

Adaptive Learning and the Transmission of Government Spending Shocks in the Euro Area

Ewoud Quaghebeur*

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Abstract

This paper analyses the transmission of government spending shocks in an estimated dynamic stochastic general equilibrium model for the euro area when agents use forecasting models updated by the Kalman filter to form expectations. Based on the marginal likelihood criterion, there is evidence in favour of this Kalman filter learning mechanism relative to rational expectations. Moreover, under Kalman filter learning, the transmission of government spending shocks varies over time. This variation stems from the adjustment of the beliefs of the agents on their forecasting model. Hence, this adjustment process provides an endogenous explanation for time-varying government spending multipliers. We find that, in contrast to rational expectations, the responses of private consumption to a government spending shock is positive for most periods in the sample. Moreover, the government spending multiplier for output is substantially larger under learning than under rational expectations.

1 Introduction

Recently there has been a renewed interest in the idea that changes in expectations are an important source of business cycle fluctuations — see for example Beaudry and Portier (2007), Eusepi and Preston (2011), and Jaimovich and Rebelo (2009). One important issue is the role of expectations for the transmission mechanism of government spending shocks. In particular, it is well understood that the macroeconomic effects of these shocks crucially depend on private-sector expectations. Bachmann and Sims (2012), for instance, show that the response of consumer expectations to a positive government spending shock explains the main part of the expansionary output effect in times of slack.

*Ghent University, Department of Social Economics and Study Hive for Economic Research and Public Policy Analysis (SHERPPA). Correspondence to Ewoud.Quaghebeur@UGent.be. Sint-Pietersplein 6, B-9000 Ghent, Belgium. Phone: +32 9 264.34.87. I wish to thank Raf Wouters, Freddy Heylen, Selien De Schryder, Punnoose Jacob, Gert Peersman, Céline Poilly, Martin Iseringhausen, and the members of the Macro Research Group of Ghent University for constructive comments and suggestions. Any remaining errors are my own. Most of this research has been carried out while I visited the National Bank of Belgium who provided financial support for this research. The views expressed in this paper are my own and do not necessarily reflect the views of the National Bank of Belgium.

The analysis of government spending shocks in structural macroeconomic models is almost invariably developed under the assumption that agents hold rational expectations (see Coenen et al., 2012, for a review of the literature). This paper goes beyond rational expectations and considers agents who have limited information and must form expectations based on estimated forecasting models. In particular, we estimate a medium-scale Dynamic Stochastic General Equilibrium (DSGE) model for the euro area where agents form expectations using adaptive learning.

This paper demonstrates that the learning model, in contrast to the rational expectations model, is able to capture time variations in the macroeconomic responses to government spending shocks. Consequently, learning behaviour provides an endogenous explanation for time-varying government spending multipliers documented in several recent, more data-driven studies — see e.g. Auerbach and Gorodnichenko (2012, 2013), Kirchner et al. (2010), and Pereira and Lopes (2014). We show how variation in the belief coefficients of the agents generates time variation in the transmission of government spending shocks in the euro area. Importantly, in contrast to the time-varying parameter Vector Autoregression (VAR) studies, such as Kirchner et al. (2010), and Pereira and Lopes (2014), this variation does not stem from random changes in the structural parameters. Time variation in the transition of shocks is induced by a learning process along which agents revise their expectations about the future. In general, government spending multipliers may vary across many dimensions. A growing literature assesses the dependency of multipliers on the stance of monetary policy and the amount of slack in the economy. This paper contributes to the literature by highlighting the expectations channel as an additional driver for time variation in the spending multiplier.

Several authors have studied the relationship between adaptive learning and fiscal policy. Evans et al. (2009) investigate the effect of learning for the dynamics of anticipated and unanticipated changes in government spending in an endowment economy and the Ramsey model. Mitra et al. (2013) extend the analysis by looking at the dynamics in a Real Business Cycle (RBC) model. Hollmayr and Matthes (2015) study a permanent change in government expenditure in an RBC model with a rich fiscal sector. Gasteiger and Zhang (2014) examine the effect of adaptive learning for the impact of discretionary tax changes in a Ramsey model with variable labour supply. Giannitsarou (2006) studies the transitional dynamics of a capital tax cut under learning in a stochastic growth model.

This paper contributes to this learning literature on fiscal policy in several ways. Firstly, as explained above, this paper provides an endogenous explanation for time-varying government spending multipliers based on time-varying expectations.

Secondly, we use Bayesian estimation techniques to estimate a model for the euro area under different assumptions about how expectations are formed. The Bayesian approach allows us to test the rational expectations model against adaptive learning models, based on the marginal likelihood criterion. We find that the baseline learning model fits the data substantially better than the rational expectations benchmark.

Finally, in contrast to the aforementioned authors, this paper considers a medium-scale DSGE model similar to Christiano et al. (2005) and Smets and Wouters (2007) with a number of model features such as sticky prices and wages that are necessary to capture the persistence in the euro area data. The results

presented here confirm earlier findings that these features crucially affect the impact of learning on the dynamics of government spending shocks. In particular, Quaghebeur (2013) shows that, in the first year and a half after the shock, government spending can crowd *in* private consumption when agents use the learning mechanism. By contrast, the rational expectations model predicts a substantial drop in private consumption after the shock. Moreover, on impact a co-movement between real wage and hours worked occurs.

The remainder of the paper is organised as follows. The next section discusses the log-linearised equations of the DSGE model that we estimate. Section 3 defines the rational expectations equilibrium of the model. In Section 4 we present the Kalman filter learning set-up. Section 5 discusses the estimation approach and the prior and posterior distributions of the model parameters. In this section we also show the time-varying belief coefficients of the agents' forecasting model. In Section 6 we discuss the dynamics of a government spending shock under learning and under rational expectations. The time variation in the learning mechanism, allows us to present impulse responses for each quarter in the sample. The present-value multipliers for output, private consumption, and private investment on impact and at longer horizons are presented in Section 7. Section 8 discusses the robustness of our results with respect to the sample period and the specification of the learning mechanism. The last section concludes.

2 The Model Economy

In this section, we describe the linearised version of the model.¹ Our model is a medium-scale DSGE model similar to that of An and Schorfheide (2007), Christiano et al. (2005), and Smets and Wouters (2007). We assume that technological progress is non-stationary. Therefore, all real trending variables are divided by the level of technology. Throughout the paper, hatted variables denote log-deviations from the steady state. Barred variables refer to steady state values.

The *accounting identity* is given by

$$\hat{y}_t = \left(1 - \frac{\bar{i}}{\bar{y}} - \frac{\bar{g}}{\bar{y}}\right) \hat{c}_t + \frac{\bar{i}}{\bar{y}} \hat{i}_t + \frac{\bar{g}}{\bar{y}} \hat{g}_t, \quad (1)$$

where y_t , c_t , g_t , and i_t denote period t output, private consumption, government expenditure, and gross investment.

The aggregate *production function* is given by

$$\hat{y}_t = \frac{\bar{y} + \Phi}{\bar{y}} [\alpha \hat{k}_{t-1} + \hat{z}_t + (1 - \alpha) \hat{N}_t], \quad (2)$$

where k_{t-1} is the installed capital stock, N_t is employment, and α is the elasticity of output with respect to capital. $\hat{z}_t \sim \mathcal{N}(0, \sigma_z^2)$ represents a technology shock and Φ is a fixed cost of production.²

¹The model appendix provides the full derivations of the model equations and is available upon request.

²Technology A_t follows a random walk with drift in its log: $\ln(A_t) = \ln(\gamma) + \ln(A_{t-1}) + \hat{z}_t$.

The representative household maximises expected lifetime utility. Following King et al. (1988), the utility function has the following functional form:

$$U(C_t, 1 - N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \exp\left(\frac{\sigma-1}{1+\phi} N_t^{1+\phi}\right), \quad (3)$$

with $\sigma, \phi > 0$.³

The *Euler equation for consumption* is given by

$$\hat{c}_t = E_t^* \hat{c}_{t+1} + c_1 (\hat{N}_t - E_t^* \hat{N}_{t+1}) - c_2 (\hat{R}_t - E_t^* \hat{\Pi}_{t+1}) + \hat{u}_t^b \quad (4)$$

with $c_1 = (\sigma - 1) \bar{N}^{1+\phi} \sigma^{-1}$ and $c_2 = \sigma^{-1}$. Here Π_t is the gross inflation rate and R_t is the gross nominal interest rate. $E_t^*(\cdot)$ denotes the subjective expectations of the household at time t . The disturbance term \hat{u}_t^b represents a risk premium shock à la Smets and Wouters (2007). The disturbance is assumed to obey $\hat{u}_t^b = \rho_b \hat{u}_{t-1}^b + \varepsilon_t^b$, with $\varepsilon_t^b \sim \mathcal{N}(0, \sigma_b^2)$.

Following Schmitt-Grohé and Uribe (2006), labour decisions are made by a union who represents the household and operates in a continuum of monopolistically competitive labour markets.⁴ In each market the union sets the wage and supplies enough differentiated labour to satisfy demand. Nominal wages are assumed to be sticky à la Calvo (1983). In those labour markets where the union cannot re-optimize its wage, nominal wages are indexed to productivity growth (z_t) and a weighted average of target inflation (Π_t^*) and lagged inflation (Π_{t-1}). The *real wage equation* that follows from the union's wage setting decision is given by

$$\hat{w}_t = w_1 (\phi \hat{N}_t + \hat{c}_t - \hat{w}_t) + w_2 \hat{w}_{t-1} + w_3 E_t^* \hat{w}_{t+1} + w_4 \hat{\Pi}_t + w_5 \hat{\Pi}_{t-1} + w_6 E_t^* \hat{\Pi}_{t+1} + w_7 \hat{\Pi}_t^* + \hat{u}_t^w, \quad (5)$$

with $w_1 = (1 - \theta_w) (1 - \beta \theta_w \gamma^{1-\sigma}) / [\theta_w (1 + \beta \gamma^{1-\sigma})]$, $w_2 = 1 / (1 + \beta \gamma^{1-\sigma})$, $w_3 = \beta \gamma^{1-\sigma} / (1 + \beta \gamma^{1-\sigma})$, $w_4 = -(1 + \beta \gamma^{1-\sigma} \gamma_w) / (1 + \beta \gamma^{1-\sigma})$, $w_5 = \gamma_w / (1 + \beta \gamma^{1-\sigma})$, $w_6 = \beta \gamma^{1-\sigma} / (1 + \beta \gamma^{1-\sigma})$, and $w_7 = (1 - \gamma_w) (1 - \rho_{\pi^*} \beta \gamma^{1-\sigma}) / (1 + \beta \gamma^{1-\sigma})$. According to this equation, the real wage w_t gradually adjusts to the difference between the real wage and the marginal rate of substitution between consumption and leisure ($\phi \hat{N}_t + \hat{c}_t$). The adjustment depends on the degree of wage stickiness, θ_w , and the normalised discount factor, $\beta \gamma^{1-\sigma}$. Moreover, the real wage is a function of past and expected real wages, past, current, and expected inflation, as well as current target inflation. The degree of wage indexation to past inflation relative to target inflation is determined by γ_w . Finally, \hat{u}_t^w represents a wage mark-up shock, which is assumed to evolve according to $\hat{u}_t^w = \rho_w \hat{u}_{t-1}^w - \mu_w \varepsilon_{t-1}^w + \varepsilon_t^w$, with $\varepsilon_t^w \sim \mathcal{N}(0, \sigma_w^2)$.

