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The Impact of Liquidity Constraints and Imperfect Commitment on Migration Decisions of Offspring of Rural Households

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Abstract

This paper presents a non-cooperative model of intra-household decision-making regarding investment in migration. It is shown that the combination of liquidity constraints and imperfect commitment are a source of underinvestment in migration. More precisely, we highlight that, if remittances are unenforceable as a repayment for parent's contribution in migration transaction costs, then both migrant and parent's liquidity constraints, rather than household's liquidity constraint as a whole, matter in determining the investment decision. Besides, the insurance motive for remittances is shown to generate divergence of interest over the characteristics of migration. This result calls for a theoretical approach that properly takes account of potential internalization problems, which the paper intends to offer. Plausibility checks of the model are provided by comparative statics whose outcomes are consistent with previous research on migration and remittances.

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1 Introduction

Migration has multiple implications for sending households. There are indeed many ways through which migrant's relatives are involved in both costs and benefits of migration. Regarding the former, migration outcomes depend on the ability to cover important transaction costs such as transport or administrative expenses. Besides, the level of education may increase the likelihood of successful integration in urban or foreign job markets. On the one hand, the parents are often the main contributors to all those expenditures. On the other hand, they also draw several benefits from migration, mainly by receiving remittances. The impact of remittances is larger than a simple distribution of migrant's private gains. For instance, it is often argued that these funds allow poor households to overcome liquidity constraints (Lucas (1987); Stark (1991)). They are thereby allowed to increase initially suboptimal investments in human capital (Edwards & Ureta (2003); Yang (2004); Calero et al. (2009)) or productive assets (Adams (1998)). Moreover, remittances may help the recipients to cope with transitory shocks (de la Briere et al. (2002)) or to afford health care expenditure (Amuedo-Dorantes & Pozo (2009)). In this perspective, it has been shown by Mendola (2008) that access to remittance flows has a significantly positive impact on adoption of modern but risky agricultural technologies.

Parents are therefore expected to play a crucial role in migration decisions. While, at its starting point, the economic literature on migration choices did not allow for origin household's involvement (Todaro (1969); Harris & Todaro (1970)), it is now well established that a relevant representation of the decision-making process should include the parents (Stark & Levhari (1982); Stark (1991); Hoddinott (1994)). However, the way parents' wellbeing is introduced into the analysis and the way their interaction with the prospective migrant impacts on the decision are not neutral. In particular, we argue that, due to the complexity of the parents-migrant interaction both before and after migration, a unitary model of the household is not appropriate. Moreover, we are even convinced that efficient bargaining (Hoddinott (1994)), while clearly distinguishing between parents and migrant's interests, fails to offer a complete understanding of parents-migrant interaction and its outcome. In particular, it misses potential inefficiencies, which, in a context of market imperfections, is problematic.

This paper develops a model of intra-household decision-making regarding investment in migration. Our aim is to show that the combination of liquidity constraints and imperfect commitment may be the source of rural households' under-investment in migration. In the second part of the paper, we provide a set of comparative statics results and compare them with comparative statics of the first best migration decision rule, that is what a unitary model or an efficient bargaining model would have produced.

The direct impact of the migrant's liquidity constraint is mechanical and intuitive. If she is prevented from borrowing, transaction costs may not be affordable even if migration generates large private returns. However, if initial household wealth allows it, parent's contribution could relax the migrant's constraint, provided this contribution is rewarded in the future, for instance through higher remittances. In this perspective, remittances can be seen as part of a long term migration contract with a loan component (Cox et al. (1998)). However, it can reasonably be argued that repayment of parent's contribution by the migrant is subject to ex post moral hazard. The migration decision is (most of the time) one shot and the commitment issue is therefore severe. In this paper, two alternative assumptions are nevertheless explored, either the loan is enforceable, or commitment is imperfect and it is not. The former assumption serves as a benchmark. Under the latter assumption, we need to define the incentive compatible level of remittances. In other words, a motive for the migrant to remit has to be selected. Indeed, even if a loan is not per se enforceable, remittances are a widespread and huge phenomenon. Enforcement mechanisms therefore exist. Our main point is to show that, as soon as remittances are motivated by other considerations than a loan repayment, inefficiencies follow.

Rapoport & Docquier (2006) provide a useful overview of the different theoretical explanations behind remittance flows. In addition to altruism, several selfish motives can be listed. A first motive is given by savings and investment. In the perspective of return migration, remittances are a means of transferring resources to the future. Osili (2007) provides an empirical investigation of migrants' investments in their origin country for the case of Nigeria, whereas Osili (2004) focuses more specifically on the benefits of housing investments. In those cases, the origin household acts as an agent. Exchange is a second element (Cox (1987)). Remittances may be a form of payment for services offered by the family, for instance, taking care of migrant's cattle or her household members left behind. A third explanation refers to the strategic bequest motive (Bernheim et al. (1985)): if the parents can credibly commit to disinherit their child, they are able to attract care and transfers by designing an appropriate reward or bequest function. The heirs compete for inheritance which is larger if part of remittances is invested in the family estate. The empirical relevance of this explanation has been tested and confirmed by Hoddinott (1994) and de la Briere et al. (2002). Finally, farmers being confronted with important income variability, migration can serve the purpose of diversifying family income sources (Stark & Levhari (1982)). In this perspective, remittances play the role of insurance transfers between the migrant and the parent (de la Briere et al. (2002)). The insurance motive for remittances has desirable properties for this paper. First, mutual insurance creates an additional surplus compared to migrant's strictly private returns. This surplus that embodies the collective return to migration is a potential source of imperfect internalization since it is not entirely captured neither by the migrant, nor by the parent. Second, as we show in the following section, the insurance motive is a potential source of divergence of interest over the characteristics of migration. Interacting this feature with imperfect commitment and credit constraints allows to highlight important efficiency issues.

The paper is organized as follows. Section 2 introduces the general setting of the model and the insurance motive for remittances. Section 3 is devoted to the normative analysis of the household migration outcome. Several assumptions are explored, from perfect capital markets to binding liquidity constraints, with and without enforceable loan. In section 4, we provide comparative statics with the aim of assessing the plausibility of the model. Section 5 concludes.

2 The model: mutual insurance and migration

Suppose a rural household composed of two members $i \in \{p, m\}$, namely the parent p and the prospective migrant m. In the first period of the game, both players derive a risky income $w_r \sim F_r$ from farm activities. The prospective migrant is then faced with the opportunity of internal or international migration. Migration requires that a fixed transaction cost C be spent in order to cover travel or administrative expenses, to subsist during a period of job search, etc[1]. As explained below, we allow the parent to contribute to this transaction cost thereby providing the prospective migrant with incentives to leave. Besides, migration allows to enter the urban or foreign job market and to earn its prevailing wage w_u in the second period of the game. The urban income is a random variable that follows a given distribution $w_u \sim F_u$.

[1] In addition, since the level of human capital might increase the probability of successful migration, education expenditures may be seen as belonging to this cost.

As announced earlier, we are going to deal with imperfect commitment regarding reimbursement of parent's contribution. To this end, the following subsection develops a mutual insurance framework that allows to define the incentive compatible value of remittances. Besides, the mutual insurance motive highlights a potential divergence of interest between the migrant and the parent over the characteristics of migration and the resulting necessity to care about internalization.

As is shown in the following subsection, the parent and the migrant can easily smooth their consumption path by pooling rural and urban risks together. We precisely define migration as the possibility for a household member to change her income distribution. This event has private implications since, compared to the rural income, the urban wage follows a distribution with different characteristics in terms of mean and variance as well as collective implications since it creates an insurance surplus to be shared between both players. Risk sharing is described in the following subsection where migration is supposed to have taken place.

2.1 The mutual insurance contract, collective returns to migration

We assume that risk sharing is perfectly enforceable in the sense that the contract is not subject to the ad interim participation constraint highlighted by Coate & Ravallion (1993). The ad interim constraint pertains to the incentive for any participant to the risk sharing arrangement to renege on the insurance agreement after having drawn a high income. The threat of exclusion has to deter deviation in the sense that the short term gains that the participant derives would she refuse to grant the insurance transfer to others have to be outweighed by the utility cost associated to the loss of access to insurance in the future. In other words, fulfilling the ad interim constraint amounts to requiring that the contract be incentive compatible. As shown by Coate & Ravallion (1993), this requirement limits the scope for risk sharing and hence the insurance surplus. In our model, risk sharing is supposed to be efficient[2]. And, even under this assumption, we highlight inefficiencies in the migration decision. It follows that, if we further impose this restriction on risk sharing, potential returns to migration are a fortiori imperfectly internalized by the other household members. Put differently, efficient risk sharing is the best-case scenario in terms of collective returns to migration and should lead the other household members to contribute to transaction costs.

Turning to the formal setting, let $y = w_u + w_r$ denote the aggregate household income after migration, with

$$\begin{aligned} E(y) &= \mu_u + \mu_r, \\ Var(y) &= \sigma_u^2 + \sigma_r^2 + 2\rho\sigma_u\sigma_r, \end{aligned} \tag{1}$$

where μ_u and μ_r are the expected urban and rural income, σ_u^2 and σ_r^2 denote their respective variances and where ρ is correlation between urban and rural earnings. Assume the parent and the migrant have homogeneous CARA (constant absolute risk aversion) preferences and let η stand for the coefficient of absolute risk aversion. Also, define $\tilde{w}_u$ and $\tilde{w}_r$ as the certainty equivalents of the urban and rural income, respectively. Lemma 1 describes the collective returns to migration.

