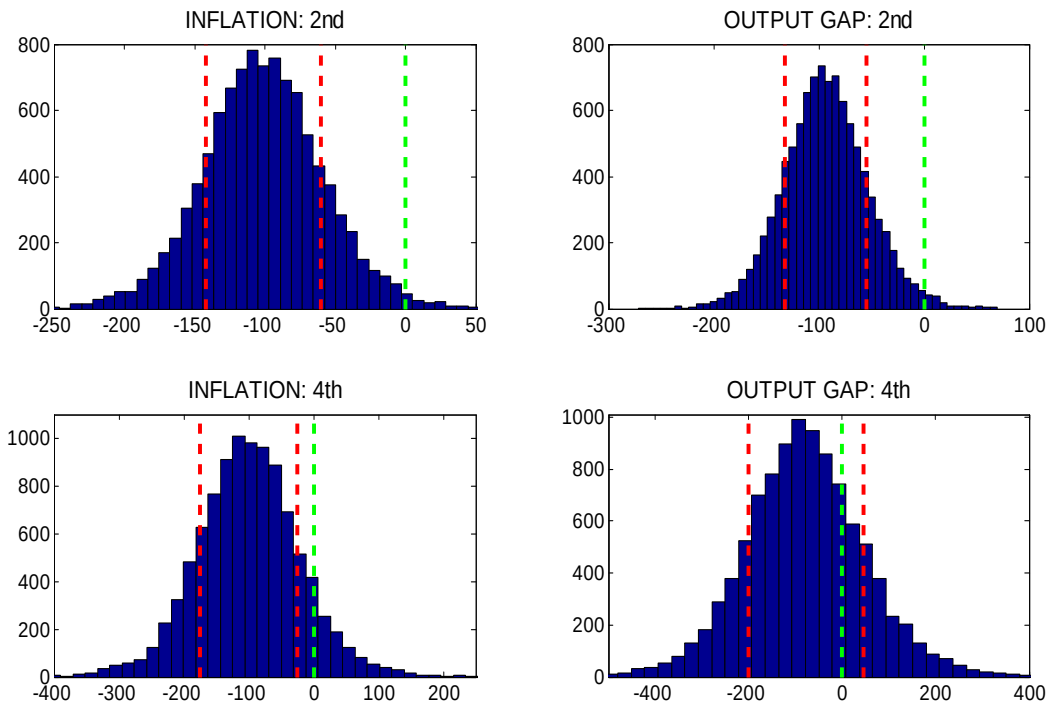

Figure 4: **Estimated bias.** Bias computed as percentage deviation of the SVAR response with respect to the DNK (true) response. Computation of the densities based on 10,000 draws of the structural parameters of the DNK model.

