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INFORMAL INSURANCE WITH ENDOGENOUS GROUP SIZE

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Informal Insurance with Endogenous Group Size*

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Abstract

We present a theory of endogenous formation of insurance groups which combines heterogeneity on agents' risk aversion under asymmetric information and lack of enforceability of contracts. Income sharing inside the group is decided by majority voting and the size of the group adjusts to this decision through participation constraints. At equilibrium, all group members agree on the same imperfect level of income sharing, which yields a constrained-efficient equilibrium. Comparative statics on the risk faced by the community provide interesting results. A mean preserving spread of income implies more income sharing and a larger group size. New members, and possibly even old members may be better off, while non-members are worse-off. These results have relevant policy implications

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1 Introduction

In rural areas of developing countries, people are exposed to substantial income risks due to large fluctuations of agricultural productions and prices, weather shocks, incomplete markets, ... The lack of access to formal insurance and credit markets leads individuals to create informal insurance agreements. These agreements may be bilateral or take the form of larger groups where households are both consumers and providers of insurance. Since no legal framework exists to enforce these agreements, they must be designed to be self-enforceable, that is, the expected benefits from becoming a group member must be, at any point in time, larger than the gains from defection.

Empirical studies of risk-sharing find evidence of partial insurance in rural communities but uniformly reject the hypothesis of full insurance at the community level (Deaton (1992), Townsend (1994), Udry (1994), Grimard (1997), Jalan & Ravallion (1999), Ligon et al. (2002), Gertler & Gruber (2002)). It appears that the major constraint explaining the failure of full insurance arises from the lack of effective enforcement mechanism to support these insurance arrangements.

In this paper, we present a theory of endogenous formation of insurance groups which reproduces the above mentioned stylized facts and discuss its normative properties. The model combines heterogeneity on agents' risk aversion under asymmetric information and lack of enforceability of contracts involving group members. Insurance services are repeated over time, and members reneging on their commitment are permanently excluded from the group. This framework gives rise to an ad interim participation constraint, first introduced by Coate & Ravallion (1993): when incomes are observed, members drawing a high income face a trade-off between the current gain of keeping the integrality of this income and the loss of all future insurance services. This constraint is more likely to be binding for agents whose risk aversion is low. Since agents' types are unobservable, and all members of the community want to benefit from insurance ex ante, the only variable on which the group has to agree is the extent of insurance provided by the group, that is, the income sharing rule. The decision-making process is modeled by majority voting. The size of the insurance group decreases since members whose participation constraint is not satisfied progressively leave the group as they draw high incomes. This change in the group composition in the early periods of the group calls for a new vote at each period as long as the group size is not stable.

We show that in the long run, all group members agree on the same imperfect level of income sharing, which maximizes their utility subject to the participation constraint. This equilibrium is constrained efficient, that is, given the participation constraint, no agent can be made better-off without reducing the utility of one agent. Indeed, decreasing income sharing in order to ensure that the participation constraint of new members would be satisfied reduces the utility of current members. A comparative statics analysis with respect to an increase in the risk faced by the community is also provided. Interestingly, a mean preserving spread (MPS further) has a positive effect on both the level of the sharing rule and the size of the group, which unambiguously improves the quality of insurance and allows more agents to become members. Since a mean preserving spread increases the group size, three groups of agents emerge and are affected in different ways. First, the effect of a MPS has an ambiguous impact on the welfare of the agents who were already members prior to its introduction. Second, the "new members", who are the agents who enter the group thanks to the MPS, enjoy a higher welfare thanks to the increase in risk. Finally, the third group of agents who are still out of the insurance group is worse-off after the introduction of the MPS. Therefore, while the income risk increases, some agents' welfare may be improved. These results stress that one has to be careful about introducing risk-reducing policies without accounting for the adverse effect they might have on the stability of local communities.

Related literature

Important contributions have studied risk-sharing agreements in rural societies by accounting for the lack of contract enforceability, starting with Coate & Ravallion (1993). The space of possible efficient contracts limited to stationary transfers considered by Coate & Ravallion (1993) has been expanded to dynamic efficient arrangements (Kocherlakota (1996), Ligon et al. (2002)), maintaining the crucial and detrimental role of ad interim participation constraints.

All these models conclude that the extent of insurance provided by the group is limited. However, they consider the size of the group and the characteristics of its members as exogenous. This paper contributes to the literature by endogenizing both the extent of insurance provided by the group and the size of the group itself under asymmetric information. The model takes account that the benefits from risk-sharing derive not only from the extent of risk-sharing but also from the overall size of the insurance arrangement while maintaining the role of the lack of contract enforceability. More precisely, there is a trade-off to the quality of insurance: it can be improved by increasing both the group size and the extent of risk-sharing. However, due to the lack of contract enforceability, an increase in the extent of risk-sharing results in a smaller size of the insurance group. Only few theoretical papers have studied the endogenous formation of insurance groups. Relying upon an argument of coalition-proofness, Genicot & Ray (2003) prove the existence of an upper bound on the size of insurance groups and show that the degree of risk-sharing is non-monotonic in the level of uncertainty or need for insurance in the community. Bold (2009) focuses on the optimal contract on the set of history-dependant contracts that are robust with respect to coalition deviations. In this paper, the problem of deviations by coalitions is not present since at equilibrium, all group members agree on the same sharing rule level, which maximizes their utility.[1]

Other considerations may also be advanced to explain the group size limits as cast, kinship, or informational problems. Murgai et al. (2002) suggest another explanation to the boundaries of mutual insurance groups and the quality of insurance within these groups based on costs of group formation and transaction costs.

[1] This simplification arises from the assumption on agents' mean-variance preferences, which introduces separability and leads all members to only focus on minimizing the income variance despite their differences in risk aversion.

This paper provides relevant insights for policy makers since it shows that policies reducing income variance at the community level have ambiguous impacts on welfare. Such policies should therefore be accompanied by measures protecting non-members of insurance groups, since their numbers are likely to increase.

Section 2 presents the model and describes the equilibrium. Comparative statics are provided in Section 3 and Section 4 concludes.

2 The model

The economy is composed of a finite number of agents i where $i \in \{1, ...N\}$ with $N \geq 2$. Each agent lives for an infinity of periods $t \in \{1, 2, ..., +\infty\}$ and draws at each period a random income Y_{it} that is high h with probability p and low l with probability $(1-p)$. Y_{it} is independently and identically distributed across agents and periods.[2] For all $i \in \{1, ...N\}$ and $t \in \{1, 2, ..., +\infty\}$, the mean and the variance of the income are

$$E(Y_{it}) = ph + (1-p)l \equiv \mu, \tag{1}$$

$$Var(Y_{it}) = p(1-p)(h-l)^2 \equiv \sigma^2. \tag{2}$$

Agents are heterogeneous in risk aversion. This captures various sources of heterogeneity in terms of preferences but also in terms of capacities to mitigate risk, for example by diversifying production, or to face shocks thanks to consumption smoothing mechanisms. Agents' coefficient of absolute risk aversion a is private information and follows a distribution $F(a)$ inside the population. Agents' instantaneous preferences are defined over the mean and the variance of their income. Agent i at period t has instantaneous utility:

$$u_{it}(C_{it}) = E(C_{it}) - a_i Var(C_{it}), \tag{3}$$

where C_{it} is the agent's disposable income which is entirely consumed at each period.[3] In other words, if this agent is not a member of the insurance group, $C_{it} = Y_{it}$, whereas if he/she is a member of the group, $C_{it} = I_{it}$, where I_{it} is the post-transfer -*insured*- income defined by the following sharing rule. At each period t, each member i concedes a proportion $\alpha \in [0, 1]$ of his/her income Y_i and the sum of all contributions is equally split. Without loss of generality, this technology is assumed costless. Formally,

$$I_{it} = (1-\alpha)\, Y_{it} + \frac{1}{n}\sum_{j=1}^{n} \alpha Y_{jt} = Y_{it}\left(1 - \alpha\left(1 - \frac{1}{n}\right)\right) + \alpha \sum_{j \neq i} \frac{Y_{jt}}{n},$$

[2] The zero correlation assumed here is just a convenient simplification. Indeed, correlation needs not be nil for the surplus of mutual insurance to exists.