Lemma 1 ***The insurance surplus****: Under CARA preferences, if two persons with given random incomes share risk efficiently, then the sum of the certainty equivalents of their income after transfers does not depend*

[2] As appears below, efficient risk sharing does not imply complete insurance, nor for the migrant, neither for the parent. What is meant here is that the parent and the migrant cannot do better in terms of risk pooling and that the insurance surplus is maximized. As a consequence, potential inefficiencies in the migration decision do not result from incomplete insurance but are caused by other factors highlighted in the paper.

on the distribution of the insurance surplus. This aggregate certainty equivalent writes $\tilde{y} \approx \tilde{w}_u + \tilde{w}_r + \chi$, *where* $\chi = \frac{\eta}{4}\left(\sigma_r^2 + \sigma_u^2 - 2\rho\sigma_r\sigma_u\right)$ *is always positive.*

Proof. To begin with, notice that efficient risk sharing corresponds to full income pooling. Each player earns one half of the aggregate income y. Under such a sharing rule, idiosyncratic shocks are perfectly insured and the residual income fluctuation is only due to covariate shocks, that is the variability of the aggregate income. In addition, let the migrant transfer a lump sum payment r to the parent. This variable determines the distribution of the surplus between both stakeholders. With this sharing rule, expected utilities are given by

$$\begin{aligned} U_m &= Eu\left(\frac{1}{2}y - r\right) = u\left(\frac{1}{2}E(y) - r - \tilde{\pi}_{y/2}\right), \\ U_p &= Eu\left(\frac{1}{2}y + r\right) = u\left(\frac{1}{2}E(y) + r - \tilde{\pi}_{y/2}\right), \end{aligned}$$

where $\tilde{\pi}_{y/2}$ is the risk premium associated to the random income $\frac{1}{2}y$. Making use of Pratt's approximation and of the assumption of CARA preferences and since $Var(y/2) = \frac{1}{4}Var(y)$,

$$\tilde{\pi}_{y/2} \approx \frac{\eta}{8}Var(y). \tag{2}$$

where η is absolute risk aversion. Isolating the transfer r,

$$\begin{aligned} r &= u^{-1}(U_p) - \frac{1}{2}E(y) + \tilde{\pi}_{y/2}, \\ r &= -u^{-1}(U_m) + \frac{1}{2}E(y) - \tilde{\pi}_{y/2}. \end{aligned}$$

Equalizing the two equations and rearranging gives the expression of the aggregate certainty equivalent[3]:

$$u^{-1}(U_p) + u^{-1}(U_m) = \tilde{y} = E(y) - 2\tilde{\pi}_{y/2},$$

which does not depend on the transfer r. Certainty equivalents are therefore transferable in this context. Making use of (1) and (2), one can write

$$\tilde{y} \approx \mu_u + \mu_r - \frac{\eta}{4}\left(\sigma_u^2 + \sigma_r^2 + 2\rho\sigma_r\sigma_u\right).$$

Finally, given that

$$\begin{aligned} \tilde{w}_u &\approx \mu_u - \frac{\eta}{2}\sigma_u^2, \\ \tilde{w}_r &\approx \mu_r - \frac{\eta}{2}\sigma_r^2, \end{aligned}$$

[3] This expression can also be used to derive the Pareto frontier of this problem:

$$U_p(U_m) = u\left\{\tilde{y} - u^{-1}(U_m)\right\}.$$

We can check that this function is indeed decreasing and concave:

$$\begin{aligned} \frac{\partial U_p}{\partial U_m} &= -u'\frac{\partial u^{-1}(U_m)}{\partial U_m} < 0, \\ \frac{\partial^2 U_p}{\partial U_m^2} &= u''\left(\frac{\partial u^{-1}(U_m)}{\partial U_m}\right)^2 - u'\frac{\partial^2 u^{-1}(U_m)}{\partial U_m^2} < 0. \end{aligned}$$

we end up with

$$\tilde{y} \approx \tilde{w}_u + \tilde{w}_r + \chi,$$

where $\chi = \frac{\eta}{4}\left(\sigma_r^2 + \sigma_u^2 - 2\rho\sigma_r\sigma_u\right)$ is the insurance surplus.

The second step consists in showing that $\chi \geq 0$. The insurance surplus χ is decreasing in ρ which is intuitive since the less incomes are correlated, the larger are the gains from risk sharing. In this perspective, the worst-case scenario for insurance is perfect positive correlation. Even in this case ($\rho = 1$), the insurance surplus is

$$\chi = \frac{\eta}{4}\left(\sigma_r^2 + \sigma_u^2 - 2\sigma_r\sigma_u\right) = \frac{\eta}{4}\left(\sigma_r - \sigma_u\right)^2 \geq 0.$$

This means that even if incomes are perfectly correlated and unless variances are identical, there is always room for mutual insurance since the person who has the more stable income can always insure the other. ■

The aggregate certainty equivalent and hence the collective returns to migration are then the sum of the certainty equivalents of both rural and urban incomes and a surplus drawn from efficient risk pooling. Since we know the value of the surplus that migration allows to generate ($\tilde{y} \approx \tilde{w}_u + \tilde{w}_r + \chi$), we now turn to the issue of the distribution of this surplus. In the paper, two assumptions are explored: either the parent is able to specify remittances in an enforceable migration contract, in such a case, this amount is going to determine the distribution of the migration surplus, or she is not able to do so. Under the latter assumption, we need to determine the equilibrium distribution of the gains. To this end, we adopt the Nash bargaining solution.

We argue that the Nash bargaining solution is relevant in this game, for at least two reasons. First, a "take it or leave it" set-up in which the whole surplus is captured by one player would not have appropriately translated this situation which looks like a bilateral monopoly. Indeed, on the one hand, the parent needs the migrant since only the latter can provide the former with insurance against idiosyncratic rural risks. On the other hand, enforceable insurance contracts are the easiest to conclude within the household where information flows quite efficiently and where family ties can act as an enforcement device. Second, the Nash solution is consistent with the non-cooperative nature of the game. Indeed, it has been shown that the limiting outcome of strategic bargaining models (alternating offers games, Rubinstein (1982)), obtained when the length of a single offer period tends to zero, is precisely the Nash bargaining solution (Binmore et al. (1986)).

The Nash bargaining program writes

$$\underset{r}{Max}\ \left[u\left(\tilde{w}_u + \chi - r\right) - u\left(\tilde{w}_u\right)\right]\left[u\left(\tilde{w}_r + r\right) - u\left(\tilde{w}_r\right)\right],$$

where r are remittances sent by the migrant[4]. The exit options are given by the value of the urban and rural incomes if they are consumed in autarky, that is without risk sharing, by the migrant and the parent, respectively. Appendix 1 shows that, if the utility function is specified as exponential CARA:

$$u\left(x\right) = -e^{-\eta x},$$

then equilibrium remittances are given by

$$r^* = \frac{1}{2}\chi. \tag{3}$$

These are the remittances that can be expected at equilibrium if any other arbitrary amount is not enforceable. As already mentioned, we are going to deal with the two alternative assumptions of enforceable vs

[4] r is not remittances strictly speaking since it does not correspond to a monetary transfer. We are indeed dealing with certainty equivalents.

unenforceable remittances. Equation (3) describes what is meant by unenforceable remittances. Hence, at equilibrium,

$$U_m^* = u\left(\tilde{w}_u + \frac{1}{2}\chi\right) \tag{4}$$

$$U_p^* = u\left(\tilde{w}_r + \frac{1}{2}\chi\right) \tag{5}$$

In this subsection, migration is assumed to have taken place. We are therefore dealing with a subgame of the complete setting. Within this subgame, we have derived the equilibrium distribution of returns to migration and we are now able to highlight some of its comparative statics properties. Proposition 1 introduces a preliminary result related to equilibrium remittances and highlights the possibility of diverging interests between the parent and the migrant over the characteristics of migration.

Proposition 1 ***Diverging interests over the characteristics of migration:***

1. *If remittances are sent for an insurance motive, then the parent does not care about the expected urban wage (μ_u), while migrant's utility increases with μ_u: $\frac{\partial U_p^*}{\partial \mu_u} = 0$; $\frac{\partial U_m^*}{\partial \mu_u} > 0$.*
2. *There exists a range of values for the correlation coefficient such that parent's utility increases with the urban income variance $\left(\sigma_u^2\right)$ while migrant's utility is decreasing in σ_u^2: if $\rho \in \left(-3\frac{\sigma_u}{\sigma_r}, \frac{\sigma_u}{\sigma_r}\right)$, then $\frac{\partial U_p^*}{\partial \sigma_u^2} > 0$; $\frac{\partial U_m^*}{\partial \sigma_u^2} < 0$.*

Proof. These results are obtained by simple derivatives of equations (4) and (5) with respect to the mean μ_u and variance σ_u^2 of the urban income. ■

The first part of Proposition 1 tells us that the expected urban wage is strictly private return for the migrant. This return is not internalized by the parent. As shown below, if the migrant suffers from liquidity constraints and if she cannot credibly commit to remit more than the Nash level, she might be unable to induce the parent to contribute to transaction costs. If returns to migration are essentially private, migration, while optimal, might not take place. In addition, consistently with previous theoretical findings (de la Briere et al. (2002)), we find that, if remittances are sent for an insurance purpose, they should not increase with the mean income of the migrant[5].

The second part of Proposition 1 shows that preferences over the characteristics of migration and in particular over the urban income variance may be opposite. Parent's utility may increase with urban income variance, while the converse is true for the migrant. It follows that, between two migration opportunities, the parent might prefer to support the one in which the migrant is worse off (see below). Notice that this result always holds if σ_u is at least as large as σ_r.

There are two channels through which urban income variability impacts on equilibrium utility levels. There is an efficiency effect and a distribution effect. The former stems from the impact of σ_u^2 on the insurance surplus:

$$\frac{\partial \chi}{\partial \sigma_u^2} > 0 \iff \rho < \frac{\sigma_u}{\sigma_r}.$$

[5] This prediction is in contradiction with empirical evidence of a positive link between migrant's income and remittances (Lucas & Stark (1985); Hoddinott (1994); Funkhouser (1995); Brown (1997); Sinning (2009)). However, in reality, different motives to remit coexist in a unique decision on the amount transferred. For instance, the strategic bequest motive predicts such a relationship (de la Briere et al. (2002)) since migrant's disposable income positively impacts on her ability to compete with siblings for inheritance. Besides, remittances motivated by investment should also increase with income (Osili (2004); Osili (2007)). Finally, altruism goes in the same direction (Lucas & Stark (1985)).

If the correlation coefficient is sufficiently low, a higher variability of the urban income allows to better offset the shocks on rural earnings. It might therefore increase the benefits from risk sharing for both players. The distribution effect is more intuitive. An increase in urban risks deteriorates the migrant's disagreement payoff $\left(\frac{\partial \tilde{w}_u}{\partial \sigma_u^2} < 0\right)$, thereby improving the parent's bargaining position. If the efficiency effect is not large enough, the migrant is going to be worse off due to a loss of bargaining power. More precisely,

$$\frac{\partial}{\partial \sigma_u^2}\left(\tilde{w}_u + \frac{1}{2}\chi\right) < 0 \iff \rho > -3\frac{\sigma_u}{\sigma_r}.$$

With these divergences of interest, the migration decision is unlikely to result from an efficient aggregation of individual preferences. From this perspective, joint utility maximization is an oversimplifying representation of the migration decision. We need to disaggregate the decision process in a non-cooperative framework. This is the standpoint adopted in this paper.

Having described the payoffs under migration, we now introduce the complete timing of the game.

2.2 General setting

The complete timing of the game is as follows. Under unenforceable loan,

1. The parent commits to her contribution t to the migration transaction cost C.

2. The prospective migrant decides whether to migrate or not, $j \in \{0, 1\}$. If she does, she invests the fixed transaction cost C and receives parent's contribution t.

3. If migration has been chosen in stage 2, the migrant earns the urban wage $w_u \sim F_u$ and bargains with the parent over a mutual insurance contract. The payoffs are determined by the Nash allocation described in the preceding subsection. Otherwise, both players get the rural income.

