Microfinance, Social Capital, and For-Profit Lending

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Abstract

This paper compares the behavior of for-profit microfinance lenders with market power with the benchmark model of a competitive or non-profit lender. The environment is one where borrowers are collateral-poor but group lending can leverage the social capital that borrowers might have among themselves. We show that even though a for-profit lender can exploit the borrowers’ social capital to extract higher rents under joint liability contracts, such contracts are still preferred by borrowers to individual liability under quite general conditions. Furthermore, more social capital makes borrowers strictly better off, despite the lender’s exploitation of it. We consider several extensions, including modeling social capital endogenously, and allowing a competitive equilibrium with free entry of for-profit borrowers.

1 Introduction

The recent controversy about the activities of microfinance institutions (MFIs) in the Indian state of Andhra Pradesh and elsewhere in the world has stirred debate about for-profit lending and mission drift in the microfinance industry. As in standard economic models, it seems natural to be particularly concerned about for-profit lending when combined with concentration of market power in the hands of a small number of commercial lenders. This raises a sharp contrast with the microfinance literature thus far which has typically assumed lenders to be non-profits or to operate in a perfectly competitive market. While the success of MFIs across the world has been tremendous over the last two decades, culminating in the Nobel Peace Prize for the Grameen Bank and its founder Dr. Muhammad Yunus, “for their efforts to create economic and social development from below”, these recent controversies have cast a shadow on the industry.

The main critique is that MFIs are making profits on the backs of the poor, which seemingly contradicts the original purpose of the MFI movement, namely making capital accessible to the poor to lift them out of poverty. This critique is acknowledged within the MFI sector. For example, Muhammad Yunus argues that the shift from non-profit

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‡For a review see, for example, Ghatak and Guinnane (1999).
to for profit, with some institutions going public, led to aggressive marketing and loan
collection practices in the quest for profits to serve the shareholders equity. Through this
he argues microcredit “[gave] rise to its own breed of loan sharks.” This has led to calls
for tougher regulations on the MFI sector, which some have argued might stifle the sector
and is akin to throwing the baby out with the bathwater.

For researchers these events raise many interesting questions. Is it possible that the
celebrated innovative lending methods (such as a group-based lending mechanisms) can
be a potent tool of rent extraction in the hands of a for-profit lender with market power?
Is it possible that the borrowers’ social capital that MFIs are thought to leverage to relax
borrowing constraints for collateral-poor borrowers might also be a resource that a lender
can tax? In this paper we analyze the behavior of a monopoly lender offering individual
and joint liability loans, henceforth denoted IL and JL respectively.

In our model, a for-profit lender with market power (in the simplest case, a monop-
olist) can extract rents from borrowers that are positively related to the level of social
capital that these borrowers share. We assume that being in a borrowing group enables
borrowers to observe one another’s output realizations, while the lender is unable to ob-
serve output. Under joint liability contracts, borrowers leverage their social capital to
write incentive-compatible mutual insurance agreements amongst themselves in a manner
akin to the mechanism highlighted in Besley and Coate (1995). The stronger the borrow-
ers’ social ties, the higher the interest payment their agreement can insure. Exploiting
this, the lender can increase interest rates when social ties are strong, and furthermore
benefits from the higher repayment probability when borrowers mutually insure. The
lender de facto outsources part of the enforcement of the loan contract to the borrowers
themselves. However, perhaps surprisingly, the borrowers are always at least as well off
under “exploitative” joint liability as individual liability. This follows from the fact that
there are two relevant incentive compatibility constraints under joint liability. The first,
which we term the “group incentive condition” (GIC) is identical in joint and individual
liability lending and must be satisfied for repayment to be incentive compatible for the
borrower(s) taken as a group (under individual liability, the borrower is the group). The
second, which we term the “individual incentive condition” (IIC) coincides trivially with
the GIC under individual liability, but differs under joint liability and must be satisfied
for one borrower to be willing to repay on behalf of her partner should her partner suffer a
low output realization. Since the IIC involves repayment of two loans under joint liability
it will tend to be tighter than the GIC, leading to lower interest rates under joint liability.

We extend the model to study investments in social capital by the lender and analyze
the consequences of increased competition in our framework. We also discuss modeling
social capital as an endogenous variable. We show that, perhaps counterintuitively, the
monopolist may actually create social capital, even as he taxes away its benefits. We go
on to discuss the somewhat restrictive assumptions that would need to be made in order
for the lender’s actions to actually harm social capital.

The theory is a model of lending with limited liability and no collateral where the
source of friction is weak enforcement along the lines of Besley and Coate (1995), Rai and

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3 Abhijit Banerjee (MIT), Pranab Bardhan (Berkeley), Esther Duflo (MIT), Erica Field (Harvard),
Dean Karlan (Yale), Asim Khwaja (Harvard), Dilip Mookherjee (Boston), Rohini Pande (Harvard),
Raghuram Rajan (Chicago) urge caution in an op-ed in the Indian Express, 26th November 2010, acces-
Sjöström (2004) and Bhole and Ogden (2010). The main departure in our core model is that the lender is a profit-maximizing monopolist. We compare the outcomes against what we would obtain with the benchmark in the literature thus far, a non-profit or perfectly competitive lender satisfying a zero profit constraint. McIntosh and Wydick (2005) also look at variation in competition, but they focus on non-profit lenders. The main effect they highlight is that the lenders’ client-maximizing objectives cause them to cross-subsidize within their pool of borrowers and when competition eliminates rents on profitable borrowers, it is likely to yield a new equilibrium in which poor borrowers are worse off.

Besley and Coate (1995) is the seminal paper in this literature, and shows how joint liability can induce informal insurance within borrowing groups, with lucky borrowers helping their unlucky partners with repayment when needed. They show a trade-off between improved repayment through mutual insurance, and a perverse effect of joint liability, that sometimes a group may default en masse even though one member would have repaid had they received an individual liability loan. Introducing social sanctions, they show how these can help alleviate this perverse effect by making full repayment incentive compatible in more states of the world.

Rai and Sjöström (2004) and Bhole and Ogden (2010) are important recent contributions to the literature, both of which solve a simple lending problem close to ours for “efficient” or near-efficient contracts. In both papers the lender is unable to observe the borrowers’ output realizations. Rai and Sjöström use a mechanism design approach, adding a message game to the standard repayment structure, interpreted as borrowers “cross-reporting” on one another. In doing so, the lender is able to extract the borrowers’ private information and thus only the weakest possible punishments are used on the equilibrium path. Importantly, the lender’s formal sanctions are exogenously given and can be large relative to the borrowers’ output realizations. Bhole and Ogden point out that cross-reporting does not seem to be widely used, and that lenders may shy away from the antagonism it might create within groups. They instead ask what would be the most efficient contract, allowing for partial repayment, in the absence of cross-reporting, and further assume that the lender’s sanctions are restricted to simple dynamic incentives: probabilistic termination of future lending to the borrower. This enables them to derive a “flexible joint liability” contract which is nearly as efficient as the Rai and Sjöström mechanism. Both papers assume borrowers cannot use social sanctions against one another.

We depart from Rai and Sjöström (2004) and Bhole and Ogden (2010) in three key ways. Firstly, as already mentioned our lender is a profit-maximizing monopolist, while both of these papers assume the lender earns zero profits, effectively maximizing borrower utility. Secondly, we incorporate social capital into the framework, and show how this might be abused by the lender. Thirdly, we step back from the contractual flexibility of these papers and limit the lender’s freedom to a choice of interest rate which must be repaid in full and the use of either a joint liability or individual liability clause in the contract. The lender’s formal sanctions are strict dynamic incentives: complete termination in the event of failure to repay one loan (individual liability) or both loans (joint liability).

We do not think the more flexible mechanism design-based approach is an uninteresting one, but we stick to simple contracts for two reasons. First of all, giving the lender

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Tedeschi (2006) also presents a model where strict dynamic incentives are relaxed in the form of a “punishment phase” in the usual repeated-game sense, during which loans are denied.
only two degrees of freedom considerably simplifies the analysis, and yet does not pre-
vent us drawing some quite striking conclusions that we believe would not substantially 
change with more flexibility. Secondly, it seems to us that in practice lenders use rela-
tively simple contracts, typically with strict dynamic incentives and little willingness to 
accept partial repayment. Why and the extent to which this is the case is an interesting 
and open question, perhaps it is simply costly to persuade borrowers to sign and abide 
by a contract with a lot of “small print”, perhaps lenders do feel the concerns about 
cross-reporting mechanisms, perhaps what goes on in practice is different to what is on 
paper. Indeed there is evidence that lenders were using additional coercive loan recovery 
methods in the run-up to the Andhra Pradesh crisis of 2010/11. There may be concerns 
about loan officer fraud (theft of funds) if partial repayment is permitted.

In an extension we unpack the “black box” of social capital by allowing for it to be an 
endogenous variable. Many papers treat social sanctions as exogenously given. However 
the endogenous nature of social capital has been pointed out by among others Karlan 
(2007) and Cassar and Wydick (2010). Feigenberg et al. (2011) attempt to identify the 
causal effect of altering loan repayment frequency on mutual insurance, claiming that 
more frequent meetings can foster the production of social capital and critically may 
lead to more informal insurance within the group, which is exactly the channel upon 
which we focus. Feigenberg et al. also highlight that peer effects are important for 
loan repayment even without explicit joint liability through informal insurance, and that 
these effects are decreasing in social distance. We model social capital as the value of 
a two-player repeated game played with a “friend” in the community, that can credibly 
be destroyed in response to some deviation. This setup is by no means novel, it was 
alluded to by Besley and Coate (1995) and is also mentioned by Fukuyama (2000), among 
others. Guttman (2010) presents a related model where agents play a trust game and 
a microfinance repayment game in the presence of untrustworthy types, where agents 
can build reputation in both games. He shows how these can support one another, with 
trustworthy types able to reveal their type more quickly.

In our model, the lender is able to exploit the borrowers’ social capital by leverag-
ing informal insurance agreements formed within borrowing groups, in particular when 
joint liability lending is used. A common claim is that the use of joint liability lending 
is dying out, following the Grameen Bank and BancoSol among others to move to in-
dividual liability lending (see, for example, Armendáriz de Aghion and Morduch 2010). 
However, whether or not JL is falling out of fashion, it is still widely used as indicated 
by Table 1 below. This groups 663 institutions that reported to the Microfinance Infor-
mation Exchange (MIX) dataset in 2009 according to those that have some or no joint 
liability and/or individual liability loans in their portfolios. Unfortunately this part of 
the dataset is too new to allow us to analyze trends, but we do observe that 12.2% of 
lenders exclusively offer joint liability loans and 57.9% offer at least some joint liability 
loans.

Moreover, this data does not include the important Self-Help Group (SHG) movement 
in India. The results we present are framed in the familiar context of microfinance 
institutions as lenders, but are equally applicable to a bank that makes joint liability 
loans to SHGs. Thus we believe our results are very relevant to thinking about market 
power in microfinance, especially following the recent events in India, discussed in the 
next section.

The plan of the paper is as follows. In the next section we briefly discuss the recent 
crisis in the microfinance sector in India and highlight some of the facts that motivate
Table 1: Number of Institutions by Lending Methodology. JL/no JL denotes institutions that report some/no joint liability lending, likewise for IL (individual liability). Institutions denoted “No IL” and “No JL” are “Village Banks” that use very large borrowing groups but are in practice very close to joint liability lending.

<table>
<thead>
<tr>
<th></th>
<th>No JL</th>
<th>JL</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>No IL</td>
<td>20</td>
<td>81</td>
<td>101</td>
</tr>
<tr>
<td></td>
<td>(3.0%)</td>
<td>(12.2%)</td>
<td>(15.2%)</td>
</tr>
<tr>
<td>IL</td>
<td>259</td>
<td>303</td>
<td>562</td>
</tr>
<tr>
<td></td>
<td>(39.1%)</td>
<td>(45.7%)</td>
<td>(84.8%)</td>
</tr>
<tr>
<td>Total</td>
<td>279</td>
<td>384</td>
<td>663</td>
</tr>
<tr>
<td></td>
<td>(42.1%)</td>
<td>(57.9%)</td>
<td>(100%)</td>
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our theoretical analysis. In section 3 we lay down the basic model, analyzing the choice of contracts a non-profit lender who maximizes borrower welfare in section 3.4 and a for-profit monopolist in 3.5. In section 4 we work out two various extensions, modeling social capital as endogenous in 4.1 and the effects of competition in 4.2. We also briefly discuss the effects of relaxing some of our key assumptions. In section 4.6 we analyze the use of coercive enforcement methods. Section 5 concludes.