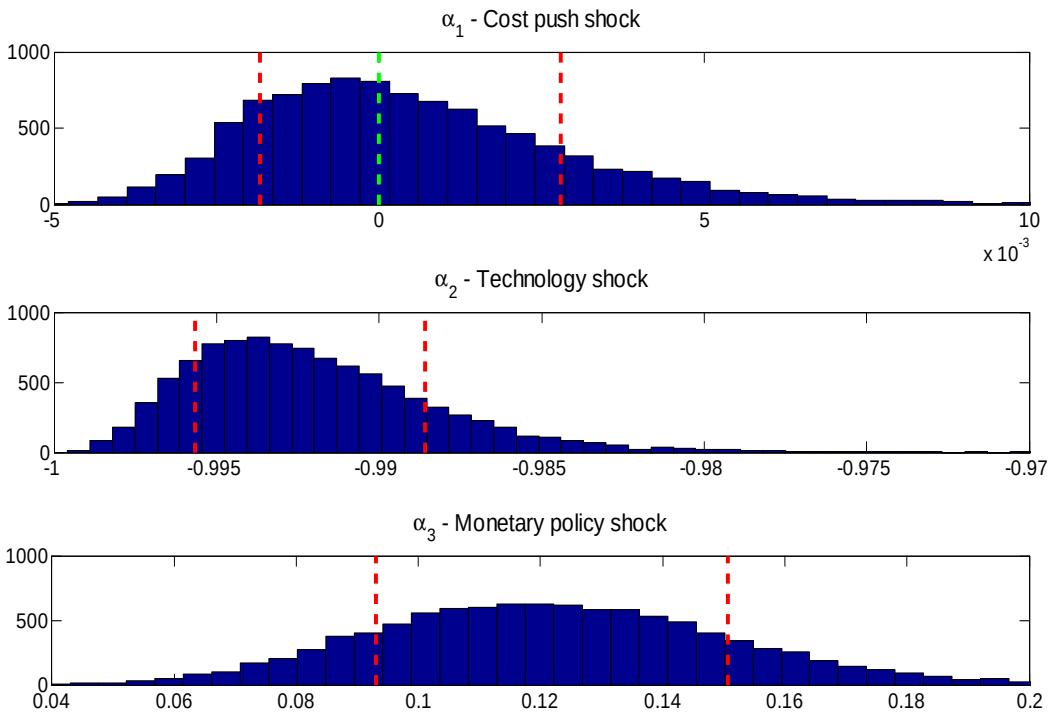

Figure 5: **Densities of the weights of the true structural shocks in the Cholesky-monetary policy "shock".** Computation of the densities based on 10,000 draws of the structural parameters of the DNK model. The mapping from the structural parameters to the coefficients plotted in the Figure is described in the text.

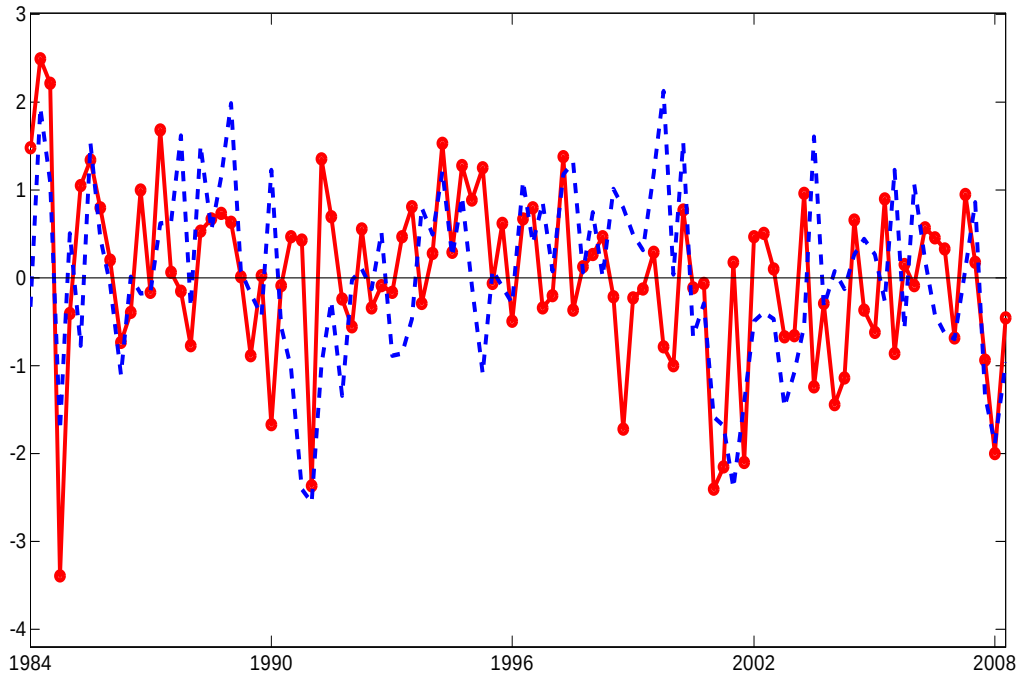

Figure 6: **DNK vs. Cholesky-SVAR monetary policy shock (standardized).** Red circled line: DNK monetary policy shock (smoothed estimate, parameters calibrated at their posterior modes); Blue dotted line: Cholesky-SVAR monetary policy shock (conditional on smoothed estimates of our DNK model's shocks, and constructed as explained in the text, see eq. (9)).