The migration decision is one shot implying that, if remittances are unenforceable, in the sense that any arbitrary amount cannot be enforced, they should be zero would the game end at stage two. The third stage of the game is the reduced form of an infinitely repeated mutual insurance game. Its repeated nature makes enforceable the amount of remittances corresponding to the distribution of the insurance surplus, what we have called incentive compatible remittances. In addition, it is worth noting that, since the parent is the first player, her promise of contribution to transaction costs is supposed to be credible. There are indeed no commitment problem at this stage since the parent transfers this amount directly in cash when travel expenditures have to be covered. On the other hand, for the same reason, the migrant cannot receive this transfer if she does not migrate.

Under the alternative assumption that a loan, and therefore any amount of remittances, is enforceable, the sequence is the following:

1. The parent designs a migration contract that specifies her contribution t to the migration transaction cost C, on the one hand, the amount of remittances r, on the other hand.

2. The prospective migrant decides whether to migrate and accept the contract or to refuse it, $j \in \{0, 1\}$. If she accepts, she invests the fixed transaction cost C and receives parent's contribution t.

3. If migration has been chosen in stage 2, the migrant earns the urban wage $w_u \sim F_u$ and remits[6] r as specified in the migration contract. Otherwise, both players get the rural income.

Without migration, expected utilities over the two periods of consumption write:

$$\begin{aligned} U_{m0} &= u\left(\tilde{w}_r - s_{m0}\right) + \delta u\left(\tilde{w}_r + \kappa s_{m0}\right), \\ U_{p0} &= u\left(w_i + \tilde{w}_r - s_{p0}\right) + \delta u\left(\tilde{w}_r + \kappa s_{p0}\right). \end{aligned}$$

With migration, this gives

$$\begin{aligned} U_{m1} &= u\left(\tilde{w}_r + t - C - s_{m1}\right) + \delta u\left(\tilde{w}_u + \chi - r + \kappa s_{m1}\right), \\ U_{p1} &= u\left(w_i + \tilde{w}_r - t - s_{p1}\right) + \delta u\left(\tilde{w}_r + r + \kappa s_{p1}\right). \end{aligned}$$

where δ is the discount factor, κ the interest rate, w_i initial household wealth and where s_{ij} denotes player i's savings in state of the world j, that is with or without migration. For the sake of tractability, we assume that[7] $\kappa = \delta^{-1}$. With this assumption, optimal savings are easily derived since they simply equalize consumption between the two periods. In the developments below, if players lack access to capital markets, we assume they are prevented from borrowing. However, without migration, one can readily show that, even under borrowing constraints, optimal savings are interior (non-negative) and imply

$$U_{m0} = (1+\delta)\, u\left(\tilde{w}_r\right), \tag{6}$$

$$U_{p0} = (1+\delta)\, u\left\{\tilde{w}_r + (1+\delta)^{-1} w_i\right\}. \tag{7}$$

The remaining analytical developments are organized as follows. The first objective is to show that, in a context of divergence of interest, internalization and hence inefficiency issues result from the combination of imperfect commitment and liquidity constraints. The following section is devoted to a normative analysis aimed at highlighting these inefficiencies and their source. Second, in section 4, we provide a positive analysis of the game with comparative statics with respect to household as well as migration characteristics.

3 A normative assessment of the migration decision rule

Five cases are successively explored: perfect capital markets; constrained migrant, unconstrained parent with and without enforceable loan and finally, constrained migrant and parent with and without enforceable loan. The first best migration decision rule is adopted by the household under the first two sets of assumptions, while decision-making proves inefficient in the remaining cases. However, we highlight that, under perfect commitment, an intra-family loan allows to reach a second best option if both the parent and the migrant are constrained.

At this point, a caveat is important to mention. The insurance motive has been chosen to define incentive compatible remittances. However, one could argue that, if capital markets allow to finance the investment in migration and hence to smooth consumption over time, they could as well be used to absorb income shocks. This possibility is ruled out in what follows in order to distinguish intertemporal allocation of consumption

[6] In this case also, risks are efficiently shared and the insurance surplus appears, see the developments below.

[7] Notice that such a relationship would be the outcome of a perfect capital market in which consummers have homogeneous discount factors.

from income variability within the two periods of the game. Since perfect capital markets are unrealistic in rural developing countries settings, the first purpose they serve in our framework is to provide benchmark cases allowing to highlight the impact of liquidity constraints. Second, since we are solving the game backward, incentive compatible remittances are to some extent exogenous values at this stage. It follows that the payoffs could easily be replaced by other values and, in particular, by remittances resulting from other motives, such as investment or strategic bequest.

3.1 Perfect capital markets

If the household has access to perfect capital markets, the migration decision is unaffected by cash constraints and proves efficient. This case is provided as benchmark. Since, with access to external capital, the intra-family loan is unnecessary, remittances are replaced by their incentive compatible values.

3.1.1 Migrant's decision

Incentive compatible remittances are given by equation (3). Besides, if the migrant has access to capital markets, she can smooth consumption over the two periods of time even if she migrates. Put together, Nash remittances and optimal savings, or borrowing in this case, give

$$U_{m1} = (1+\delta)\, u\left\{(1+\delta)^{-1}\left[\tilde{w}_r + t - C + \delta\left(\tilde{w}_u + \frac{1}{2}\chi\right)\right]\right\}. \tag{8}$$

Define $\Omega_m = U_{m1} - U_{m0}$ as the utility difference between migration and status quo and let $\tilde{t}$ denote the level of parental contribution that leaves the migrant indifferent between migration and status quo. Making use of (8) and (6),

$$\Omega_m\left(\tilde{t}\right) = 0 \iff \tilde{t} = C - \delta\left(\tilde{w}_u + \frac{1}{2}\chi - \tilde{w}_r\right). \tag{9}$$

Decision to migrate is taken by m if the parent commits to any contribution $t \geq \tilde{t}$. m opts for migration provided parental contribution fills the gap between the present value of her private return to migration and the fixed transaction cost. The private return is composed of the certainty equivalent of the urban income and the share of the insurance surplus captured by the migrant net of the opportunity cost, that is the certainty equivalent of the agricultural income.

3.1.2 Parent's decision

Similarly for the parent, Nash remittances and optimal borrowing translates in

$$U_{p1} = (1+\delta)\, u\left\{(1+\delta)^{-1}\left[(1+\delta)\,\tilde{w}_r + w_i - t + \delta\frac{1}{2}\chi\right]\right\}. \tag{10}$$

Notice that the parent also takes a binary decision. Indeed, on the one hand, any contribution strictly lower than $\tilde{t}$ is wasted since it does not lead to migration. On the other hand, the parent cannot draw additional gains by contributing above the indifference threshold. It follows that the parent chooses her contribution t in $\left\{0, \max\left\{0, \tilde{t}\right\}\right\}$. Suppose $\tilde{t} < 0$. In such a case, migrant's private returns to migration are larger than the transaction cost. It is therefore optimal from the parent's point of view to free ride on the costs of migration and to benefit from it afterwards through mutual insurance. This result holds under all the alternative assumptions studied below. If private returns are large enough, internalization is not an issue. In the relevant cases, $\tilde{t} > 0$ and the outcome of household decision-making is interesting to analyze. Suppose $\tilde{t} > 0$. Then the parent chooses between paying the threshold contribution $\tilde{t}$ thereby triggering off migration and status quo. Define $\Omega_p = U_{p1} - U_{p0}$. Making use of (10) and (7),

$$\Omega_p \geq 0 \iff \delta \frac{1}{2} \chi - \tilde{t} \geq 0.$$

The parent supports migration if and only if the present value of remittances outweighs the required contribution to transaction costs. Substituting for the threshold contribution (9), we obtain

$$\Omega_p \geq 0 \iff C \leq C^* = \delta \left(\tilde{w}_u + \chi - \tilde{w}_r \right).$$

Migration takes place if and only if the transaction cost is not larger than the present value of the collective returns to migration. This is the first best decision rule since all the costs and benefits are internalized. It is quite interesting to realize that even in a non-cooperative setting with unenforceable remittances, the outcome proves efficient. The reason for this comes from migrant's ability to make the parent perfectly internalize his wellbeing through $\tilde{t}$. If private returns are high, $\tilde{t}$ is accordingly lower. In turn, the parent takes the final decision by perfectly taking care of her own welfare, obviously. C^* can be interpreted as the household's optimal willingness to pay for migration.

3.2 Constrained migrant, unconstrained parent, enforceable loan

Given that we assume enforceable loan, the timing of the game is slightly different. As introduced above, the parent designs a complete migration contract that specifies how costs and returns to migration are shared. Solving backward, we begin by taking this contract as given.

3.2.1 Migrant's decision

Suppose the migrant does not have access to capital markets and suppose her borrowing constraint is binding in case of migration: $s^*_{m1} = 0$. In this case, expected utility with migration writes

$$U_{m1} = u \left(\tilde{w}_r + t - C \right) + \delta u \left(\tilde{w}_u + \chi - r \right).$$

The migrant is therefore indifferent between migration and status quo if and only if

$$\begin{aligned} \Omega_m \left(\tilde{t} \right) &= u \left(\tilde{w}_r + \tilde{t} - C \right) + \delta u \left(\tilde{w}_u + \chi - r \right) - (1 + \delta) u \left(\tilde{w}_r \right) = 0 && (11) \\ &\iff \frac{1}{1+\delta} u \left(\tilde{w}_r + \tilde{t} - C \right) + \frac{\delta}{1+\delta} u \left(\tilde{w}_u + \chi - r \right) = u \left(\tilde{w}_r \right) && (12) \\ &\iff (1+\delta)^{-1} \left[\tilde{w}_r + \tilde{t} - C + \delta \left(\tilde{w}_u + \chi - r \right) \right] - \phi_m = \tilde{w}_r, \end{aligned}$$

where ϕ_m is the risk premium[8] associated to a lottery where the migrant would earn an amount $\tilde{w}_r + \tilde{t} - C$ with probability $\frac{1}{1+\delta}$ and an amount $\tilde{w}_u + \chi - r$ with probability $\frac{\delta}{1+\delta}$. Indeed, the sum of the weights on

[8] If the migrant is infinitely risk averse, the certainty equivalent of this lottery is the smallest amount, hence at worse the risk premium is the difference between expected consumption and first period consumption:

$$\begin{aligned} \phi_m &= \frac{1}{1+\delta} \left[\tilde{w}_r + \tilde{t} - C + \delta \left(\tilde{w}_r + \chi - r \right) \right] - \left(\tilde{w}_r + \tilde{t} - C \right) \\ &= \frac{\delta}{1+\delta} \left(\tilde{w}_u + \chi - r - \tilde{w}_r - \tilde{t} + C \right). \end{aligned}$$

At the other extreme, under risk neutrality, $\phi_m = 0$.

utilities on the left hand side of equation (12) is one. This expression, while similar to expected utility of a lottery with a binary outcome, represents ex ante utility derived from a distorted consumption path. Since the migrant is averse to fluctuations of her consumption over time, such a distortion implies a cost embodied by ϕ_m. This variable captures the extent to which liquidity constraints are binding. The more consumption is distorted, that is the more first period consumption is reduced by the burden of the investment in migration transaction costs, the higher ϕ_m. Isolating $\tilde{t}$,

$$\tilde{t} = C - \delta\left(\tilde{w}_u + \chi - r - \tilde{w}_r\right) + (1+\delta)\,\phi_m.$$

As in the previous case, the migrant has to be compensated for the gap between the transaction cost and her private return, but in addition, she needs to be compensated for the cost of a distorted consumption path stemming from liquidity constraints.