³Here $C_t = c_t A_t$, where A_t is the level of technology and c_t is detrended private consumption. In the remainder of the paper, we work with detrended real variables.

⁴As in Schmitt-Grohé and Uribe (2006), our formulation of the labour market assures that labour supply and consumption are identical across households, even if the utility function (3) is non-separable in consumption and employment.

The *optimality conditions for investment and the capital stock* are given by

$$\hat{i}_t = i_1 (\hat{i}_{t-1} - \hat{z}_t) + (1 - i_1) E_t^* \hat{i}_{t+1} + i_2 \hat{Q}_t + \hat{u}_t^i, \quad (6)$$

$$\hat{Q}_t = - \left(\hat{R}_t - E_t^* \hat{\Pi}_{t+1} - \sigma \hat{u}_t^b \right) + \beta \gamma^{-\sigma} \left[\bar{r}^k E_t^* \hat{r}_{t+1}^k + (1 - \delta) E_t^* \hat{Q}_{t+1} \right], \quad (7)$$

where $i_1 = 1 / (1 + \beta \gamma^{1-\sigma})$, $i_2 = 1 / [(1 + \beta \gamma^{1-\sigma}) s'' \gamma^2]$, δ is the physical rate of depreciation and Q_t is Tobin's Q .⁵ As in Christiano et al. (2005) investment is subject to adjustment costs. The cost parameter $1/s''$ is the elasticity of investment with respect to a one percent temporary increase in the current price of installed capital. The disturbance \hat{u}_t^i represents an investment-specific shock and obeys $\hat{u}_t^i = \rho_i \hat{u}_{t-1}^i + \varepsilon_t^i$, with $\varepsilon_t^i \sim \mathcal{N}(0, \sigma_i^2)$.

The *stock of physical capital* evolves according to

$$\hat{k}_t = k_1 (\hat{k}_{t-1} - \hat{z}_t) + (1 - k_1) \hat{i}_t + k_2 \hat{u}_t^i, \quad (8)$$

with $k_1 = (1 - \delta) / \gamma$ and $k_2 = (1 + \beta \gamma^{1-\sigma}) s'' \gamma^2 \bar{i} / \bar{k}$.

Similar to the wage setting decision of the union, intermediate goods producers set nominal prices according to a Calvo (1983) mechanism. Firms that cannot re-optimize their price, index their old price to a weighted average of target inflation ($\hat{\Pi}_t^*$) and lagged inflation ($\hat{\Pi}_{t-1}$). Optimal price setting gives rise to the following specification of the *new Keynesian Phillips curve*:

$$\hat{\Pi}_t = \pi_1 \widehat{MC}_t + \pi_2 \hat{\Pi}_{t-1} + \pi_3 E_t^* \hat{\Pi}_{t+1} + \pi_4 \hat{\Pi}_t^* + \hat{u}_t^\pi, \quad (9)$$

with $\pi_1 = (1 - \theta_p) (1 - \beta \theta_p \gamma^{1-\sigma}) / [\theta_p (1 + \beta \gamma^{1-\sigma} \gamma_p)]$, $\pi_2 = \gamma_p / (1 + \beta \gamma^{1-\sigma} \gamma_p)$, $\pi_3 = \beta \gamma^{1-\sigma} / (1 + \beta \gamma^{1-\sigma} \gamma_p)$, and $\pi_4 = (1 - \gamma_p) (1 - \rho_\pi \beta \gamma^{1-\sigma}) / (1 + \beta \gamma^{1-\sigma} \gamma_p)$. The inflation rate is a function of the real marginal cost MC_t , a price mark-up disturbance \hat{u}_t^π , past and expected future values of actual inflation, and target inflation. The coefficients of the inflation equation depend on the Calvo parameter of price stickiness, θ_p , the degree of price indexation to past inflation, γ_p , and the normalised discount factor, $\beta \gamma^{1-\sigma}$. The price mark-up disturbance follows the exogenous process $\hat{u}_t^\pi = \rho_\pi \hat{u}_{t-1}^\pi - \mu_\pi \varepsilon_{t-1}^\pi + \varepsilon_t^\pi$, with $\varepsilon_t^\pi \sim \mathcal{N}(0, \sigma_\pi^2)$. The ARMA(1,1) structure is designed to capture the high-frequency component of inflation.

Cost minimisation by the intermediate goods producers yields the following *labour demand equation* and *equation for the rental rate of capital*:

$$\hat{w}_t = \hat{MC}_t + \alpha (\hat{k}_{t-1} - \hat{N}_t) + \hat{z}_t, \quad (10)$$

$$\hat{r}_t^k = \hat{MC}_t + (\alpha - 1) (\hat{k}_{t-1} - \hat{N}_t) + \hat{z}_t. \quad (11)$$

⁵Tobin's Q is defined as q_t / λ_t , where q_t is the Lagrangian multiplier with respect to the capital accumulation rule and λ_t the Lagrangian multiplier with respect to the household's (real) budget constraint. See the model appendix for more details.

The central bank sets the nominal interest rate according to the following *generalised Taylor rule*:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \hat{\Pi}_t^* + \rho_R (\hat{\Pi}_t^* - \hat{\Pi}_{t-1}^*) + (1 - \rho_R) [\phi_\pi (\hat{\Pi}_t - \hat{\Pi}_t^*)] + \phi_{\Delta y} \Delta \hat{y}_t + \hat{u}_t^r. \quad (12)$$

Analogous to De Graeve et al. (2009) the monetary authority gradually adjusts the interest rate in response to the ‘‘inflation gap’’ $\hat{\Pi}_t - \hat{\Pi}_t^*$. The coefficient ϕ_π controls the responsiveness of the nominal interest rate to the inflation gap. The degree of interest rate smoothing is governed by the parameter ρ_R . In addition, we allow the interest rate to react to changes in the growth rate of output, with sensitivity parameter $\phi_{\Delta y}$. Following Cogley et al. (2010), the inflation target $\hat{\Pi}_t^*$ is time-varying and evolves according to $\hat{\Pi}_t^* = \rho_{\pi^*} \hat{\Pi}_{t-1}^* + \varepsilon_t^{\pi^*}$, with $\varepsilon_t^{\pi^*} \sim \mathcal{N}(0, \sigma_{\pi^*}^2)$. The monetary policy shock \hat{u}_t^r evolves according to $\hat{u}_t^r = \rho_r \hat{u}_{t-1}^r + \varepsilon_t^r$, with $\varepsilon_t^r \sim \mathcal{N}(0, \sigma_r^2)$.

The fiscal authority finances expenditure through lump-sum taxes. *Real government expenditure* evolves according to

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g. \quad (13)$$

3 Rational Expectations Equilibrium

We begin with the standard case of rational expectations as a benchmark to compare against the adaptive learning model. In the rational expectations case, agents have full knowledge of the structure of the economy. In the next section, we will relax this assumption and consider a learning mechanism where agents form expectations based on a small forecasting model.

Note that the linear model can be represented as

$$\mathbf{A}_0 \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{A}_1 \begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} + \mathbf{A}_2 E_t^* \mathbf{y}_{t+1} + B_0 \varepsilon_t = \text{constant}, \quad (14)$$

where \mathbf{y}_t is the column vector of log-linearised endogenous variables and \mathbf{w}_t is the column vector of log-linearised shocks. The vector \mathbf{y}_t contains six endogenous state variables $\mathbf{y}_t^s = [\hat{l}_t; \hat{k}_t; \hat{\Pi}_t; \hat{R}_t; \hat{w}_t; \hat{y}_t]$ and seven forward-looking variables $\mathbf{y}_t^f = [\hat{c}_t; \hat{l}_t; \hat{N}_t; \hat{\Pi}_t; \hat{Q}_t; \hat{r}_t^k; \hat{w}_t]$. The vector \mathbf{w}_t consists of the eight stochastic processes in the model.

When agents have rational expectations, the dynamics of the model are characterised by the following rational expectations equilibrium (REE):

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} = \boldsymbol{\mu} + \mathbf{T} \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{R} \varepsilon_t. \quad (15)$$

4 Adaptive Learning

Following Branch and Evans (2006), Sargent (1999), Sargent and Williams (2005), Sargent et al. (2006), and Slobodyan and Wouters (2012a), agents form expectations using forecasting models updated by the

Kalman filter.

For every forward-looking variable y_j^f , with $j = 1, 2, \dots, 7$, agents use the following forecasting model

$$y_{j,t}^f = \mathbf{X}_{j,t-1}^T \boldsymbol{\beta}_{j,t-1} + u_{j,t}. \quad (16)$$

In the learning literature this equation represents the Perceived Law of Motion (PLM). In the baseline specification of our model, the data matrix $\mathbf{X}_{j,t-1}$ contains a constant and the endogenous state variables of the model: the capital stock (\hat{k}_{t-1}), the nominal interest rate (\hat{R}_{t-1}), output (\hat{y}_{t-1}), the real wage rate (\hat{w}_{t-1}), investment (\hat{i}_{t-1}), and the inflation rate ($\hat{\Pi}_{t-1}$). By including all the endogenous state variables in the data matrix, our approach applies only a modest departure from rational expectations. We assume however that agents cannot access values of exogenous processes \mathbf{w}_t . Section 8 discusses the robustness of our results across alternative specifications of the forecasting models (16). In that section we show that our baseline specification results in the largest improvement of the marginal likelihood vis-à-vis the rational expectations model.

Let $\boldsymbol{\beta}_t$ denote the vector of stacked regression coefficients $\boldsymbol{\beta}_{j,t}$. Following Sargent and Williams (2005), agents believe that the vector $\boldsymbol{\beta}_t$ evolves according to a random walk process

$$\text{vec}(\boldsymbol{\beta}_t) = \text{vec}(\boldsymbol{\beta}_{t-1}) + \mathbf{v}_t. \quad (17)$$

The shocks \mathbf{v}_t are i.i.d. with covariance matrix \mathbf{V} . The random walk assumption is widely used in time-varying parameter Vector Autoregression (VAR) studies.

Equation (17) reflects the agents' view that the coefficients in their forecasting rules are not stable, but drift over time. As argued by Sargent and Williams (2005), among others, this assumption assures that the variation in the agents' beliefs does not die out. Hence, this set-up it is a natural way of accomplishing so-called "perpetual learning". A motivation for this set-up is that it allows agents to be alert to structural changes, because the Kalman filter learning algorithm discounts past data. In other words, this approach implicitly assumes that recent observations contain more accurate information about the current forecasting coefficients than past observations. Especially in a context where structural changes occur from time to time, this seems a reasonable assumption. Another justification is that the discounting of past data can be seen as a way to formalise finite-memory forecasting by the agents.

An alternative way to achieve "perpetual learning" behaviour is to consider a constant-gain variant of recursive least squares – see, for example, Milani (2007) and Orphanides and Williams (2007). In this paper, the Kalman filter is preferred over constant-gain algorithm based on the finding of Sargent and Williams (2005) that, although both algorithms have the same asymptotic behaviour, the Kalman filter converges much faster than the constant-gain algorithm.