[3] The mean-variance utility function is an approximation of all $CARA$ utility functions. Indeed, using the Arrow-Pratt approximation of the risk premium, we can write the certainty equivalent as $C_X \approx E(X) - \frac{1}{2}A(X)Var(X)$, where $A(X)$ is the Arrow-Pratt measure of absolute risk aversion.

where $i, j \in \{1, ...n\}$ and n is the number of agents committing to the insurance group.[4] The mean income under mutual insurance is unchanged, while the variance of income is smaller than $Var(Y_{it})$:

$$\begin{aligned} E(I_{it}) &= ph + (1-p)l \equiv \mu \\ Var(I_{it}) &= \Phi(\alpha, n)\, \sigma^2, \end{aligned}$$

where

$$\begin{aligned} \Phi(\alpha, n) &= \left(1 - \alpha\left(1 - \frac{1}{n}\right)\right)^2 + (n-1)\frac{\alpha^2}{n^2} \\ &= 1 - \frac{n-1}{n}\alpha(2-\alpha) \\ &\in [0; 1]. \end{aligned} \tag{4}$$

Note that for any $\alpha \in [0, 1]$, the larger the size of the insurance group, the lower the variance.[5] This stems from the fact that the transfer received by each contributor, $\frac{1}{n}\sum_{j=1}^{n} \alpha Y_{jt}$, is less variable when the community size increases:

$$\begin{aligned} \frac{\partial \Phi}{\partial n} &= -\frac{\alpha(2-\alpha)}{n^2} \\ &< 0. \end{aligned} \tag{5}$$

Also note that the variance of the post-transfer income is decreasing in α and is minimized when $\alpha = 1$:

$$\begin{aligned} \frac{\partial \Phi}{\partial \alpha} &= -2\frac{n-1}{n}(1-\alpha) \\ &\leq 0. \end{aligned} \tag{6}$$

Since both arguments unambiguously decrease the income variance and therefore increase utility, one might wonder why, in the absence of ex ante moral hazard, the insurance network is not composed of the whole community and does not apply perfect sharing: $n = N$, $\alpha = 1$.[6] The reason thereof is due to the fact that the insurance scheme is repeated over time. This repetition gives rise to the so-called ad interim constraint: some agents might have an incentive to renege on their membership obligations once they observe their income draw. Indeed, this case will emerge if the current gain of not sharing income exceeds all the future gains of mutual insurance. Let us analyze this problem in detail by first describing the ad interim stage.

[4] In this model, n is endogenous.

[5] Note that there is no reduction in the variance ($\Phi = 1$) if the insurance group is composed of a single individual ($n = 1$) or no income is shared ($\alpha = 0$). Also, if perfect sharing is applied, $\Phi(1, n) = \frac{1}{n}$.

[6] Under ex ante moral hazard, the distribution of incomes is endogenous and insurance might have a negative effect on agents' effort to obtain a high income.

2.1 The ad interim participation constraint

At the ad interim stage, an agent observes the income he/she has drawn, but not the other agents' income, which he/she considers as random variables. Formally, agent i observes the drawn income $y \in \{l, h\}$, so that the post-transfer income at the ad interim stage if i respects the agreement is:

$$I_{it}^{AI} = (1-\alpha) y + \frac{\alpha}{n} y + \frac{\alpha}{n} \sum_{j \neq i} Y_{jt},$$

where the first two terms are constant and the last one is a sum of random variables. Therefore, the agent's contemporaneous utility at the ad interim stage is determined by

$$\begin{aligned} E\left(I_{it}^{AI}\right) &= y + \left(1 - \frac{1}{n}\right) \alpha (\mu - y), && (7) \\ Var\left(I_{it}^{AI}\right) &= (n-1) \frac{\alpha^2}{n^2} \sigma^2. && (8) \end{aligned}$$

On the other hand, if the agent does not respect the agreement, his/her consumption does not depend on the other members' income drawings and is therefore certain:

$$\begin{aligned} Y_{it}^{AI} &= y, \\ E\left(Y_{it}^{AI}\right) &= y, \\ Var\left(Y_{it}^{AI}\right) &= 0. \end{aligned}$$

In other words, because at the ad interim stage agents only observe their own income, respecting the insurance agreement instead of breaking it increases a member's income variance by $(n-1) \frac{\alpha^2}{n^2} \sigma^2$. Also, respecting the agreement allows the agent to benefit from a net transfer of $\left(1 - \frac{1}{n}\right) \alpha (\mu - y)$, which is positive if the agent had drawn a low income, and negative if he/she had drawn a high income. Since agents drawing a high income both face a higher variance and are net contributors to the group, they might be tempted to renege on their obligations. In order to limit the extent of this phenomenon the group applies a trigger strategy, which consists in permanently excluding transgressors from all future insurance possibilities. Therefore, the actualized total utility at the ad interim stage of period t_0 for an agent who observes an income y and decides to break the agreement (B) writes:

$$\begin{aligned} U_i^{AI}(B; y) &= u(Y_{it_0}^{AI}) + \sum_{t=t_0+1}^{\infty} \delta^t u_{it}(Y_{it}) \\ &= y + \frac{\delta}{1-\delta} \left(\mu - a_i \sigma^2\right), \end{aligned}$$

where $\delta \in [0,1]$ is a discount factor. Conversely, the actualized total utility of an agent who always *respects* the informal agreement (R) writes:

$$\begin{aligned} U_i^{AI}(R;y) &= u\left(I_{it_0}^{AI}\right) + \sum_{t=t_0+1}^{\infty} \delta^t u_{it}(I_{it}) \\ &= y + \frac{\delta}{1-\delta}\left(\mu - a_i \Phi \sigma^2\right) - a_i (n-1) \frac{\alpha^2}{n^2}\sigma^2 \\ &\quad + \frac{n-1}{n}\alpha(\mu - y). \end{aligned} \tag{9}$$

We can now formally define the ad interim participation constraint, which states that the utility of respecting the agreement is larger than the utility of breaking it:

$$\begin{aligned} U_i^{AI}(R;y) &\geq U_i^{AI}(B;y) \\ &\Longleftrightarrow \\ a_i \sigma^2 \left(\frac{\delta}{1-\delta}(1-\Phi) - (n-1)\frac{\alpha^2}{n^2}\right) &\geq \frac{n-1}{n}\alpha(y-\mu). \end{aligned}$$

This condition highlights the existence of three terms. The two terms on the left hand side pertain to the income variance and are independent of the income drawn. They indicate that respecting the agreement implies a trade-off between reducing the income variance of all future periods and increasing the contemporaneous one. It can be shown that the first effect dominates the second for all n and α under the sufficient condition that agents exhibit a reasonable level of patience ($\delta > 1/3$). The sign of the right hand side depends on the income drawn and represents the expected transfer that the agent offers to the group. Clearly, if $y = l$, this transfer is negative, so that the participation constraint is always satisfied (under the above mentioned restriction on δ). Having described all the effects, one can simplify and rewrite the participation constraint under the form of a lower bound on risk aversion, for the only relevant case where $y = h$:

$$a_i \geq \Theta \equiv \left[p(h-l)\left(\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n}\right)\right]^{-1}. \tag{10}$$

Therefore, given n and α, the number of agents whose ad interim constraint is respected, n_R, is:

$$n_R = 1 + (N-1)(1 - F(\Theta)). \tag{11}$$

As we have seen from the previous equations, agents who are the most eager to enter the insurance group and respect the agreement are those who are the most averse to risk. We also know that the larger the group size and the larger the sharing rule, the better for all agents, provided that all members respect the transfer rule. However, setting a large level of income sharing has a negative impact on the number of agents who can credibly commit to always respect the group's transfer rule. The next subsection describes the mechanism behind the group formation.