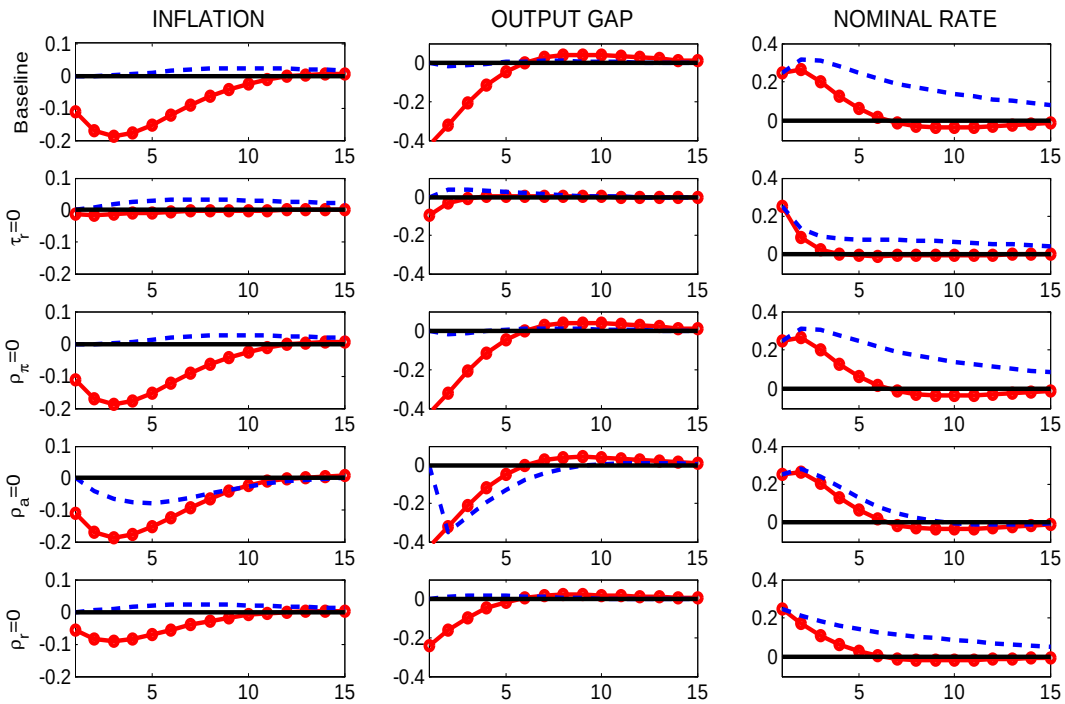

Figure 7: **DNK- vs. VAR-consistent impulse responses: Alternative calibrations.** Population moments simulated by setting N=100,000.