3.2.2 Parent's decision

Contrary to the previous case, the parent designs a complete migration contract and, in particular, chooses the amount of remittances that maximizes her utility. Applying the implicit function theorem to (11) allows to derive migrant's reaction:

$$\frac{\partial \tilde{t}}{\partial r} = \delta \frac{u'\left(\tilde{w}_u + \chi - r\right)}{u'\left(\tilde{w}_r + \tilde{t} - C\right)}. \tag{13}$$

This equation describes the link between parameters (remittances and parent's contribution to transaction costs) of different migration contracts which leave the migrant indifferent. Since t is granted in the first period whereas r is transferred in the second period, it defines (the opposite of) migrant's marginal rate of substitution between first and second period consumptions. This marginal rate of substitution is lower than the discount factor δ as long as the migrant is cash constrained. This is intuitive since, in this case, the migrant derives a higher marginal utility from first period consumption. To increase both her contribution t and remittances r amounts for the parent to grant a loan contract to the migrant. As we are going to show, if the parent has access to capital markets or is wealthy enough, this allows to compensate for the migrant's lack of access.

Parent's returns to migration can by written as a function of remittances:

$$U_{p1} = (1+\delta)\,u\left\{(1+\delta)^{-1}\left[(1+\delta)\,\tilde{w}_r + w_i - \tilde{t}\,(r) + \delta r\right]\right\},$$

where the parent is assumed to be able to smooth consumption between periods and where it has been taken into account that the indifference contribution $\tilde{t}$ depends on remittances. The first order condition with respect to remittances gives

$$\begin{aligned} \frac{\partial U_{p1}}{\partial r} &= u'\{.\}\left[\delta - \frac{\partial \tilde{t}}{\partial r}\right] = 0 \\ \Longleftrightarrow & \quad \frac{\partial \tilde{t}}{\partial r} = \delta \iff \tilde{w}_u + \chi - r^* = \tilde{w}_r + \tilde{t} - C. \end{aligned} \tag{14}$$

Optimal remittances for the parent are such that migrant's consumption is stable over time. In other words, the optimal migration contract, by including a loan component, acts as a substitute to the credit market.

Substituting for equilibrium remittances, (11) rewrites

$$\Omega_m\left(\tilde{t}\right) = 0 \iff \tilde{t} = C. \tag{15}$$

Finally, parent decides to support migration if and only if the present value of remittances compensates for her contribution:

$$\Omega_p \geq 0 \iff \delta r^* - \tilde{t} \geq 0.$$

Making use of (14) and (15),

$$\Omega_p \geq 0 \iff C \leq C^* = \delta \left(\tilde{w}_u + \chi - \tilde{w}_r \right).$$

This decision rule is again efficient. Hence, first best is reached provided the parent has interior savings and is able to enforce a loan contract thereby overcoming migrant's lack of access to credit. In this perspective, remittances act as a repayment for the initial investment made by the family on the migrant. However, unlike the mutual insurance contract in which transfers are granted on a regular basis, this loan contract is one shot and lasts a long period of time before settlement. It might therefore suffer from a severe commitment constraint. For this reason, the following cases deal with unenforceable loan.

3.3 Constrained migrant, unconstrained parent, unenforceable loan

3.3.1 Migrant's decision

If the migrant has corner savings, the expression of the threshold contribution $\tilde{t}$ is the same as in the previous case, except that remittances, are replaced by their Nash (unenforceable) value (3), that is

$$\tilde{t} = C - \delta \left(\tilde{w}_u + \frac{1}{2}\chi - \tilde{w}_r \right) + (1+\delta)\,\phi_m,$$

where ϕ_m is the utility loss originating from liquidity constraints.

3.3.2 Parent's decision

Parent decides to support migration and to pay this threshold contribution if and only if the present value of Nash remittances is high enough:

$$\begin{aligned} \Omega_p \quad &\geq \quad 0 \iff \delta\frac{1}{2}\chi - \tilde{t} \geq 0 \\ &\iff \quad C \leq C^c = \delta \left(\tilde{w}_u + \chi - \tilde{w}_r \right) - (1+\delta)\,\phi_m. \end{aligned}$$

In the case of constrained migrant and unenforceable loan, household's willingness to pay for migration is reduced. The decision rule is inefficient in the sense that if $C \in (C^c, C^*]$, then an optimal migration opportunity is not sized. This is due to the inability for the migrant to make the parent internalize her private returns to migration. This inability follows from cash constraints. On the one hand, parent's contribution is precisely needed to afford transaction costs in case of shortage of liquidity. But, on the other hand, since the parent internalizes migrant's private returns through $\tilde{t}$, the migrant would even be ready to borrow in order to require from the parent a lower contribution, which she is prevented from doing.

3.4 Constrained migrant, constrained parent, unenforceable loan

According to the remark raised at the beginning of this section, this case is the most relevant. Indeed, if savings are at a corner for both the migrant and the parent, there is no buffer stock available in the second period to absorb income shocks. The benefits from mutual insurance are therefore fully relevant. In addition, this setting is the most realistic.

3.4.1 Migrant's decision

Again, migrant's decision is given by

$$\tilde{t} = C - \delta\left(\tilde{w}_u + \frac{1}{2}\chi - \tilde{w}_r\right) + (1+\delta)\,\phi_m. \tag{16}$$

3.4.2 Parent's decision

If parent has corner savings, her expected utility with migration writes

$$U_{p1} = u\left(w_i + \tilde{w}_r - t\right) + \delta u\left(\tilde{w}_r + r\right).$$

Substituting for Nash remittances, parent decides to support migration if and only if

$$\begin{aligned}
\Omega_p &= u\left(w_i + \tilde{w}_r - \tilde{t}\right) + \delta u\left(\tilde{w}_r + \frac{1}{2}\chi\right) - (1+\delta)\,u\left\{\tilde{w}_r + (1+\delta)^{-1} w_i\right\} \geq 0 \\
\iff & \frac{1}{1+\delta}u\left(w_i + \tilde{w}_r - \tilde{t}\right) + \frac{\delta}{1+\delta}u\left(\tilde{w}_r + \frac{1}{2}\chi\right) \geq u\left\{\tilde{w}_r + (1+\delta)^{-1} w_i\right\} \\
\iff & (1+\delta)^{-1}\left[(1+\delta)\,\tilde{w}_r + w_i - \tilde{t} + \delta\frac{1}{2}\chi\right] - \phi_p \geq \tilde{w}_r + (1+\delta)^{-1} w_i \\
\iff & (1+\delta)^{-1}\left(\delta\frac{1}{2}\chi - \tilde{t}\right) - \phi_p \geq 0
\end{aligned}$$

where ϕ_p is the risk premium associated to a lottery where the parent would earn an amount $w_i + \tilde{w}_r - \tilde{t}$ with probability $\frac{1}{1+\delta}$ and an amount $\tilde{w}_r + \frac{1}{2}\chi$ with probability $\frac{\delta}{1+\delta}$. This variable embodies the utility loss due to a distorted consumption path and captures the extent to which the parent is liquidity constrained. Substituting for parent's threshold contribution (16), we end up with

$$\Omega_p \geq 0 \iff C \leq C^{cc} = \delta\left(\tilde{w}_u + \chi - \tilde{w}_r\right) - (1+\delta)\left(\phi_m + \phi_p\right).$$

This expression represents household's willingness to pay for migration under liquidity constraints and unenforceable loan. When both the migrant and the parent have corner savings, two additional costs pertaining to the distortion of the consumption path over time appear in the decision rule. At the end of this analysis, we obtain the following relationships comparing the household's willingness to pay for migration under unenforceable loan with the first best willingness to pay:

$$C^{cc} < C^{c} < C^{*}.$$

This means that if the burden of the initial investment in transaction costs cannot be absorbed by borrowing or by household wealth, the willingness to pay for migration is suboptimal, put differently, the decision rule is inefficient. Liquidity constraints play a major role, but, as show in the following subsection, they impact could be mitigated would the loan be enforceable.

3.5 Constrained migrant, constrained parent, enforceable loan

If parent has corner savings, her expected utility with migration writes

$$U_{p1} = u\left(w_i + \tilde{w}_r - t\right) + \delta u\left(\tilde{w}_r + r\right).$$

If a loan component can be included in the migration contract, then remittances can be adjusted by the parent. The first order condition with respect to remittances is given by

$$\begin{aligned}\frac{\partial U_{p1}}{\partial r} &= -u'\left(w_i+\tilde{w}_r-\tilde{t}\left(r^*\right)\right)\frac{\partial \tilde{t}}{\partial r}+\delta u'\left(\tilde{w}_r+r^*\right)=0\\ &\iff \frac{u'\left(c_{m2}\right)}{u'\left(c_{m1}\right)}=\frac{u'\left(c_{p2}\right)}{u'\left(c_{p1}\right)},\end{aligned}\tag{17}$$

where we have made use of equation (13) and where the c_{it}'s denote migrant and parent's consumption levels before and after migration. The ratio of marginal utilities of the two periods of consumption belongs to the interval $[0,1]$ and is strictly lower than 1 as soon as the liquidity constraint is binding. The more severe the distorsion between the two consumption levels, the lower this ratio. If the parent is also constrained, she is unable to entirely absorb migrant's cash constraint and smooth her consumption over time. The optimality condition[9] (17) indicates that, in such circumstances, the level of remittances, and hence the terms of the loan contract, have to be such that the burden of the cash constraint is evenly distributed between the migrant and the parent. We can also show that this particular level of remittances maximizes the household's willingness to pay for migration for a given household wealth. By definition, the maximal amount of transaction cost C^{cc} that the household can afford is such that the parent is indifferent between migration and status quo[10].

$$\Omega_p=u\left(w_i+\tilde{w}_r-\tilde{t}\left(C^{cc},r\right)\right)+\delta u\left(\tilde{w}_r+r\right)-\left(1+\delta\right)u\left\{\tilde{w}_r+\left(1+\delta\right)^{-1}w_i\right\}=0,$$

where $\tilde{t}\left(C^{cc},r\right)$ is implicitly defined by equation (11). In order to maximize C^{cc} with respect to r, we apply the implicit function theorem to the latter expression and find the first order condition:

$$\begin{aligned}\frac{dC^{cc}}{dr} &= \frac{-u'\left(w_i+\tilde{w}_r-\tilde{t}\left(C,r\right)\right)\frac{\partial \tilde{t}}{\partial r}+\delta u'\left(\tilde{w}_r+r\right)}{u'\left(w_i+\tilde{w}_r-\tilde{t}\left(C,r\right)\right)\frac{\partial \tilde{t}}{\partial C}}=0\\ &\iff \frac{u'\left(c_{m2}\right)}{u'\left(c_{m1}\right)}=\frac{u'\left(c_{p2}\right)}{u'\left(c_{p1}\right)}.\end{aligned}$$

In a situation of missing capital markets, household wealth might be too low as to finance transaction costs while, at the same time, smoothing migrant and parent consumption paths. In such a context, we show that a loan granted by the parent to the migrant allows to reach a second best by optimally distributing the burden of the investment. However, if a loan component is unenforceable as part of the migration contract, this particular level of remittances can coincide with incentive compatible remittances only by accident. Therefore, the household's willingness to pay for migration is systematically lower under unenforceable loan, that is when the commitment issue is properly taken into account. Notice that this argument applies to any kind of incentive to remit. In other words, if remittances are motivated by other considerations than the recovery of the family loan strictly speaking, this second best cannot be reached. Indeed, the probability that the amount remitted satisfies condition (17) tends to zero. For instance, the parent cannot make use of the bequest argument to enforce a second best migration contract since competition for bequest between siblings follows its own logic and will result in another equilibrium level of remittances.