2.2 Group formation

We have seen that a large group size is desirable, since this improves risk spreading. However, depending on the level of the sharing rule, some agents might choose to renege on their obligations towards the group when they draw a high income. When this is the case, the sanction they undergo is permanent exclusion from the group. The key to group formation is therefore to find the optimal trade-off between the group's size and its sharing rule.

Preferences of agents are unobservable, so that it is not possible to exclude agents from the group before they choose not to respect the insurance contract. Therefore, for any degree of α at the first period, the whole community enters the group. However, as soon as incomes are drawn at the ad interim stage, a fraction of the community leaves the group. On average, the number of agents who don't respect the contract and leave the group at the ad interim stage of the first period equals $p(N - n_R)$, that is, the fraction of agents who receive a high income p times the $N - n_R$ agents whose ad interim participation constraint is not satisfied. More generally, the expected number of agents who leave the group at period $t \geq 1$ is $p(1-p)^{t-1}(N - n_R)$. Conversely, the number of agents remaining in the group at period t is $n_R + (N - n_R)(1-p)^t$ on average. As t becomes sufficiently large, the size of the group converges towards its stable level n_R with probability 1.[7] Since n_R itself depends on n, the equilibrium size, which is noted n^*, is defined formally by the following implicit function:

$$n^* = n_R(\alpha, n^*). \tag{12}$$

Since preferences are unobservable, and the composition and size of the group follows a process of successive exits, the group's only decision variable is the sharing rule α that is imposed to all its members. The effect of the sharing rule on the steady state group size is obtained by applying the implicit function theorem to equation (12):

$$\frac{\partial n^*}{\partial \alpha} = \frac{\partial n_R}{\partial \alpha} / \left(1 - \frac{\partial n_R}{\partial n}\right).$$

An increase in α decreases n_R, but the net effect on n^* is negative only if the increase in n_R due to an increase in n is not too strong. Indeed, a low level of sharing decreases the incentives to participate in the insurance group, but on the other hand, the participation constraint, which defines n_R, is more severely binding if α is high. The actual group size is a result of this tension. This is stated in Lemma 1:

Lemma 1 *The steady state size of the insurance group n^* decreases when α increases if and only if $\frac{\partial n_R}{\partial n} < 1$.*

Proof. The proof and a discussion of the condition that $\frac{\partial n_R}{\partial n} < 1$ are provided in the appendix. ∎

[7] Note that the higher p, the higher the speed of convergence.

Prior to defining the equilibrium characteristics of the group, let us describe how preferences of members are aggregated. Since all agents of the community are willing to benefit from the insurance provided by the group, the whole community at the first stage has to agree on the level of α which will be applied. The aggregation of preferences is modeled by majority voting. Since the composition of the group is likely to change over the first periods of the group's life, a new vote is organized at the beginning of each period, until the group composition is stable.

Proposition 1 defines the characteristics of the group once it is stable, that is, when all members of the group satisfy the participation constraint.

Proposition 1 *Under the necessary condition that $\frac{\partial n_R}{\partial n} < 1$, the steady-state group characteristics are determined by α^* and $n^*(\alpha^*)$, where α^* is such that*

$$\frac{\partial \Phi}{\partial \alpha} + \frac{\partial \Phi}{\partial n}\frac{\partial n^*}{\partial \alpha} = 0. \tag{13}$$

Proof. We proceed by first solving the voting game at the first period and show how the changes in group composition affect the votes of the subsequent periods. To solve the voting game at period 1, one has to define the preferred α of each agent. Let us start with the most risk averse agent of the community. He/she will choose a level of sharing which maximizes his/her utility as a permanent member of the insurance group, taking into account both the direct effect of α on variance reduction and its indirect effect on the reduction of the steady-state/stable group size.[8] Let us define this formally. The utility of an agent who always respects the sharing rule (R) is

$$U_i(R) = \frac{\mu - a_i \Phi(\alpha, n^*)\sigma^2}{1-\delta}.$$

Since we focus on the steady-state group size (12) and agents anticipate the effect of α on n^*, the preferred sharing level of the most risk averse agent α^* must be such that

$$\begin{aligned} \frac{dU_i(R)}{d\alpha} &= \frac{\partial U_i(R)}{\partial \alpha} + \frac{\partial U_i(R)}{\partial n}\frac{\partial n^*}{\partial \alpha} \\ &= \frac{\partial U_i(R)}{\partial \Phi}\left(\frac{\partial \Phi}{\partial \alpha} + \frac{\partial \Phi}{\partial n}\frac{\partial n^*}{\partial \alpha}\right) \\ &= 0. \end{aligned} \tag{14}$$

Note that the second order condition is satisfied as shown in the appendix. The preferred level α^* defines a lower bound on risk aversion $\Theta(\alpha^*)$ above which the participation constraint is satisfied and a stable group size defined by (12). Interestingly, all the agents whose risk aversion is larger than $\Theta(\alpha^*)$ also have a preferred α equal to α^*. Indeed, for all i such that $a_i \geq \Theta(\alpha^*)$, the preferred α does not depend on risk aversion since α^* is given by equation (13), which does not depend on risk aversion. In other words, there is a mass n_R whose preferred α is α^*. If the

[8] For simplicity, we neglect the dynamics of group size in the initial stages and focus on the steady-state group size n^*.

distribution of risk aversion in the community is such that these $n^*(\alpha^*)$ agents form a majority (that is $n^*(\alpha^*)/N \geq 1/2$), the group's sharing rule is set at α^* at period 1.
What happens if these agents don't form a majority at period 1? We then have to analyze the preferred α of agents $j \in \{n^*(\alpha^*)+1, ..., N\}$, that is, agents whose a_j is lower than $\Theta(\alpha^*)$. These agents know that at α^*, they will exclude themselves from the group as soon as they draw a high income. If the discount factor is sufficiently large, so that the future benefits of mutual insurance are sufficiently important, the preferred α of these agents is lower than α^* and is set so as to make their ad interim participation constraint binding. By doing so, they will always benefit from the insurance provided by the group.[9] The voting problem at stage 1 is therefore solved by the median voter in the profile of preferred α's $\left\{\alpha^*, \alpha^*, ...\alpha^*, \alpha^*_{n^*(\alpha^*)+1}, ..., \alpha^*_N\right\}$.
Having solved the voting problem at period 1, let us now analyze the subsequent periods. As time passes, agents whose a is lower than $\Theta(\alpha^*_m)$ leave the group. This change in group composition calls for a new vote on the level of the sharing rule. Since only the least risk averse agents leave the group, the profile of preferred α's shrinks at the benefit of those who prefer α^*, which eventually form a majority. In other words, if voting is repeated, as agents leave the group, the steady-state group characteristics are determined by α^* and n^*. ■

Note that the implicit equation defining α^* which is presented in Proposition 1 can be equivalently stated in the following way

$$\frac{\partial n^*}{\partial \alpha} = \frac{\partial n}{\partial \alpha}|_{Var(I_{it})\text{ cst}}.$$

This equation has a clear interpretation: at α^*, the marginal effect of α on the group size at the steady state must be equal to the marginal rate of substitution of α for n which keeps the variance of the insured income constant.

Another important remark concerns possible deviations by coalitions. Clearly, since all group members unanimously have the same preferred alpha, the group is not subject to such deviations. This result is a direct implication of the assumption on agents' mean-variance preferences, which introduces separability and leads all members to only focus on minimizing the income variance despite their differences in risk aversion.

Agents who left the group are not part of our focus, but it is nonetheless interesting to notice that they may form another group among themselves. The information revealed by the exclusion is limited: they all know that leaving the first group shows a type (risk-aversion) below the level required by the participation constraint at the time of the exit. Since the participation constraint of the first group evolves monotonically, re-entry is never an issue. The problem faced by those who are excluded in this process is very similar to the original problem of the population, but with a strictly smaller interval of types. This interval being composed of low levels of risk-aversion, the equilibrium sharing rate will be lower in the second group than in the first one. Society as a whole

[9] The preferred α of these agents is discussed formally in the Appendix.

ends up being stratified in groups ordered by insurance coverages and attitudes towards risk.