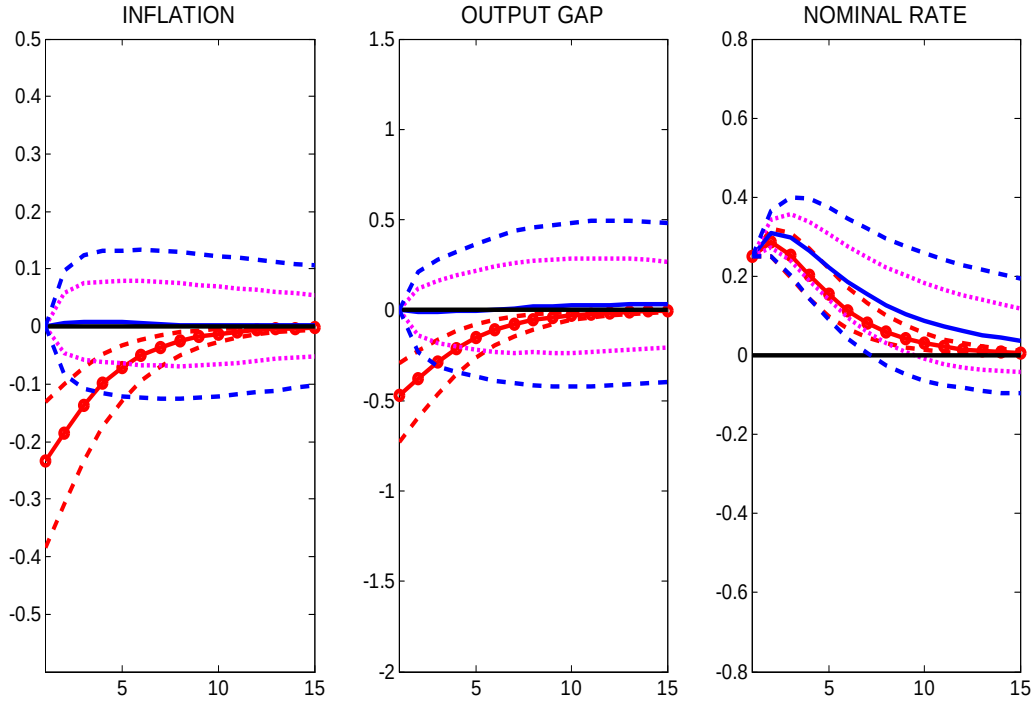

Figure 8: **Partial indexation model.** Circled red line: DNK Bayesian mean impulse response; Dashed red lines: 90% credible sets. Solid blue line: VAR mean impulse response; Dashed blue lines: 90% confidence bands; Magenta dotted lines: 68% confidence bands. Moments computed over the impulse response function distributions simulated by drawing 10,000 realizations of the vector of parameters of the DNK model, which is also used to generate the pseudo-data to feed the SVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: Inflation, output gap, nominal rate). SVARs estimated with two lags.

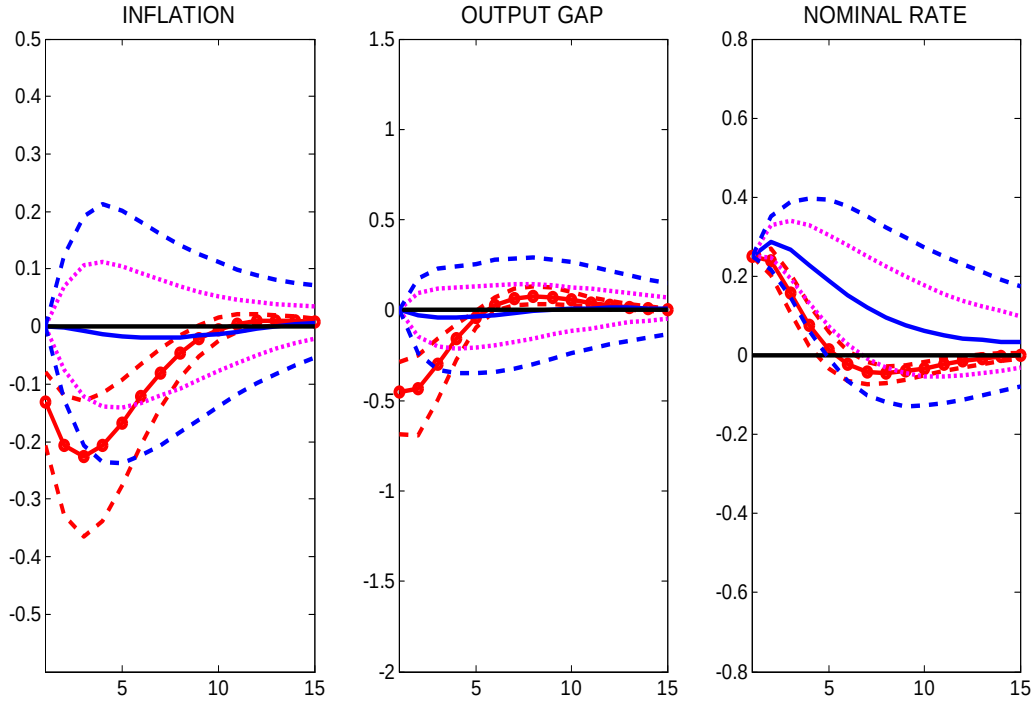

Figure 9: **Hybrid IS curve.** Circled red line: DNK Bayesian mean impulse response; Dashed red lines: 90% credible sets. Solid blue line: VAR mean impulse response; Dashed blue lines: 90% confidence bands; Magenta dotted lines: 68% confidence bands. Moments computed over the impulse response function distributions simulated by drawing 10,000 realizations of the vector of parameters of the DNK model, which is also used to generate the pseudo-data to feed the SVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: Inflation, output gap, nominal rate). VAR estimated with two lags.