2 Motivation: The Andhra Pradesh Crisis

The microfinance sector in India has expanded in the last 10 years into one of the largest microfinance industries in the world. However it has only very recently started to diversify its funding sources, away from borrowing and donations and towards equity capital. This went along with a sequence of MFIs changing legal status towards more regulated legal forms. The years 2008-2009 saw a total of more than $200 Million in venture capital deals, with big investment outlets buying stakes in Indian microfinance institutions. These deals corresponded to more than 10% of the total market in 2008 according to data from the MIX market. The climax was the initial public offering of SKS India, the largest Indian microfinance institution, just before the onset of the crisis in summer 2010.5

The influx of capital from profit-oriented banks and venture capital funds has sparked debates on the sustainability of an institution’s objective to address poverty while trying to satisfy the demand to generate returns for investors. Muhammad Yunus, one of the early critics, writes in the New York Times:

“To ensure that the small loans would be profitable for their shareholders, such banks needed to raise interest rates and engage in aggressive marketing and loan collection... The kind of empathy that had once been shown toward borrowers when the lenders were non-profits disappeared. The people whom microcredit was supposed to help were being harmed. ... Commercialization has been a terrible wrong turn for microfinance, and it indicates a worrying “mission drift” in the motivation of those lending to the poor. Poverty should be eradicated, not seen as a money-making opportunity.”6

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The recent Indian crisis gave rise to all these concerns once again. It started out with a sequence of media reports on excessive interest rates and of harsh recovery methods, that some believe drove borrowers to suicide. This was followed by public outcry and politicians urging borrowers to not repay their loans. Eventually, it resulted in a political response that put a halt to all microfinance operations in Andhra Pradesh through an ordinance by the state government. Since then, the sector has been cut off from the single most important source of funding: borrowing from formal banks. The latter refrain from lending as loan repayment rates are still very low and political risk is high. This Indian experience is one that has been observed in many other countries with a developed microfinance sector. Among many others, in 2009, Nicaragua saw a politically motivated “No Pago” (I am not paying) movement and the microfinance sector in Bolivia experienced a similar crisis in 1999 and 2000. The Indian experience is thus not a special case. However it highlighted a lot of open questions regarding the role of regulation, customer protection, multiple lending and market power. In this paper we focus only on the latter, and when we incorporate competition in an extension it will be in a simple form that does not allow for overborrowing.

In India, the five biggest microfinance institutions account for more than 50% of the market and lending operations are highly concentrated in a few states in the south of the country. We do not know precisely whether and to what extent institutions can exercise their market power. David Roodman has used non-disaggregated data and observes that market concentration in India does not seem to be at an unhealthy level. However, there are many caveats with these approaches as there is a lack of spatial data to get sensible estimates as to how locally competitive the sector really is.

![Figure 1: Correlation between profit margin and rural share of total operations, by for-profit status. Data from MIX market.](image)

There is also some suggestive evidence about the extent of for-profit vs non-profit
activity in an area and how dense social networks are, which motivates the role of social capital and its possible exploitation by a monopoly lender as a key element of our theoretical analysis. Using cross-sectional data we find that for-profits are more concentrated in rural areas where one could argue there is more social capital. Figure 1 is a scatterplot of profit margins plotted over a variable that measures the proportion of loans (by number, not loan size) that this MFI classifies as carried out in rural areas. The data comes from the MIX market and covers 39 institutions in 2009 which serve 15.7 million borrowers with an overall portfolio of $2.8 billion. We observe that the correlation appears to be positive for for-profit microfinance providers in India, while it is negative for MFIs that are non-profits. If we hypothesize that social ties will tend to be closer in rural areas, then with many caveats of course, this is at least consistent with higher operating costs in rural areas, offset by higher rent-extraction by for-profit lenders when social ties are stronger, which is a key prediction of our model when lenders have market power.

3 The Model

We assume that there are is a set of risk neutral agents or “borrowers”, each of whom has access to an independent stochastic production function that produces $R$ units of output with probability $p \in (0,1)$ and zero otherwise. The outside option of the borrower is also zero. Borrowers cannot save, and so all output is consumed each period. They also have no assets, and so must borrow 1 unit of output to finance their production. In addition, there is limited liability and no collateral, so borrowers can repay only up to their income in a given period. Each borrower’s output is observable to all other borrowers but not verifiable by any third party, so the lender cannot write state-contingent contracts. The only such contracts that borrowers can write are with one another and these are enforced by social sanctions (to be defined later). Borrowers have infinite horizons and discount the future with factor $\delta \in (0,1)$.

There is a single lender who may be a non-profit who sets interest rates to maximize borrower welfare subject to a zero-profit condition, or alternatively a for-profit who uses his market power when choosing interest rates. The lender’s opportunity cost of funds is $\omega \geq 1$ per unit. We assume that the projects yield a strictly positive expected social surplus: $pR > \omega$.

For most of the analysis we will assume for simplicity that the lender has a fixed capacity that is small relative to the number of potential borrowers. The implication is that if a lender-borrower relationship breaks up he can always costlessly find another borrower to replace the current one, and so will be indifferent to terminating her contract at the end of the period. This allows us to simplify the for-profit lender’s objective to maximizing the per-period profit from a given borrower. In section 4.3 we relax this assumption, allowing the lender to maximize the discounted sum of lifetime profits from a borrower.

We assume that it is very costly for the lender to observe borrowers’ output and there is no cross-reporting. Lenders use dynamic incentives as in, for example, Bolton and Scharfstein (1990). Following much of the microfinance literature we focus attention on

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8For-profit status is as reported by MIX market. Profit margin is an accounting measure computed as Net Operating Income/Financial Revenue and can be positive even for non-profits. The “rural share” is computed from the self reported number of “rural” loans relative to all loans for the MFI, and is only available for the 39 institutions plotted. Note that this is not weighted by loan size which is typically not reported although it seems that where reported, rural loans are smaller.
individual liability (IL) or joint liability (JL) contracts. The IL contract is a standard debt contract that specifies a gross repayment $r$ if the borrower is able or willing to repay. Otherwise the borrower is considered to be in default, which is punished by termination of that borrower’s lending relationship. Under JL, pairs of borrowers receive loans together and unless both loans are repaid in full, both borrowers’ lending relationships are terminated. The lender can choose which type of contract is offered, namely, individual or joint liability and the interest rate, $r$, which is then fixed for the duration of the lending relationship.

3.1 Social Capital

It is well-known and understood how formal sanctions such as JL can induce borrowers to engage in a form of informal insurance. In order to prevent the group from being cut off from future finance, a borrower may willingly repay the loan of her partner whose project was unsuccessful. Besley and Coate (1995) also pointed out the role of social sanctions in enhancing such behavior, whereby borrowers put pressure on one another to provide such assistance.

Formally, we assume that in addition to the surplus generated within a borrowing group, pairs of borrowers possess some exogenous social capital. By standard logic, the threat of destruction of social capital may make it possible to sustain cooperation in various spheres of life where otherwise cooperation would not be incentive-compatible.

We want to explore how a for-profit lender with market power might take advantage of the borrowers’ social capital. Borrowers have an informational advantage over the lender (they can observe one another’s output), which enables them to write contingent contracts amongst themselves. If stronger social ties enable borrowers to better enforce such contracts, then the lender may be able to exploit this, essentially outsourcing part of the loan enforcement to the borrowers themselves.

There are a number of alternative models of social capital that we could use. We take a bilateral and parallel view of social ties whereby each pair of individuals in the village shares some social capital worth $S$ in discounted lifetime utility, which is specific to the pair. We think of this loosely as the value of a friendship. If individuals can credibly threaten to destroy their social capital, they can leverage it to enforce informal contracts amongst themselves. Here we explore how such threats might enforce an informal insurance arrangement within a borrowing group, such that one member assists their partner when their partner suffers a low output realization.

For the majority of the analysis we will assume that $S$ is exogenous, constant within the community and that each individual has a large number, say, $n$ friends or candidate borrowing partners, valued additively at $nS$. Thus each friendship that breaks up represents a loss of $S$. Typically we will only need to track the relationship with the current borrowing partner. $S$ can be interpreted in a number of ways. It could simply be the utility derived from a friendship, the lifetime value of interactions with that person. Alternatively, it could represent the value of some other informal contract(s) also dependent

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9In a related paper, de Quidt et al. (2011), we show how social sanctions can enable borrowers to mutually insure repayments even without an explicit joint liability clause. Such contracts are useful when there are states of the world in which a joint liability group would default but an individually liable borrower could repay (for example, one partner is successful but cannot afford to repay both loans). Due to the simple stochastic production function we use in this paper, such states do not occur, so no lender will choose to write contracts that induce implicit joint liability.
on that relationship, for example a reciprocal childcare or insurance arrangement. We discuss how $S$ might be endogenous, and variations on the other assumptions in section 4. There, we model social interactions as a repeated game that generates a surplus, and distinguish between coordination games and “opportunism” (prisoner’s dilemma) games.

To avoid hold-up type problems whereby one individual uses the threat of social sanction to extort resources from the other, we also assume a “no hold-up” social norm whereby the friendship is automatically destroyed (it is discovered that $S = 0$) if such behavior is attempted. After all, such friends are not worth having.

We note here that there are many other ways we could model social sanctions. For example it could be that deviants can be excluded from some village public good by a collective punishment or cut off from a transfer or trading network. Greif (1993) and Bloch et al. (2008) explore social punishments such as these. In fact, much of our analysis would be identical with community-level sanctions. We discuss similarities and differences in section 4.5. We favor the bilateral approach because it is simpler and focuses attention on within-group dynamics, but we are agnostic as to which form of sanction is more important.

In reducing social interactions to a single variable, $S$, we inevitably miss key channels through which social capital influences day-to-day life in developing countries. However, we believe the concept as we model it has empirical relevance. Our conjecture is that rural communities would tend to have higher $S$ than urban ones, since informal contracting and coping mechanisms play more important roles in rural communities. This paper highlights one way in which these can interact with microfinance.

It is true that the more friends one has, potentially the lower the value of any individual friend. Provided all borrowers have access to a large stock of friends, and $S$ depends on more than just the number of alternative friends available, this is not a problem for the analysis. Indeed, to the extent that we believe rural areas may be characterized by stronger social ties than urban areas, this may work in our favor. The individual mobility, population density and greater access to the machinery of formal contracting associated with urban living may indeed reduce the importance of any particular bilateral relationship.

3.2 Group dynamics

We assume that under JL borrowers form borrowing groups of two individuals $i \in \{1, 2\}$, which are dissolved upon default. The two borrowers in a group may be able to reach an agreement to informally insure one another’s repayments. While it is plausible that these informal agreements could stretch outside the boundaries of the group, for example encompassing a collection of groups, for simplicity we assume that this is not possible, perhaps because borrowers’ output realizations or borrowing and repayment behavior are only observable to other borrowers within their group.\footnote{A simple motivation could be that there are two possible independent investments in the village, say farming and rickshaw-driving, and all investments of the same type receive the same output realization each period. Then the optimal insurance group contains an equal number of each project.}

Informal insurance takes the following form. After the loan contract has been signed, the borrowers agree a repayment rule that specifies how much each borrower must repay contingent on the state, possibly enforced by a social sanction (destruction of $S$) that punishes certain deviations from the rule. Such a rule might be “both borrowers only
repay their own loans,” or “borrower 1 pays both loans whenever she can, otherwise both default.”

Once the rule has been agreed, repayment is a simultaneous-move game with no refunds. This means that if one borrower repays and the other does not under JL, the group is considered to be in default but the first borrower does not receive her installment back. For simplicity we impose five intuitive restrictions on the set of possible repayment rules, expressed as an Assumption below.

**Assumption 1 Repayment rules must:**

1. be feasible (a borrower cannot repay more than her output in any state);
2. be incentive compatible, given the loan contract and social sanctions;
3. be stationary (identical each period and can only condition on the current state);
4. divide the per-period expected surplus equally within the group;
5. maximize the joint surplus of the group, conditional on 1,2,3,4.

Requirements 1 and 2 are obvious, and 3 is a useful simplification that enables us to formulate the repayment game as a stationary one. Requirement 4 allows us to analyze the problem using a representative borrower since all incentive constraints will be symmetric.

Requirement 5 has three important implications. Firstly, social sanctions are never carried out in equilibrium. If a sanction is carried out, it must be because its threat was not sufficient to prevent the deviation that occurred. Then an alternative contract that does not punish this specific deviation would be welfare improving since it does not involve the destruction of social capital (clearly this changes if borrowers’ output is only imperfectly observed by their partners). Secondly, borrowers will never repay any loan if \( r \) exceeds the representative borrower’s discounted continuation value of the lending arrangement. This forms an important constraint on the lender that we term the “Group Incentive Constraint” (GIC), because it must be in the combined interests of the group to repay. Thirdly, if the continuation value does exceed \( r \), the borrowers will choose a rule that achieves the highest possible incentive-compatible repayment rate, since repayment always increases joint surplus. An example of a symmetric repayment rule that maximizes the repayment probability is “both repay when both projects succeed, \( i \in \{1,2\} \) repays both loans if only \( i \) succeeds, and defaults if both fail.” The rule will use the threat of social sanctions if such threats can improve the repayment probability, for example by making an insurance scheme incentive compatible.

### 3.3 Loan contracts

In this section we derive the key constraints when there is a single lender in the market. We then go on to derive the choice of contract and interest rate when that lender is a benevolent non-profit or a profit-maximizing monopolist.

Since there is a single lender, the dynamic incentives are strict: contract termination is permanent. In a later extension in section 4.2, we apply the framework to a model of competition between profit-maximizing lenders, where a borrower cut off by one lender might go on to obtain a loan from another.
Moreover, the borrowers’ participation constraint is always satisfied, since if the interest rate is too high the borrowers can simply default immediately. Thus the lender’s problem that we analyze is to choose the contract abiding by the various constraints on repayment.