Let us conclude this section by normatively assessing the equilibrium.

Proposition 2 *The steady-state equilibrium is constrained-efficient.*

Proof. We have to show that α^* is such that the welfare of an agent cannot be improved without decreasing another agent's utility given the enforceability and informational constraints. First, nobody inside the group can be made better-off as all members agree on the same preferred level α^*. Non-members could clearly be better-off by entering the group, but this would require to decrease α to make their participation constraint satisfied. In that case, the $n^*(\alpha^*)$ members would be worse off since α^* maximizes $U(R)$. ■

3 Comparative statics

The aim of this section is to highlight the effects of the parameters on the degree of income pooling and the size of the insurance group. Indeed, the quality of insurance that is achieved in a given group is characterized by both characteristics of the group.

3.1 Mean Preserving Spread

In order to isolate the effect of an increase in the uninsured income risk on the quality of insurance while holding the other parameters of the distribution of Y, namely μ and p, fixed, we apply the following transformation to h and l:

$$\begin{aligned} h &= \mu + \sigma\sqrt{\frac{1-p}{p}}, \\ l &= \mu - \sigma\sqrt{\frac{p}{1-p}}. \end{aligned}$$

By doing so, the distribution of Y depends on the parameters μ, σ^2, p, which are independent of each other. In other words, we can now analyze the effect of a mean preserving spread, that is, an increase in σ while holding the mean μ (and the distribution of income probabilities) constant. Proposition 3 describes the effect of a MPS on the characteristics of the group at the steady-state.

Proposition 3 *A mean preserving spread of Y has a positive impact on both α^* and $n^*(\alpha^*)$:*

$$\begin{aligned} \frac{d\alpha^*}{d\sigma}|_{\mu\ cst} &> 0 \\ \frac{dn^*}{d\sigma}|_{\mu\ cst} &> 0. \end{aligned}$$

Proof. The proof is provided in the appendix. ■

Therefore, when the pre-transfer income risk increases, the size of the insurance group increases and group members share a higher proportion of income. The impact of a higher risk therefore implies a reinforcement of communities.

Let us discuss why the MPS leads to an increase in both α^* and n^*, which may seem surprising at first since n^* is decreasing in α. To see this, first recall that an increase in the risk of the pre-transfer income incites group members to increase the quality of insurance, that is, to decrease Φ. Increasing income sharing α is one way to reach this objective. However, we know that increasing α tightens the participation constraint, which may lead to a decrease in the number of members. However, an increase in σ also has the effect of loosening the participation constraint, since non-members are more reluctant to face a larger uninsured risk. At equilibrium, the net effect on the group size is positive.

The following corollary states that since both effects go in the same direction at equilibrium, the quality of insurance is unambiguously improved. This does not mean however that the variance of the insured income I_{it} is lower.

Corollary 1 *A mean preserving spread of Y has a positive impact on the quality of the insurance, that is, $\frac{d\Phi(\alpha^*, n^*(\alpha^*))}{d\sigma} < 0$. The net effect on the variance of the insured income, $\Phi\sigma^2$, is ambiguous.*

Proof. Applying Proposition 3, the proof of the corollary is straightforward since Φ is decreasing in both α and n. A formal proof is provided in the Appendix. ■

The reason why the net effect on the variance of the insured income is ambiguous is the following. In addition to the reaction of members through α, some of the agents whose participation constraint was not satisfied now enter the group and contribute to improve the insurance quality. It is therefore unclear whether the insured income variance should increase or not.

Let us conclude this section on the mean preserving spread by discussing its normative implications.

3.2 Normative analysis of the impact of a mean preserving spread

Since a mean preserving spread increases the group size, three groups of agents are relevant for this normative analysis.

First, the agents who were already members prior to the introduction of the MPS. A direct implication of Corollary 1 is that the effect of a mean preserving has an ambiguous impact on the welfare of these "old members".

Second, the "new members" are the agents who entered the group thanks to the MPS. Interestingly, the MPS may have a positive impact on the new members' welfare.

Third, agents who are still out of the insurance group. Obviously, these agents are worse-off after the introduction of the MPS.

Proposition 4 *The effect on welfare of a marginal mean preserving spread of Y*

1. *is ambiguous for "old members" of the group*

2. *is positive for "new members"*

3. *is negative for agents who remain out of the group.*

Proof. The proof of part 1 is a direct implication of Corollary 1. Part 2 comes from the fact that new members benefit from a discrete decrease in risk since they now benefit from insurance. This discrete decrease always dominates the marginal increase in variance.[10] Part three is straightforward. ■

Let us conclude this normative analysis by a numerical simulation showing that the MPS can effectively increase the utility of "old members". Let us define the following parameters' values: $N = 120$, $\mu = 1$, $h = 2$, $\delta = 0.8$, and the support of the uniform distribution for risk aversion is $[a, b] = [1, 2]$. The following table provides the most salient results for these parameter values.

For an agent $a_i = 1,81$;	when $\sigma^2 = 0,15$;	$U_i = 4,97$;
	when $\sigma^2 = 0,12$;	$U_i = 3,64$.
For an agent $a_i = 2$;	when $\sigma^2 = 0,15$;	$U_i = 4,96$;
	when $\sigma^2 = 0,12$;	$U_i = 4,92$.

First note that in this example, a new member is clearly better off with the increase in σ. Indeed, for $\sigma^2 = 0.12$, an agent with a risk aversion coefficient of 1.81 is out of the group and has a utility level of 3.64. By increasing σ^2 to 0.15, this agent's participation constraint is now satisfied and he enters the group, which provides him with a utility level of 4.97.

Second, and most interesting is the case of the agent with a risk aversion level of 2, that is the most risk averse agent of the community, who already was a member for $\sigma^2 = 0.12$ and had a utility level of 4.92. This level increases to 4.96 when the variance increases to 0.15.

4 Conclusion

This paper provides a theory of endogenous formation of insurance groups in rural communities with heterogeneity on agents' risk aversion with asymmetric information and lack of contract enforceability. Insurance services are repeated over time, and members reneging on their commitment are permanently excluded from the group which gives rise to ad interim participation constraints. Agents vote on the level of income to be shared inside the insurance group, taking into account the fact that a higher sharing rule tightens participation constraints and therefore reduces the size of the group.

We show that in the long run, all group members agree on the same sharing rule, which maximizes their utility subject to the participation constraint. This equilibrium is constrained efficient, that

[10] A formal proof of this argument is provided in the appendix.

is, given the participation constraint, no agent can be made better-off without reducing the utility of one agent. Interestingly, a mean preserving spread has a positive effect on both the level of the sharing rule and the size of the group, which unambiguously improves the quality of insurance and allows more agents to become members. Since a mean preserving spread increases the group size, three groups of agents emerge and are affected in different ways. First, the effect of a MPS has an ambiguous impact on the welfare of the agents who were already members prior to its introduction. Second, the "new members" who are the agents who entered the group thanks to the MPS. Interestingly, the MPS has a positive impact on their welfare. While the third group of agents who are still out of the insurance group is worse-off after the introduction of the MPS. Therefore, while the income risk increases, some agents' welfare may be improved. These results stress that one has to be careful about introducing risk-reducing policies without accounting for the adverse effect they might have on the stability of local communities.