We can write the forecasting model in the following SURE format:

$$\begin{aligned} \begin{bmatrix} y_{1,t}^f \\ y_{2,t}^f \\ \vdots \\ y_{m,t}^f \end{bmatrix} &= \begin{bmatrix} \mathbf{X}_{1,t-1} & 0 & \cdots & 0 \\ 0 & \mathbf{X}_{2,t-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}_{m,t-1} \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \vdots \\ \beta_{m,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{bmatrix} \\ &\Leftrightarrow \mathbf{y}_t^f = \mathbf{X}_{t-1} \boldsymbol{\beta}_{t-1} + \mathbf{U}_t, \end{aligned} \quad (18)$$

where we denote the (non-diagonal) covariance matrix of the regression errors \mathbf{U}_t by Σ .⁶

The Kalman filter provides the optimal estimate of the belief coefficients $\boldsymbol{\beta}_t$ conditional on information up to period $t-1$.⁷ The filter is described by the following two equations:

$$\boldsymbol{\beta}_{t+1|t} = \boldsymbol{\beta}_{t|t-1} + \mathbf{K}_t \left[\mathbf{y}_t^f - \mathbf{X}_{t-1}^T \boldsymbol{\beta}_{t|t-1} \right], \quad (19)$$

$$\mathbf{P}_{t+1|t} = (\mathbf{I} - \mathbf{K}_t \mathbf{X}_{t-1}^T) \mathbf{P}_{t|t-1} + \mathbf{V}, \quad (20)$$

where $\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{X}_{t-1} \left[\mathbf{X}_{t-1}^T \mathbf{P}_{t|t-1} \mathbf{X}_{t-1} + \Sigma \right]^{-1}$ is the Kalman gain.

At the end of period t , the realised value of \mathbf{y}_t^f is used to update the estimate of $\boldsymbol{\beta}_t$ based on the new information. Following equation (19), the Kalman gain \mathbf{K}_t determines the weight assigned to the new information when forming the new estimate $\boldsymbol{\beta}_{t+1|t}$.

To obtain the dynamics under Kalman filter learning, the estimate $\boldsymbol{\beta}_{t|t-1}$ from equation (19) is substituted for $\boldsymbol{\beta}_{t-1}$ in equation (18) to generate $E_t^* \mathbf{y}_{t+1} = \mathbf{X}_t \boldsymbol{\beta}_{t|t-1}$. We can insert the expression for $E_t^* \mathbf{y}_{t+1}$ in the linear approximation of the model (14) to obtain the following actual law of motion under learning

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} = \boldsymbol{\mu}_t + \mathbf{T}_t \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{R}_t \boldsymbol{\varepsilon}_t. \quad (21)$$

For the initial Kalman filter recursion, we need to specify the initial belief coefficients $\boldsymbol{\beta}_{1|0}$, the associated covariance matrix $\mathbf{P}_{1|0}$, the parameter covariance matrix \mathbf{V} , and the covariance matrix of the regression errors, Σ . We follow the approach of Slobodyan and Wouters (2012a) and use the theoretical moment matrices of the rational expectations equilibrium to derive the initial beliefs. In particular, since the OLS estimator is unbiased, we let $\boldsymbol{\beta}_{1|0} = \bar{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}}_{OLS} = E(\mathbf{X}^T \mathbf{X})^{-1} E(\mathbf{X}^T \mathbf{y}^f)$. It follows that the covariance matrix $\Sigma = E[\mathbf{U} \mathbf{U}^T] = E\left[(\mathbf{y} - \mathbf{X} \boldsymbol{\beta})(\mathbf{y} - \mathbf{X} \boldsymbol{\beta})^T\right]$. Furthermore, $\mathbf{P}_{1|0}$ and \mathbf{V} are both taken to be proportional to the covariance matrix of the GLS estimator, which is an efficient estimator of the SURE model (18). Hence, $\mathbf{P}_{1|0} = \sigma_0 (\mathbf{X}^T \Sigma^{-1} \mathbf{X})^{-1}$ and $\mathbf{V} = \sigma_v (\mathbf{X}^T \Sigma^{-1} \mathbf{X})^{-1}$.

In this paper, we consider *Euler equation* learning put forward by Evans and Honkapohja (2001) and assume that agents make one-step ahead forecasts, following e.g. Giannitsarou (2006) and Slobodyan and Wouters (2012a,b). By contrast, Evans et al. (2009), Mitra et al. (2013) and Gasteiger and Zhang

⁶Note that the regression errors $u_{j,t}$ are linear combinations of the innovations $\boldsymbol{\varepsilon}_t$ to the stochastic processes \mathbf{w}_t .

⁷See Ljung and Söderström (1986) or Kim and Nelson (1999), for example.

(2014) consider an *infinite horizon* learning scheme developed by Preston (2005). Under this learning scheme, agents must make forecasts about forward-looking variables into the infinite future. In the *infinite horizon* approach the inter-temporal budget constraint is explicitly used when deriving agents' expectations, whereas in the *Euler equation* approach only the flow budget constraint is used. In this paper, we focus on the latter approach and leave the interesting issue of the effects of the learning type on the transmission of government spending shocks for future work.⁸

5 Bayesian Estimation

5.1 Data and Observation Equations

We estimate the model with euro area quarterly data from 1970Q2 to 2013Q4.⁹ Figure 1 plots the time series used in the estimation. We use seven macroeconomic variables: the short-term nominal interest rate (r_t^{obs}) and the log differences of per capita real government consumption (g_t^{obs}), per capita real GDP (y_t^{obs}), per capita real consumption (c_t^{obs}), per capita real investment (i_t^{obs}), the real wage (w_t^{obs}), and the GDP deflator (Π_t^{obs}). The corresponding observation equations are

$$y_t^{obs} = \hat{y}_t - \hat{y}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \quad (22)$$

$$c_t^{obs} = \hat{c}_t - \hat{c}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \quad (23)$$

$$i_t^{obs} = \hat{i}_t - \hat{i}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \quad (24)$$

$$g_t^{obs} = \hat{g}_t - \hat{g}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \quad (25)$$

$$w_t^{obs} = \hat{w}_t - \hat{w}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \quad (26)$$

$$\Pi_t^{obs} = \hat{\Pi}_t + 100(\bar{\Pi} - 1), \quad (27)$$

$$r_t^{obs} = \hat{R}_t + 100(\bar{R} - 1), \quad (28)$$

where $100(\gamma - 1)$ is the common quarterly trend growth rate of real GDP, consumption, investment, capital, wages, and government spending, $100(\bar{\Pi} - 1)$ is the quarterly steady-state inflation rate, and $100(\bar{R} - 1)$ is the quarterly steady-state nominal interest rate.

5.2 Prior Distributions

The choice of the prior distributions is summarised in the left panel of Table 1. The prior distributions of the structural parameters are as follows. The prior standard deviation of most structural parameters is 0.1. For ϕ , σ , and s'' , however, we allow for a larger standard deviation, ensuring a rather large domain

⁸For a discussion of these two adaptive learning approaches see Honkapohja et al. (2013).

⁹These data are extracted from the 16th update of the Area Wide Model database compiled by Fagan et al. (2005). Following Smets and Wouters (2003), the observations in the 1970s are used as a training sample and do not enter into the calculation of the marginal likelihood.

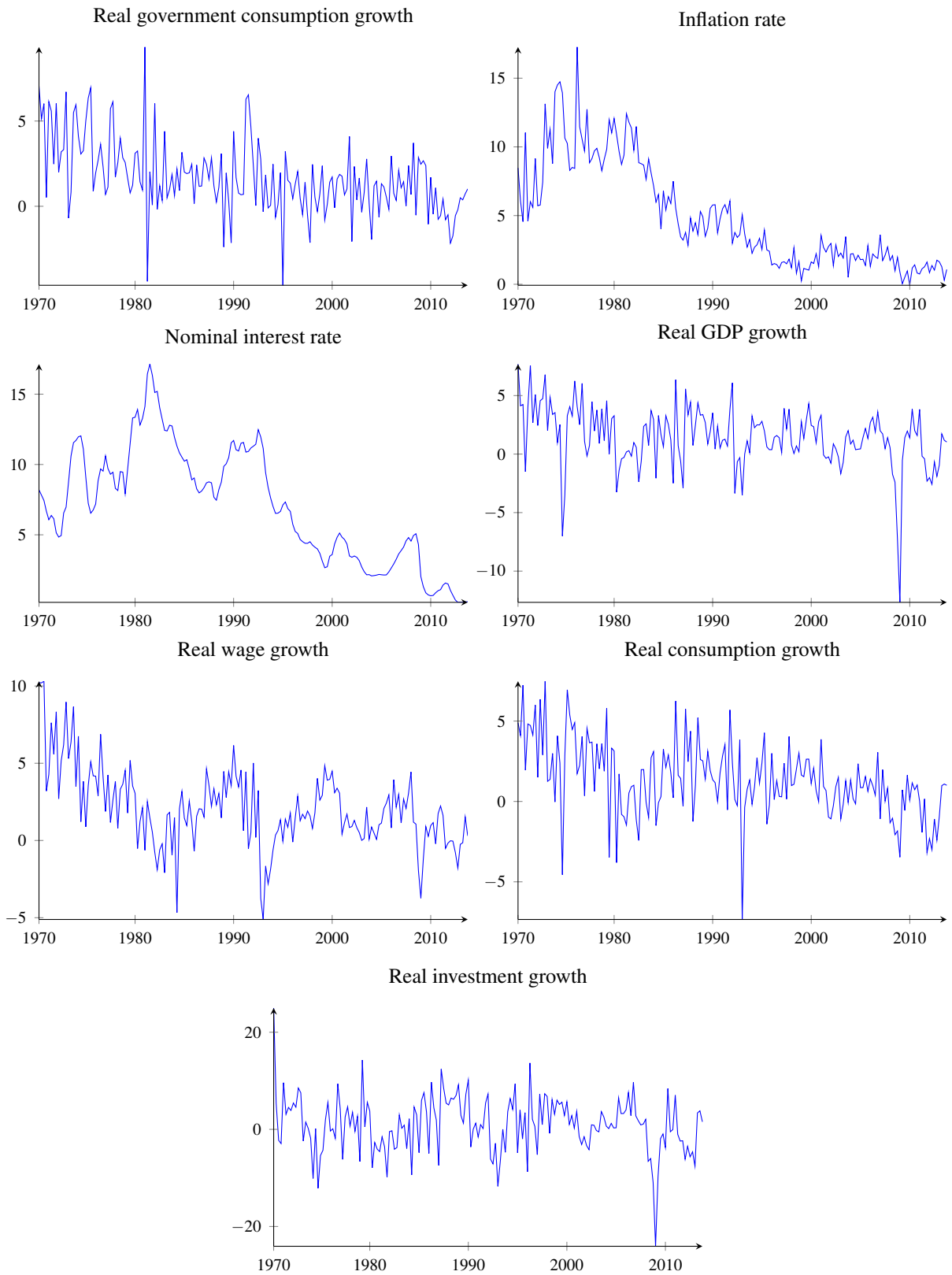


Figure 1: Data used in the estimation. For ease of interpretation, the variables are converted to an annual basis. In the estimation, however, we use quarterly data.

for these parameters. The steady-state inflation rate is assumed to follow a gamma distribution with a mean of 2 percent on an annualised basis. The priors for the degree of indexation to past inflation, γ_p and γ_w , and for the MA parameters in the mark-up processes, μ_π and μ_w , are described by a beta distribution with mean 0.5. Following Smets and Wouters (2003), the degree of risk aversion, σ , has a normal distribution with mean 1.5 and standard deviation 0.37, and the Calvo probabilities, θ_p and θ_w , have a beta distribution with mean 0.75 and standard deviation 0.05. The Frisch elasticity of labour supply, ϕ , is assumed to follow a normal distribution with mean 2 and standard deviation 0.25. The autoregressive coefficient of the stochastic processes are all assumed to follow a beta distribution with mean 0.5, except for the coefficient of standard monetary policy shock, ρ_r . The latter has a prior mean of 0.25 to have a clear separation from the inflation target shock. Based on Smets and Wouters (2007) the prior distribution for the investment adjustment cost parameter, s'' , is normal with mean 4 and standard deviation 1.5.