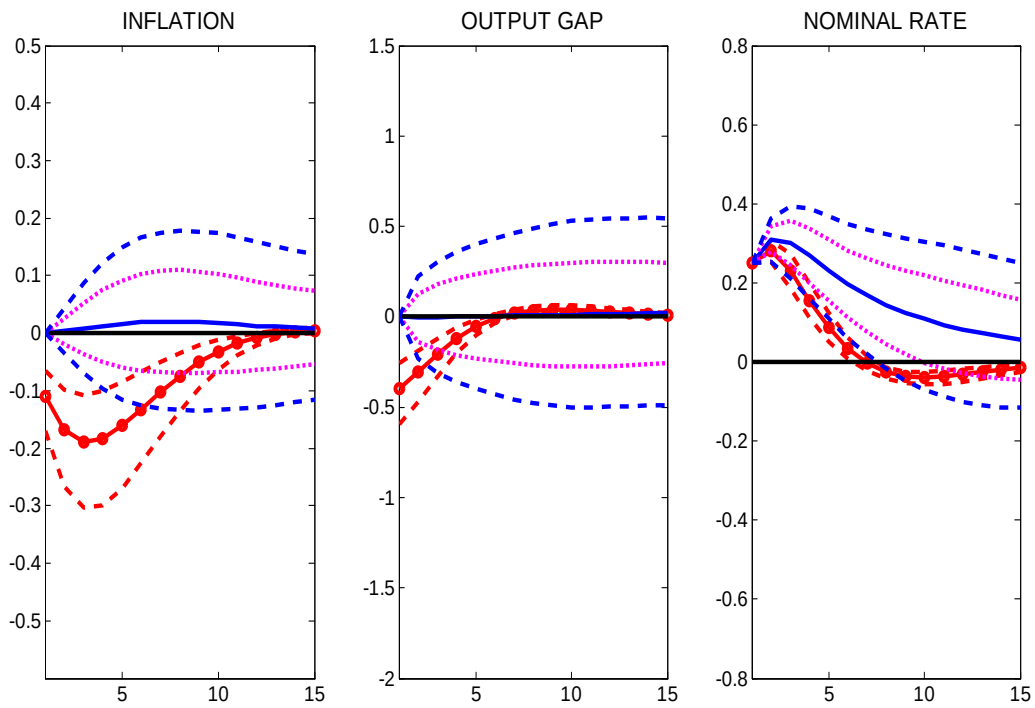

Figure 10: **Perron-Wada piecewise linear output trend.** Circled red line: DNK Bayesian mean impulse response; Dashed red lines: 90% credible sets. Solid blue line: VAR mean impulse response; Dashed blue lines: 90% confidence bands; Magenta dotted lines: 68% confidence bands. Moments computed over the impulse response function distributions simulated by drawing 10,000 realizations of the vector of parameters of the DNK model, which is also used to generate the pseudo-data to feed the SVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: Inflation, output gap, nominal rate). SVARs estimated with two lags. The output trend is linear, but allows for a break in 1973:I.