As final remark, in case of liquidity constraints affecting the household as a whole, the impact of imperfect commitment can be identified as follows. If a loan component is assumed to be enforceable, then the migration decision is only affected by the average or aggregate cash constraint of the household since individual

[9] This condition ensures the tangency between the migrant and the parent indifference curves in the space of the parameter of the migration contract (t,r).

[10] Recall that the migrant is always indifferent since the parent is the first player and captures the whole surplus.

constraints can be shared. In this case, a unitary model of the household allowing for cash constraints would have produced similar predicitions. If, on the contrary, we take the more realistic case of unenforceable loan, the migration outcome depends on the household cash constraint as well as on its composition. More precisely, for a given level of household wealth, investment in migration is expected to decrease if the cash is unevenly distributed among household members.

In order to conclude this section, let us briefly summarize the main results. We have seen that, as expected, perfet capital markets lead to first best decision-making. If capital markets are missing, first best can be reached under a twofold condition. On the one hand, household wealth has to be sufficiently large as to finance the transaction cost while, at the same time, smoothing the household members' consumption paths over time. On the other hand, intra-household loan contracts have to be perfectly enforceable as such. If one of those two conditions is not satisfied, the migration decision proves inefficient. Finally, for any given level of household wealth, imperfect commitment prevents the parent and migrant from sharing the burden of the investment optimally. This leads to underinvestment.

4 Comparative statics of the migration decision rule

4.1 Household characteristics

Comparative statics of household's willingness to pay for migration and hence of its likelihood to send a migrant with respect to its initial characteristics are summarized in the following propositions. Since the parameters pertain to the mutual insurance setting, we derive comparative statics of the double liquidity constraints and imperfect commitment case (C^{cc}) for which the insurance motive is fully relevant. The comparative statics of the first best decision rule are provided as benchmark.

Proposition 2 *Comparative statics with respect to household wealth:*

1. *Household's likelihood to send a migrant increases with initial household wealth if and only if the parent has corner savings. Under CARA preferences:*

$$\frac{dC^{cc}}{dw_i} = \frac{\delta}{1+\delta}\left(1-\alpha_p\right),$$

where $\alpha_p = \frac{u'\left(\tilde{w}_r + \frac{1}{2}\chi\right)}{u'\left(w_i + \tilde{w}_r - \tilde{t}\right)} \in [0,1]$ *is equal to 1 if parent's savings are interior.*

2. *The optimal migration decision rule is not affected by household wealth:*

$$\frac{dC^*}{dw_i} = 0.$$

Proof. See Appendix 2. ■

This result is quite intuitive. An increase in household wealth allows to relax liquidity constraints. If migration was a continuous rather than a binary choice variable, $\frac{dC^{cc}}{dw_i}$ would be marginal propensity to invest in migration out of household wealth. This propensity belongs to the interval $\left[0, \frac{\delta}{1+\delta}\right]$ and is therefore always lower than 1, meaning that a fraction of the increase in wealth is allocated to first period consumption. If capital markets were perfect, household wealth should not influence migration decisions.

Proposition 3 *Comparative statics with respect to the mean rural income:*

1. *Household's likelihood to send a migrant decreases with the mean rural income. However, under liquidity constraints and unenforceable remittances, the more constrained the migrant, the lower this reduction. Under CARA preferences,*

$$\frac{dC^{cc}}{d\mu_r} = -\delta\alpha_m,$$

where $\alpha_m = \frac{u'\left(\tilde{w}_u + \frac{1}{2}\chi\right)}{u'\left(\tilde{w}_r + \tilde{t} - C\right)} \in [0,1]$ *is strictly lower than 1 if migrant's savings are at a corner. In addition, the larger the difference between first and second period consumption levels, the lower* α_m.

2. *The impact of the mean rural income on optimal willingness to pay for migration is given by*

$$\frac{dC^{*}}{d\mu_r} = -\delta.$$

Proof. See Appendix 2. ■

The mean rural income is relevant in the migrant's decision, since it belongs to her opportunity cost, but is not relevant in the parent's decision. The migrant compares the two income distributions and is more likely to migrate, other things equal, if the rural income is low. However, migrant's wellbeing is imperfectly reflected in household decision-making. As already stressed, migrant's liquidity constraints prevent the parent from internalizing properly migrant's utility. This mechanism is embodied by $\alpha_m < 1$, which distinguishes the expression of comparative statics in the cash constraint and imperfect commitment case from its first best counterpart.

Proposition 4 *Comparative statics with respect to the agricultural risk:*

1. *Under liquidity constraints and unenforceable remittances, the impact of rural income variance on household's willingness to pay for migration is given by*

$$\frac{dC^{cc}}{d\sigma_r^2} = \frac{1}{2}\delta\left[\alpha_p\frac{\partial\chi}{\partial\sigma_r^2} + \alpha_m\left(\eta + \frac{\partial\chi}{\partial\sigma_r^2}\right)\right].$$

A sufficient condition for a positive effect of agricultural risk on migration is that the insurance surplus χ *increases with the agricultural risk:* $\frac{\partial\chi}{\partial\sigma_r^2} \geq 0 \implies \frac{dC^{cc}}{d\sigma_r^2} \geq 0$.

2. *The impact of rural income variance on optimal willingness to pay for migration is*

$$\frac{dC^{*}}{d\sigma_r^2} = \delta\left(\frac{\eta}{2} + \frac{\partial\chi}{\partial\sigma_r^2}\right).$$

Proof. See Appendix 2. ■

Let us begin by interpreting comparative statics of the first best decision rule, that is the second part of the proposition. Effects of agricultural risks on migration decisions are twofold. First, from the migrant's point of view, for a given distribution of the urban income, the higher the variance of the rural income, the higher the incentive to migrate. Due to this effect, the parameter of absolute risk aversion η appears in the expression in isolation from the insurance surplus χ. The latter is precisely the channel of the second effect. If correlation between rural and urban incomes is not too important[11], the higher the variability of the rural income, the larger the benefits of mutual insurance. The first part of the proposition describes the impact of agricultural risks on migration under liquidity constraints and imperfect commitment. The effects are similar

[11] It can be shown that $\frac{\partial\chi}{\partial\sigma_r^2} > 0 \iff \rho < \frac{\sigma_r}{\sigma_u}$.

except that the impacts on individual returns are weighted by α_p and α_m. Individual utilities are imperfectly internalized in the household decision making process. And the more her cash constraints are binding, the less an individual's wellbeing is integrated in the decision of the household[12]. It follows that liquidity constraints and internalization issues are intrinsically related to one another.

4.2 Characteristics of migration

The characteristics of the urban income distribution define the migration opportunity. Their impact on the decision rule are summarized in the following propositions.

Proposition 5 *Comparative statics with respect to the mean urban income:*

1. *Household's likelihood to send a migrant increases with the mean urban income. However, under liquidity constraints and unenforceable remittances, this effect is a decreasing function of migrant's liquidity constraints. Under CARA preferences,*
$$\frac{dC^{cc}}{d\mu_u} = \delta\alpha_m.$$
2. *The impact of the mean urban income on optimal willingness to pay for migration is given by*
$$\frac{dC^*}{d\mu_u} = \delta.$$

Proof. See Appendix 2. ■

The mean urban income is part of migrant's private returns. The higher the urban wage, other things equal, the higher the incentive to migrate. However, household decision-making is less sensitive to migrant's private returns if migrant's incentives to migrate are impeded by liquidity constraints.

Proposition 6 *Comparative statics with respect to the urban income variance:*

1. *Under liquidity constraints and unenforceable remittances, the impact of urban income variance on household's willingness to pay for migration is given by*
$$\frac{dC^{cc}}{d\sigma_u^2} = \frac{1}{2}\delta\left[\alpha_p\frac{\partial\chi}{\partial\sigma_u^2} + \alpha_m\left(\frac{\partial\chi}{\partial\sigma_r^2} - \eta\right)\right].$$
2. *The impact of urban income variance on optimal willingness to pay for migration is*
$$\frac{dC^*}{d\sigma_u^2} = \delta\left(\frac{\partial\chi}{\partial\sigma_u^2} - \frac{\eta}{2}\right).$$
3. *There exist cases in which household's willingness to pay for migration increases while the aggregate returns decrease:* $\frac{dC^{cc}}{d\sigma_u^2} > 0$ *and* $\frac{dC^*}{d\sigma_u^2} < 0$.

[12] This model is a principal-agent model. The parent is the first player and always chooses a contribution such that the migrant is indifferent between migration and status quo. Therefore, technically, he entirely reaps the collective and private returns to migration. Hence, when we say that household decision imperfectly internalizes each member's wellbeing, we speak about the composition of the surplus not about its distribution.

Proof. See Appendix 2. ■

Again, the urban income variance impacts on migrant's private returns as well as on collective returns through the insurance surplus. On the one hand, the higher the urban risk, the lower the private returns. On the other hand, if correlation between rural and urban earnings is high enough[13], the urban risk might also be detrimental to the insurance surplus. However, in the opposite case $\left(\frac{\partial \chi}{\partial \sigma_u^2} > 0\right)$ and depending on the respective internalization weighs, α_p and α_m, it is possible to have cases in which the likelihood of migration increases with the urban income variance, while, at the same time, the migrant is worse off and the collective returns are lower. This result relies on two elements. First, if the impact of urban risk on the insurance surplus is positive but low in magnitude, the migrant might be worse off since this effect does not outweighs the negative impact on her private return. Hence, utilities derived from migration may evolve in opposite directions between the migrant and the parent. If one adds severe cash constraints for the migrant, the internalization of her wellbeing in household decision-making (α_m) might be low enough compared to α_p to produce the above-mentionned result.