For the monetary policy parameters we adopt analogue priors as those used by De Graeve et al. (2009). The degree of interest rate smoothing, ρ_R , has a beta distribution with a mean of 0.75 and a standard deviation of 0.1. We adopt normal priors for the Taylor rule coefficients ϕ_π and $\phi_{\Delta y}$ with typical mean values. Following Cogley et al. (2010), we calibrate the autocorrelation of the inflation target shock to 0.985 so that it captures low-frequency movements in inflation. In consideration of the downward trend in the inflation data, the inflation target is thus designed to capture the gradual disinflation in the euro area over the past decades.

The scale parameters σ_0 and σ_v driving the learning dynamics follow a gamma distribution. It is standard to assume that the variance-covariance of shocks to the belief coefficients is smaller than the variance-covariance of the measurement errors of the forecasting models (18), i.e. $\mathbf{V} \ll \Sigma$ (see Sargent et al., 2006, for instance). Therefore we set the prior mean of σ_v to a small number (0.004) relative to the prior mean of σ_0 (0.04).

The standard deviations of the structural shocks are assumed to follow inverse-gamma distributions with two degrees of freedom. The prior mean for the standard deviation of the inflation target shock is taken from Smets and Wouters (2003).

A few parameters are kept fixed in the estimation procedure. The quarterly trend growth rate of real GDP is set to the mean growth rate of GDP in the sample. The discount factor $100(\beta^{-1} - 1)$ is set to 1 percent on an annualised basis. The output elasticity with respect to capital, α , is set to 0.33. The quarterly rate of physical capital depreciation, δ , is set to 0.025 so that the annual depreciation rate is 10 percent. The steady-state ratio of government spending to GDP is set to 0.20. The parameters $\bar{\epsilon}_p$ and $\bar{\epsilon}_w$, governing the price and wage mark-up, are clearly not identified by the data. We follow Rabanal and Rubio-Ramírez (2008) and set $\bar{\epsilon}_p = 0.2$ and $\bar{\epsilon}_w = 0.1$. Finally, the fixed cost in the production, Φ , is calibrated so that steady-state profits in the intermediate goods sector are zero.

5.3 Estimation Results

In this section, we present the estimation results for different assumptions regarding the formation of expectations. In particular, we compare the rational expectations results with those of the learning model.

Table 1 and Table 2 present the posterior estimates of the model parameters and the log marginal likelihood for the rational expectations model and the learning model. Based on the marginal likelihood criterion, it is clear that there is substantial evidence in favour of the learning mechanism relative to rational expectations. This result is in line with the findings of Milani (2007) and Slobodyan and Wouters (2012a,b), who also conclude that adaptive learning significantly improves the fit of DSGE models.

Table 1 and Table 2 demonstrate that the modelling assumption of expectations affects some of the parameter estimates. The estimated price and wage stickiness and the investment adjustment costs, for instance, are lower under Kalman filter learning. This is consistent with the finding of Milani (2007). Learning introduces endogenous persistence in the model such that other sources of persistence are no longer required to match the inertia in the data. The posterior mode for the degree of interest rate smoothing increases from 0.62 under rational expectations to 0.84 under learning.

In the learning model, agents update the regression coefficients (i.e. “beliefs”) of their forecasting model using the Kalman filter. This updating procedure generates important variation of those coefficients over time. Figure 2 illustrates this. For every forward-looking variable, the figure shows the belief coefficients of the respective forecasting model. The shaded grey areas indicate euro area recession dates. As explained in Section 4, each forecasting model is a function of a constant, the nominal interest rate, inflation, investment, the capital stock, output, and the real wage rate. It is clear that most coefficients vary a lot over time, especially the coefficients with respect to the interest rate and the inflation rate. Moreover, agents seem to adjust their belief coefficients, and thus the implied expectations, quite substantially during recession periods.

6 Dynamics of a Government Spending Shock

Since the belief coefficients in the forecasting models of the agents vary over time, the transmission of shocks in the model will do so as well. Figure 3 plots the estimated responses of some key macroeconomic variables to a government spending shock of one percent of GDP. The law of motion under learning (cf. equation (21)) allows us to plot the impulse responses for each quarter. For ease of comparison, the impulse responses of the rational expectations model are plotted at the beginning of the sample.

The responses of output to the government spending shock are always positive on impact. For most periods, the learning model also generates a positive effect on private consumption, although the effect becomes slightly negative in the 1990s and the first half of the 2000s. The positive impact on consumption is in sharp contrast with the negative effect under rational expectations. Under rational expectations, agents are completely forward-looking and fully incorporate the negative wealth effect of higher future taxes. By contrast, under Kalman filter learning agents do not take this negative wealth effect directly

Parameter	Description	Prior distribution			Rational expectations model			Kalman filter learning model		
		Type	Mean	Std.	Mean	Mode	90% HPD interval	Mean	Mode	90% HPD interval
<i>Structural parameters</i>										
γ_p	Indexation of prices to past inflation	B	0.5	0.1	0.277	0.267	[0.239,0.317]	0.4268	0.4298	[0.414,0.436]
γ_w	Indexation of wages to past inflation	B	0.5	0.1	0.378	0.378	[0.298,0.459]	0.5135	0.5211	[0.505,0.525]
θ_p	Degree of nominal price rigidity	B	0.75	0.05	0.86	0.86	[0.837,0.874]	0.7612	0.761	[0.756,0.764]
θ_w	Degree of nominal wage rigidity	B	0.75	0.05	0.685	0.685	[0.658,0.723]	0.648	0.6502	[0.645,0.653]
$100(\bar{\Pi} - 1)$	Quarterly steady-state inflation rate	G	0.5	0.1	0.544	0.544	[0.44,0.654]	0.696	0.6928	[0.687,0.706]
ρ_R	Degree of interest rate smoothing	B	0.75	0.1	0.618	0.618	[0.587,0.655]	0.836	0.8388	[0.831,0.84]
ϕ	Inverse Frisch elasticity of labour supply	N	2	0.25	2.275	2.275	[2.145,2.49]	1.954	1.9482	[1.945,1.967]
ϕ_π	Taylor rule inflation rate coefficient	N	1.5	0.1	1.063	1.063	[1.055,1.088]	1.519	1.5175	[1.514,1.528]
$\phi_{\Delta y}$	Taylor rule output growth coefficient	N	0.125	0.05	0.13	0.13	[0.103,0.153]	0.0728	0.0692	[0.0658,0.0789]
σ	Degree of risk aversion	G	1.5	0.37	1.181	1.181	[1.104,1.258]	1.0744	1.0847	[1.06,1.086]
s''	Investment adjustment cost parameter	N	4	1.5	2.003	2.003	[2,2.452]	5.3082	5.3135	[5.293,5.318]
ρ_b	Risk premium shock AR coefficient	B	0.5	0.2	0.633	0.633	[0.555,0.809]	0.7289	0.7262	[0.725,0.734]
ρ_g	Government expenditure AR coefficient	B	0.5	0.2	0.997	0.997	[0.992,0.999]	0.9943	0.995	[0.993,0.995]
ρ_π	Price mark-up shock AR coefficient	B	0.5	0.2	0.42	0.42	[0.305,0.53]	0.6379	0.6322	[0.629,0.647]
ρ_τ	Monetary policy shock AR coefficient	B	0.25	0.1	0.489	0.489	[0.416,0.528]	0.4802	0.4816	[0.474,0.488]
ρ_i	Investment shock AR coefficient	B	0.5	0.2	0.051	0.051	[0.0157,0.0929]	0.1025	0.1071	[0.0894,0.111]
ρ_w	Wage mark-up AR coefficient	B	0.5	0.2	0.992	0.992	[0.989,0.995]	0.967	0.9606	[0.961,0.972]
μ_w	Wage mark-up shock MA coefficient	B	0.5	0.2	0.788	0.788	[0.698,0.852]	0.7073	0.7048	[0.701,0.716]
μ_π	Price mark-up shock MA coefficient	B	0.5	0.2	0.331	0.331	[0.235,0.459]	0.6368	0.635	[0.631,0.643]
σ_0	Scale of $\beta_{1 0}$ covariance matrix $\mathbf{P}_{1 0}$	G	0.04	0.03	—	—	—	0.0124	0.012	[0.0093,0.0152]
σ_v	Scale of belief covariance matrix \mathbf{V}	G	0.004	0.003	—	—	—	0.0109	0.0106	[0.0106,0.0114]
<i>Marginal likelihood</i>										
							-928.64			
							-892.57			

Note: B represents beta, G gamma, IG inverse gamma, and N normal.

Table 1: Prior and posterior distributions of the model parameters under Kalman filter learning and under rational expectations.

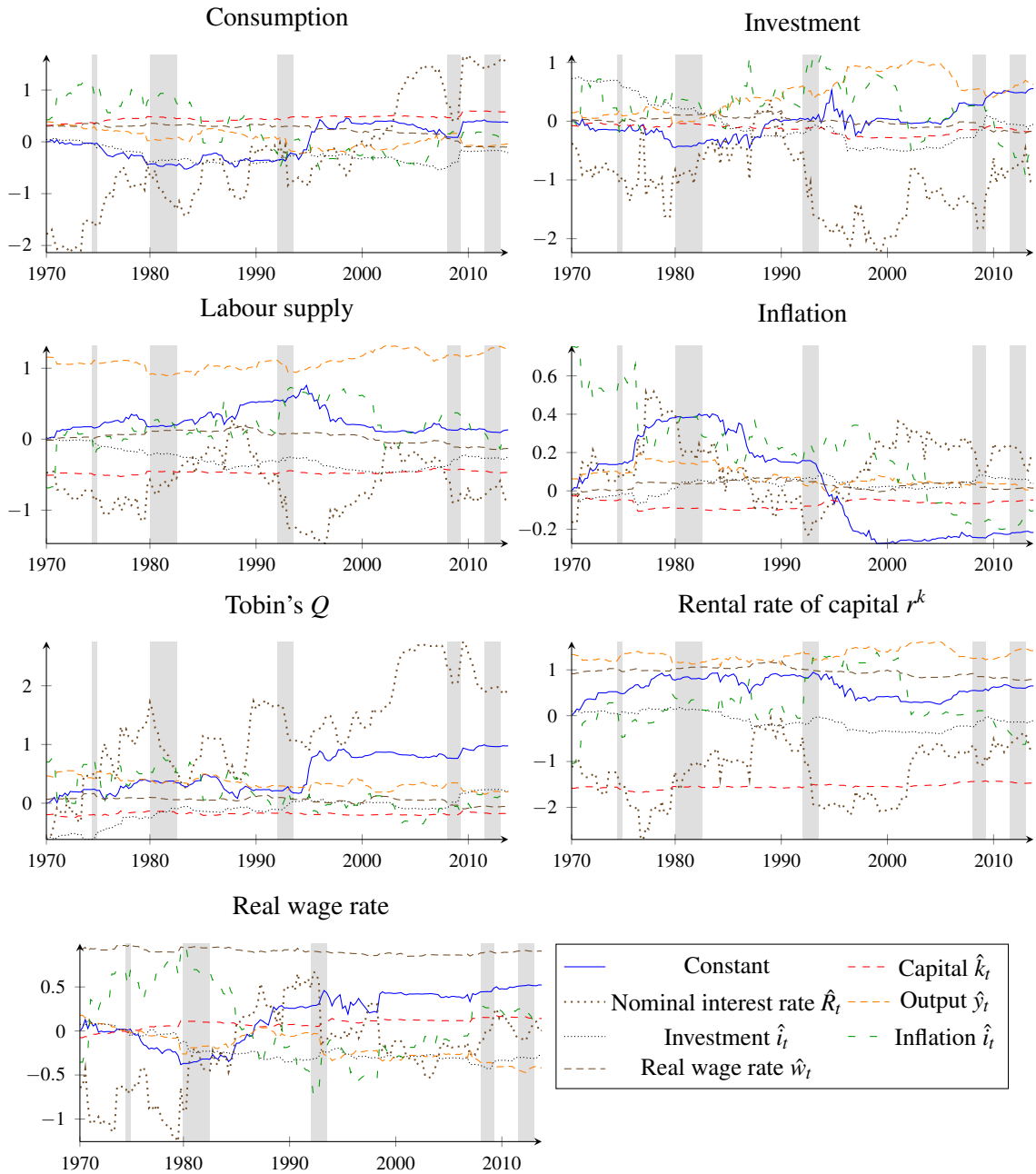


Figure 2: Estimated beliefs in the forecasting models of the agents. The shaded grey areas are quarters of recessions as defined by the CEPR Euro Area Business Cycle Dating Committee