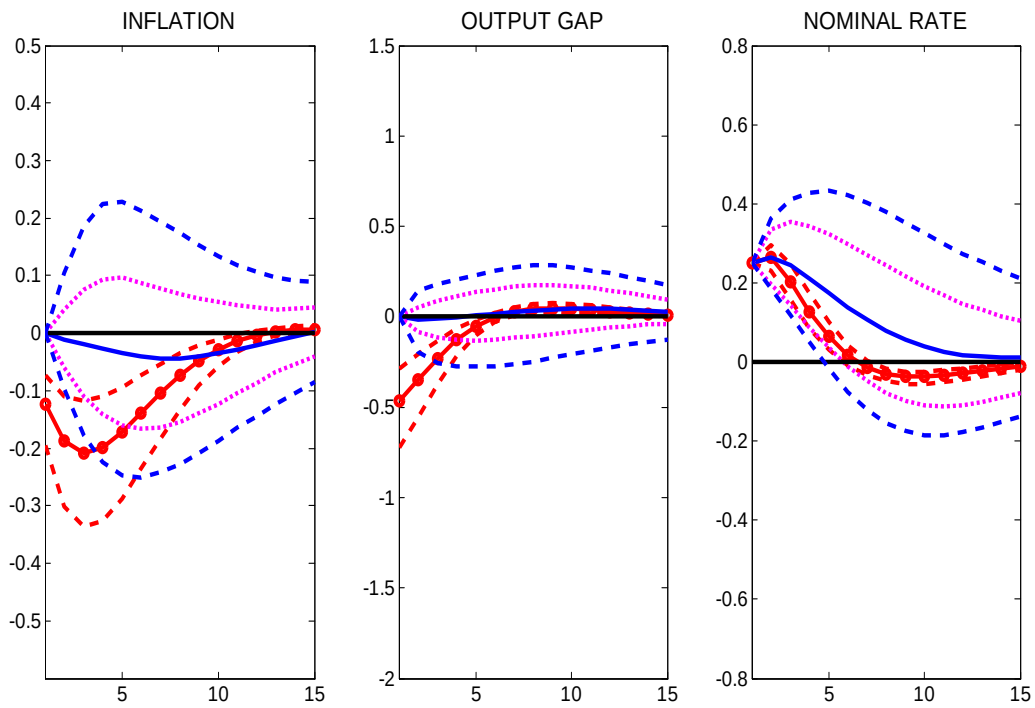

Figure 11: **Optimal lag-selection.** Circled red line: DNK Bayesian mean impulse response; Dashed red lines: 90% credible sets. Solid blue line: VAR mean impulse response; Dashed blue lines: 90% confidence bands; Magenta dotted lines: 68% confidence bands. Moments computed over the impulse response function distributions simulated by drawing 10,000 realizations of the vector of parameters of the DNK model, which is also used to generate the pseudo-data to feed the SVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: Inflation, output gap, nominal rate). SVARs estimated with the number of lags suggested, per each vector, by the Schwarz criterion.

Param.	*Interpretation*	*Priors*	*Posterior Means* [5h,95th]
β	Discount factor	*Calibrated*	0.99 [–]
σ	Risk aversion	$Normal(3,1)$	4.07 [2.88,5.25]
v^{-1}	Frisch elasticity	*Calibrated*	1 [–]
κ	Slope of the NKPC	*Calibrated*	0.1275 [–]
τ_π	T. Rule, Inflation	$Normal(1.5,0.3)$	2.12 [1.79,2.46]
τ_y	T. Rule, Output gap	$Gamma(0.3,0.2)$	0.40 [0.25,0.56]
τ_R	T. Rule, Inertia	$Beta(0.5,0.285)$	0.84 [0.80,0.88]
ρ_a	AR tech. shock	$Beta(0.5,0.285)$	0.89 [0.85,0.94]
ρ_π	AR cost-push shock	$Beta(0.5,0.285)$	0.67 [0.52,0.84]
ρ_R	AR mon. pol. shock	$Beta(0.5,0.285)$	0.42 [0.27,0.57]
σ_a	Std. tech. shock	$InvGamma(1.5,0.2)$	2.64 [1.86,3.40]
σ_π	Std. cost-push. shock	$InvGamma(0.35,0.2)$	0.23 [0.18,0.28]
σ_R	Std. mon. pol. shock	$InvGamma(0.35,0.2)$	0.12 [0.11,0.14]

Table 1: **Bayesian estimates of the benchmark model.** 1984:I-2008:II U.S. data. Prior densities: Figures indicate the (mean,st.dev.) of each prior distribution. Posterior densities: Figures reported indicate the posterior mean and the [5th,95th] percentile of the estimated densities. Details on the estimation procedure provided in the text. Marginal likelihood computed via Laplace approximation.

	INFLATION	OUTPUT GAP
2nd qtr. ahead	−101.80 [−142.17,−60.04]	−98.33 [131.99,−54.58]
4th qtr. ahead	−100.41 [−175.25,−25.18]	−95.69 [−200.60,44.91]

Table 2: **DNK vs. SVAR Impulse Response Functions: Estimated Bias.** The Table reports the means and [5th,95th] percentiles of the distributions of the percentage deviation of the VAR responses with respect to the DNK responses.