5 Appendix

5.1 Proof and discussion of Lemma 1

Lemma 1 states that

$$\begin{aligned}\frac{\partial n^*}{\partial \alpha} &= \frac{\partial n_R}{\partial \alpha} / \left(1 - \frac{\partial n_R}{\partial n}\right) \\ &< 0 \iff \frac{\partial n_R}{\partial n} < 1.\end{aligned}$$

To see why, it suffices to show that

$$\begin{aligned}\frac{\partial n_R}{\partial \alpha} &= -\left(N-1\right) F'\left(\Theta\right) \Theta_\alpha \\ &\leq 0\end{aligned}$$

since

$$\begin{aligned}\Theta_\alpha &\equiv \frac{\partial \Theta}{\partial \alpha} = \frac{1}{p\left(h-l\right)} \left[\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n}\right]^{-2} \left(\frac{\delta}{1-\delta} + \frac{1}{n}\right) \\ &> 0.\end{aligned}$$

The participation constraint will be respected by a lower number of agents if the share of income which members transfer to the pool increases (provided some agents in the community have a risk aversion level equal to Θ).

Let us now discuss the denominator. The participation constraint will be respected by a larger number of agents if the size of the insurance group increases:

$$\begin{aligned}\frac{\partial n_R}{\partial n} &= -\left(N-1\right) F'\left(\Theta\right) \Theta_n \\ &\geq 0,\end{aligned}$$

since

$$\begin{aligned}\Theta_n &\equiv \frac{\partial \Theta}{\partial n} = -\frac{\alpha}{n^2} \frac{1}{p\left(h-l\right)} \left[\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n}\right]^{-2} \\ &< 0.\end{aligned}$$

Let us finally discuss the relevance of the condition that $\frac{\partial n_R}{\partial n} < 1$. This condition is the least likely to hold is when α is large and n is small. Let us therefore compute $\frac{\partial n_R}{\partial n}$ for $\alpha = 1$ and $n = 2$. In this particular case, $\frac{\partial n_R}{\partial n}$ may be written as

$$\frac{\partial n_R}{\partial n} = \Omega \frac{\left(1-\delta\right)^2}{\left(3\delta - 1\right)^2},$$

where

$$\Omega = \frac{\left(N-1\right) F'(\Theta)}{p\left(h-l\right)}.$$

If $F'(\Theta) = 0$, the condition is always trivially satisfied. If Θ takes one of the possible values in the distribution, posing $F'(\Theta) = 1/N$ which boils down to ignore distributional effects by imposing a discrete uniform distribution on risk aversion, one obtains that

$$\begin{aligned} \frac{\partial n_R}{\partial n} &= \frac{(N-1)}{Np\,(h-l)} \frac{(1-\delta)^2}{(3\delta-1)^2} \\ &< 1 \text{ if } \frac{(1-\delta)^2}{(3\delta-1)^2} < p\,(h-l)\,. \end{aligned}$$

This condition is satisfied when agents are sufficiently patient (δ close to 1) or if the probability of drawing a high income and the difference between high and low incomes are high. This means that agents will be willing to build a mutual insurance group if they are sufficiently patient.

5.2 Second order condition for α^*

The second order condition is always satisfied:

$$\begin{aligned} \frac{d^2 U_i\,(R)}{d\alpha^2} &= \frac{\partial U_i\,(R)}{\partial \Phi} \begin{pmatrix} \frac{\partial^2 \Phi}{\partial \alpha^2} + 2\frac{\partial^2 \Phi}{\partial n \partial \alpha}\frac{\partial n^*}{\partial \alpha} + \frac{\partial^2 \Phi}{\partial n^2}\left(\frac{\partial n^*}{\partial \alpha}\right)^2 \\ +\frac{\partial \Phi}{\partial n}\left(\frac{\partial^2 n^*}{\partial \alpha^2} + \frac{\partial^2 n^*}{\partial \alpha \partial n}\frac{\partial n^*}{\partial \alpha}\right) \end{pmatrix} \\ &< 0 \text{ since } \begin{cases} \frac{\partial U_i(R)}{\partial \Phi} < 0 \\ \begin{pmatrix} \overset{(+)}{\frac{\partial^2 \Phi}{\partial \alpha^2}} + 2\overset{(-)}{\frac{\partial^2 \Phi}{\partial n \partial \alpha}}\overset{(-)}{\frac{\partial n^*}{\partial \alpha}} + \overset{(+)}{\frac{\partial^2 \Phi}{\partial n^2}}\overset{(+)}{\left(\frac{\partial n^*}{\partial \alpha}\right)^2} \\ +\overset{(-)}{\frac{\partial \Phi}{\partial n}}\left(\overset{(-)}{\frac{\partial^2 n^*}{\partial \alpha^2}} + \overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial n}}\overset{(-)}{\frac{\partial n^*}{\partial \alpha}}\right) \end{pmatrix} > 0 \end{cases}. \end{aligned}$$

Indeed, the signs of the terms which compose the second term are:

$$\begin{aligned} \frac{\partial^2 \Phi}{\partial \alpha^2} &= \frac{2}{n}(n-1) \\ &> 0 \\ \frac{\partial^2 \Phi}{\partial n \partial \alpha} &= -\frac{2}{n^2}(1-\alpha) \\ &< 0 \\ \frac{\partial \Phi}{\partial n} &= -\frac{1}{n^2}\alpha\,(2-\alpha) \\ &< 0, \\ \frac{\partial^2 \Phi}{\partial n^2} &= \frac{2}{n^3}\alpha\,(2-\alpha) \\ &> 0, \end{aligned}$$

and under the assumption that risk aversion follows a uniform distribution $U[a,b]$,

$$\begin{aligned}
\frac{\partial n^*}{\partial \alpha} &= \frac{\frac{\partial n_R}{\partial \alpha}}{1-\frac{\partial n_R}{\partial n}} \\
&< 0 \iff \frac{\partial n_R}{\partial n} < 1, \\
\frac{\partial^2 n^*}{\partial \alpha^2} &= \frac{\frac{\partial^2 n_R}{\partial \alpha^2}\left(1-\frac{\partial n_R}{\partial n}\right)+\frac{\partial n_R}{\partial \alpha}\frac{\partial^2 n_R}{\partial n \partial \alpha}}{\left(1-\frac{\partial n_R}{\partial n}\right)^2} \\
&< 0 \iff \frac{\partial n_R}{\partial n} < 1 \\
\frac{\partial^2 n^*}{\partial \alpha \partial n} &= \frac{\frac{\partial^2 n_R}{\partial n \partial \alpha}\left(1-\frac{\partial n_R}{\partial n}\right)+\frac{\partial n_R}{\partial \alpha}\frac{\partial^2 n_R}{\partial n^2}}{\left(1-\frac{\partial n_R}{\partial n}\right)^2} \\
&> 0
\end{aligned}$$

with

$$\begin{aligned}
\frac{\partial n_R}{\partial \alpha} &= -\Omega \frac{1}{n}\frac{1}{p(h-l)(1-\delta)}\frac{\delta(n-1)+1}{\left(\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right)^2} \\
&< 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 n_R}{\partial \alpha^2} &= -2\Omega \frac{1}{p(h-l)}\frac{1}{n^2(1-\delta)^2}\frac{(\delta(n-1)+1)^2}{\left(\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right)^3} \\
&< 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 n_R}{\partial n^2} &= -2\frac{\alpha}{n^3}\Omega\frac{1}{p(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-3}\left(\frac{\delta}{1-\delta}(2-\alpha)\right) \\
&< 0,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 n_R}{\partial \alpha \partial n} &= \Omega \frac{1}{p(h-l)(1-\delta)}\frac{(\alpha(1-\delta)+n(2+\alpha)\delta)}{\left(n\left(\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right)\right)^3} \\
&> 0.
\end{aligned}$$

where

$$\Omega = \frac{N-1}{b-a}.$$

5.3 Preferred α of agents who don't respect the ad interim constraint for $\alpha = \alpha^*$

Two opposite forces are at work in the determination of these agents' preferred α. On the one hand, if they know they will have to leave the group for $\alpha = \alpha^*$, they might as well prefer a larger α

and hope to benefit from a high insurance as long as they draw low incomes. On the other hand, they might prefer a level of α strictly lower than α^* in order to be able to satisfy the ad interim constraint and permanently benefit from the insurance group. Formally, choosing a lower α means to choose the level which makes their participation constraint binding, that is

$$\begin{aligned} a_j &= \Theta\left(\alpha_j^*\right) \equiv \left[p\left(h-l\right)\left(\frac{\delta}{1-\delta}(2-\alpha_j^*)-\frac{\alpha_j^*}{n}\right)\right]^{-1}, \\ \alpha_j^* &= \frac{1}{\frac{\delta}{1-\delta}+\frac{1}{n}}\left(2\frac{\delta}{1-\delta}-\frac{1}{a_j p\left(h-l\right)}\right). \end{aligned}$$

so that the utility of a respectful agent j writes

$$U_j\left(R\right)=\frac{\mu-a_j\Phi\left(\alpha_j^*,n_R\left(\alpha_j^*\right)\right)\sigma^2}{1-\delta}.$$