### 3.3.1 Individual Liability

We first model behavior under IL, where borrowers do not insure one another. Suppose the borrower always repays her loan when successful, i.e. with probability $p$. If she defaults, the lender terminates her contract. Therefore her utility is:

$$\begin{align*}
V_{IL} &= p(R - r_{IL}) + \delta p V_{IL} \\
&= p(R - r_{IL}) \quad \frac{1}{1 - \delta p}.
\end{align*}$$

(1)

Throughout this paper we consider three key constraints on the lender. The first is the Feasibility Condition (FC), that is it must be feasible for the borrower(s) to make any repayments they are called upon to make. In this case, the FC is simply $R \geq r$.

The second constraint is the Group Incentive Constraint (GIC). For IL lending, the group is simply the individual borrower. This constraint requires that the continuation utility of the whole group ($\delta V_{IL}$) must exceed the cost of repayment this period, $r_{IL}$. Thus we require $\delta V_{IL} \geq r_{IL}$, or $\delta p R \geq r_{IL}$. From this equation we see an equivalent interpretation of this constraint: the expected net benefit of delaying a strategic default by one period must be non-negative. It should be clear from this interpretation that the GIC will be the same whatever type of contract is used. Notice also, that in the case of IL, the GIC is strictly tighter than the FC. We define $r_{GIC}$ as the interest rate at which the GIC binds:

$$r_{GIC} \equiv \delta p R.$$

The GIC is required for the dynamic incentives in the contract to induce repayment. Since this requires the lender to leave some surplus to the borrowers, their participation constraint $V_{IL} \geq 0$ will always be slack.

The third key constraint is that the highest repayment expected of the borrower is incentive compatible at the individual level (which implies incentive compatibility of all other possible repayments, since the borrowers are risk neutral). For simple IL the individual and group are identical and the borrower only ever repays $r_{IL}$, or defaults, so it must be incentive compatible to pay $r_{IL}$. Thus the IIC under IL is equivalent to the GIC.

Since the GIC is always the tightest constraint under IL, IL lending can earn non-negative profits provided expected repayment at the interest rate that binds the GIC, equal to $pr_{GIC}$ exceeds the opportunity cost of funds, $\omega$. To use IL lending as a benchmark, we retain this throughout as an assumption.

**Assumption 2** $\delta p^2 R \geq \omega$.

Notice that this assumption implies that the projects yield strictly positive social surplus, i.e. $pR > \omega$. 

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3.3.2 Joint liability

Under JL, the borrowers’ contracts are terminated unless the total repayment is $2r^{JL}$ in any period. Therefore both borrowers will default whenever they cannot jointly make this repayment.

We ignore any equilibria under which borrowers do not mutually insure one another’s repayments. The reason for this is that without mutual insurance, the maximum possible repayment probability is $p^2$, i.e. repayment only when both borrowers are successful, and therefore lower than under IL. Such a contract would not be used since IL can deliver higher borrower welfare (in the non-profit case) or higher profits (the monopoly case).\footnote{It is not possible that the interest rate could be higher to compensate for the lower repayment probability, since we know the GIC is identical.}

Instead suppose that borrowers are able to mutually insure, and therefore both loans are repaid whenever at least one borrower is successful. Thus the repayment probability is $1 - (1 - p)^2 = p(2 - p)$ which we define as:

$$q \equiv p(2 - p)$$

Since borrowers have the same per-period expected utility under our assumptions about the agreed repayment rule, their expected repayment must be $qr^{JL}$ each per period. Thus we have:

$$V^{JL} = pR - qr^{JL} + \delta q V^{JL} = \frac{pR - qr^{JL}}{1 - \delta q}. \quad (2)$$

Under this repayment rule, the highest payment a borrower is called upon to make is $2r^{JL}$, so the FC is $R \geq 2r^{JL}$.

Unless the opportunity cost of repaying $2r^{JL}$ is smaller than the joint benefit, $2\delta V^{JL}$, the borrowers will agree a repayment rule “default always”, since this rule maximizes the group’s total welfare. Thus for borrowers to be willing to repay at all, it must be that the GIC holds. This is $\delta V^{JL} \geq r^{JL}$ or $\delta pR \geq r^{JL}$, or simply $r_{GIC} \geq r^{JL}$, just as under simple IL. A simple intuition for the GIC here is that it prevents the lender playing off the borrowers against one another, using their social sanctions to extract repayments that are not beneficial for either borrower.

When considering incentive compatibility of the repayment rule, we note that if borrower $j$’s partner is repaying her own loan, then borrower $i$’s repayment will be incentive compatible by the GIC. Thus we only need to check incentive compatibility of repaying $2r^{JL}$ when the partner’s project fails, which guarantees that all lower payments are also incentive-compatible. If the repayment rule specifies this repayment, failure to make it will be punished by the destruction of $S$ if necessary. The IIC is

$$\delta(V^{JL} + S) \geq 2r^{JL}$$

or

$$\frac{\delta[pR + (1 - \delta q)S]}{2 - \delta q} \geq r^{JL}. \quad (3)$$

Let us denote the interest rate at which the JL IIC binds as:

$$r_{IIC}^{JL}(S) \equiv \frac{\delta[pR + (1 - \delta q)S]}{2 - \delta q}.$$
Unlike under IL, any one of the three constraints could potentially bind. To simplify the problem, we make the following assumption:

**Assumption 3** \( \delta p \leq \frac{1}{2} \).

For example, \( \delta \leq 0.7 \), and \( p \leq 0.7 \) would satisfy this assumption. This guarantees that the GIC is tighter than the FC, thus we can ignore the FC. This simplifying assumption is not required for our key results to go through (it simply determines whether we compare the FC and IIC or GIC and IIC). We discuss the implications of relaxing it in section 4.3.

**Observation 1** \( r_{JL}^{IIC}(0) < r_{GIC} \), so the IIC always binds in the neighborhood of \( S = 0 \).

JL can be used profitably provided that revenue when the tightest of GIC and IIC binds exceeds the opportunity cost of capital. In other words:

\[
q \min \{ r_{GIC}, r_{JL}^{IIC}(S) \} \geq \omega. \tag{4}
\]

Naturally, the higher is \( S \), the more likely this condition will hold, but it is feasible for it to hold even for \( S = 0 \). Now we define a threshold at which \( r_{GIC} \) and \( r_{JL}^{IIC}(S) \) intersect.

\[
\bar{S} \equiv pR.
\]

For all \( S < \bar{S}, r_{JL}^{IIC}(S) < r_{GIC} \), otherwise \( r_{JL}^{IIC}(S) \geq r_{GIC} \). Now if \( S \geq \bar{S} \), then \( 1 \) is guaranteed, since the maximum chargeable interest rate will be \( r_{GIC} = \delta pR \) and \( q\delta pR > \omega \) by Assumption 2 and \( q > p \). If \( S < \bar{S} \), we require \( qr_{JL}^{IIC}(S) \geq \omega \) or

\[
q \frac{\delta[pR + (1 - \delta q)S]}{2 - \delta q} \geq \omega. \tag{5}
\]

This allows us to derive a threshold value of \( S, \hat{S} \), such that for all \( S \geq \hat{S} \), JL lending can earn non-negative profits.

\[
\hat{S} \equiv \max \left\{ 0, \frac{(2 - \delta q)\omega - (2 - p)\delta p^2 R}{\delta q(1 - \delta q)} \right\}.
\]

Clearly, \( \hat{S} < \bar{S} \), since positive profits are guaranteed at \( \bar{S} \). As simple sufficient condition for \( \hat{S} = 0 \) (i.e. JL is always profitable) is

\[
1 + \delta p - 2\delta \leq 0. \tag{6}
\]

### 3.4 Non-profit lender

Now we characterize equilibrium contracts when there is only one benevolent non-profit lender who maximizes borrower utility subject to a zero profit constraint. This benchmark is identical to the standard “competitive” lender in most models of microcredit, with lenders earning zero profits in equilibrium (e.g. [Ghatak and Guinnane 1999]). We distinguish here between the non-profit lender and competitive lending by profit-maximizers because when we introduce competition to the model in section 4.2, we will show that the implications can be quite different.
We assume that the non-profit lender maximizes the surplus of the borrowers subject to a zero profit condition. To do this, we work out the zero-profit interest rate for IL and JL then compare to find the contract that achieves the highest borrower welfare.\footnote{This is by no means the only possible model of non-profit lending. In another paper close in spirit to ours, McIntosh and Wydick (2005) model a non-profit lender maximizing outreach, the number of clients reached. We could give many other candidate examples. We stick with this particular version since it is identical to the motivation of the lender in Rai and Sjöström (2004) and Bhole and Ogden (2010) and also by definition achieves the constrained first best.}

The zero-profit interest rates and borrower utilities under IL and JL are:

\[
\hat{r}^{IL} = \frac{\omega}{p} > \hat{r}^{JL} = \frac{\omega}{q} \\
\hat{V}^{IL} = \frac{pR - \omega}{1 - \delta p} < \hat{V}^{JL} = \frac{pR - \omega}{1 - \delta q}
\]

where the inequalities follow from \( q > p \). As \( \hat{V}^{JL} > \hat{V}^{IL} \) the nonprofit always wants to use JL if it can break even doing so. The following proposition is immediate:

**Proposition 1** With a nonprofit lender, borrowers are strictly better off with JL than IL. The lender offers JL at \( \hat{r}^{JL} \) whenever \( S \geq \hat{S} \), otherwise he offers IL at \( \hat{r}^{IL} \). Borrower utility under nonprofit lending is

\[
\hat{V} = \begin{cases} 
\hat{V}^{IL} & S < \hat{S} \\
\hat{V}^{JL} & S \geq \hat{S} 
\end{cases}
\]

The following observation is useful:

**Observation 2** \( \hat{V} \) is the maximum borrower utility from access to microfinance achievable in this framework.

That is, if JL lending is feasible then under a non-profit lender the borrower receives the highest expected payoff. This happens because JL relaxes the incentive constraint, and thereby raises the repayment rate for a given interest rate. Since under a non-profit lender all welfare gains accrue to borrowers in equilibrium, this leads to a lower interest rate.

### 3.5 For-profit monopolist lender

While the competitive lender maximizes borrower welfare subject to a zero profit condition, a lender with monopoly power chooses the interest rate and contract type to maximize profits. We assume that the lender maximizes current period profits from any given borrower.\footnote{As discussed in the introduction, we could imagine a lender who maximizes the discounted sum of profits earned from a given borrower. For example if borrower recruitment or replacement is costly or impossible, it is beneficial to retain existing borrowers. On the other hand, if the lender has limited lending capacity relative to demand then new borrowers may be easy to find to replace ejected borrowers. The second case motivates our assumption that the lender only maximizes current-period profits from each borrower, since this is the simplest. We relax this assumption in an extension in section 4.3.} Such a monopolist will charge the highest possible interest rate under each contract, subject to the tightest of the relevant constraints, GIC and IIC. He then
simply picks the contract that maximizes expected revenue. This is because the opportunity cost of funds is identical under both, and IL is guaranteed to earn non-negative profits by Assumption 2.

The monopolist’s interest rates and borrower utility are:

\[ \tilde{r}_{IL} = r_{GIC} \]

\[ \tilde{r}_{JL}(S) = \min \{ r_{GIC}, r_{JL}^{IIC}(S) \} \]

\[ \tilde{V}_{IL} = \frac{pR - p r_{GIC}}{1 - \delta p} = pR \]

\[ \tilde{V}_{JL}(S) = \frac{pR - q \min \{ r_{GIC}, r_{JL}^{IIC}(S) \} }{1 - \delta q} \geq pR. \]

When the IIC is binding the JL interest rate is increasing in \( S \), and this is always the case in the neighborhood of \( S = 0 \) by Observation 1. Figure 2 plots \( r_{GIC} \) and \( r_{JL}^{IIC} \) against \( S \).

![Figure 2: Interest rates](image)

The lender’s per-period, per-borrower profits for contract type \( i \), interest rate \( r \) and repayment probability \( \pi \) are

\[ \Pi^i = \pi r - \omega. \]

He uses JL contracts when revenue (\( \pi r \)) is higher under JL, i.e. when

\[ q r_{JL}^{IIC}(S) \geq p r_{IL}. \]  

(7)

It is clear that for \( S \geq \tilde{S} \), the lender will choose JL, since the interest rate is the same as under IL but the repayment probability is \( q > p \). Now consider \( S < \tilde{S} \). Since \( r_{JL}^{IIC}(S) \) is increasing in \( S \), we can derive a threshold \( \hat{S} \) such that the monopolist offers JL for all \( S \geq \hat{S} \). We can solve \( pr_{GIC} = q r_{JL}^{IIC}(\hat{S}) \) to find \( \hat{S} \) which gives:

\[ \hat{S} \equiv \max \left\{ 0, \frac{p^2 R (1 + \delta p - 2\delta)}{(2 - p)(1 - \delta q)} \right\}. \]

Condition (6), which was sufficient but not necessary for the JL to break even (and thus be offered by the nonprofit) for all \( S \geq 0 \), is necessary and sufficient for the monopolist to offer JL for all \( S \geq 0 \). This is because of the following observation.
Observation 3 \( \hat{S} \geq \bar{S} \), with the relation strict if \( \delta p^2 R > \omega \) and \( p > \delta q \). Therefore, the monopolist offers JL over a (weakly) smaller range of \( S \) than the non-profit lender.