Parameter	Prior distribution			Rational expectations model			Kalman filter learning model		
	Type	Mean	Std.	Mean	Mode	90% HPD interval	Mean	Mode	90% HPD interval
σ_b	IG	0.1	2	0.046	0.046	[0.027,0.11]	0.7	0.7	[0.7,0.71]
σ_g	IG	0.1	2	0.18	0.18	[0.16,0.2]	0.19	0.19	[0.19,0.2]
σ_i	IG	0.1	2	0.98	0.98	[0.87,1.07]	1.15	1.15	[1.14,1.16]
σ_{π^*}	IG	0.02	2	0.026	0.026	[0.02,0.038]	0.061	0.061	[0.059,0.064]
σ_{π}	IG	0.1	2	0.15	0.15	[0.14,0.18]	0.2	0.2	[0.19,0.2]
σ_r	IG	0.1	2	0.15	0.15	[0.14,0.17]	0.1	0.1	[0.1,0.11]
σ_w	IG	0.1	2	0.31	0.31	[0.27,0.35]	0.42	0.42	[0.4,0.43]
σ_z	IG	0.1	2	0.83	0.83	[0.76,0.92]	0.84	0.84	[0.83,0.84]

Note: B represents beta, G gamma, IG inverse gamma, and N normal.

Table 2: Prior and posterior distributions of the standard deviations of the shocks under Kalman filter learning and under rational expectations.

into account when forming their beliefs. Hence, the learning mechanism provides an intuitive explanation for the positive response of consumption found in several empirical studies such as Burriel et al. (2010), for the euro area, and Blanchard and Perotti (2002), Fatás and Mihov (2001), Galí et al. (2007), and Perotti (2004), for the United States.¹⁰

The impact responses of output are high in the beginning of the sample but decline in the 1980s and 1990s. This declining trend over the sample is particularly interesting since it is in accordance with the empirical evidence found in Kirchner et al. (2010). If we examine the components of aggregate demand more closely, the downward trend is mainly driven by a decreasing impact of government spending on private consumption.

An important observation is the persistent medium- and long-term adverse effect of a government spending shock on private demand in the 1970s and – to a lesser extent – the beginning of the 1980s. In that period, time variation in private sector expectations may be driven by the oil shock in 1973 and the early 1980s recession. In response to the oil shock several European countries increased nominal short-term interest rates. The oil shock also triggered a significant drop in GDP growth over the course of 1974 (see Figure 1). In the model, both the rise in the nominal interest rate and the drop in output and investment, led agents to revise their expectations according to the Kalman filter learning mechanism. Figure 2 shows how the coefficients in the agents' forecasting models were updated in those years. Similarly, that graph illustrates how movements in the observed data series before and during the recession of the early 1980s, led to significant shifts in agents' belief coefficients. Especially, the coefficients for the nominal interest rate and the inflation rate were revised substantially during that period.

The response of private investment is always positive on impact. This is in line with the euro area estimates of Burriel et al. (2010) and Kirchner et al. (2010). On the other hand, this finding is in sharp

¹⁰Empirical evidence regarding the response of private consumption to a government spending shock is not conclusive, however. Studies using the structural vector autoregression method usually find that private consumption increases after a positive government spending shock. The narrative method of Ramey and Shapiro (1998), on the other hand, typically finds the opposite. Perotti (2008) provides an extensive discussion of the two methods.

contrast with the empirical evidence for the United States. Blanchard and Perotti (2002) and Mountford and Uhlig (2009), for instance, find a decline in investment in response to a positive government spending shock. Burriel et al. (2010) relate this difference between the euro area and the US to the reaction of the nominal interest rate. They find that in the euro area the nominal interest rate reacts more gradually to a government spending shock. This slower interest rate increase may dampen the adverse effects on private demand. Indeed, the bottom left panel of Figure 3 depicts a hump-shaped reaction of the nominal interest rate to the spending shock for most of the sample periods. Another observation is that, similar to Kirchner et al. (2010), the responses are in general relatively small. However, in the beginning of the 1980s government spending had an important crowding-out effect on private investment in the medium and long run. On the other hand, from the second half of the 1980s onwards, government spending shocks had positive effects on private investment, both in the short and long run.

Turning now to the labour market variables, the impulse response functions of Figure 3 show that a government spending shock has a sizeable effect on employment throughout the whole sample. Consistent with most theoretical and empirical evidence, the employment effect on impact is always positive.¹¹ At the long horizon, the response of employment is close to zero from the second half of the 1980s onwards.

The reaction of real wages displays a downward trend. In the first half of the sample, the wage rate goes up by 0.26 to 0.68 percent on impact. Towards the end of the sample, however, the initial response of the real wage is close to zero. Real wages adjust only gradually to the government spending shock. Especially from the second half of the 1980s onwards, the positive effects of government spending shocks on real wages are very persistent.

7 The Government Spending Multiplier

7.1 Results

Learning behaviour by the agents in our model generates endogenous variation in the government spending multipliers. Figure 4 shows how the present-value multipliers for output, private consumption, and investment vary over time. We calculate the multipliers on impact and at one, four, and eight years after the shock. Following Mountford and Uhlig (2009), at quarter $t = \{1970Q2, \dots, 2013Q4\}$ the present-value multiplier for variable X over a k -period horizon is calculated as

$$\frac{PV(\Delta X)}{PV(\Delta G)} \Big|_t = \frac{\sum_{s=0}^k (\bar{R}/\bar{\Pi})^{-s} X_{t+s}}{\sum_{s=0}^k (\bar{R}/\bar{\Pi})^{-s} G_{t+s}} \frac{1}{\bar{G}/\bar{X}}, \quad (29)$$

where X_{t+s} is the response of variable X at period $t+s$, G_{t+s} is government spending at period $t+s$, $\bar{R}/\bar{\Pi}$ is the steady state gross real interest rate, and \bar{G}/\bar{X} is the steady state government expenditure to X ratio.

The upper left panel of Figure 4 shows the impact multiplier of government spending for output. The

¹¹See Perotti (2008) for a survey of the evidence on the effects of government spending shocks on labour market outcomes.

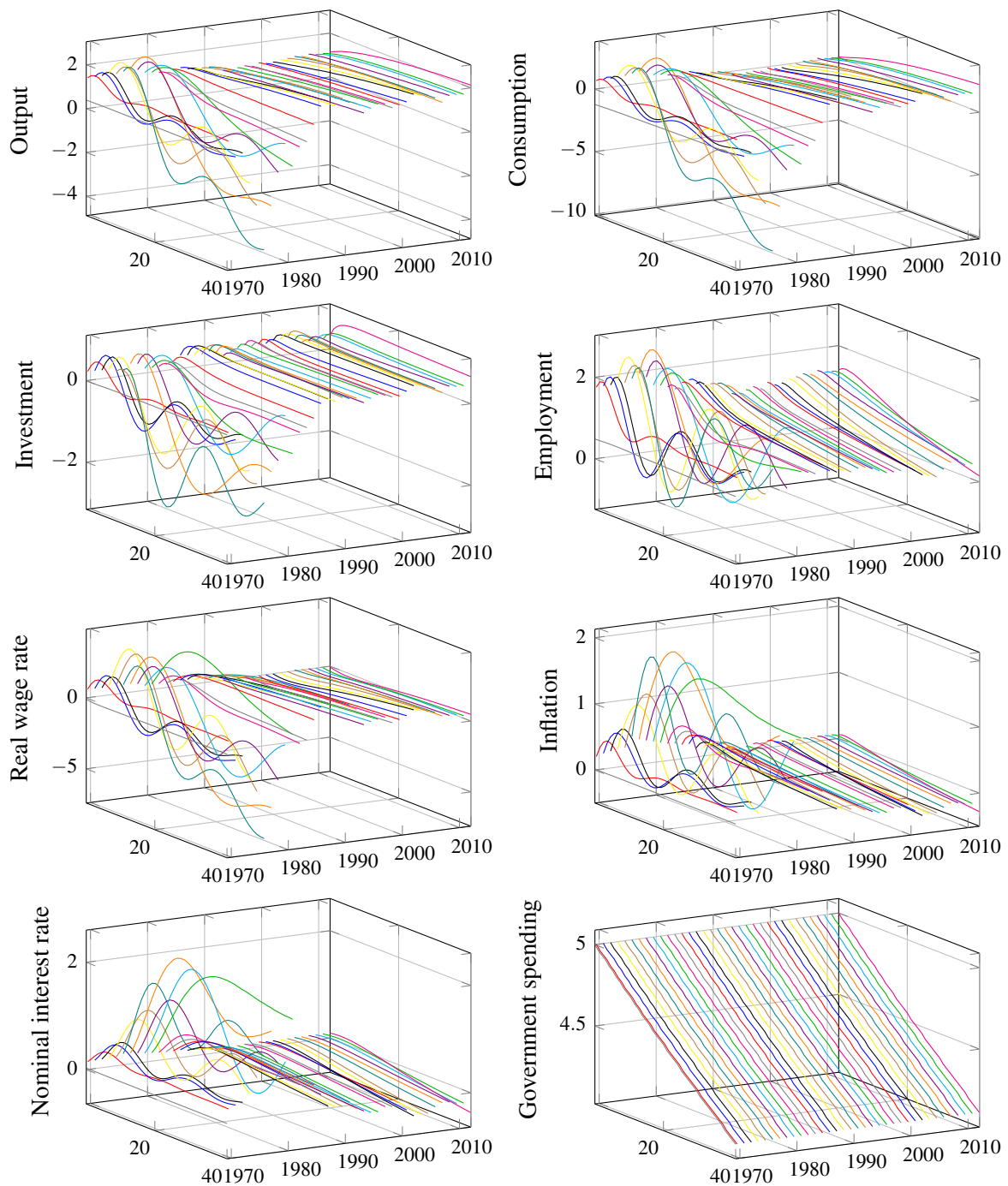


Figure 3: Impulse responses to an increase in government spending of 1% of GDP. Pseudo impulse responses are reported since the beliefs are held constant during the transition of the shock. The impulse responses at the beginning of the sample are those of the rational expectations model. The impulse response functions are calculated at the posterior mode and measured in percentage deviations from steady-state.

multiplier under learning ranges from 0.91 to 1.56. By contrast, the multiplier under rational expectations is only 0.43.¹² In particular, the impact multiplier under learning is at its highest level in the first half of the sample. It peaks in the last quarter of 1977 and the second quarter of 1986, at a value of 1.56, and 1.31, respectively. During the 1980s and first half of the 1990s the multiplier declines. In the second quarter of 1994 the impact multiplier reaches an absolute minimum of 0.91. However, an important observation is the improvement in the multiplier over the past two decades. In particular, the impact multiplier on output reaches a local maximum of 1.17 in the 2008-2009 recession.