On the other hand, starting from the assumption that the agent would not be able to satisfy the ad interim constraint, his preferred α maximizes U_j, where

$$\begin{aligned} U_j &= pU_B+\left(1-p\right)u_R+\left(1-p\right)\delta\left(pU_B+\left(1-p\right)u_R\right) \\ &\quad +\left(1-p\right)^2\delta^2\left(pU_B+\left(1-p\right)u_R\right)+... \\ &= \left(pU_B+\left(1-p\right)u_R\right)\sum_{t=1}^{+\infty}\left(\left(1-p\right)\delta\right)^{t-1} \\ &= \frac{1}{1-\left(1-p\right)\delta}\left(pU_B+\left(1-p\right)u_R\right) \end{aligned}$$

Only u_R is affected by α, and the value which maximizes u_R is such that

$$\begin{aligned} \frac{n-1}{n}\left(\mu-l\right) &= 2a_i\frac{n-1}{n^2}\alpha'\sigma^2 \\ \alpha' &= \frac{n\left(\mu-l\right)}{2a_i\sigma^2}, \end{aligned}$$

so that the maximal U_j is

$$\begin{aligned} &\frac{1}{1-\left(1-p\right)\delta}\left(\begin{array}{c} p\left(h+\frac{\delta}{1-\delta}\left(\mu-a_i\sigma^2\right)\right) \\ +\left(1-p\right)\left(l+\frac{\left(\mu-l\right)^2\left(n-1\right)}{4\sigma^2 a_i}\right) \end{array}\right) \\ \Leftrightarrow\ &\frac{1}{1-\left(1-p\right)\delta}\left(\begin{array}{c} \mu+p\left(\frac{\delta}{1-\delta}\left(\mu-a_i\sigma^2\right)\right) \\ +\left(1-p\right)\left(\frac{\left(\mu-l\right)^2\left(n-1\right)}{4\sigma^2 a_i}\right) \end{array}\right). \end{aligned}$$

the agent whose risk aversion is lower than $\Theta\left(\alpha^*\right)$ prefers a lower α_j^* to a higher α_j' iff

$$\frac{\mu-a_j\Phi_j^*\sigma^2}{1-\delta}>\frac{1}{1-\left(1-p\right)\delta}\left(\begin{array}{c} \mu+p\left(\frac{\delta}{1-\delta}\left(\mu-a_i\sigma^2\right)\right) \\ +\left(1-p\right)\left(\frac{\left(\mu-l\right)^2\left(n-1\right)}{4\sigma^2 a_i}\right) \end{array}\right).$$

One can see that the LHS is divided by $\left(1-\delta\right)$, whereas the RHS is divided by $1-\left(1-p\right)\delta$. Therefore, if δ and p are sufficiently large, the LHS always dominates the RHS.

5.4 Comparative Statics

Let us assume for the comparative statics that the distribution of risk aversion is approximated by a continuous uniform distribution with support $[a; b]$. One can then rewrite n_R as

$$\begin{aligned} n_R &= 1 + (N-1)\left(1 - \frac{\Theta - a}{b-a}\right) \\ &= -\Omega\Theta + N + (N-1)\,\frac{a}{b-a}, \end{aligned}$$

where

$$\Omega = \frac{N-1}{b-a}.$$

The partial derivatives on n^* are:

$$\begin{aligned} \frac{\partial n^*}{\partial \alpha} &= \frac{\frac{\partial n_R}{\partial \alpha}}{1 - \frac{\partial n_R}{\partial n}} < 0, \\ \frac{\partial^2 n^*}{\partial \alpha^2} &= \frac{\frac{\partial^2 n_R}{\partial \alpha^2}\left(1 - \frac{\partial n_R}{\partial n}\right) + \frac{\partial n_R}{\partial \alpha}\frac{\partial^2 n_R}{\partial n \partial \alpha}}{\left(1 - \frac{\partial n_R}{\partial n}\right)^2} < 0, \\ \frac{\partial^2 n^*}{\partial \alpha \partial n} &= \frac{\frac{\partial^2 n_R}{\partial n \partial \alpha}\left(1 - \frac{\partial n_R}{\partial n}\right) + \frac{\partial n_R}{\partial \alpha}\frac{\partial^2 n_R}{\partial n^2}}{\left(1 - \frac{\partial n_R}{\partial n}\right)^2} > 0, \end{aligned}$$

with

$$\begin{aligned} \frac{\partial n_R}{\partial \alpha} &= -\Omega\Theta_\alpha < 0, \\ \frac{\partial^2 n_R}{\partial \alpha^2} &= -\Omega\Theta_{\alpha\alpha} < 0, \\ \frac{\partial^2 n_R}{\partial \alpha \partial n} &= -\Omega\Theta_{n\alpha} > 0, \\ \frac{\partial n_R}{\partial n} &= -\Omega\Theta_n > 0 \text{ but } < 1, \\ \frac{\partial^2 n_R}{\partial n^2} &= -\Omega\Theta_{nn} < 0. \end{aligned}$$

since

$$\begin{aligned}
\Theta_n &\equiv \frac{\partial\Theta}{\partial n} = -\frac{\alpha}{n^2}\frac{1}{p\,(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-2} \\
&< 0, \\
\Theta_{nn} &\equiv \frac{\partial^2\Theta}{\partial n^2} = 2\frac{\alpha}{n^3}\frac{\left(\frac{\delta}{1-\delta}(2-\alpha)\right)}{p\,(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-3} \\
&> 0, \\
\Theta_\alpha &\equiv \frac{\partial\Theta}{\partial \alpha} = \frac{\left(\frac{\delta}{1-\delta}+\frac{1}{n}\right)}{p\,(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-2} \\
&> 0, \\
\Theta_{\alpha\alpha} &\equiv \frac{\partial^2\Theta}{\partial \alpha^2} = 2\frac{\left(\frac{\delta}{1-\delta}+\frac{1}{n}\right)^2}{p\,(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-3} \\
&> 0, \\
\Theta_{n\alpha} &\equiv \frac{\partial^2\Theta}{\partial \alpha\partial n} = -\frac{1}{n^2}\frac{\left(\frac{\delta}{1-\delta}(2+\alpha)+\frac{\alpha}{n}\right)}{p\,(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-3} \\
&< 0.
\end{aligned}$$

We know that α^* is such that

$$\Psi\,(\alpha, n^*, \theta) \equiv \frac{\partial\Phi\,(\alpha,n^*)}{\partial\alpha}+\frac{\partial\Phi\,(\alpha,n^*)}{\partial n}\frac{\partial n^*}{\partial\alpha}=0,$$

where

$$\frac{\partial n^*}{\partial\alpha}=\frac{\frac{\partial n_R}{\partial\alpha}}{1-\frac{\partial n_R}{\partial n}}<0,$$

and

$$\begin{aligned}
\frac{\partial\Phi}{\partial n} &= -\frac{\alpha(2-\alpha)}{n^2}<0, \\
\frac{\partial\Phi}{\partial \alpha} &= -2\frac{n-1}{n}(1-\alpha)\leq 0, \\
\frac{\partial^2\Phi}{\partial\alpha\partial n} &= -2\frac{1}{n^2}\,(1-\alpha)<0, \\
\frac{\partial^2\Phi}{\partial n^2} &= -2\frac{1}{n^3}\alpha(2-\alpha)<0, \\
\frac{\partial^2\Phi}{\partial \alpha^2} &= 2\frac{n-1}{n}>0.
\end{aligned}$$