This is the source of inefficiency in the model. The monopolist is unable to extract sufficiently high rents when \( S \) is small, and so offers IL, when JL would be welfare-enhancing. The effect is particularly stark here, where the lender puts no weight on future profits from this borrower (which benefit from the higher repayment probability under JL), but as we shall see in section 4.3, the result holds also for non-myopic lenders.

Borrower utility under the monopolist therefore depends on \( S \) in two ways. Firstly, higher \( S \) makes the lender more likely to use JL, and secondly, when the lender is using JL, and the IIC is binding, higher \( S \) leads to a higher interest rate. We summarize the results in the following proposition.

**Proposition 2** With a monopolist lender, borrowers are weakly better off with JL than IL, strictly if \( S < \tilde{S} \). The lender offers JL at \( \tilde{r}^{JL}(S) \) whenever \( S \geq \tilde{S} \), otherwise he offers IL at \( \tilde{r}^{IL} = r_{GIC} \). Borrower utility with a monopolist lender is

\[
\tilde{V}(S) = \begin{cases} 
\tilde{V}^{IL} & S < \tilde{S} \\
\tilde{V}^{JL}(S) & S \geq \tilde{S}
\end{cases}
\]

**Proof.** We have already shown the lender's contract choice rule and threshold \( \tilde{S} \), which gives us \( \tilde{V}(S) \) immediately. The claim that borrowers are better off under JL follows from \( \tilde{V}^{JL} = pR \) and \( \tilde{V}^{JL}(S) \geq pR \) with equality at \( S = \tilde{S} \).

This result is somewhat surprising and deserves some discussion. Under IL the lender has no ability to exploit the borrowers’ social capital, while he does under JL. He can choose the interest rate to induce the borrowers to mutually insure and repay more often. He can increase the interest rate as their social capital increases. However, he will never make them worse off under JL than IL. The lender must always abide by the GIC, which binds under IL, but under JL the IIC may be tighter, reducing the interest rate and making the borrowers better off.

This result is relevant to policy, because it raises questions about what exactly is “exploitative” behavior by the lender. The use of group lending to leverage borrowers’ social capital has been criticized and hailed as an important motivation for lenders to move toward individual loans. In our model, a monopolist using joint liability is bad for borrowers, but he is even worse with individual liability.

### 3.6 Profits and borrower welfare

Returning to the benchmark model, we are now able to analyze the welfare of borrowers and the lender’s profits for all values of \( S \) under IL and JL. For the purposes of this discussion we will distinguish between \( V \), the borrowers’ utility derived purely from access to microfinance lending, and their total welfare that also takes into account the direct payoffs from social capital.\(^{15}\)

\(^{14}\) In fact, borrowers will be better off under JL for a large class of stochastic production functions including, for example, the uniform and exponential distributions. A proof is given in the Appendix for the interested reader.

\(^{15}\) Whenever we consider borrower welfare we will only count the utility from social capital of the specific pair in the borrowing group, and not their other social interactions. This is because the results thus far do not rely on the borrowers having more than one “friend”. Assuming “many friends” as we in fact do throughout simply strengthens the result in Proposition 3.
We have already established that when the lender is a non-profit, borrower utility is independent of \( S \) except for a jump at \( \tilde{S} \) when the lender is able to offer JL. With a monopolist, for \( \tilde{S} \leq S < \hat{S} \) the lender offers JL and the IIC is binding, so \( r \) is increasing and \( V \) decreasing in \( S \). For \( S < \tilde{S} \) the lender offers IL and \( V \) is independent of \( S \). For \( S \geq \hat{S} \) the lender offers JL but the GIC binds so again \( V \) is independent of \( S \).

The lender’s profits follow a similar pattern. Under IL, profits do not depend on \( S \). Under JL, profits are strictly increasing in \( S \) for all \( S < \hat{S} \), and constant for \( S \geq \hat{S} \). Profits are strictly higher under JL for \( S > \tilde{S} \), otherwise they are higher under IL. We collect these observations into Figure 3 below. Panel 1 shows an example where \( \tilde{S} = 0 \), so the lender always offers JL contracts. In Panel 2, \( \tilde{S} > 0 \) so the borrowers receive IL for some values of \( S \).

Summarizing, we see that the monopolist can extract weakly greater rents as \( S \) increases. Meanwhile, the borrowers have the most to gain from microfinance when \( S \) is close to but greater than \( \hat{S} \). Communities with lower social capital are excluded from the full benefits of JL, while those with higher social capital end up paying higher interest rates.

However, the story is more positive when we consider borrower welfare as a whole. Focusing on the utility the borrower derives purely from the specific friendship in question, i.e. the social capital \( S \) plus the borrowing relationship built upon that social capital, we obtain the following proposition.

**Proposition 3** With a monopolist lender who exploits social capital, borrower welfare, measured as \( \hat{W}(S) = \hat{V}(S) + S \) is strictly increasing in \( S \).

**Proof.** For \( S < \tilde{S} \) or \( S \geq \hat{S} \), \( \hat{W}'(S) = 1 \). \( \hat{W}(S) \) increases discontinuously at \( \tilde{S} \). For \( S \in [\tilde{S}, \hat{S}] \), \( \hat{W}'(S) = 1 - \frac{\delta q}{1 - \delta q} > 0 \). ■

This proposition shows that although the lender “taxes” the benefits of social capital, he cannot tax away all of them and hence more social capital is always beneficial to the borrowers.

Now we compare borrower welfare under non-profit and monopoly for-profit lending, i.e. \( \hat{W}(S) - \hat{W}(S') = \hat{V}(S) - \hat{V}(S') \). We define this object as \( \triangle V(S) \). We obtain the following

\[
\triangle V(S) = \begin{cases} 
\hat{V}^{IL} - \hat{V}^{IL} = \frac{\delta p R - \omega}{1 - \delta p} & S \in [0, \tilde{S}) \\
\hat{V}^{JL} - \hat{V}^{JL} = \frac{\delta p R - \omega}{1 - \delta q} & S \in [\tilde{S}, \hat{S}) \\
\hat{V}^{JL} - \hat{V}^{JL}(\hat{S}) = \frac{(\delta p R - \omega)(1 - \delta q)(1 - \delta q)}{1 - \delta q} & S \geq \hat{S} 
\end{cases}
\]

The intervals \([0, \tilde{S})\) and \([\tilde{S}, \hat{S})\) may be empty, and if the latter is empty, the former is guaranteed to be. Note that \( \triangle V(S) \) is non-monotonic: it increases discretely to a maximum at \( \hat{S} \), decreases discretely at \( \tilde{S} \), then increases gradually until \( \hat{S} \) where it reaches its maximum again. It is straightforward to show that \( \triangle V(S) \) is minimized at \( S \in [0, \tilde{S}) \). Otherwise, we would have \( \triangle V(\hat{S}) - \triangle V(0) < 0 \). But this can be written as \( \hat{V}^{JL} - \hat{V}^{IL} + \hat{V}^{IL} - \hat{V}^{JL}(S) \). Simplifying, and using the fact that \( p_{GIC}^{JL} = q_{IL}^{JL}(\hat{S}) = \delta p R \) we obtain \( \frac{\delta p (1 - p)}{(1 - \delta p)(1 - \delta q)} \), which is nonnegative by Assumption 2. We graph \( \triangle V(S) \) in Figure 4.
3.7 Efficiency

Lastly we turn to aggregate (borrower and lender) welfare from microfinance. For comparability we discount the lender’s profits according to the borrowers’ discount rate, so the net present value of a profits from a contract with repayment probability $\pi$ is $\Pi_\delta(S) = \frac{\pi r - \omega}{1 - \delta} - \delta \pi$.

**Definition 1** An efficient contract maximizes $V(S) + \Pi_\delta(S)$.

Aggregate welfare under non-profit lending is trivially $\hat{V}$ since the lender earns zero profits, and as already noted is the maximum attainable subject to the zero profit condition. Therefore the aggregate welfare loss under monopoly lending is $\Delta V(S) - \Pi_\delta(S)$. We obtain:

$$\Delta V(S) - \Pi_\delta(S) = \begin{cases} 0 & S \in [0, \hat{S}) \\ \frac{\delta p (1 - p) (p R - \omega)}{(1 - \delta p) (1 - \delta q)} & S \in [\hat{S}, \tilde{S}) \\ 0 & S \geq \tilde{S} \end{cases}$$

**Proposition 4** Monopoly for-profit lending is inefficient when $S \in [\hat{S}, \tilde{S})$. In this interval the non-profit offers JL but the for-profit monopolist offers IL since it earns him higher profits.
4 Extensions

4.1 Endogenous social capital

The analysis so far has assumed that $S$, the value of social ties that can credibly be broken as a social sanction, is exogenous. In this section we discuss a simple way to model $S$ endogenously. Since social capital plays no role under IL, for simplicity in this section we will assume that equation (6) is satisfied, so the lender always offers JL contracts ($\hat{S} = \tilde{S} = 0$).

Suppose each period, each pair of friends plays a “social game” with payoffs according to the following payoff matrix.

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$s, s$</td>
<td>$as, bs$</td>
</tr>
<tr>
<td>D</td>
<td>$bs, as$</td>
<td>$0, 0$</td>
</tr>
</tbody>
</table>

Table 2: Social game payoffs

We assume $a + b < 2$, so $(C, C)$ is Pareto dominant, and that $a \leq 0$ so that $(D, D)$ is always a stage-game Nash equilibrium. Therefore it is always a credible threat to switch to $(D, D)$ in response to some deviation in the social game or in the repayment game. Specifically, we assume that borrowers use trigger strategies: if any player deviated in the social game or failed to make a repayment specified in the repayment rule, they switch to $(D, D)$ in all subsequent periods.

We distinguish between two ranges for $b$. If $b \leq 1$, a *coordination game*, there are two pure strategy Nash equilibria, $(C, C)$ and $(D, D)$. If $b > 1$, a prisoner’s dilemma or *opportunism game*, there is a unique stage-game Nash equilibrium $(D, D)$.

**Definition 2** Social capital is the discounted lifetime utility of playing $(C, C)$ in the social game in every period, i.e. $S = \frac{s}{1 - s}$.

We will refer to pairs who expect to play $(C, C)$ in future periods as “friends”, and those who do not (who effectively have $S = 0$) as “non-friends”.

Figure 4: The loss of borrower welfare under monopoly lending compared with non-profit lending
The analysis thus far is thus motivated by assuming \((C, C)\) can be sustained in sub-game perfect equilibrium (SPE) under trigger strategies. This is always true in the coordination game, and holds in the opportunism game for \(b \leq \frac{1}{1-\delta}\).

**Definition 3** Social capital is sustainable if \(b \leq \frac{1}{1-\delta}\).

### 4.1.1 Timing

Suppose social capital is sustainable. A key consideration for the interaction of the social and repayment games is the timing of output realizations and actions in the games each period. There are two basic setups we could consider, distinguishing whether the social game is played before or after output is realized.

First suppose the social game is a cooperation game. By the design of the trigger strategies, whatever the timing it is always a best response to play \(C\) in the current period if \((C, C)\) was played last period, so the social and repayment games will not interact in the current period. Thus we focus attention on the opportunism game.

If the social game is played before outputs are realized, the borrowers cannot condition on output realizations, so this is the simplest case. If social capital is sustainable, \((C, C)\) was played in the last period and nobody deviated from the repayment rule, they will play \(C\). Otherwise, they switch to the \((D, D)\) equilibrium. Thus deviations are punished in the subsequent period by loss of \(S\), exactly as modeled so far.

Now suppose the social game is played after outputs are realized. In this case, a borrower might condition their play in the social game on the state. A successful borrower with an unsuccessful partner might be tempted to default on her insurance obligations and ensure the subsequent social sanction (switching to \((D, D)\) in the social game in future periods). Since the social game is an opportunism game she would simultaneously or preemptively play \(D\) in the social game this period. \(S\) is destroyed either way, but by playing \(D\) this period she gains \((b - 1)s > 0\).

It is easy to formalize this logic. When both partners are successful or unsuccessful nothing changes: the GIC guarantees repayment in the former, so social sanctions do not need to be threatened, and default without social sanctions is guaranteed in the latter. Suppose then that one partner is successful and the other unsuccessful, and the successful one expects her partner to play \(C\) in the social game. It is irrelevant whether the social game is played before or simultaneously with the repayment game, since her partner must play “default” in the repayment game. We know the successful partner will never play \((D, \text{repay})\) since social capital is sustainable. She plays \((C, \text{repay})\) instead of \((D, \text{default})\) if

\[
\delta(V + S) - 2r + s \geq bs
\]

This is equivalent to the original IIC, replacing \(S\) with “effective social capital”

\[
E \equiv S\left[1 - \frac{(1 - \delta)(b - 1)}{\delta}\right].
\]

It is easy to show using \(1 < b \leq \frac{1}{1-\delta}\) that \(E \in [0, S]\).

---

\(^{16}\)We ignore the possibility that the social game is played after the repayment game, i.e. at the end of the period, since this corresponds with appropriate discounting to the beginning-of-period case.

\(^{17}\)Since she knows the play in all subsequent periods will be \((D, D)\), it is no longer a best response to play \(C\) this period.
The modified IIC using $E$ in place of $S$ must be satisfied for the borrowers to mutually insure, in other words, $r \leq r_{IIC}^{IL}(E)$. Otherwise, the borrowers will agree not to use social sanctions at all, since insurance would not be incentive compatible anyway. Thus, if the social game is played after outputs are realized, we can simply rewrite the model replacing $S$ with $E$ in all of the constraints.