Proposition 7 *Comparative statics with respect to correlation between rural and urban earnings:*

1. *Under liquidity constraints and unenforceable remittances, the impact of correlation between rural and urban earnings on household's willingness to pay for migration is given by*

$$\frac{dC^{cc}}{d\rho} = \frac{\alpha_p + \alpha_m}{2} \delta \frac{\partial \chi}{\partial \rho} < 0.$$

2. *The impact of correlation between rural and urban earnings on optimal willingness to pay for migration is*

$$\frac{dC^{*}}{d\rho} = \delta \frac{\partial \chi}{\partial \rho} < 0.$$

Proof. See Appendix 2. ■

Since correlation only matters for mutual insurance, the parent and the migrant are symmetrically affected by it. The impact is intuitive, if correlation increases, the desirability of migration for insurance purposes is reduced.

5 Concluding remarks

As widely acknowledged, returns to migration go beyond migrant's earnings. The benefits drawn by sending households take multiple forms, such as a reduction of land pressure, advantages in terms of information about urban or foreign job opportunities, accommodation facilities for schoolchildren, etc. However, among those benefits, remittances are maybe the most important channel through which the sending household shares in the benefits from migration. In this paper, the collective returns to migration are modelled as benefits from mutual insurance. The insurance surplus is created by pooling rural and urban risks together and is therefore, as the examples mentioned above, distinct from migrant's purely private returns. Moreover, we have shown that the mutual insurance motive may generate divergence of interest between the migrant and the parent over the characteristics of migration, namely the risk associated to the migration income. With this distinction between private and collective gains and with this possibility of divergence of interest, household

[13] $\frac{\partial \chi}{\partial \sigma_u^2} > 0 \iff \rho < \frac{\sigma_u}{\sigma_r}$.

decision-making cannot prove efficient unless individual gains are properly internalized. While joint utility maximization or efficient bargaining (Hoddinott (1994)) assume internalization, a non-cooperative approach allows to assess it.

We have presented a model of migration choice under liquidity constraints and imperfect commitment in order to highlight the sources of imperfect internalization. The way we model intra-household decision to invest in migration seems plausible since our comparative statics results are consistent with previous work on migration and remittances. First, migration is positively related to household wealth (Hoddinott (1994)). Notice that this is not the outcome of the strategic bequest motive, rather, our explanation stems from the fact that household wealth relaxes liquidity constraints. Second, plausibility checks of the model are also given by the following results. From a strictly individual point of view, the propensity to migrate is shown to depend on the expected wage gap between rural and urban earnings (Todaro (1969); Larson & Mundlak (1997)) and is also shown to be higher if it allows a reduction of personal risks (Stark & Levhari (1982)).

Turning to normative issues that are more central in the paper, we highlight that internalization problems potentially go in both directions: on the one hand, the migrant has to take account of the whole collective returns, on the other hand, the parent has to allow for migrant's private returns. In our model, instruments of internalization exist. Indeed, on the one hand, our setting takes account of the important migration transaction costs and allows the parent to contribute to it. On the other hand, the migrant sends remittances that could serve as a reward for parent's contribution. However, those instruments are potentially impeded by the combination of individual liquidity constraints and imperfect commitment. As far as the former is concerned, could the migrant credibly commit to remit more, she would be able to induce the parent to contribute more, thereby relaxing her liquidity constraint. We have shown that first best can be reached if the parent has interior savings and is able to enforce a migration contract with a loan component. Regarding parent's liquidity constraints, if savings are at a corner, that is if household wealth does not allow to support the burden of the investment, then the migrant imperfectly internalizes parent's returns. It follows that conditions for efficiency are twofold, important household wealth and enforceable loan. As intuition suggests, those conditions are unlikely to be simultaneously fulfilled in developing countries. In most cases, household's willingness to pay for migration therefore appears to be inefficiently low. In a policy perspective, one could therefore expect that programs promotting access to capital in rural areas may have as side effect an increase in rural-urban or even international migration flows.

Finally, the outcome of our model can be related to the broad theoretical and empirical literature on household decision. As widely acknowledged, for a given aggregate household income, the pattern of expenditures is significantly influenced by each spouse's earning power[14]. It follows that, each household member's wellbeing is reflected in household decision-making[15] according to her relative income. This paper translates this result, pertaining to consumption, in the case of an investment decision under imperfect capital markets. Migration is indeed an investment subject to cash constraints[16]. In the migration story, we have shown

[14] This is documented empirically (Browning et al. (1994); Woolley (2003); Duflo & Udry (2003)). From a theoretical point of view, this fact is predicted by Grossbard-Shechtman (1984) (p. 878) in her model about the interactions between labor and marriage markets. This prediction of a link between the expenditure pattern and individual incomes can also be found in Chiappori (1988) and Chiappori (1992)'s collective setting or in its interesting extension by Basu (2006).

[15] Notice that, according to Grossbard (ming)'s definition of models of decision-making in households, an individual household member's decision can be regarded, and therefore analyzed, as a decision originating from the household. Hence, individual consumption, even as the outcome of "private" optimization enters this category, a fortiori migration, since we have shown that migration costs and benefits are eventually shared.

[16] An important difference between Chiappori's collective setting and this model is worth mentioning. In the collective setting,

that household decision integrates each household member's welfare, not according to her income stream, but, more precisely, according to her capacity to provide liquidity at the time of the investment. It follows that the strength of each member's own liquidity constraint matters in the decision-making process. As an implication, it is misleading to consider only household's aggregate cash constraint in explaining the level of investment (in this case, the migration outcome) since the "distribution of the constraint" is an important factor. This result is very similar to the observation that aggregate household budget does not allow to predict consumption patterns accurately. It might therefore be interesting to apply a similar setting to other investment decisions that a household can take in contexts where cash is a scarce resource and where household members' preferences may diverge, for instance to education and child labor.

References

Adams, Richard H, J. (1998). Remittances, investment, and rural asset accumulation in pakistan. *Economic Development and Cultural Change*, 47(1), 155–73.

Amuedo-Dorantes, C. & Pozo, S. (2009). New evidence on the role of remittances on healthcare expenditures by mexican households. *Review of Economics of the Household*, (pp. 1–30). 10.1007/s11150-009-9080-7.

Basu, K. (2006). Gender and say: a model of household behaviour with endogenously determined balance of power. *Economic Journal*, 116(511), 558–580.

Bernheim, B. D., Shleifer, A., & Summers, L. H. (1985). The strategic bequest motive. *Journal of Political Economy*, 93(6), 1045–76.

Binmore, K., Rubinstein, A., & Wolinsky, A. (1986). The nash bargaining solution in economic modelling. *RAND Journal of Economics*, 17(2), 176–188.

Brown, R. P. C. (1997). Estimating remittance functions for pacific island migrants. *World Development*, 25(4), 613–626.

Browning, M., Bourguignon, F., Chiappori, P.-A., & Lechene, V. (1994). Income and outcomes: A structural model of intrahousehold allocation. *Journal of Political Economy*, 102(6), 1067–96.

Calero, C., Bedi, A. S., & Sparrow, R. (2009). Remittances, liquidity constraints and human capital investments in ecuador. *World Development*, 37(6), 1143 – 1154.

Chiappori, P.-A. (1988). Rational household labor supply. *Econometrica*, 56(1), 63–90.

Chiappori, P.-A. (1992). Collective labor supply and welfare. *Journal of Political Economy*, 100(3), 437–67.

Coate, S. & Ravallion, M. (1993). Reciprocity without commitment : Characterization and performance of informal insurance arrangements. *Journal of Development Economics*, 40(1), 1–24.

Cox, D. (1987). Motives for private income transfers. *Journal of Political Economy*, 95(3), 508–46.

it is assumed that the household always engages in efficient decision-making, while our framework precisely highlights inefficient investments. If the investment was optimal, its level would not depend on each member's contribution, since there is only one investment level that can be said to be optimal. Rather, only the distribution of the benefits could depend on it.

Cox, D., Eser, Z., & Jimenez, E. (1998). Motives for private transfers over the life cycle: An analytical framework and evidence for peru. *Journal of Development Economics*, 55(1), 57–80.

de la Briere, B., Sadoulet, E., de Janvry, A., & Lambert, S. (2002). The roles of destination, gender, and household composition in explaining remittances: an analysis for the dominican sierra. *Journal of Development Economics*, 68(2), 309–328.

Duflo, E. & Udry, C. (2003). *Intrahousehold Resource Allocation in Côte D'ivoire: Social Norms, Separate Accounts and Consumption Choices*. Working Papers 857, Economic Growth Center, Yale University.

Edwards, A. C. & Ureta, M. (2003). International migration, remittances, and schooling: evidence from el salvador. *Journal of Development Economics*, 72(2), 429–461.

Funkhouser, E. (1995). Remittances from international migration: A comparison of el salvador and nicaragua. *The Review of Economics and Statistics*, 77(1), 137–46.

Grossbard, S. (forthcoming). *Independent individual decision-makers in household models and the New Home Economics*. Household Economic Behaviors. Springer.

Grossbard-Shechtman, S. A. (1984). A theory of allocation of time in markets for labour and marriage. *Economic Journal*, 94(376), 863–82.

Harris, J. R. & Todaro, M. P. (1970). Migration, unemployment & development: A two-sector analysis. *American Economic Review*, 60(1), 126–42.

Hoddinott, J. (1994). A model of migration and remittances applied to western kenya. *Oxford Economic Papers*, 46(3), 459–76.

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica*, 58(1), 53–73.

Larson, D. & Mundlak, Y. (1997). On the intersectoral migration of agricultural labor. *Economic Development and Cultural Change*, 45(2), 295–319.

Lucas, R. E. B. (1987). Emigration to south africa's mines. *American Economic Review*, 77(3), 313–30.

Lucas, R. E. B. & Stark, O. (1985). Motivations to remit: Evidence from botswana. *Journal of Political Economy*, 93(5), 901–18.

Mendola, M. (2008). Migration and technological change in rural households: Complements or substitutes? *Journal of Development Economics*, 85(1-2), 150–175.

Osili, U. O. (2004). Migrants and housing investments: Theory and evidence from nigeria. *Economic Development and Cultural Change*, 52(4), 821–49.

Osili, U. O. (2007). Remittances and savings from international migration: Theory and evidence using a matched sample. *Journal of Development Economics*, 83(2), 446–465.

Rapoport, H. & Docquier, F. (2006). *The Economics of Migrants' Remittances*, volume 1 of *Handbook on the Economics of Giving, Reciprocity and Altruism*, chapter 17, (pp. 1135–1198). Elsevier.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, 50(1), 97–109.