We also know that

$$\begin{aligned}
\frac{\partial\Psi}{\partial\alpha} &= \overset{(+)}{\frac{\partial^2\Phi}{\partial\alpha^2}}+2\overset{(-)}{\frac{\partial^2\Phi}{\partial n\partial\alpha}}\overset{(-)}{\frac{\partial n^*}{\partial\alpha}}+\overset{(+)}{\frac{\partial^2\Phi}{\partial n^2}}\overset{(+)}{\left(\frac{\partial n^*}{\partial\alpha}\right)^2} \\
&\quad +\overset{(-)}{\frac{\partial\Phi}{\partial n}}\left(\overset{(-)}{\frac{\partial^2 n^*}{\partial\alpha^2}}+\overset{(+)}{\frac{\partial^2 n^*}{\partial\alpha\partial n}}\overset{(-)}{\frac{\partial n^*}{\partial\alpha}}\right) \\
&> 0.
\end{aligned}$$

Therefore, the effect of any parameter θ on α^* is the following:

$$\frac{d\alpha^*}{d\theta} = -\frac{\frac{\partial \Psi}{\partial \theta}}{\frac{\partial \Psi}{\partial \alpha}},$$

so that

$$sign\left(\frac{\partial \alpha^*}{\partial \theta}\right) = sign\left(-\frac{\partial \Psi}{\partial \theta}\right),$$

where

$$\begin{aligned}\frac{\partial \Psi}{\partial \theta} &= \overset{(0)}{\frac{\partial^2 \Phi}{\partial \alpha \partial \theta}} + \overset{(-)}{\frac{\partial^2 \Phi}{\partial n \partial \alpha}} \overset{(?)}{\frac{\partial n^*}{\partial \theta}} + \left(\overset{(0)}{\frac{\partial^2 \Phi}{\partial n \partial \theta}} + \overset{(+)}{\frac{\partial^2 \Phi}{\partial n^2}} \overset{(?)}{\frac{\partial n^*}{\partial \theta}} \right) \overset{(-)}{\frac{\partial n^*}{\partial \alpha}} \\ &\quad + \overset{(-)}{\frac{\partial \Phi}{\partial n}} \left(\overset{(?)}{\frac{\partial^2 n^*}{\partial \alpha \partial \theta}} + \overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial n}} \overset{(?)}{\frac{\partial n^*}{\partial \theta}} \right)\end{aligned}$$

with

$$\begin{aligned}\frac{\partial n^*}{\partial \theta} &= -\frac{-\frac{\partial n_R}{\partial \theta}}{\left(1 - \frac{\partial n_R}{\partial n}\right)}, \\ \frac{\partial^2 n^*}{\partial \alpha \partial \theta} &= \frac{\frac{\partial^2 n_R}{\partial \alpha \partial \theta}\left(1 - \frac{\partial n_R}{\partial n}\right) + \frac{\partial n_R}{\partial \alpha} \frac{\partial^2 n_R}{\partial n \partial \theta}}{\left(1 - \frac{\partial n_R}{\partial n}\right)^2}.\end{aligned}$$

The effect of any parameter θ on n^* is

$$\frac{dn^*}{d\theta} = \overset{(?)}{\frac{\partial n^*}{\partial \theta}} + \overset{(-)}{\frac{\partial n^*}{\partial \alpha}} \overset{(?)}{\frac{\partial \alpha^*}{\partial \theta}}.$$

5.4.1 Proof of proposition 3

We write h and l as functions of μ, σ and p:

$$\begin{aligned} h &= \mu + \sigma \sqrt{\frac{1-p}{p}}, \\ l &= \mu - \sigma \sqrt{\frac{p}{1-p}}, \\ p(h-l) &= \sigma p', \end{aligned}$$

where $p' = \frac{\sqrt{p}}{\sqrt{1-p}}$. The effect of σ on Ψ is

$$\begin{aligned}\frac{\partial \Psi}{\partial \sigma} &= \overset{(0)}{\frac{\partial^2 \Phi}{\partial \alpha \partial \sigma}} + \overset{(-)}{\frac{\partial^2 \Phi}{\partial n \partial \alpha}} \overset{(+)}{\frac{\partial n^*}{\partial \sigma}} + \left(\overset{(0)}{\frac{\partial^2 \Phi}{\partial n \partial \sigma}} + \overset{(+)}{\frac{\partial^2 \Phi}{\partial n^2}} \overset{(+)}{\frac{\partial n^*}{\partial \sigma}} \right) \overset{(-)}{\frac{\partial n^*}{\partial \alpha}} \\ &\quad + \overset{(-)}{\frac{\partial \Phi}{\partial n}} \left(\overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial \sigma}} + \overset{(+)}{\frac{\partial^2 n^*}{\partial \alpha \partial n}} \overset{(+)}{\frac{\partial n^*}{\partial \sigma}} \right) \\ &< 0\end{aligned}$$

with

$$\begin{aligned}
\frac{\partial n^*}{\partial \sigma} &= \frac{\overset{(+)}{\frac{\partial n_R}{\partial \sigma}}}{\left(1-\frac{\partial n_R}{\partial n}\right)} \\
&> 0, \\
\frac{\partial^2 n^*}{\partial \alpha \partial \sigma} &= \frac{\overset{(+)}{\frac{\partial^2 n_R}{\partial \alpha \partial \sigma}} \overset{(+)}{\left(1-\frac{\partial n_R}{\partial n}\right)} + \overset{(-)}{\frac{\partial n_R}{\partial \alpha}} \overset{(-)}{\frac{\partial^2 n_R}{\partial n \partial \sigma}}}{\left(1-\frac{\partial n_R}{\partial n}\right)^2} \\
&> 0,
\end{aligned}$$

since

$$\begin{aligned}
\frac{\partial^2 n_R}{\partial \alpha \partial \sigma} &= -\Omega \Theta_{\alpha\sigma} > 0, \\
\text{with } \Theta_{\alpha\sigma} &= -\sigma^{-1}\Theta_\alpha < 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial n_R}{\partial \sigma} &= -\Omega \Theta_\sigma > 0, \\
\text{with } \Theta_\sigma &= -\sigma^{-1}\frac{1}{p\,(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-1} < 0,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 n_R}{\partial n \partial \sigma} &= -\Omega \Theta_{n\sigma} < 0, \\
\text{with } \Theta_{n\sigma} &= \frac{\alpha}{n^2}\sigma^{-1}\frac{1}{p\,(h-l)}\left[\frac{\delta}{1-\delta}(2-\alpha)-\frac{\alpha}{n}\right]^{-2} > 0.
\end{aligned}$$

It implies that

$$\begin{aligned}
\frac{\partial \alpha^*}{\partial \sigma} &= -\frac{\frac{\partial \Psi}{\partial \sigma}}{\frac{\partial \Psi}{\partial \alpha}} \\
&> 0.
\end{aligned}$$