**Proposition 5** When the social game is played after output is realized, the JL IIC must be modified to reflect the weaker “effective” social sanctions available. As a result, the lender earns weakly lower profits and the borrowers weakly higher utility under JL. However, if $E < \tilde{S}$, the lender will not offer JL contracts at all.

For the remainder of the analysis we will assume the order of play within each period is as follows, and thus will not need to use the notion of effective social capital.

1. Social game is played.
2. Outputs are realized.
3. Repayment game is played.
4. Contracts terminated and social sanctions carried out according to the contract terms and repayment rule.

### 4.1.2 Creation of social capital

There are two obvious ways in which lending might lead to creation of social capital, in the sense that borrowers who were playing $D$ in the social game switch to playing $C$. Firstly, lending could act as a coordinating device. It could be that the social game is a pure coordination one, but until the borrowers are grouped together in a lending group they are not able to coordinate on playing $(C, C)$. This seems to fit well with the findings of Feigenberg et al. (2011), where groups that meet more frequently seem to build more social capital.

The second, more interesting channel for social capital creation is that borrowers may have an incentive to create $S$ in order to enforce informal insurance in a joint liability group. To see this, suppose initially social capital was unsustainable. In other words, $b > \frac{1}{\delta^2}$, so trigger strategies are not sufficient to enforce the Pareto optimal equilibrium in the social game.

Recall that provided the lender charges $r \leq r_{IIC}^{IL}(0)$ under JL the borrowers would be able to sustain mutual insurance even without social capital. We have assumed that equation (6) is satisfied, which means that the monopolist prefers this contract to IL. However, by increasing the interest rate, such that mutual insurance would not be possible without social capital, he might induce the borrowers to create social capital for the duration of the borrowing agreement. Cooperation incentives in the social game are improved once the loan contract is agreed, because now if a borrower deviates in the social game – destroying the group’s social capital – they will no longer be able to sustain mutual insurance and therefore the repayment probability will decrease.

To analyze whether the lender can induce the borrowers to create social capital, we need to add a little notation. Suppose the interest rate has been fixed at $r$ under JL. We denote by $V(r, q)$ the value function of a borrower in a group able to informally insure,
repaying with probability $q$, and by $V(r, p^2)$ if not, therefore repaying with probability $p^2$.

Consider a borrowing group for whom the interest rate has been fixed at $r > r_{IIC}(0)$. The borrowers know that their partner will play $D$ for sure in all periods after the borrowing group breaks up (no trigger strategy can sustain cooperation), so the unique equilibrium is $(D, D)$ from this point on. However, let us suppose that for the duration of the borrowing relationship, borrowers can agree to play $(C, C)$, switching to $(D, D)$ following a deviation as usual. We will now check under what conditions such an agreement would be self supporting. If the borrowers are now able to mutually insure using their new social capital, repaying with probability $q$, the social game has expected payoff $\frac{s_1 - \delta q}{1 - \delta q}$ in net present value.

Assume that borrowers are able to mutually insure with their new social capital. If a borrower plays $C$ in the social game, her utility is $V(r, q) + \frac{s_1 - \delta q}{1 - \delta q}$ while if she plays $D$, earning instantaneous payoff $bs$ and continuation value zero in the social game, her utility is $V(r, p^2) + bs$, since once the social capital is destroyed, mutual insurance will no longer be possible.\(^{18}\) She prefers to play $C$ if

$$\frac{pR - qr}{1 - \delta q} - \frac{pR - p^2r}{1 - \delta p^2} \geq s \left( b - \frac{1}{1 - \delta q} \right).$$

Rearranging, we obtain:

$$\delta pR - s \left( b - \frac{1}{1 - \delta q} \right) \left( 1 - \delta p^2 \right) \left( 1 - \delta q \right) \geq r$$

This equation defines a new constraint on the lender: the interest rate must be sufficiently low to ensure borrowers will gain enough from mutual insurance to be willing to cooperate in the social game. We term this the Social Incentive Constraint (SIC) and define

$$r_{SIC}(s) \equiv \delta pR - s \left( b - \frac{1}{1 - \delta q} \right) \left( 1 - \delta p^2 \right) \left( 1 - \delta q \right) \frac{2p(1 - p)}{2p(1 - p)}.$$

Note that $r_{SIC}(s) \leq r_{GIC}$ since by assumption, $b > \frac{1}{1 - \delta}$ which implies $b > \frac{1}{1 - \delta q}$. Therefore $r_{SIC}$ is decreasing in $s \left( b - \frac{1}{1 - \delta q} \right)$, which is the borrower’s gain from deviating in the social game.

Now that we have a condition for the borrowers to cooperate in the social game, we need to check that the social capital generated is sufficient to sustain mutual insurance. But this of course is our standard IIC, adjusted to reflect that social capital is now worth $\frac{s_1 - \delta q}{1 - \delta q}$. We shall call this the Social IIC (SIIC) and define

$$r_{SIIC}(s) \equiv \frac{\delta [pR + s]}{2 - \delta q}.$$

If the monopolist wishes to induce the borrowers to mutually insure by creating social capital, he will set $r = \min\{r_{SIIC}(s), r_{SIC}(s)\}$. Alternatively, he can simply set $r = r_{IIC}(0)\(^{19}\)$ and the borrowers will mutually insure, but will not be able to create

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\(^{18}\)Note the key difference with borrower welfare $W(S)$ in section 3.6 is that here the social capital is always lost when the group breaks up.

\(^{19}\)We can safely ignore the GIC since $r_{IIC}(0) < r_{GIC}$. 

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social capital if this interest rate is larger than \( \min\{r_{SIIC}(s), r_{SIC}(s)\} \). Since he achieves repayment probability \( q \) either way, he simply picks the highest interest rate, i.e.

\[
\hat{r}^{JL}(s) = \max\{r_{IIC}^{JL}(0), \min\{r_{SIIC}(s), r_{SIC}(s)\}\}.
\]

Note that \( r_{SIIC}(s) > r_{IIC}^{JL}(0) \) for all \( s > 0 \). Therefore it is easy to see that the lender creates social capital if and only if \( r_{SIC}(s) > r_{IIC}^{JL}(0) \). This reduces to:

\[
\frac{2\delta p^2 R(1-p)}{(1-\delta p^2)(2-\delta q)} > s \left( b - \frac{1}{1-\delta q} \right).
\]

It is not surprising, given the results proved so far, that when the monopolist acts to create social capital it makes the borrowers strictly better off in the process. To see this, we compare borrower welfare with and without social capital creation. Without social capital creation, borrower welfare is

\[
V(\hat{r}^{JL}(0), q) = \frac{2pR}{2} - \delta q
\]

With social capital creation, borrower welfare is

\[
W = pR - q \min\{r_{SIIC}(s), r_{SIC}(s)\} + s\frac{1}{1-\delta q}.
\]

Substituting the definition of \( r_{SIIC}(s) \) we obtain

\[
W \geq \frac{2(pR + s)}{2-\delta q}
\]

with equality when \( r_{SIIC}(s) \leq r_{SIC}(s) \). This is strictly greater than \( V(\hat{r}^{JL}(0), q) \) for all \( s > 0 \), so when the lender increases the interest rate to induce borrowers to create social capital, he actually makes them strictly better off.

**Proposition 6** The monopolist creates social capital if and only if the borrower’s gain from deviating in the social game, \( s \left( b - \frac{1}{1-\delta q} \right) \), is strictly smaller than \( \frac{2\delta p^2 R(1-p)}{(1-\delta p^2)(2-\delta q)} \). Doing so always makes the borrowers strictly better off compared with no social capital creation.

Now suppose the lender is a non-profit. If he charges \( \hat{r}^{JL} = \frac{\omega}{q} \) the borrowers will not create social capital. This is because \( \hat{r}^{JL} \leq r_{IIC}^{JL}(0) \) since we have assumed [6] holds. In order for the borrowers to have an incentive to cooperate in the social game it must be that \( r > r_{IIC}^{JL}(0) \), otherwise there is no fall in \( V \) when social capital is destroyed. Therefore, if \( \min\{r_{SIIC}(s), r_{SIC}(s)\} > r_{IIC}^{JL}(0) \), the nonprofit charges \( r_{IIC}^{JL}(0) + \epsilon \) for some small \( \epsilon \), to induce the borrowers to create social capital. He maximizes borrower welfare this way, but must make strictly positive profits in the process!

In this discussion, social capital is created only within the borrowing group and only lasts for the duration of the borrowing relationship, since it is unsustainable without a borrowing relationship. A simple extension would be to assume that social capital can “stick” in the following sense: we could assume that \( b \) decreases over time as \((C,C)\) is played, that is the social game becomes closer to the coordination version rather than the opportunism version. A motivation would be that borrowers “learn to cooperate” in some sense. This effect still might not be enough to induce agents to begin to cooperate in the social game independently, but a few periods as borrowers might be sufficient for the social capital to stick and outlast the borrowing relationship. Similarly, if these effect can spill over to other relationships, social capital could be strengthened outside the group as well.
4.1.3 Destruction of social capital

It seems natural that since the lender “taxes” the borrowers’ social capital, he might undermine it in some way. It turns out that to obtain such an outcome requires quite strong assumptions that we believe to be rather implausible. Since the borrowers must have social capital to destroy, we assume that social capital is sustainable (in the absence of a lender). Since $V(S) + S$ is strictly increasing by Proposition 3, borrowers will only ever take actions to undermine social capital when they have short-term incentives to do so, i.e. in an opportunism social game.

The non-profit lender will never undermine social capital, since he only ever leverages $S$ to make the borrowers better off by switching from IL to JL when $S \geq \bar{S}$. For the monopolist to undermine social capital, we would need to assume that the lender observes social capital at the group level and that borrowers learn of the lender’s entry to the community in advance of the lender committing to $r$. The lender needs to observe $S$ at the pair level, otherwise he cannot condition on it so the borrowers would never have an incentive to take actions to undermine it. The borrowers would need to learn of entry before interest rates are set or they would not be able to take any action to influence the interest rate. Even with these conditions, we do not feel that undermining is particularly intuitive. Firstly, we note that such an outcome does not benefit the lender, since his interest rate will fall. He can avoid undermining social capital by deliberately ignoring his information on the groups’ social capital, setting interest rates on at the community level for example. Therefore we feel the social capital creation story is the more plausible one in our context.

However, if we extend the model then it is possible to show an intuitive way in which a monopolist’s taxing of the benefits of social capital can lead to under-investment on the part of borrowers. Suppose $x_i \in [0, X]$ is a one-shot investment decision that costs $x_i$ to player $i$ and that is taken at the beginning of the repeated social interaction game. Let $X = x_1 + x_2$ and let the cooperation payoff of each player in all subsequent periods be positively affected by the sum of these investments: $s(X)$ is strictly increasing and strictly concave in $X$. It follows that $S$ is increasing in $X$ as well. That is, as far as the two borrowers are concerned, the contributions yield benefits that have a public goods aspect. To abstract from complications, we assume that the borrowers can choose these investments cooperatively (and symmetrically) and do so to maximize joint surplus. In the absence of the possibility of a lender entering the scene, borrower’s will choose an investment level $X^*$ given by:

$$2S'(X^*) = 1.$$ 

Now, suppose we allow the monopolist lender to in the first period but after the investment decisions are undertaken. Suppose for simplicity that $S(X) < \bar{S}, \forall X$. Since the lender increases his interest rate in response to higher $S$, the borrowers will choose an investment level $\hat{X}$ such that:

$$2 \left[ 1 - \frac{\delta q}{2 - \delta q} \right] S'(\hat{X}) = 1.$$ 

Clearly, $\hat{X} < X^*$. However, as before if the lender anticipates this response and can act like a Stackelberg leader by pre-committing to an interest rate, the borrowers’ choice of

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20 We also need to relax the “many equivalent friends” assumption in favor of some kind of sparse friendship group. A sparse friendship group is needed because the lender can always induce borrowers to bring their most valued friend to the group, thus if friendship groups are dense the borrowers would have to undermine a large number of friendships to influence the interest rate.
X can no longer influence the interest rate and so there will not be underinvestment. In fact, the lender might be able to induce overinvestment since as argued before social capital is also valuable for sustaining mutual insurance.

4.1.4 Lender investments in social capital

Suppose now that the lender could make a one-off costly investment that increases $s$. Here we give a brief non-formal discussion of what this might imply. When discussing costly investments in social capital we have in mind the results of Feigenberg et al. (2011) who show that increasing the meeting frequency of borrowing groups seems to lead to social capital creation: borrowers interact more on an informal basis and are more likely to help one another when in difficulties. Increasing meeting frequency would require a change in our modeling framework, but we can imagine other programs that might improve ties between borrowers.

If the lender is a nonprofit and able to recover his costs through higher interest rates he would invest in $s$ since he maximizes borrower welfare. However, he may be constrained from making the optimal investment from the borrowers’ perspective since he cannot collect payments from the borrowers that violate the relevant incentive and feasibility conditions.