The present-value multipliers for output at medium and long horizons are shown in the upper right panel of Figure 4. The medium and long-run multipliers vary considerably in the 1970s. Eight years after the shock, the output multipliers becomes negative for most periods between 1975Q1 and 1979Q4. From the 1980s onwards, however, the multiplier is always positive in the first eight years after the shock. However, for most of the periods the long-run multiplier (eight years after the shock) is clearly positive and lies around 1.5 in the last two decades. Looking at the entire sample, the long-run multiplier reaches maximum values of in 1980Q1 (2.57), 1990Q1 (1.87), and 2013Q2 (1.92). On the whole, the medium- and long-term multipliers for output display significant time variation.

The second row of panels in Figure 4 shows the government spending multipliers for private consumption. The multiplier on impact and after one year follows the same pattern over time as the output multiplier. The multiplier is positive most of the time, but declines sharply in the 1980s and the beginning of the 1990s. The positive consumption multipliers for a considerable part of the sample are in sharp contrast to the negative rational expectations multiplier of -0.58 . On impact, the consumption multiplier under learning ranges range from -0.15 in the second quarter of 1995 to 0.48 in the last quarter of 1977. The consumption multiplier generated by our learning model is on average 0.11, which is smaller than the estimate of 0.48 obtained by Burriel et al. (2010). In the most recent years of the sample (2000–2013), the impact multiplier lies around 0.03. At longer horizons, the multiplier varies a lot during the 1970s, but is always positive since the second quarter of 1984. In the last two decades of the sample, the long-run multiplier is on average 0.38.

The present-value multipliers for investment are plotted in the bottom panels of Figure 4. The impact multipliers are always positive and range from 0.03 in the first quarter of 1982 to 0.13 in the second quarter of 2005. The present-value multiplier one year after the shock is also always positive. At long horizons, the multiplier is negative for most of the quarters in the 1970s and beginning of the 1980s. However, from the second quarter of 1983 onwards, the present-value of the long-run multiplier (eight years after the shock) is always positive. In the recent two decades, the long-run multiplier is close to 0.16.

¹²The multiplier under rational expectations refers to the multiplier evaluated at the posterior mode of the rational expectations model (see Table 1). An important observation is that the different size of the government spending multiplier does not stem from differences in the estimated structural parameters between the learning model and the rational expectations model. Evaluated at the posterior mode of the learning model, the output multiplier under rational expectations is 0.39, which is clearly below the values under adaptive learning and close to 0.43, i.e. the multiplier evaluated at the posterior mode of the rational expectations model.

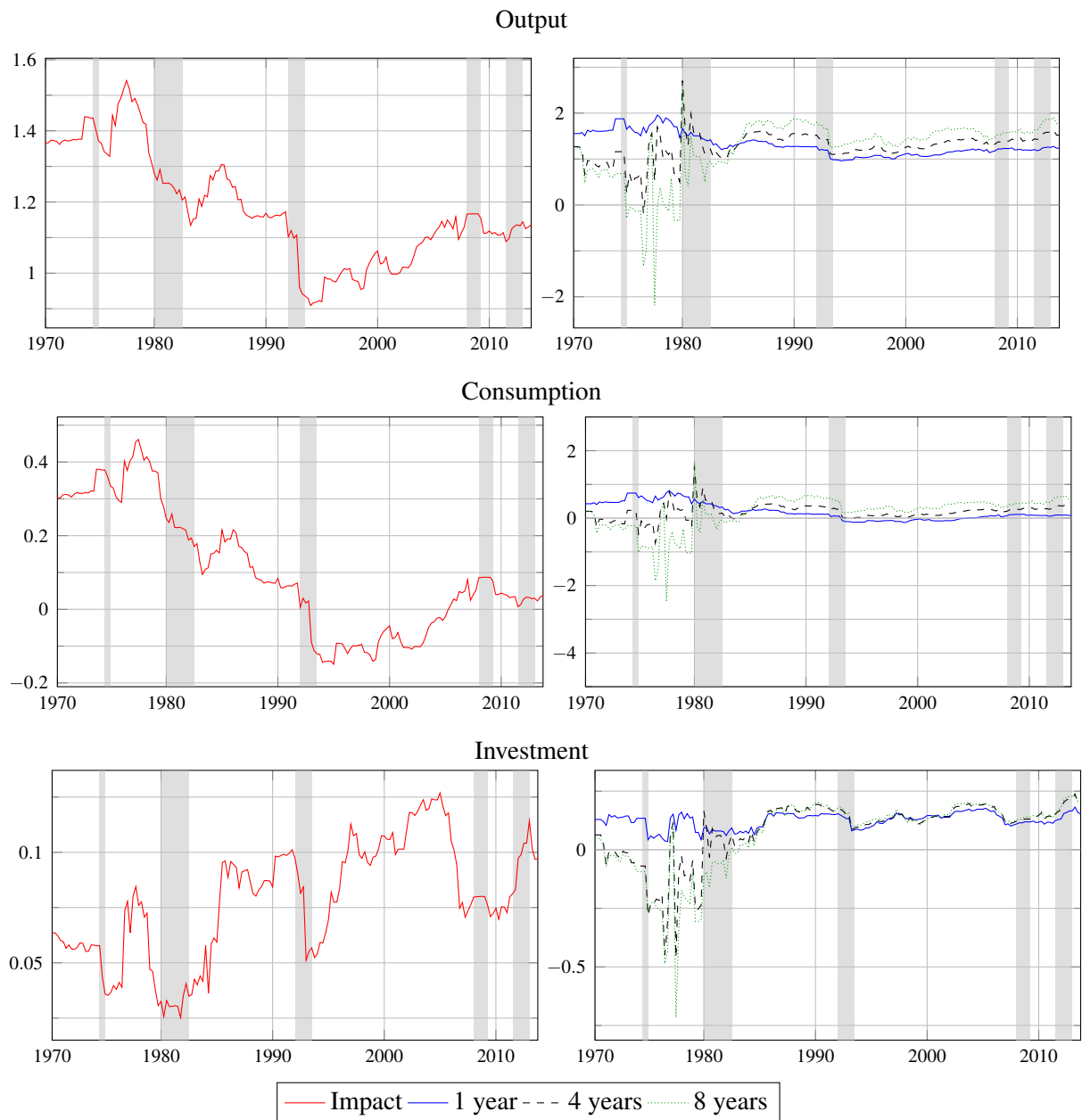


Figure 4: Present value government spending multipliers at selected horizons. The shaded grey areas are quarters of recessions as defined by the CEPR Euro Area Business Cycle Dating Committee.

7.2 Discussion

We now turn to discussion of the size and time variation of the government spending multipliers in the learning model. The intuition for the higher multiplier in that model, is that the future effects of the government spending shock are not fully anticipated. Most importantly, this leads to a different reaction of private consumption and, hence, overall economic activity. Two effects are key for understanding the difference with the rational expectations model. First, the negative wealth effect of future higher taxes is not fully anticipated. Under rational expectations, this effect almost inevitably leads to a drop in private consumption. Under adaptive learning, the consumption response may be smaller or even reversed if the forecasting model under-estimates this negative wealth effect. Second, rational consumers anticipate a rise in future real interest rates. This motivates households to postpone consumption. Again, if the path of future interest rates is not correctly forecasted under learning, the consumption response may differ from rational expectations.

Generally speaking, the effects of a government spending shock heavily depend on the expectations agents have about the future effects of the shock. In the learning model, these expectations vary over time, which results in endogenous variation in the government spending multiplier. Figure 5 illustrates how changes in expectations contributed to the time variation of the multipliers for output and consumption. The figure focusses on the contribution of expected future consumption ($E_t^*C_{t+1}$) and expected future inflation ($E_t^*\Pi_{t+1}$) because the updating of the forecasting models for these two variables had a significant effect on the evolution of the multipliers.¹³ This is not surprising, as equation (4) shows that the agents' consumption choice is directly based on these expectations.

The dashed red lines depict the multipliers when the belief coefficients in the forecasting model for consumption are held fixed to their 1980Q1 values. Consequently, in this scenario expected future consumption $E_t^*C_{t+1}$ after the government spending shock will not change over the period 1980Q1–2013Q4. Comparing this counterfactual scenario with the baseline leads to three findings. First, the peak in the output and consumption multiplier in the second quarter of 1986 can be attributed to higher consumption expectations after the government spending shock. Second, downward revisions of expected consumption can explain the negative consumption multipliers in the 1990s and the first half of the 2000s. If the belief coefficients in the consumption forecasting model would not have been updated, the consumption multiplier was close to zero in that period. Third, the updating of the belief coefficients can explain the (slight) increase of the multipliers in the last two decades of the sample.

The dotted brown lines show the multipliers when the beliefs in the forecasting model for inflation are held fixed. It is clear that the updating of the inflation forecasting model downplayed the impact of government spending on consumption and output. If the forecasting model would not have been updated, the output multiplier at the end of the sample (2013Q4) would have been almost identical to the multiplier in 1980Q1.

A growing literature investigates the dependency of the government spending multiplier on the stance

¹³Figure 8 on page 29 of Appendix A also shows the impact multipliers when the beliefs in the forecasting models of the other forward-looking variables are held constant.

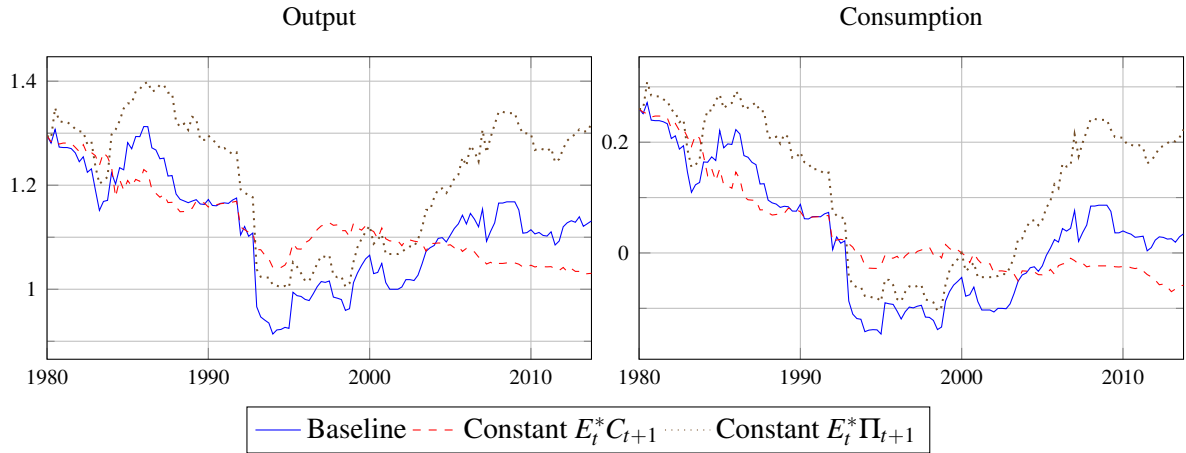


Figure 5: Impact multipliers for counterfactual evolutions of the belief parameters.

of the business cycle. Auerbach and Gorodnichenko (2012, 2013), for example, find that the U.S. government spending multiplier on output is considerably larger in recessions than in expansions. However, Owyang et al. (2013) do not find evidence for this state dependency. Looking at the evolution of the multipliers in Figure 4, the learning model does not always generate higher multipliers during recession periods (grey shaded areas), although the output multiplier reaches a local maximum in the 2008-2009 recession. Moreover, agents seem to have adjusted their beliefs quite substantially during the recession of the early 1990s which has led to a smaller multiplier.