The effect of σ on n^* is

$$
\begin{aligned}
\frac{dn^*}{d\sigma} &= \overset{(+)}{\frac{\partial n^*}{\partial \sigma}} + \overset{(-)}{\frac{\partial n^*}{\partial \alpha}}\overset{(+)}{\frac{\partial \alpha^*}{\partial \sigma}} \\
&= \frac{1}{\frac{\partial \Psi}{\partial \alpha}}\left(1-\frac{\partial n_R}{\partial n}\right)^{-1}\left(\frac{\partial n_R}{\partial \sigma}\frac{\partial \Psi}{\partial \alpha}-\frac{\partial n_R}{\partial \alpha}\frac{\partial \Psi}{\partial \sigma}\right) \\
&= \frac{1}{\frac{\partial \Psi}{\partial \alpha}}\left(1-\frac{\partial n_R}{\partial n}\right)^{-1}
\left(\begin{array}{c}
\overset{(+)}{\tfrac{\partial n_R}{\partial \sigma}}\left(\begin{array}{c}
\overset{(+)}{\tfrac{\partial^2 \Phi}{\partial \alpha^2}} + 2\overset{(-)}{\tfrac{\partial^2 \Phi}{\partial n \partial \alpha}}\overset{(-)}{\tfrac{\partial n^*}{\partial \alpha}} + \overset{(+)}{\tfrac{\partial^2 \Phi}{\partial n^2}}\overset{(+)}{\left(\tfrac{\partial n^*}{\partial \alpha}\right)^2} \\
+\overset{(-)}{\tfrac{\partial \Phi}{\partial n}}\left(\overset{(-)}{\tfrac{\partial^2 n^*}{\partial \alpha^2}} + \overset{(+)}{\tfrac{\partial^2 n^*}{\partial \alpha \partial n}}\overset{(-)}{\tfrac{\partial n^*}{\partial \alpha}}\right)
\end{array}\right) \\
-\tfrac{\partial n_R}{\partial \alpha}\left(\begin{array}{c}
\overset{(0)}{\tfrac{\partial^2 \Phi}{\partial \alpha \partial \sigma}} + \overset{(-)}{\tfrac{\partial^2 \Phi}{\partial n \partial \alpha}}\overset{(+)}{\tfrac{\partial n^*}{\partial \sigma}} \\
+\left(\overset{(0)}{\tfrac{\partial^2 \Phi}{\partial n \partial \sigma}} + \overset{(+)}{\tfrac{\partial^2 \Phi}{\partial n^2}}\overset{(+)}{\tfrac{\partial n^*}{\partial \sigma}}\right)\overset{(-)}{\tfrac{\partial n^*}{\partial \alpha}} \\
+\overset{(-)}{\tfrac{\partial \Phi}{\partial n}}\left(\overset{(+)}{\tfrac{\partial^2 n^*}{\partial \alpha \partial \sigma}} + \overset{(+)}{\tfrac{\partial^2 n^*}{\partial \alpha \partial n}}\overset{(+)}{\tfrac{\partial n^*}{\partial \sigma}}\right)
\end{array}\right)
\end{array}\right) \\
&= \frac{1}{\frac{\partial \Psi}{\partial \alpha}}\left(1-\frac{\partial n_R}{\partial n}\right)^{-1}
\left(\begin{array}{c}
\overset{(+)}{\overset{(+)}{\left(\overset{(+)}{\tfrac{\partial n_R}{\partial \sigma}}\overset{(+)}{\tfrac{\partial^2 \Phi}{\partial \alpha^2}}\left(1-\tfrac{\partial n_R}{\partial n}\right)^2\right)}} \\
+\overset{(+)}{\left(\overset{(+)}{\tfrac{\partial n_R}{\partial \sigma}}\overset{(-)}{\tfrac{\partial^2 \Phi}{\partial n \partial \alpha}}\overset{(-)}{\tfrac{\partial n_R}{\partial \alpha}}\overset{(+)}{\left(1-\tfrac{\partial n_R}{\partial n}\right)}\right)} \\
+\overset{(-)}{\tfrac{\partial \Phi}{\partial n}}\left(1-\tfrac{\partial n_R}{\partial n}\right)\overset{(-)}{\left(\overset{(+)}{\tfrac{\partial n_R}{\partial \sigma}}\overset{(-)}{\tfrac{\partial^2 n_R}{\partial \alpha^2}} - \tfrac{\partial n_R}{\partial \alpha}\overset{(+)}{\tfrac{\partial^2 n_R}{\partial \alpha \partial \sigma}}\right)} \\
+\overset{(-)}{\tfrac{\partial \Phi}{\partial n}}\tfrac{\partial n_R}{\partial \alpha}\overset{(+)}{\left(\overset{(+)}{\tfrac{\partial n_R}{\partial \sigma}}\overset{(+)}{\tfrac{\partial^2 n_R}{\partial n \partial \alpha}} - \tfrac{\partial n_R}{\partial \alpha}\overset{(-)}{\tfrac{\partial^2 n_R}{\partial n \partial \sigma}}\right)}
\end{array}\right) \\
&> 0.
\end{aligned}
$$

Since

$$
\begin{aligned}
\frac{\partial n_R}{\partial \sigma} &= \Omega \left(p' \left(\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n} \right) \right)^{-1} \sigma^{-2} \\
&> 0 \\
\overset{(+)}{\frac{\partial n_R}{\partial \sigma}} \overset{(-)}{\frac{\partial^2 n_R}{\partial \alpha^2}} - \overset{(-)}{\frac{\partial n_R}{\partial \alpha}} \overset{(+)}{\frac{\partial^2 n_R}{\partial \alpha \partial \sigma}} &= -\frac{1}{n^2 (p')^2} \frac{\Omega^2}{\sigma^3} \frac{1}{(1-\delta)^2} \frac{((1-\delta)+\delta n)^2}{\left(\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n} \right)^4} \\
&< 0. \\
\overset{(+)}{\frac{\partial n_R}{\partial \sigma}} \overset{(+)}{\frac{\partial^2 n_R}{\partial n \partial \alpha}} - \overset{(-)}{\frac{\partial n_R}{\partial \alpha}} \overset{(-)}{\frac{\partial^2 n_R}{\partial n \partial \sigma}} &= 2 \frac{1}{n^2 (p')^2} \frac{\Omega^2}{\sigma^3} \frac{\delta}{(1-\delta)} \frac{1}{\left(\frac{\delta}{1-\delta}(2-\alpha) - \frac{\alpha}{n} \right)^4} \\
&> 0.
\end{aligned}
$$

5.4.2 Proof of corollary 1

The effect of a mean preserving spread of Y on the insured income risk is

$$
\begin{aligned}
\frac{d\left(\Phi \sigma^2\right)}{d\sigma} &= \sigma^2 \frac{d\Phi}{d\sigma} + 2\sigma\Phi \\
&< 0 \Leftrightarrow -\frac{d\Phi}{d\sigma} > \frac{2\Phi}{\sigma},
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{d\Phi}{d\sigma} &= \overset{(-)}{\frac{\partial \Phi}{\partial \alpha}} \overset{(+)}{\frac{\partial \alpha^*}{\partial \sigma}} + \overset{(-)}{\frac{\partial \Phi}{\partial n}} \left(\frac{\partial n^*}{\partial \sigma} + \overset{(+)}{\frac{\partial n^*}{\partial \alpha}} \frac{\partial \alpha^*}{\partial \sigma} \right) \\
&< 0.
\end{aligned}
$$

5.4.3 Proof of proposition 4

First note that when the pre-transfer income variance is higher, more agents are part of the insurance group since $\frac{dn^*}{d\sigma} > 0$. To see an improvement of the welfare of these agents as the income variance increases, their welfare of being out of the agreement before the variance increase

$$
U_i\left(B, \sigma'\right) = \frac{\mu - a_i \sigma'^2}{1-\delta},
$$

has to be smaller than the welfare of being part of the group with a higher uninsured income variance

$$
U_i\left(R, \sigma\right) = \frac{\mu - a_i \Phi \sigma^2}{1-\delta}
$$

with

$$
\begin{aligned}
\sigma &= \sigma' + \Delta\sigma, \\
\Phi &= \Phi\left(\alpha^*\left(\sigma\right), n^*\left(\alpha^*\left(\sigma\right)\right)\right).
\end{aligned}
$$

We have

$$\begin{aligned}
\frac{\mu - a_i \sigma'^2}{1-\delta} &< \frac{\mu - a_i \Phi \sigma^2}{1-\delta} \\
&\Leftrightarrow \sigma'^2 > \Phi \sigma^2 \\
&\Leftrightarrow \Phi\left(\alpha^*, n^*\right) < \left(1 - \frac{\Delta\sigma}{\sigma}\right)^2.
\end{aligned}$$

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