Meanwhile, provided $s < \bar{s}$, the monopolist may also make costly investments in $s$. Doing so could enable him to switch from IL to JL lending and increase his interest rate under JL. From the borrowers’ perspective, however, he is unlikely to make the optimal investment since he only considers the effect on his profits and does not internalize the effect on borrower welfare.

4.2 Modeling competition

In this extension we apply the core model to modeling competition in the lending market, with a setup analogous to Shapiro and Stiglitz (1984). In our simple setup competition affects the key constraints through the probability of a defaulting borrower being able to obtain a loan from another lender. For example, if this probability is high then it is clear that there are no dynamic incentives to repay a loan (the GIC will always be violated). The analysis thus far and in much of the literature assumes this probability is zero, so default leads to full exclusion from future borrowing from any lender. We take the opposite approach here. A defaulting borrower has her contract terminated but is free to rematch with another lender in a subsequent period.

There exists a unit mass of potential borrower pairs and mass $l$ of lenders. Each period, a borrower may be “matched”, borrowing from a single lender, or “unmatched”, waiting to find a willing lender.

We could model lenders and communication between lenders in a number of ways. To avoid the complication of tracking the individual histories of borrowers, we shall focus on two extreme cases. On the one hand, there could be a strong credit bureau or a single large lender who can track borrowers’ histories and exclude those who have previously defaulted. This coincides with the case analyzed so far. Alternatively, we could imagine a large number of small lenders who do not communicate, or alternatively that lenders are “forgetful”, and might approve a loan to someone who previously defaulted.

Consider the latter case, which we shall refer to as “competition”. Formally, there is a set of lenders, each of whom have one or more “branches”, capable of serving two
IL borrowers or one JL pair. For simplicity, we use a pair of borrowers as the basic unit of population, and we assume this population has mass 1. We denote by \( l \) total capacity or scale of the industry in terms of pairs of borrowers. Critically, there is no communication of borrowers’ histories between lenders or between branches of a given lender. As a result, a borrower who defaults at one branch may go on to borrow from another branch shortly afterwards. If \( l < 1 \) there will be rationing in the credit market: not all borrowers can obtain a loan, if \( l > 1 \) then some lenders will have excess capacity.

Every borrower has a large number of potential partners, with each of whom they share social capital \( S \). Any borrower who breaks a social tie (losing utility worth \( S \)) can always find another borrowing partner with the same social capital. Each period, branches with available capacity simultaneously and publicly post a contract type and interest rate, then unmatched borrowers are uniformly randomly matched to lenders, starting with the most favorable offers, until all capacity is filled or all unmatched borrowers are matched. Since branches are atomistic the probability of a borrower rematching to a branch at which she previously defaulted is zero, so an unmatched borrower’s matching probability does not depend on her history. Loans are then made at the agreed interest rate and contract type, the repayment game is played, and any defaulters have their contract terminated, rejoining the pool of unmatched borrowers.

**Observation 4** There will always be credit rationing in equilibrium.

Suppose not, i.e. suppose all willing borrowers can obtain a loan. This eliminates all dynamic repayment incentives (the GIC cannot be satisfied), so all borrowers default. This cannot be an equilibrium since lenders or branches are better off exiting the market. Therefore we only need to consider \( l < 1 \).

For an arbitrary loan contract which delivers repayment probability \( \pi \), fraction \( 1 - \pi \) matched borrowers will default each period and have their contracts terminated. Therefore, each period \( l \) pairs receive a loan, then \( l(1 - \pi) \) will be terminated, leaving a total pool of \( l(1 - \pi) + 1 - l = 1 - l\pi \) unmatched pairs. Each unmatched pair then has probability \( \frac{l(1-\pi)}{1-l\pi} \) of obtaining one of the now-vacant spots next period.

Since there is rationing, every branch will operate at full capacity every period, irrespective of the contract offered (subject of course to the borrowers’ participation constraint which we know will always be slack). Therefore each branch can act as a local monopolist, offering the more profitable of IL and JL at the highest \( r \) that satisfies the (modified) GIC and IIC. Since the borrowers’ dynamic incentives are weakened by the possibility of future rematching, the contract choice and interest rate will differ from the full monopoly case.

Denote by \( V \) and \( U \) the lifetime expected utility of a matched and unmatched borrower respectively. An unmatched borrower becomes matched this period with the appropriate probability, or receives 0 and has to wait until next period for another chance to match again. We have:

\[
U = \frac{l(1-\pi)}{1-l\pi} V + \left[ 1 - \frac{l(1-\pi)}{1-l\pi} \right] \delta U
\]

\[
= \frac{l(1-\pi)}{l(1-\pi) + (1-\delta)(1-l)} V < V.
\]

\footnote{The industry could be a single lender with mass \( l \) branches, mass \( l \) lenders with one branch each, or something else in between.}
Thus, as in Shapiro and Stiglitz (1984), we have equilibrium credit rationing as a borrower motivating device. We define \( \phi(l, \pi) \equiv \frac{l(1-\pi)}{(1-\pi)+(1-\delta)(1-l)} \), \( \phi_l > 0, \phi_\pi < 0 \). Typically we will denote this object simply by \( \phi \) to reduce notational clutter. Thus we have:

\[
V = pR - \pi r + \delta[\pi V + (1 - \pi)U]
\]

and using \( U = \phi V \):

\[
V = \frac{pR - \pi r}{1 - \delta[\pi + (1 - \pi)\phi]}.
\]

4.2.1 Modifying the constraints

Now we turn attention to the key constraints to enable us to solve the lender’s problem. The FCs are unchanged. The GIC is:

\[
\delta V - r \geq \delta \phi V
\]

It is now clear that when \( l = 1 \), and thus \( \phi = 1 \), the GIC cannot be satisfied for any positive interest rate. Substituting for \( \phi \) and rearranging, we can find the interest rates at which the GIC binds under IL and JL respectively:

\[
r_{\text{GIC}}^{\text{IL}}(l) \equiv \frac{1 - l}{1 - lp} \delta pR
\]

\[
r_{\text{GIC}}^{\text{JL}}(l) \equiv \frac{1 - l}{1 - lq} \delta pR.
\]

Since the repayment probability is larger under JL than under IL, the GIC is more slack under JL than IL for a given \( l \), whereas previously the conditions coincided exactly. The reason is that for a given \( l \) there will be fewer unmatched available slots each period under JL (less churn), so \( \phi \) is smaller under JL for a given \( l \).

The IIC for IL is the same as the GIC. The IIC for JL is now:

\[
\delta[V + S] - 2r \geq \delta \phi(l, q)V
\]

which we can solve for the interest rate at which the IIC binds:

\[
r_{\text{IIC}}^{\text{IL}}(S, l) \equiv \frac{\delta[(1 - l)pR + [l(1 - q) + (1 - l)(1 - \delta q)]S]}{2l(1 - q) + (2 - \delta q)(1 - l)}.
\]

The JL GIC and IIC intersect at:

\[
\bar{l}(S) \equiv \min \left\{ 0, \frac{pR - S}{pR - qS} \right\} \leq 1.
\]

For all \( l < \bar{l}(S) \), the IIC is binding, otherwise the GIC binds.

4.2.2 Modifying the main assumptions

Thus far we have assumed \( l \) fixed. Now we allow for free entry, either through existing lenders opening new branches or new lenders entering. Since there is no communication between branches and Bertrand-style price competition both cases are equivalent. Under competition, we can no longer guarantee for all \( l \) that IL will be profitable, since the GIC
is tighter as \( l \) increases so there is always a threshold \( l^{IL*} \) above which IL ceases to be profitable.

Assumption (3) implied that the GIC was tighter than the FC under JL, which requires \( \delta p \leq \frac{1}{2} \). Since the GIC is weakly tighter under competition, we can continue to ignore the FC.

By the same arguments as before each branch will charge \( \hat{r}^{IL}(l) = r^{IL}_{GIC}(l) \) under IL and 
\[
\hat{r}^{JL}(S, l) = \min\{r^{JL}_{GIC}(l), r^{JL}_{IIC}(S, l)\}
\]
under JL. JL is more profitable if 
\[
q\hat{r}^{JL}(S, l) \geq p\hat{r}^{IL}(l).
\]
If \( \max\{q\hat{r}^{JL}(S, l), pr^{IL}(l)\} < \omega \), the branch closes.

4.2.3 Equilibrium

With free entry, branches will open until they are earning zero profits. This enables us to solve for the equilibrium market scale.

**Proposition 7** In an equilibrium with free entry, lenders will make zero profits but there will always be credit rationing. IL is used in equilibrium if and only if \( S < \tilde{S} \), with \( \tilde{S} \) given by
\[
\tilde{S} \equiv \max \left\{ 0, \frac{\omega (1 + \delta p - 2\delta)\omega - (\delta p^2 R - \omega)}{\delta q (1 + \delta p - 2\delta)\omega + (1 - \delta p)\delta p R} \right\}.
\]
If \( S \geq \tilde{S} \) JL is used. The equilibrium market scale is
\[
l^*(S) = \begin{cases} 
\delta p^2 R - \omega, & S < \tilde{S} \\
\delta q (1 + \delta p - 2\delta)\omega - (1 - \delta)\omega - (1 - \delta p)\delta p R, & S \in [\tilde{S}, \frac{\omega}{\delta q}], \\
\delta q (1 + \delta p - 2\delta)\omega, & S \geq \frac{\omega}{\delta q}.
\end{cases}
\]
\( l^*(S) \) is strictly increasing in \( S \) for \( S \in [\tilde{S}, \frac{\omega}{\delta q}] \).

The proof is given in the Appendix.

Proposition 7 shows that market scale is broadly increasing, and hence credit rationing decreasing in \( S \), since social capital relaxes the JL IIC and hence increases the surplus that can be attained through lending.

4.2.4 Welfare

We know that market scale, \( l \), is increasing in social capital. It is also clear that the utility of borrowers, \( V \) is increasing in \( S \): lenders make zero profits but higher \( S \) implies less credit rationing so the probability of a rematch after a contract is terminated is higher.

However, it is perhaps more interesting to consider aggregate utility from microfinance, that is the utility of borrowers and unmatched potential borrowers. We can write this as the weighted sum:
\[
Z = lV + (1 - l)U = V(l + (1 - l)\phi)
\]
Substituting for \( \phi \) and using \( \pi r = \omega \) we obtain
\[
Z = \frac{l[(1 - \pi) + (1 - \delta)(1 - l)] pR - \omega}{l(1 - \pi) + (1 - \delta \pi)(1 - l) 1 - \delta}.
\]
Again it is easy to see and obvious that \( Z \) is increasing in \( l \), for a given repayment probability \( \pi \). However at \( l^*(\tilde{S}) \) the lenders switch from offering \( IL \) to \( JL \) contracts, and \( \pi \) increases discontinuously from \( p \) to \( q \), and this actually reduces aggregate utility. The reason is that when the repayment probability increases, the probability of an unmatched borrower or “outsider” finding a match decreases substantially, making her worse off, and this more than offsets the gains for the matched “insiders”. From this point on, higher \( S \) again leads to increases in \( l \) until a maximum at \( S = \frac{\omega}{\delta q} \).

**Proposition 8** Aggregate utility from borrowing, \( Z \), is constant in \( S \) for \( S < \tilde{S} \), decreases discontinuously at \( \tilde{S} \), then increases to its maximum at \( S = \frac{\omega}{\delta q} \).

The proof is given in the Appendix.

### 4.2.5 Benefits of competition

So far we have assumed that due to the absence of information sharing between branches, the dynamic repayment incentives from contract termination threats are weakened under competition, and credit rationing is guaranteed in equilibrium. Now we compare with the alternative extreme: full information sharing with strict dynamic incentives: defaulting borrowers can never borrow again. This could be because of a credit bureau that shares borrowers’ histories between branches and lenders, or because the market is captured by one large lender whose branches are able to communicate. Either way, the strict dynamic incentives will mean that there will be no credit rationing as opening new branches does not erode incentives. Free competitive entry or a single large nonprofit will yield a zero-profit equilibrium identical to our non-profit lender. Alternatively, the market might be captured by a single for-profit or cartel equivalent to our monopolist lender. In the following discussion we will indicate the credit rationing or “competitive” equilibrium with a \( C \), the credit bureau equilibrium with \( B \) and the monopolist with \( M \).

The key tradeoff here is between credit rationing and stricter punishment. When \( S \) is small, the market scale may be so small in the competitive equilibrium that borrowers would be better off in the credit bureau or monopolist equilibria. As \( S \) increases, we know that \( Z_C \) is flat initially, decreases discontinuously at \( \tilde{S} \), then increases to its maximum at \( S = \frac{\omega}{\delta q} \). \( Z_B \) will be equal to \( \hat{V} \), as all borrowers are able to borrow, and increases once discontinuously at \( \tilde{S} \). \( Z_M \) is flat, increases discontinuously at \( \tilde{S} \), then declines to its starting value at \( \bar{S} \).

Due to the various discontinuities and \( S \) thresholds to track, comparing these three is not straightforward. However, it is easy to construct simple examples that show that the orderings of \( Z_C \) and \( Z_B \) and \( Z_C \) and \( Z_M \) are ambiguous (\( Z_B \geq Z_M \) always), even for large \( S \). To illustrate our basic point, therefore, we will work with a simple example.