Several contributions to the literature highlight alternative driving forces behind the time variation in the government spending multipliers. Christiano et al. (2011) and Woodford (2011), for instance, show that the government spending multiplier is large when the zero lower bound on nominal interest rates is binding. Other empirical studies explain variation in spending multipliers based on other factors such as changes in private debt overhang (Bernardini and Peersman, 2015), asset market participation (Bilbiie et al., 2008), and the composition of government spending (Kirchner et al., 2010). A formal evaluation of the importance of learning behaviour relative to these other factors is an important direction for future research.

8 Robustness analysis

In this section we check the robustness of our results with respect to the choice of the agents' forecasting models and the choice of the sample period.¹⁴

¹⁴The time series of the impact multipliers for the alternative model specifications are depicted in Figure 9 on page 30 of Appendix A.

8.1 Alternative learning schemes

Table 3 reports the marginal likelihood of our model for different assumptions regarding the formation of expectations. In Figure 6 the distribution of the impact multipliers for output, consumption, and investment under the different assumptions are summarised by box-plots.

Autoregressive model In the baseline model agents believe that the regression coefficients β_t in their forecasting models follow a random walk process. Although this is a common specification in the literature, Slobodyan and Wouters (2012a) specify the dynamics in β_t as a vector autoregressive process

$$vec(\beta_t - \bar{\beta}) = \mathbf{F}vec(\beta_{t-1} - \bar{\beta}) + \mathbf{v}_t, \quad (30)$$

where $\mathbf{F} = \rho\mathbf{I}$, with $\rho \leq 1$. As before, the shocks \mathbf{v}_t are i.i.d. with covariance matrix \mathbf{V} . The Kalman filter equations (19) and (20) then become

$$\left(\beta_{t|t-1} - \bar{\beta}\right) = \mathbf{F}\left(\beta_{t-1|t-1} - \bar{\beta}\right), \quad (31)$$

$$\mathbf{P}_{t|t-1} = \mathbf{F} \cdot \mathbf{P}_{t-1|t-1} \cdot \mathbf{F}^T + \mathbf{V}, \quad (32)$$

$$\beta_{t|t} = \beta_{t|t-1} + \mathbf{K}_t \left[\mathbf{y}_t^f - \mathbf{X}_{t-1}^T \beta_{t|t-1} \right], \quad (33)$$

$$\mathbf{P}_{t|t} = (\mathbf{I} - \mathbf{K}_t \mathbf{X}_{t-1}^T) \mathbf{P}_{t|t-1}. \quad (34)$$

Since the learning parameters σ_0 , σ_v and ρ are not jointly identified, Slobodyan and Wouters (2012a) fix σ_0 and σ_v to some plausible values and estimate the autoregressive parameter ρ using a uniform prior over $[0, 1]$. As a robustness exercise, we follow the same approach and fix σ_0 and σ_v to the values in Slobodyan and Wouters (2012a). In contrast to the previous authors, this autoregressive specification of the belief process does not improve the marginal likelihood of the model relative to the random walk specification of the baseline model. Comparing the box-plots in the second panel of Figure 6 with those of the baseline model, this alternative specification does not affect the distribution of the multipliers a lot, although it slightly reduces the time variation. Moreover, the posterior estimates and impulse response functions to a government spending shock, are very similar to the baseline model.

Alternative perceived law of motion We also experimented with a different specification of the forecasting model. Recall that the agents use the forecasting model $y_{j,t}^f = \mathbf{X}_{j,t-1}^T \beta_{j,t-1} + u_{j,t}$, also known as the Perceived Law of Motion (PLM). In the baseline model, the data vector $\mathbf{X}_{j,t-1}^T$ is the same for every forecasting model and consists of the capital stock, the nominal interest rate, output, the real wage rate, investment, and inflation. Hence, the PLM consists of all the endogenous state variables of the model. The last row of Table 3 shows the marginal likelihood of the model when private consumption is added to the PLM. We find that the baseline model outperforms this alternative specification in terms of marginal likelihood. However, this alternative PLM has an important impact on the distribution of the government spending multipliers. The right panel of Figure 6 shows that consumption multiplier in this model is

Model specification	Marginal likelihood
<i>Rational expectations equilibrium</i>	−928.64
<i>Kalman filter learning</i>	
– Baseline model	−892.57
– Autoregressive model	−893.40
– PLM with consumption	−913.95

Table 3: Log marginal likelihood of different model specifications.

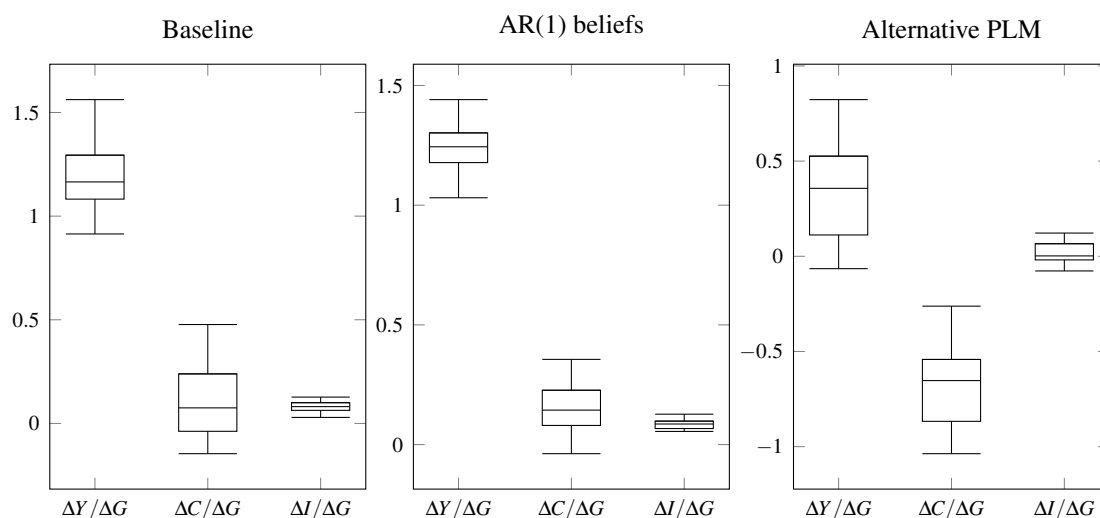


Figure 6: Box-plots for the impact multipliers for different model specifications. The bottom and top of the box are the first and third quartiles. The whiskers represent the minimum and maximum multipliers over the sample.

negative, leading to a significantly lower value of the output multiplier.

8.2 Alternative sample periods

As a robustness exercise, we also estimated the model for different sub-periods: the “Pre-EMU” period (1970Q2–1998Q4), the “post-EMU” period (1999Q1–2013Q4), and the “Great Moderation” (1984Q1–2007Q4). Table 4 compares the marginal likelihood under Kalman filter learning and rational expectations for four different sample periods.¹⁵ Notice that in every sample period there is strong evidence for the learning model relative to the rational expectations benchmark in terms of marginal likelihood.

First, we compare the estimation results for the period before and after the introduction of the euro. The “Pre-EMU” and “Post-EMU” rows in Table 4 show that in both periods Kalman filter learning improves on the rational expectations model in terms of marginal likelihood. Second, in a supplementary estimation we restrict the sample to the “Great Moderation” period (1984Q1–2007Q4) to check if the

¹⁵See Table 5 and Table 6 on pages 31 and 32 for the posterior modes of the structural parameters over the different sub-periods.

Sample period		Rational expectations	Kalman filter learning
Pre-EMU	1970Q2–1998Q4	−594.3	−544.82
Post-EMU	1999Q1–2013Q4	−470.66	−451.64
Great Moderation	1984Q1–2007Q4	−715.88	−585.14
Entire sample	1970Q2–2013Q4	−928.64	−892.57

Table 4: Log marginal likelihood under Rational Expectations and under Kalman Filter Learning for different sample periods.

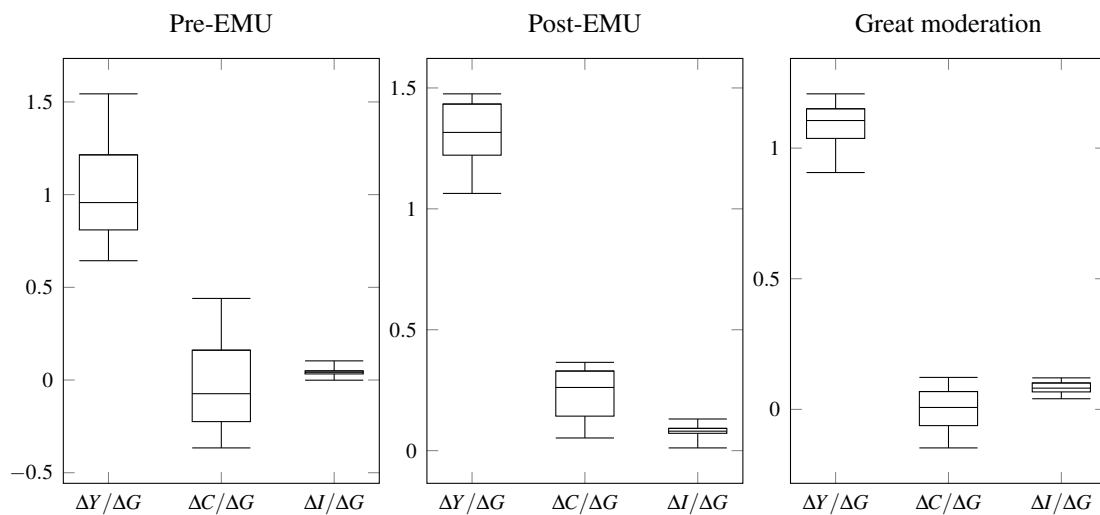


Figure 7: Box-plots for the impact multipliers for different sample periods. The bottom and top of the box are the first and third quartiles. The whiskers represent the minimum and maximum multipliers over the sample.

results are not blurred by the high output and inflation volatility in the 1970s and the non-standard monetary policy measures in the aftermath of the financial and economic crisis of 2008. Also for this sub-period Kalman filter learning significantly improves the marginal likelihood of the model. In summary, Kalman filter learning improves upon the rational expectations model in sub-periods considered.

From the box-plots in Figure 7 (and the time series in Figure 9 on page 30 of Appendix A) we can see that the pre-EMU estimation leads to somewhat lower multipliers towards the end of the pre-EMU period. On the other hand, the multipliers in the post-EMU period are estimated somewhat higher than in the full sample. The estimation of the “Great Moderation” period leads to an evolution of the multipliers that is very similar to the full sample estimation. Generally speaking, the learning model also generates time variation in the multipliers within each sub-sample. This observation suggests that the time variation in the full sample does not only stem from structural differences between the sub-samples.