Sinning, M. (2009). Determinants of savings and remittances: empirical evidence from immigrants to germany. *Review of Economics of the Household*, (pp. 1–23). 10.1007/s11150-009-9082-5.

Stark, O. (1991). *The migration of labor / Oded Stark*. B. Blackwell, Cambridge, Mass., USA ; Oxford, UK.

Stark, O. & Levhari, D. (1982). On migration and risk in ldcs. *Economic Development and Cultural Change*, 31(1), 191–96.

Todaro, M. P. (1969). A model for labor migration and urban unemployment in less developed countries. *American Economic Review*, 59(1), 138–48.

Woolley, F. (2003). *Control over Money in Marriage*, chapter 5. Marriage and the Economy. Cambridge University Press.

Yang, D. (2004). International migration, human capital, and entrepreneurship: Evidence from philippine migrants exchange rate shocks. *University of Michigan Working Paper Series*, (531).

6 Appendix 1: Nash remittances under exponential CARA utility

The Nash bargaining program writes

$$\underset{r}{Max}\ \left[u\left(\tilde{w}_u+\chi-r\right)-u\left(\tilde{w}_u\right)\right]\left[u\left(\tilde{w}_r+r\right)-u\left(\tilde{w}_r\right)\right].$$

The first order condition is

$$u'\left(\tilde{w}_u+\chi-r^*\right)\left[u\left(\tilde{w}_r+r^*\right)-u\left(\tilde{w}_r\right)\right]=\left[u\left(\tilde{w}_u+\chi-r^*\right)-u\left(\tilde{w}_u\right)\right]u'\left(\tilde{w}_r+r^*\right).$$

If utility is specified as

$$u\left(x\right)=-e^{-\eta x},$$

the first order condition can be rewritten as

$$\begin{aligned}
& \eta\exp\left\{-\eta\left(\tilde{w}_u+\chi-r^*\right)\right\}\left[\exp\left(-\eta\tilde{w}_r\right)-\exp\left\{-\eta\left(\tilde{w}_r+r^*\right)\right\}\right] \\
= \quad & \left[\exp\left(-\eta\tilde{w}_u\right)-\exp\left\{-\eta\left(\tilde{w}_u+\chi-r^*\right)\right\}\right]\eta\exp\left\{-\eta\left(\tilde{w}_r+r^*\right)\right\}
\end{aligned}$$

$$\begin{aligned}
\Longleftrightarrow \quad & \exp\left(\eta r^*\right)\exp\left(-\eta\tilde{w}_u\right)\exp\left(-\eta\chi\right)\exp\left(-\eta\tilde{w}_r\right)\left[1-\exp\left(-\eta r^*\right)\right] \\
= \quad & \left[1-\exp\left(\eta r^*\right)\exp\left(-\eta\chi\right)\right]\exp\left(-\eta\tilde{w}_u\right)\exp\left(-\eta r^*\right)\exp\left(-\eta\tilde{w}_r\right) \\
\Longleftrightarrow \quad & \exp\left(\eta r^*\right)\exp\left(-\eta\chi\right)\left[1-\exp\left(-\eta r^*\right)\right]=\left[1-\exp\left(\eta r^*\right)\exp\left(-\eta\chi\right)\right]\exp\left(-\eta r^*\right) \\
\Longleftrightarrow \quad & \exp\left(2\eta r^*\right)\exp\left(-\eta\chi\right)-\exp\left(\eta r^*\right)\exp\left(-\eta\chi\right)=1-\exp\left(\eta r^*\right)\exp\left(-\eta\chi\right)
\end{aligned}$$

$$\begin{aligned}
\Longleftrightarrow \quad & \exp\left\{\eta\left(2r^*-\chi\right)\right\}=1 \\
\Longleftrightarrow \quad & 2r^*-\chi=0 \Longleftrightarrow r^*=\frac{1}{2}\chi.
\end{aligned}$$

7 Appendix 2: Comparative statics of the migration decision rule

7.1 Preliminary remarks

7.1.1 Liquidity constraints and unenforceable remittances

Recall that, if parent's savings are at a corner and if remittances are unenforceable, she is indifferent between contributing or not if and only if

$$\Omega_p = u\left(w_i + \tilde{w}_r - \tilde{t}\right) + \delta u\left(\tilde{w}_r + \frac{1}{2}\chi\right) - (1+\delta)\, u\left\{\tilde{w}_r + (1+\delta)^{-1} w_i\right\} = 0, \tag{18}$$

where $\tilde{t}$ is implicitly defined by

$$\Omega_m\left(\tilde{t}\right) = u\left(\tilde{w}_r + \tilde{t} - C\right) + \delta u\left(\tilde{w}_u + \frac{1}{2}\chi\right) - (1+\delta)\, u\left(\tilde{w}_r\right) = 0, \tag{19}$$

if migrant's savings are at a corner. Now, taking any parameter θ and applying the implicit function theorem to (18), we have that

$$\frac{dC^{cc}}{d\theta} = -\frac{d\Omega_p}{d\theta}\left(\frac{d\Omega_p}{dC}\right)^{-1},$$

where

$$\frac{d\Omega_p}{d\theta} = \frac{\partial\Omega_p}{\partial\theta} + \frac{\partial\Omega_p}{\partial\tilde{t}}\frac{\partial\tilde{t}}{\partial\theta}.$$

The impact of a given parameter θ on the migration outcome results from the aggregation of its impact on parent's willingness to contribute Ω_p and its impact on migrant's private returns through $\tilde{t}$. In addition, we now that

$$\frac{\partial\Omega_p}{\partial\tilde{t}} = \frac{\partial\Omega_p}{\partial C} = -u'\left(w_i + \tilde{w}_r - \tilde{t}\right),$$

since

$$\frac{\partial\tilde{t}}{\partial C} = 1.$$

Hence

$$\begin{aligned}\frac{dC^{cc}}{d\theta} &= -\left(\frac{\partial\Omega_p}{\partial\theta} + \frac{\partial\Omega_p}{\partial C}\frac{\partial\tilde{t}}{\partial\theta}\right)\left(\frac{\partial\Omega_p}{\partial C}\right)^{-1} \\ &= -\frac{\partial\Omega_p}{\partial\theta}\left(\frac{\partial\Omega_p}{\partial C}\right)^{-1} - \frac{\partial\tilde{t}}{\partial\theta}.\end{aligned} \tag{20}$$

Equation (20) will be used for comparative statics. Other useful relationships are given by the following lemma:

Lemma 2 *Under CARA preferences,*

$$\begin{aligned}\Omega_p &= 0 \implies u'\left(w_i + \tilde{w}_r - \tilde{t}\right) + \delta u'\left(\tilde{w}_r + \frac{1}{2}\chi\right) - (1+\delta)\, u'\left\{\tilde{w}_r + (1+\delta)^{-1} w_i\right\} = 0, \\ \Omega_m &= 0 \implies u'\left(\tilde{w}_r + \tilde{t} - C\right) + \delta u'\left(\tilde{w}_u + \frac{1}{2}\chi\right) - (1+\delta)\, u'\left(\tilde{w}_r\right) = 0.\end{aligned}$$

Proof. Let us take the latter expression. The proof of the former is similar.

$$u'\left(\tilde{w}_r + \tilde{t} - C\right) + \delta u'\left(\tilde{w}_u + \frac{1}{2}\chi\right) - (1+\delta)\, u'\left(\tilde{w}_r\right) = 0$$

$$
\begin{aligned}
&\iff \quad \frac{1}{1+\delta}u'\left(\tilde{w}_r+\tilde{t}-C\right)+\frac{\delta}{1+\delta}u'\left(\tilde{w}_u+\frac{1}{2}\chi\right)=u'\left(\tilde{w}_r\right) \\
&\iff \quad u'\left\{(1+\delta)^{-1}\left[\tilde{w}_r+\tilde{t}-C+\delta\left(\tilde{w}_u+\frac{1}{2}\chi\right)\right]-\phi_m\right\}=u'\left(\tilde{w}_r\right) \\
&\iff \quad (1+\delta)^{-1}\left[\tilde{w}_r+\tilde{t}-C+\delta\left(\tilde{w}_u+\frac{1}{2}\chi\right)\right]-\phi_m=\tilde{w}_r \\
&\iff \quad u\left\{(1+\delta)^{-1}\left[\tilde{w}_r+\tilde{t}-C+\delta\left(\tilde{w}_u+\frac{1}{2}\chi\right)\right]-\phi_m\right\}=u\left(\tilde{w}_r\right) \\
&\iff \quad \Omega_m=0.
\end{aligned}
$$

The point is that ϕ_m, the risk premium, is equal to the precautionary premium in case of exponential CARA preferences. The definition of the precautionary premium (Kimball (1990)), here denoted by ψ is given by the following relationship where Y is a random variable:

$$
Eu'(Y)=u'\{E(Y)-\psi\}.
$$

Kimball (1990) also provides an approximation of the precautionary premium similar to Pratt's approximation of the risk premium:

$$
\psi \approx -\frac{u'''}{u''}\frac{Var(Y)}{2},
$$

where $-\frac{u'''}{u''}$ is the coefficient of absolute prudence as defined by Kimball (1990). The final step is given by the fact that absolute prudence and absolute risk aversion are equal under CARA preferences (Kimball (1990), p.65). ■

7.1.2 First best decision rule

The comparative statics of the optimal willingness to pay for migration are readily given by the derivative of

$$
C^*=\delta\left(\tilde{w}_u+\chi-\tilde{w}_r\right),
$$

where $\chi=\frac{\eta}{4}\left(\sigma_r^2+\sigma_u^2-2\rho\sigma_r\sigma_u\right)$, with respect to the relevant parameter.

7.2 Proof of Proposition 2: household wealth w_i

Two elements are needed in order to use equation (20): $\frac{\partial\Omega_p}{\partial w_i}$ and $\frac{\partial\tilde{t}}{\partial w_i}$. Beginning by the latter,

$$
\frac{\partial\tilde{t}}{\partial w_i}=0.
$$

The former is given by

$$
\frac{\partial\Omega_p}{\partial w_i}=u'\left(w_i+\tilde{w}_r-\tilde{t}\right)-u'\left\{\tilde{w}_r+(1+\delta)^{-1}w_i\right\}.
$$

Hence, substituting these elements in (20), we find

$$
\frac{dC^{cc}}{dw_i}=1-\frac{u'\left\{\tilde{w}_r+(1+\delta)^{-1}w_i\right\}}{u'\left(w_i+\tilde{w}_r-\tilde{t}\right)}.
$$

Making use of Lemma 2,

$$\begin{aligned}\frac{dC^{cc}}{dw_i} &= 1-\frac{u'\left(w_i+\tilde{w}_r-\tilde{t}\right)+\delta u'\left(\tilde{w}_r+\frac{1}{2}\chi\right)}{(1+\delta)\,u'\left(w_i+\tilde{w}_r-\tilde{t}\right)} \\ &= \frac{\delta}{1+\delta}\left(1-\alpha_p\right),\end{aligned}$$

where $\alpha_p=\frac{u'\left(\tilde{w}_r+\frac{1}{2}\chi\right)}{u'\left(w_i+\tilde{w}_r-\tilde{t}\right)}$.