**Proposition 9** Let \( p = \delta q \) and \( \delta p^2 R = \omega \). Then \( \hat{S} = \tilde{S} = \tilde{S} = 0 \), so \( JL \) is offered in any equilibrium. \( Z_B \) is positive and constant, \( Z_M \) is positive and decreasing in \( S \) to a minimum at \( S = \tilde{S} \) and \( Z_C \) is zero for \( S = 0 \), increases to a maximum at \( S = \frac{\omega}{\delta q} = \delta p R \), and is strictly greater than \( Z_M \) and \( Z_C \) for sufficiently large \( S \).

The proof is given in the Appendix, but the result is easily visualized in Figure 5.
4.3 Non-myopic lender

Until now the lender has ignored future profits because borrowers can be costlessly replaced, so there is no benefit to retaining a particular individual. For example, in our model of competition each period a lender can only serve two borrowers at a time and there is a large pool of identical replacements. However, if recruiting new borrowers is costly or impossible, it becomes valuable for the lender to retain existing clients. We model this in a simple reduced-form way by assuming that profits from a given borrower are discounted with factor $\beta \in [0, 1)$. The more costly it is to recruit new borrowers, the larger will be $\beta$. Now the lender’s discounted profits per borrower are

$$\Pi_\beta = \frac{\pi r - \omega}{1 - \beta \pi}.$$

The only ingredient of the benchmark model that will change is the monopolist lender’s contract choice. The constraints and thus interest rates for a given contract as a function of $S$ remain the same. The nonprofit lender earns zero per-period profits so is unaffected, while the monopolist now prefers JL whenever $\frac{\pi^JL(S) - \omega}{1 - \beta p} \geq \frac{\pi^IL(S) - \omega}{1 - \beta p}$. We can solve this condition for a new $\tilde{S}(\beta)$ which is the value of $S$ at which the lender switches from IL to JL. This is:

$$\tilde{S}(\beta) \equiv \max \left\{0, \frac{p^2 R(1 + \delta p - 2\delta)(1 - \beta p)(2 - \delta q)(\delta p^2 R - \omega)}{(2 - p)(1 - \delta q)} \right\}.$$

Observation 5 $\tilde{S}(\beta)$ is weakly decreasing in $\beta$, strictly when $\delta p^2 R > \omega$.

The implication of this observation is that as $\beta$ increases, the monopolist offers JL over a larger range of values of $S$. Since we know borrowers are strictly better off under JL for all $S < \tilde{S}$ this improves borrower welfare. A policy implication of this result is that policies or regulation that restrict the lender’s capacity relative to demand, or that make

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22 The model given here is more consistent with a lender who is unencumbered by capacity constraints and thus can lend to the whole market, so terminating a borrower leads to a loss of future profits, or alternatively a search model where a lender may have to wait before spare capacity is filled. An alternative model might apply a fixed cost whenever a new borrower is recruited, which similarly increases the value of a higher repayment probability from the lender’s perspective.
the lender discount the future more heavily (for example due to the perceived threat of future harsh regulations) may cause $\beta$ to decrease and thus cause the lender to switch to contracts that are worse for the borrowers.

However, even with a non-myopic lender, monopoly lending is still inefficient for some values of $S$ according to the definition of efficiency given in Definition 1.

**Proposition 10** The monopolist lender is inefficient relative to the nonprofit, since he offers JL for a weakly smaller range of values of $S$, strictly when $\tilde{S}(\beta) > 0$ and $\delta p^2 R > \omega$.

The proof is given in the Appendix. The intuition is clear: JL is the aggregate welfare maximizing contract, and the nonprofit switches to JL as soon as it breaks even. The monopolist only switches once the discounted profits from JL coincide with those from IL, which will typically require a higher level of social capital.

### 4.4 Allowing the feasibility constraint to bind

Assumption 3 implied that the GIC was always tighter than the FC under JL (the GIC is always tighter under IL), thus reducing the number of constraints we needed to track. In this section we discuss the implications of reversing this assumption, i.e. we replace Assumption 3 with

**Assumption 4** $\delta p > \frac{1}{2}$

Now we need to introduce a new interest rate, $r_{FC}$, which is the highest possible rate at which mutual insurance is feasible under JL. This is

$$r_{FC} \equiv \frac{R}{2}.$$ 

As a result, we now have

$$\tilde{r}^{JL}(S) = \min\{r_{FC}, r_{IIC}^{JL}(S)\}$$

The FC and IIC intersect at a new $\tilde{S} = \max\{0, \frac{(2-\delta q - 2\delta p)R}{2(1-\delta q)}\} < \bar{S}$. If $\bar{S} = 0$, the IIC never binds and therefore social capital does not influence the interest rate or contract choice.

The nonprofit lender offers JL whenever he can break even doing so. However it is no longer guaranteed that the IIC will be binding at low levels of $\tilde{S}$, so we need to check that JL can earn non-negative profits at least when $S$ is large. Provided he can break even when the FC binds, requiring $\frac{qR}{2} \geq \omega$, he will offer JL when the IIC is sufficiently slack, i.e. $S \geq \tilde{S}$. The monopolist will offer JL provided it is more profitable than IL. We have shown that this requires $S \geq \bar{S}$ for the IIC to be sufficiently slack. JL will be offered for $S \geq \tilde{S}$ if it is weakly more profitable than IL when the FC is binding (and thus weakly more profitable than IL when the IIC is binding for $S \geq \tilde{S}$). This requires $q_{r_{FC}} \geq p_{r_{GIC}}$ or $\delta p \leq \frac{2-p}{2}$.

Finally we observe that we can now take the strict form of Proposition 2, i.e. the borrowers are now strictly better off under JL. The reason is that the FC is strictly tighter than the GIC, and therefore borrower utility under JL will be strictly higher than $pR$, the utility under IL.

We can now ignore both Assumptions 3 and 4 and collect the general results in the following Proposition.
Proposition 11

- The nonprofit lender offers JL if and only if \( S \geq \hat{S} \) and \( qR \geq 2\omega \), otherwise he offers IL. The interest rate is \( \hat{r}^{IL} \) under IL and \( \hat{r}^{JL} \) under JL. Borrowers are strictly better off under JL than IL.

- The monopolist lender offers JL contracts if and only if \( S \geq \tilde{S} \) and \( \delta p \leq 2 - p^2 \). The interest rate is \( r^{GIC} \) under IL and \( \min \{ r^{GIC}, r^{FC}, r^{JL}_{IC}(S) \} \) under JL. Borrowers are strictly better off under JL for \( S < \bar{S} \) when \( \delta p \leq \frac{1}{2} \), and strictly better off under JL for all \( S \) when \( \delta p > \frac{1}{2} \). Otherwise they are indifferent between IL and JL.

4.5 Community-level social sanctions

In this paper we take a particular bilateral view of social capital. By so doing, we can focus on interactions between a pair of borrowers without concerning ourselves with the affairs of society as a whole. An alternative view is that social sanctions are enacted at the community (or some intermediate sub-community) level. For example, someone who fails to abide by a social norm may find themselves ostracized by the community as a whole. A classic example is Greif (1993). In our context, it could be that there is some village public good (access to a well, marketplace or social events) to which access can be restricted to anyone considered to have a “bad reputation”. Access is worth \( S \). Then in our framework, sanctions become one-sided: a borrower who deviates from the mutual insurance agreement is cut off from the public good.

This alters nothing in the basic model: all value functions, constraints and results are unchanged. The only situations in which the form of sanctions will matter is where borrowers might re-form a new borrowing group after a friendship breaks up. For example, in the discussion of destruction of social capital we noted that “dense” friendship groups will not be susceptible to undermining since a large number of friendships would need to be dissolved before the interest rate was affected. Here, we would instead need to consider what happens when ostracized borrowers attempt to form borrowing groups.

In the discussion of competition, we also assumed that if a borrower were to face a social sanction and lose her partner, she could still form a new group with a new friend also worth \( S \). This simplified things a great deal since we do not need to track the reputation of borrowers, nor do we need to track the size of the borrowing pool if ostracized borrowers can no longer borrow. The extension of this model to the community-level sanctions case we leave to future research.

4.6 Coercive enforcement

Lastly, we consider what happens when the lender has access to an additional enforcement technology we refer to as “coercive methods.” Suppose that on top of denying future credit, the lender can costlessly commit to inflict an additional non-monetary punishment \( z \) on the borrowers in the period after a default. To make our simple point, we assume only IL contracts can be used.

How does the additional sanction affect the key constraints? Let the borrower’s repayment probability be \( \pi \). The borrower’s utility function subject to feasibility, efficiency and incentive compatibility is now

\[
V = \frac{pR - \pi r - \delta (1 - \pi)z}{1 - \delta \pi}.
\]
Clearly the FC will be unchanged. The GIC (which requires that at the point of repayment the continuation value exceeds the interest payment) is \( \delta(V + z) \geq r \) which reduces to
\[
\delta pR + \delta(1 - \delta)z \geq r.
\]
In the presence of coercive methods, the GIC no longer implies the borrower’s participation constraint, which is \( V \geq 0 \) or:
\[
\frac{pR - \delta(1 - \pi)z}{\pi} \geq r.
\]
(8)

The tightest IIC is identical to the GIC.

The lender can profitably lend if \( p \hat{r}(z) - \omega \geq 0 \) or:
\[
p \min \left\{ \frac{pR - \delta(1 - p)z}{p}, \delta pR + \delta(1 - \delta)z \right\} \geq \omega
\]
where the first term is \( r_{PC}(z) \) and the second is \( r_{GIC}(z) \) (which is equivalent to the GIC). The FC can never bind under IL since it is slacker than the PC. Notice that in the neighborhood of \( z = 0 \), the GIC is the tightest condition under IL.

Since coercion makes borrowers worse off under a given lending type, the competitive lender will not use it. However, suppose coercion is costless (for simplicity). Then the monopolist sets \( z \) such that \( r_{GIC}(z) = r_{PC}(z) \), i.e. such that the PC binds.

**Proposition 12** Suppose only IL contracts are available. The nonprofit lender will never use coercion, while the monopolist sets \( z = \frac{pR}{\delta} \) and \( \hat{r}(z) = pR \), such that the borrower’s PC binds. The lender’s per-period profits are therefore \( pR - \omega \), the entire surplus.

5 Conclusion

This paper was motivated by the question as to what happens if lenders are not competitive or non-profits and have market power in the context of microfinance. We focussed on the choice between standard lending contracts and those with a joint liability clause in borrowing groups of size 2. We studied the role that social capital plays, and also looked at the endogeneity of social capital with respect to investments by borrowers or the lender. The existing literature on microfinance starts with the premise that MFIs are motivated by borrower welfare and in this paper we showed that there are interesting implications for relaxing this assumption. A lender with market power can extract rents from informal insurance agreements between his borrowers, but is ultimately constrained from making those borrowers worse off in the process.

In this paper there is no “perverse effect” of joint liability, whereby one borrower who could otherwise repay, nevertheless defaults because her partner is doing so. Besley and Coate (1995) raised this issue, and we explore its consequences in detail in de Quidt et al. (2011), showing that this is the key context where informal insurance without joint liability, or “implicit joint liability”, can generate welfare improvements.

There are several related questions that are of great interest. For example, what role does the for-profit and non-profit distinction play in the context of microfinance? Is it similar to a cost-quality trade-off as in the non-profits literature (see, for example, Glaeser and Shleifer 2001)? What happens if they operate in a fairly competitive setting? Lastly, some authors are beginning to explore the importance of external funding sources
for the behavior of MFIs, see for example Ghosh and Van Tassel, 2008, 2011. Our work suggests that the effects of market power on borrower welfare may vary considerably with the importance of social capital. We believe that more work is needed to quantify the extent of competition within the sector and better understand the behavior of lenders who do not conform to a simple zero-profit condition.
Appendix

Proof of the claim in footnote \[14\]

Suppose a pair of borrowers' outputs $Y_i$ are individually distributed according to $F(Y_i)$ with joint distribution $G(Y_1, Y_2)$, symmetric but not necessarily independent. Then if (but not only if) the GIC binds under IL, the borrowers are weakly better off under JL. The GIC binds under IL if and only if

$$\delta E(Y_i)[1 - F(\delta E(Y_i))] > x[1 - F(x)]$$

for all $x < \delta E(Y_i)$.

Sufficiency: consider either an IL or JL contract with interest rate $r$ that induces repayment probability $\pi(r)$. Note that $\pi(r)$ is a choice variable of the borrowers, thus under any contract a decrease in $r$ must make the borrowers strictly better off. Borrower utility is

$$V(r) = E(Y_i) - r\pi(r)$$

If the GIC ($\delta V(r) \geq 1 - F(\delta V(r))$) binds under IL or JL, i.e. $r = \delta V(r)$, we obtain $V = E(Y_i)$ and $r = \delta E(Y)$. Since the GIC must be satisfied under JL, and borrower utility is strictly decreasing in $r$, it must be that borrowers are indifferent when the GIC binds under JL, and strictly better off under JL when it is slack, for example when the lender chooses to reduce the interest rate to induce borrowers to insure one another in more states of the world.

If the GIC ($\delta V \geq r$) is satisfied under IL, then the borrower repays whenever repayment is feasible, i.e. in every state $Y_i \geq r$. Therefore the repayment probability is $\pi(r) = 1 - F(r)$. Thus the lender solves

$$\max_r \Pi(r) = r(1 - F(r)) - \omega$$

s.t. $r \leq \delta E(Y_i)$.