9 Conclusion

In this paper, we have developed a medium-scale DSGE model with adaptive learning to investigate the transmission of government spending shocks in the euro area. In particular, agents form expectations using a forecasting model with belief coefficients that are updated using the Kalman filter. We compare the model dynamics under this learning mechanism with those under rational expectations and find that the learning mechanism significantly improves the marginal likelihood of the model. Moreover, the updating of the belief coefficients generates time variation in the macroeconomic responses to a government spending shock. Hence, the expectations channel provides an endogenous explanation for time-varying government spending multipliers. In contrast to the time-varying parameter VAR approach, this variation does not stem from some random variation in the structural parameters of the model. In fact, variation in the government spending multipliers is an endogenous outcome of the model, generated by agents learning to forecast future macroeconomic variables.

We find that the responses of output to the government spending shock are always positive on impact. For most periods, the learning model also generates a positive effect on private consumption, although the effect becomes slightly negative in the 1990s and the first half of the 2000s. The rational expectations model, on the other hand, finds a significant drop in private consumption after the shock. Hence, learning behaviour provides an explanation for the crowding-in effect of government spending on private consumption found in several empirical studies. Another difference with rational expectations, is the positive reaction of private investment to a government spending shock. This positive investment response is in line with the empirical findings for the euro area provided by Burriel et al. (2010) and Kirchner et al. (2010). In general, the responses to a government spending shock under Kalman filter learning are significantly different from those under rational expectations.

Another important observation is that Kalman filter learning generates time variation in the effects of a government spending shock, especially at the medium and long horizon. For example, although the effects on aggregate demand are always positive on impact, in the 1970s and beginning of the 1980s,

government spending shocks had significant negative effects on private demand in the medium- and long-run. On the other hand, the long-term multipliers on output, consumption and investment are always positive from the second half of the 1980s onwards.

The learning approach provides a natural explanation for time variation in the transmission of government spending shocks. Obviously, this variation could also stem from other time-varying factors not considered in our analysis. The literature provides a list of factors that may explain time variation in the fiscal transmission mechanism. A formal evaluation of the importance of learning behaviour relative to these other factors is left for future research.

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A Additional results

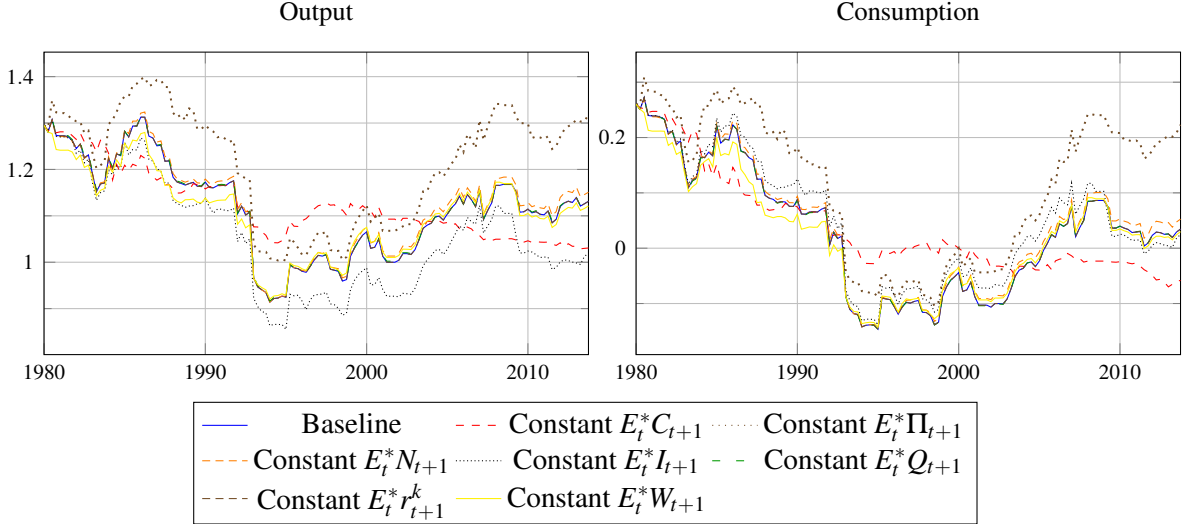


Figure 8: Impact multipliers for counterfactual evolutions of the belief parameters.

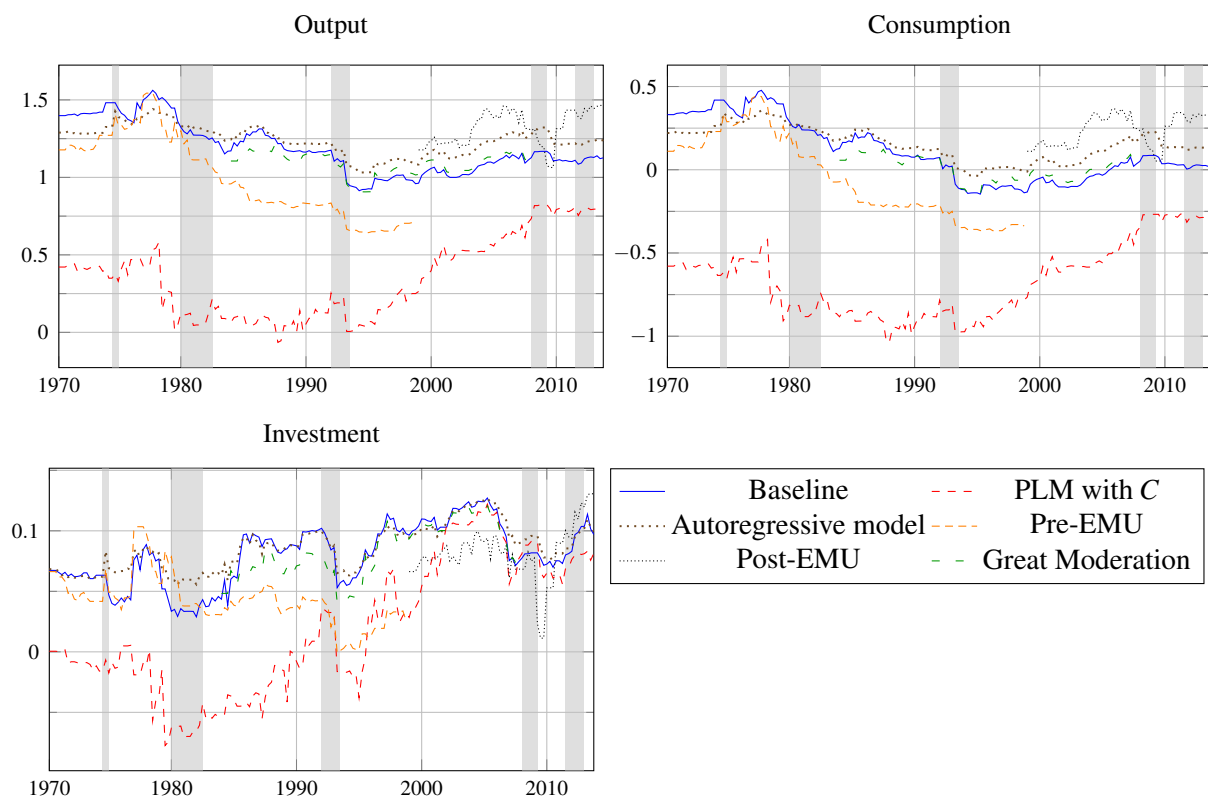


Figure 9: Impact multipliers for different model specifications and different sample periods. The shaded grey areas are quarters of recessions as defined by the CEPR Euro Area Business Cycle Dating Committee.

	pre-EMU		post-EMU		Great Moderation		Entire sample	
	1970Q2–1998Q4	1999Q1–2013Q4	1984Q1–2007Q4	1970Q2–2013Q4	KF	RE	KF	RE
<i>Structural parameters</i>								
γ_p	0.53	0.32	0.44	0.34	0.15	0.57	0.43	0.27
Indexation of prices to past inflation	0.61	0.43	0.41	0.65	0.48	0.49	0.52	0.38
γ_w	0.8	0.84	0.83	0.92	0.91	0.79	0.76	0.86
Indexation of wages to past inflation	0.69	0.7	0.7	0.73	0.78	0.69	0.65	0.69
Degree of nominal price rigidity	0.48	0.29	0.69	0.51	0.4	0.61	0.69	0.54
Degree of nominal wage rigidity	0.75	0.58	0.84	0.66	0.72	0.86	0.84	0.62
$100(\bar{\pi} - 1)$	1.78	2.59	2.06	2.71	2.59	1.95	1.95	2.27
Quarterly steady-state inflation rate	1.41	1.06	1.52	1.22	1.08	1.57	1.52	1.06
Degree of interest rate smoothing	0.07	0.13	0.12	0.22	0.15	0.067	0.069	0.13
ρ_R	1.07	1.24	1.06	0.68	1.13	1.11	1.08	1.18
Degree of Frisch elasticity of labour supply	5.34	2	5.69	4.85	6.23	5.45	5.31	2
ϕ	0.62	0.29	0.71	0.22	0.13	0.68	0.73	0.63
Taylor rule inflation rate coefficient	1	0.97	1	0.99	0.96	0.99	1	1
$\phi_{\Delta y}$	0.45	0.98	0.72	0.28	0.85	0.8	0.63	0.42
Degree of risk aversion	0.44	0.41	0.46	0.19	0.19	0.3	0.48	0.49
Investment adjustment cost parameter	0.051	0.027	0.05	0.11	0.036	0.047	0.11	0.051
Risk premium shock AR coefficient	0.77	0.86	0.88	1	0.81	0.74	0.96	0.99
Government expenditure AR coefficient	0.68	0.66	0.4	0.77	0.38	0.54	0.7	0.79
Price mark-up shock AR coefficient	0.66	0.96	0.6	0.17	0.68	0.71	0.64	0.33
Monetary policy shock AR coefficient	0.053	-	0.02	-	0.0047	-	0.012	-
Investment shock AR coefficient	0.003	-	0.019	-	0.0009	-	0.011	-
Wage mark-up shock MA coefficient	-544.82	-594.30	-451.64	-470.66	-585.14	-715.88	-892.57	-928.64
Wage mark-up shock MA coefficient								
Price mark-up shock MA coefficient								
Scale of $\beta_{1 0}$ covariance matrix $\mathbf{P}_{1 0}$								
Scale of belief covariance matrix \mathbf{V}								
<i>Marginal likelihood</i>								

Note: KF represents Kalman filter learning and RE represents rational expectations.

Table 5: Comparison of the marginal likelihoods and the posterior modes of the model parameters for different sample periods.

	pre-EMU 1970Q2–1998Q4		post-EMU 1999Q1–2013Q4		Great Moderation 1984Q1–2007Q4		Entire sample 1970Q2–2013Q4	
	KF	RE	KF	RE	KF	RE	KF	RE
σ_b	0.77	0.048	0.57	0.046	0.047	0.65	0.7	0.046
σ_g	0.19	0.18	0.2	0.18	0.16	0.16	0.19	0.18
σ_i	1.09	0.97	0.93	0.71	0.75	0.96	1.15	0.98
σ_{π^*}	0.044	0.04	0.016	0.01	0.01	0.011	0.061	0.026
σ_{π}	0.17	0.18	0.17	0.15	0.12	0.21	0.2	0.15
σ_r	0.12	0.17	0.082	0.15	0.16	0.1	0.1	0.15
σ_w	0.4	0.34	0.29	0.25	0.24	0.41	0.42	0.31
σ_z	0.76	0.82	0.86	0.83	0.73	0.71	0.84	0.83

Note: KF represents Kalman filter learning and RE represents rational expectations.

Table 6: Comparison of the posterior modes of the standard deviations of the shocks under Kalman filter learning and under rational expectations for different sample periods.