7.3 Proof of Proposition 3: mean rural income μ_r

Again, in order to use equation (20), we need to calculate $\frac{\partial\Omega_p}{\partial\mu_r}$ and $\frac{\partial\tilde{t}}{\partial\mu_r}$. The former is

$$\frac{\partial\Omega_p}{\partial\mu_r}=\frac{\partial\Omega_p}{\partial\tilde{w}_r}=u'\left(w_i+\tilde{w}_r-\tilde{t}\right)+\delta u'\left(\tilde{w}_r+\frac{1}{2}\chi\right)-(1+\delta)\,u'\left\{\tilde{w}_r+(1+\delta)^{-1}w_i\right\}=0, \tag{21}$$

by Lemma 2. Applying the implicit function theorem to (19), one obtains

$$\frac{\partial\tilde{t}}{\partial\mu_r}=\frac{\partial\tilde{t}}{\partial\tilde{w}_r}=\frac{(1+\delta)\,u'\left(\tilde{w}_r\right)-u'\left(\tilde{w}_r+\tilde{t}-C\right)}{u'\left(\tilde{w}_r+\tilde{t}-C\right)}.$$

By Lemma 2, this expression becomes

$$\frac{\partial\tilde{t}}{\partial\mu_r}=\delta\frac{u'\left(\tilde{w}_u+\frac{1}{2}\chi\right)}{u'\left(\tilde{w}_r+\tilde{t}-C\right)}.$$

Finally, using (20),

$$\frac{dC^{cc}}{d\mu_r}=-\delta\alpha_m,$$

where $\alpha_m=\frac{u'\left(\tilde{w}_u+\frac{1}{2}\chi\right)}{u'\left(\tilde{w}_r+\tilde{t}-C\right)}$.

7.4 Proof of Proposition 4: rural income variance σ_r^2

Making use of the fact that $\frac{\partial\Omega_p}{\partial\tilde{w}_r}=0$ (see equation (21)),

$$\frac{\partial\Omega_p}{\partial\sigma_r^2}=\frac{\partial\Omega_p}{\partial\tilde{w}_r}\frac{\partial\tilde{w}_r}{\partial\sigma_r^2}+\frac{\partial\Omega_p}{\partial\chi}\frac{\partial\chi}{\partial\sigma_r^2}=\frac{\partial\Omega_p}{\partial\chi}\frac{\partial\chi}{\partial\sigma_r^2}.$$

It follows that

$$\frac{\partial\Omega_p}{\partial\sigma_r^2}=\frac{1}{2}\frac{\partial\chi}{\partial\sigma_r^2}\delta u'\left(\tilde{w}_r+\frac{1}{2}\chi\right). \tag{22}$$

Besides

$$\frac{\partial\tilde{t}}{\partial\sigma_r^2}=\frac{\partial\tilde{t}}{\partial\tilde{w}_r}\frac{\partial\tilde{w}_r}{\partial\sigma_r^2}+\frac{\partial\tilde{t}}{\partial\chi}\frac{\partial\chi}{\partial\sigma_r^2}, \tag{23}$$

where

$$\frac{\partial\tilde{w}_r}{\partial\sigma_r^2}=-\frac{\eta}{2}.$$

Applying the implicit function theorem to (19), we find

$$\begin{aligned}\frac{\partial\tilde{t}}{\partial\chi} &= -\frac{1}{2}\delta\frac{u'\left(\tilde{w}_u+\frac{1}{2}\chi\right)}{u'\left(\tilde{w}_r+\tilde{t}-C\right)}, \\ \frac{\partial\tilde{t}}{\partial\tilde{w}_r} &= \frac{(1+\delta)\,u'\left(\tilde{w}_r\right)-u'\left(\tilde{w}_r+\tilde{t}-C\right)}{u'\left(\tilde{w}_r+\tilde{t}-C\right)}.\end{aligned}$$

As already noted, by Lemma 2, the latter expression becomes

$$\frac{\partial \tilde{t}}{\partial \tilde{w}_r} = \delta \frac{u'\left(\tilde{w}_u + \frac{1}{2}\chi\right)}{u'\left(\tilde{w}_r + \tilde{t} - C\right)}.$$

Substituting these expressions in (23) gives

$$\frac{\partial \tilde{t}}{\partial \sigma_r^2} = -\frac{1}{2}\delta\alpha_m \left(\eta + \frac{\partial \chi}{\partial \sigma_r^2}\right). \tag{24}$$

Finally, combining equations (20), (22) and (24),

$$\frac{dC^{cc}}{d\sigma_r^2} = \frac{1}{2}\delta \left[\alpha_p \frac{\partial \chi}{\partial \sigma_r^2} + \alpha_m \left(\eta + \frac{\partial \chi}{\partial \sigma_r^2}\right)\right].$$

7.5 Proof of Proposition 5: mean urban income μ_u

Again, two elements are needed, namely $\frac{\partial \Omega_p}{\partial \mu_u}$ and $\frac{\partial \tilde{t}}{\partial \mu_u}$. The former is zero since $\tilde{w}_u$ does not appear in Ω_p. The latter is directly derived from the application of the implicit function theorem to (19). This gives

$$\begin{aligned} \frac{dC^{cc}}{d\mu_u} &= -\frac{\partial \tilde{t}}{\partial \mu_u} = -\frac{\partial \tilde{t}}{\partial \tilde{w}_u} = \delta \frac{u'\left(\tilde{w}_u + \frac{1}{2}\chi\right)}{u'\left(\tilde{w}_r + \tilde{c} - T\right)} \\ \iff & \frac{dC^{cc}}{d\mu_u} = \delta\alpha_m. \end{aligned}$$

7.6 Proof of Proposition 6: urban income variance σ_u^2

The proof of the first part of Proposition 6 is as follows. On the one hand,

$$\frac{\partial \Omega_p}{\partial \sigma_u^2} = \frac{1}{2}\frac{\partial \chi}{\partial \sigma_u^2}\delta u'\left(\tilde{w}_r + \frac{1}{2}\chi\right). \tag{25}$$

On the other hand,

$$\frac{\partial \tilde{t}}{\partial \sigma_u^2} = \frac{\partial \tilde{t}}{\partial \tilde{w}_u}\frac{\partial \tilde{w}_u}{\partial \sigma_u^2} + \frac{\partial \tilde{t}}{\partial \chi}\frac{\partial \chi}{\partial \sigma_u^2}, \tag{26}$$

where

$$\frac{\partial \tilde{w}_u}{\partial \sigma_u^2} = -\frac{\eta}{2}.$$

Applying the implicit function theorem to (19), we find

$$\begin{aligned} \frac{\partial \tilde{t}}{\partial \tilde{w}_u} &= \delta \frac{u'\left(\tilde{w}_u + \frac{1}{2}\chi\right)}{u'\left(\tilde{w}_r + \tilde{t} - C\right)}, \\ \frac{\partial \tilde{t}}{\partial \chi} &= -\frac{1}{2}\delta \frac{u'\left(\tilde{w}_u + \frac{1}{2}\chi\right)}{u'\left(\tilde{w}_r + \tilde{t} - C\right)}. \end{aligned}$$

Substituting these expressions in (26) gives

$$\frac{\partial \tilde{t}}{\partial \sigma_u^2} = -\frac{1}{2}\delta\alpha_m \left(\frac{\partial \chi}{\partial \sigma_u^2} - \eta\right).$$

Combining the latter expression with equation (25) and applying the rule given in (20), we end up with

$$\frac{dC^{cc}}{d\sigma_u^2} = \frac{1}{2}\delta \left[\alpha_p \frac{\partial \chi}{\partial \sigma_u^2} + \alpha_m \left(\frac{\partial \chi}{\partial \sigma_u^2} - \eta\right)\right].$$

It remains to be shown that there exist cases in which we can have $\frac{dC^{cc}}{d\sigma_u^2} > 0$ together with $\frac{dC^*}{d\sigma_u^2} < 0$.

First assume that

$$\begin{aligned} \frac{dC^{cc}}{d\sigma_u^2} &> 0 \iff \alpha_p \frac{\partial \chi}{\partial \sigma_u^2} + \alpha_m \left(\frac{\partial \chi}{\partial \sigma_u^2} - \eta \right) > 0 \\ &\iff \frac{\alpha_m}{\alpha_p} \left(\eta - \frac{\partial \chi}{\partial \sigma_u^2} \right) < \frac{\partial \chi}{\partial \sigma_u^2}. \end{aligned}$$

Notice that a necessary condition for this is $\frac{\partial \chi}{\partial \sigma_u^2} > 0$. Besides,

$$\begin{aligned} \frac{dC^*}{d\sigma_u^2} &= \delta \left(\frac{\partial \chi}{\partial \sigma_u^2} - \frac{\eta}{2} \right) < 0 \iff \frac{\partial \chi}{\partial \sigma_u^2} < \frac{\eta}{2} < \eta \\ &\implies \eta - \frac{\partial \chi}{\partial \sigma_u^2} > 0. \end{aligned}$$

Hence,

$$\frac{dC^{cc}}{d\sigma_u^2} > 0 \iff \frac{\alpha_m}{\alpha_p} < \frac{\partial \chi}{\partial \sigma_u^2} \left(\eta - \frac{\partial \chi}{\partial \sigma_u^2} \right)^{-1},$$

where the right hand side is positive, as we have just shown. This condition can be fulfilled, provided α_m is sufficiently low compared to α_p.

7.7 Proof of Proposition 7: correlation between rural and urban incomes ρ

On the one hand,

$$\frac{\partial \Omega_p}{\partial \rho} = \frac{\partial \Omega_p}{\partial \chi} \frac{\partial \chi}{\partial \rho},$$

where

$$\frac{\partial \Omega_p}{\partial \chi} = \frac{1}{2} \delta u' \left(\tilde{w}_r + \frac{1}{2} \chi \right).$$

On the other hand,

$$\frac{\partial \tilde{t}}{\partial \rho} = \frac{\partial \tilde{t}}{\partial \chi} \frac{\partial \chi}{\partial \rho},$$

where

$$\frac{\partial \tilde{t}}{\partial \chi} = -\frac{1}{2} \delta \frac{u' \left(\tilde{w}_u + \frac{1}{2} \chi \right)}{u' \left(\tilde{w}_r + \tilde{t} - C \right)} = -\frac{1}{2} \delta \alpha_m,$$

as we found earlier. Making use of these informations and of equation (20), one obtains

$$\frac{dC^{cc}}{d\rho} = \frac{\alpha_p + \alpha_m}{2} \delta \frac{\partial \chi}{\partial \rho}.$$