The necessary and sufficient condition for the GIC to bind under IL follows immediately from this lender’s maximization problem, and is required for $r = \delta E(Y_i)$ to be the unique maximizer on $r \in [0, \delta E(Y_i)]$.

To show that the GIC binding is not necessary, consider a simple counterexample, where $Y_1$ and $Y_2$ are perfectly correlated, so no mutual insurance is possible. Then the interest rate and repayment probability will be identical under both IL and JL, so the borrowers will be indifferent. It is straightforward to find an example where the profit-maximizing interest rate is smaller than $\delta E(Y_i)$. For example, let $Y_i \in [0, \bar{Y}]$ and $F(x) = F(x, z) = \frac{z^x}{\bar{Y}^{\frac{x+1}{2}}} \forall z \geq x$. The profit maximizing interest rate under IL and JL is $r^* = \frac{\bar{Y}}{\sqrt{3}}$, while $E(Y_i) = \frac{2\bar{Y}}{3} > r^*$. Therefore we can find a $\delta < 1$ such that $r^* < \delta E(Y_i)$ and hence the GIC is slack at the lender’s optimum. This concludes the proof.

A simple sufficient condition for the GIC to bind is that for all $r < E(Y_i)$, $\Pi(r)$ is strictly increasing. This is satisfied for many distribution functions $F$.

Example 1 $Y_i$ is uniform on $[0, \bar{Y}]$. Then $\Pi'(r) = 1 - \frac{2r}{\bar{Y}}$ which is positive for all $r < E(Y_i) = \frac{\bar{Y}}{2}$.

Example 2 $Y_i$ is exponentially distributed with parameter $\lambda$. Then $\Pi' - \lambda r(1 - r\lambda)$ which is positive for all $r < E(Y_i) = \frac{1}{\lambda}$. 

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Proof of Proposition 7

First we prove two intermediate results.

**Lemma 1** Profits under each contract type and equilibrium profits are strictly and continuously decreasing in $l$.

**Proof.** We know that whatever contract type he chooses, the branch will satisfy the GIC and IIC for the borrowers, hence the repayment probability will not depend on $l$ and we need only consider how $l$ influences the interest rate. $r_{GIC}^I(l)$ and $r_{GIC}^J(S,l)$ are strictly and continuously decreasing for $l \in (0,1)$ for a given $S$.

Now consider $r_{GIC}^I(S,l)$, which is defined implicitly by the equation $2r_{GIC} = \delta[(1 - \phi)V(r, \phi) + S]$. Differentiating this, we obtain

$$\frac{\partial r_{GIC}^I(S,l)}{\partial l} = \frac{\partial r}{\partial l} \frac{\partial \phi}{\partial l} = -\frac{\delta(1 - \delta)}{2 - \delta(2 - q) + q} \frac{\partial \phi}{\partial l} V,$$

which is strictly negative. Therefore, $r_{GIC}^I(S,l)$ is continuous and strictly decreasing.

Combining this with the result for $r_{GIC}^J(S,l)$ we have that $\hat{r}_{JL}(S,l) = \min\{r_{GIC}^I(l), r_{GIC}^J(S,l)\}$ will be continuous and strictly decreasing.

To conclude the proof, we note that since the branch chooses the per-period revenue maximizing contract, equilibrium profits are $\Pi = \max\{p_{JL}^I(l), q_{JL}(S,l)\} - \omega$ and therefore continuous and strictly decreasing in $l$. ■

This result is useful as it enables us to analyze the type of contract that will be offered in equilibrium. Since profits are decreasing in $l$, entry will occur until the zero profit condition binds. $l^{IL*}$ satisfies $p_{JL}^I(l^{IL*}) - \omega = 0$ and analogously $l^{IL*}(S)$ satisfies $q_{JL}(S, l^{IL*}(S)) - \omega = 0$. Then we have the following result.

**Lemma 2** The equilibrium market scale is $l^*(S) = \max\{l^{IL*}, l^{IL*}(S)\}$.

**Proof.** We know that $l^*(S) \in \{l^{IL*}, l^{IL*}(S)\}$. Suppose (without loss of generality) $l^{IL*} < l^{IL*}(S)$ but $l^*(S) = l^{IL*}$. Since profits under each contract type are strictly decreasing in $l$, a branch offering JL must make strictly positive profits at $l^{IL*}$, thus branches will switch to JL. The positive profits will induce entry, contradicting the claim that $l^*(S) = l^{IL*}$. ■

Solving $p_{GIC}^I(l^{IL*}) = \omega$ we obtain $l^{IL*} = \frac{\delta p R - \omega}{\delta p R - \omega}$. Meanwhile under JL, if the GIC is binding in equilibrium $l^{IL*}(S) = \frac{\delta p R - \omega}{q(\delta p R - \omega)}$. This is the equilibrium value if $\frac{\delta p R - \omega}{q(\delta p R - \omega)} \geq \bar{l}(S)$. If the IIC is binding under JL, we obtain:

$$l^{IL*}(S) = \frac{(\delta q p R - \omega) - (1 - \delta)(\omega - \delta q S)}{q(\delta p R - \omega) - q(1 - \delta)(\omega - \delta q S)}.$$

Comparing this with $\bar{l}(S)$, we confirm that the IIC binds in equilibrium if $S < \frac{\omega}{\delta q}$. Note that $\frac{\partial l^{IL*}(S)}{\partial S} > 0$, so when JL is used in equilibrium and the IIC is binding, market scale is expanding in $S$, as social sanctions substitute for formal sanctions.

When $S \geq \frac{\omega}{\delta q}$, $l^{IL*}(S) > l^{IL*}$, so we know that JL will be used in equilibrium. Since the relation is strict, by continuity of $\hat{r}_{JL}(S,l)$ there also exist values of $S < \frac{\omega}{\delta q}$ such that
JL is used. It remains to check under what conditions IL will be used in equilibrium. JL will be used in equilibrium for all \( S \) if \( l^{IL} \leq l^{JL}(0) \), or:

\[
\delta p^2 R - \omega - \omega (1 + \delta p - 2\delta) \geq 0
\]

for which a sufficient condition is \( 1 + \delta p - 2\delta \leq 0 \), which corresponds to the sufficient condition (6) for JL to dominate IL for all \( S \) when the lender has monopoly power.

The threshold value of \( S \) below which \( IL \) is used in equilibrium is:

\[
\tilde{S} \equiv \max \left\{ 0, \frac{\omega (1 + \delta p - 2\delta) + (\delta p^2 R - \omega)}{\delta q (1 + \delta p - 2\delta) + (1 - p) \delta p R} \right\}.
\]

If (6) holds then \( \tilde{S} = 0 \), i.e. JL is always used in equilibrium. Also note that \( \tilde{S} < \frac{\omega}{\delta q} \), so there is always a region \( S \in [\tilde{S}, \frac{\omega}{\delta q}] \) over which JL is offered and the GIC is binding, and hence market scale is increasing in \( S \).

Collecting results, we have:

\[
l^*(S) = \begin{cases} 
\frac{\delta p^2 R - \omega}{p(\delta p R - \omega)} & S < \tilde{S} \\
\frac{(\delta p R - \omega) - (1 - \delta q)(\omega - \delta q S)}{q(\delta p R - \omega) - q(1 - \delta)(\omega - \delta q S)} & S \in [\tilde{S}, \frac{\omega}{\delta q}] \\
\frac{\delta p q R - \omega}{q(\delta p R - \omega)} & S \geq \frac{\omega}{\delta q}
\end{cases}
\]

**Proof of Proposition 8**

It is obvious that \( Z \) does not depend on \( S \) for \( S < \tilde{S} \) or \( S \geq \frac{\omega}{\delta q} \), since in the first case IL is offered and the GIC binds, and in the second case JL is offered and the GIC binds.

First we show the discontinuity at \( l^*(\tilde{S}) \). At this point lenders switch from IL contracts to JL contracts. Holding \( l \) constant we can simply evaluate the effect of an increase in \( \pi \) on \( Z \). Differentiating \( Z \) with respect to \( \pi \) yields

\[
\frac{dZ}{d\pi} = -\frac{l(1 - l)^2(1 - \delta)(p R - \omega)}{[l(1 - \pi) + (1 - \delta \pi)(1 - l)]^2} < 0.
\]

Therefore the discontinuous increase in \( \pi \) at \( l^*(\tilde{S}) \) leads to a discontinuous drop in \( Z \). After this, \( \pi \) is again constant so \( Z \) is strictly increasing in \( S \) as \( l \) increases.

Finally we show that when \( l \) reaches its maximum at \( S = \frac{\omega}{\delta q} \) borrowers are strictly better off than at its previous maximum when \( S < \tilde{S} \). In each case \( l = \frac{\pi \delta p R - \omega}{\pi(\delta p R - \omega)} \). Also, we know that the GIC binds, so \( V = \frac{r}{\delta(1 - \phi)} = \frac{\omega}{\delta \pi (1 - \phi)} \). Plugging these into the equation for \( Z \) and simplifying we obtain

\[
Z' = \frac{\omega}{\delta} \left[ \frac{1}{\pi} + \frac{\delta p R - \omega}{\omega (1 - \delta)} - \frac{\omega (1 - \pi)}{\pi^2 (\delta p R - \omega)} - \frac{(1 - \pi)}{\pi (1 - \delta)} \right].
\]

With this expression we can explore the effect of switching from \( S < \tilde{S} \) to \( S \geq \frac{\omega}{\delta q} \), simply by switching \( \pi \) from \( p \) to \( q \). Differentiating with respect to \( \pi \) we obtain

\[
\frac{dZ'}{d\pi} = \frac{\omega}{\delta \pi^2} \left[ \frac{(2 - \pi) \omega}{\pi (\delta p R - \omega)} + \frac{\delta}{1 - \delta} \right] > 0.
\]

Therefore switching from the GIC binding under IL to the GIC binding under JL in a zero-profit competitive equilibrium increases aggregate utility from microfinance.
Proof of Proposition 9

The restriction \( p = \delta q \) implies \( 1 + \delta p - 2\delta = 0 \), i.e. equation (6) holds with equality, so \( \delta = \frac{1}{2-p} \). Also we obtain \( \frac{\omega}{\delta q} = \delta pR \). Using the conditions, we obtain that in the competitive equilibrium, market scale will be

\[
l^*(S) = \begin{cases} \frac{2\delta pR(1-q)+p(1-p)S}{1-2p} & S < \delta pR \\ \frac{1}{1-2p} & S \geq \delta pR \end{cases}
\]

\( Z^C \) is increasing in \( S \) for \( S < \delta pR \), and is continuous. At \( S = 0 \) it also takes value zero at \( S = 0 \). We also know that \( Z^B = \frac{pR-\omega}{1-\delta q} \) which is strictly positive and constant in \( S \). Similarly, \( Z^M = \frac{pR-q^J(S)}{1-\delta q} = Z^B - \frac{q^J(S)-\omega}{1-\delta q} \), and is equal to \( Z^C \) at \( S = 0 \), strictly positive, strictly decreasing to \( S = \tilde{S} \) and continuous.

With some manipulation (and replacing \( \pi \) with \( q \) since JL is always offered) we can write \( Z^C \) as

\[
Z^C = \frac{l(1-q)(1-\delta q) + l(1-l)(1-\delta)(1-\delta q)}{l(1-q)(1-\delta) + (1-l)(1-\delta)(1-\delta q)} Z^B
\]

Evaluating this expression at \( S = \delta pR \), i.e. \( l = \frac{1}{2-p} = \delta \), the maximum possible market scale in the competition equilibrium, we obtain with some manipulation

\[
Z^C = \left[ 1 + \delta \frac{(1-q)}{2} \right] Z^B
\]

By \( Z^B = Z^M(\delta pR) + \frac{q^J(\delta pR)-\omega}{1-\delta q} > Z^M(\delta pR) \), monotonicity and continuity we confirm that \( Z^C \) must start strictly below \( Z^B \) and \( Z^M \) but cross both before \( S = \delta pR \). This is graphed in Figure 5.

Proof of Proposition 10

Recall that

\[
\hat{S} \equiv \max \left\{ 0, \frac{(2-\delta q)\omega - (2-p)\delta p^2 R}{\delta q(1-\delta q)} \right\}
\]

\[
\tilde{S}(\beta) \equiv \max \left\{ 0, \frac{p^2 R(1+\delta p - 2\delta)}{(2-p)(1-\delta q)} - \frac{\beta(1-p)(2-\delta q)(\delta p^2 R - \omega)}{\delta(1-\beta p)(2-p)(1-\delta q)} \right\}.
\]

Taking the difference between the second terms in each maximum we have:

\[
\frac{p^2 R(1+\delta p - 2\delta) - \beta(1-p)(2-\delta q)(\delta p^2 R - \omega)}{(2-p)(1-\delta q)} - \frac{(2-\delta q)\omega - (2-p)\delta p^2 R}{\delta q(1-\delta q)} \geq 0
\]

With the inequality strict for \( \delta p^2 R > 0 \). Therefore, if \( \delta p^2 R > \omega \) and \( \hat{S}(\beta) > 0 \), \( \tilde{S}(\beta) > \hat{S} \). Otherwise, the two thresholds coincide.

To match this expression to our definition of efficiency (Definition 4), we can simply set \( \beta = \delta \).
References


