Adaptive Learning and the Transmission of Government Spending Shocks in the Euro Area

Ewoud Quaghebeur

Belgian Macroeconomics Workshop, University of Namur

12 September 2017

FACULTY OF ECONOMICS AND
BUSINESS ADMINISTRATION

Adaptive Learning and the Transmission of Government Spending Shocks in the Euro Area

Ewoud Quaghebeur

Belgian Macroeconomics Workshop, University of Namur

12 September 2017

GHENT
UNIVERSITY

Motivation

Expectations

- ... are important drivers of business cycle fluctuations.

Motivation

Expectations

- ... are important drivers of business cycle fluctuations.
- ... play a key role in explaining the impact of government spending shocks.

Motivation

Expectations

- ... are important drivers of business cycle fluctuations.
- ... play a key role in explaining the impact of government spending shocks.

Example: government spending increase

Motivation

Expectations

- ... are important drivers of business cycle fluctuations.
- ... play a key role in explaining the impact of government spending shocks.

Example: government spending increase

- Scenario 1: fully forward-looking "rational" expectations

Motivation

Expectations

- ... are important drivers of business cycle fluctuations.
- ... play a key role in explaining the impact of government spending shocks.

Example: government spending increase

- Scenario 1: fully forward-looking "rational" expectations
- Scenario 2: short-sighted expectations

Motivation

Rational expectations hypothesis

Motivation

Rational expectations hypothesis

Very restrictive:

Motivation

Rational expectations hypothesis

Very restrictive:

- requires full knowledge of the structure of the model

Motivation

Rational expectations hypothesis

Very restrictive:

- requires full knowledge of the structure of the model
- time invariant transmission

Motivation

Rational expectations hypothesis

Very restrictive:

- requires full knowledge of the structure of the model
- time invariant transmission

Adaptive learning

Motivation

Rational expectations hypothesis

Very restrictive:

- requires full knowledge of the structure of the model
- time invariant transmission

Adaptive learning

- Agents estimate forecasting models to form expectations

Motivation

Rational expectations hypothesis

Very restrictive:

- requires full knowledge of the structure of the model
- time invariant transmission

Adaptive learning

- Agents estimate forecasting models to form expectations
- Kalman Filter Learning: forecasting models updated by the Kalman filter

Motivation

Rational expectations hypothesis

Very restrictive:

- requires full knowledge of the structure of the model
- time invariant transmission

Adaptive learning

- Agents estimate forecasting models to form expectations
- Kalman Filter Learning: forecasting models updated by the Kalman filter
- Time varying beliefs

Contribution and main findings

- Learning behaviour generates time-varying government spending multipliers

Contribution and main findings

- Learning behaviour generates time-varying government spending multipliers
- Bayesian estimation of a medium-scale DSGE model for the euro area

Contribution and main findings

- Learning behaviour generates time-varying government spending multipliers
- Bayesian estimation of a medium-scale DSGE model for the euro area
- Kalman filter learning improves the marginal likelihood of the model

Outline

The Model Economy

A medium-scale DSGE Model with Adaptive Learning

- Representative household works, consumes, invests, and buys government bonds
- Labour market with unions and employment agencies
- Intermediate and final good producers
- Central bank follows a generalised Taylor-rule
- Fiscal authority finances expenditure through lump-sum taxes

Detailed description

Model dynamics

The linear model can be represented as

$$\mathbf{A}_0\mathbf{y}_{t-1} + \mathbf{A}_1\mathbf{y}_t + \mathbf{A}_2 E_t^*\mathbf{y}_{t+1} + B_0\epsilon_t = cst$$

where $\mathbf{y}_t$ is the vector of log-linearised model variables.

Linearised equations

Model dynamics

The linear model can be represented as

$$\mathbf{A}_0\mathbf{y}_{t-1} + \mathbf{A}_1\mathbf{y}_t + \mathbf{A}_2 E_t^*\mathbf{y}_{t+1} + B_0\boldsymbol{\epsilon}_t = cst$$

where $\mathbf{y}_t$ is the vector of log-linearised model variables.

▸ Linearised equations

→ Rational expectations equilibrium (REE)

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{T}\mathbf{y}_{t-1} + \mathbf{R}\boldsymbol{\epsilon}_t$$

Model dynamics

The linear model can be represented as

$$\mathbf{A}_0\mathbf{y}_{t-1} + \mathbf{A}_1\mathbf{y}_t + \mathbf{A}_2 E_t^*\mathbf{y}_{t+1} + B_0\boldsymbol{\epsilon}_t = cst$$

where $\mathbf{y}_t$ is the vector of log-linearised model variables.

▸ Linearised equations

→ Rational expectations equilibrium (REE)

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{T}\mathbf{y}_{t-1} + \mathbf{R}\boldsymbol{\epsilon}_t$$

→ Adaptive learning

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \mathbf{T}_t\mathbf{y}_{t-1} + \mathbf{R}_t\boldsymbol{\epsilon}_t$$

Expectation Formation

Expectation Formation

Rational Expectations

$$E_t^* y_{t+1}^f = E_t \left(y_{t+1}^f | \boldsymbol{\Omega}_t \right)$$

Expectation Formation

Rational Expectations

$$E_t^* y_{t+1}^f = E_t \left(y_{t+1}^f | \mathbf{\Omega}_t \right)$$

Adaptive learning

$$E_t^* y_{t+1}^f = \mathbf{X}_{t-1} \beta_{t-1}$$

Expectation Formation

Rational Expectations

$$E_t^* y_{t+1}^f = E_t \left(y_{t+1}^f | \mathbf{\Omega}_t \right)$$

Adaptive learning

$$E_t^* y_{t+1}^f = \mathbf{X}_{t-1} \beta_{t-1}$$

- Kalman filter learning: beliefs β_t updated using the Kalman filter
  ▷ Sargent and Williams (2005); Slobodyan and Wouters (2012a); Branch and Evans (2006); Sargent (1999)

Agents' forecasting model

Regression model

- Seven forward-looking variables: consumption, investment, labour supply, Tobin's Q, rental rate of capital, wage rate, and inflation rate
- Baseline:

$$y^f_{j,t} = \begin{bmatrix} 1 & \hat{k}_{t-1} & \hat{R}_{t-1} & \hat{y}_{t-1} & \hat{w}_{t-1} & \hat{i}_{t-1} & \hat{\Pi}_{t-1} \end{bmatrix} \beta_{j,t-1} + u_{j,t},$$
$$j = 1, 2, \ldots, 7$$

Agents' forecasting model

Regression model

- Seven forward-looking variables: consumption, investment, labour supply, Tobin's Q, rental rate of capital, wage rate, and inflation rate
- Baseline:

$$y^f_{j,t} = \left[\begin{array}{ccccccc} 1 & \hat{k}_{t-1} & \hat{R}_{t-1} & \hat{y}_{t-1} & \hat{w}_{t-1} & \hat{i}_{t-1} & \hat{\Pi}_{t-1} \end{array}\right] \boldsymbol{\beta}_{j,t-1} + u_{j,t},$$

$j = 1, 2, \ldots, 7$

Agents' beliefs

Agents believe that the regression coefficients $\boldsymbol{\beta}_{j,t}$ evolve according to

$$vec\left(\boldsymbol{\beta}_t\right) = vec\left(\boldsymbol{\beta}_{t-1}\right) + \mathbf{v}_t, \qquad \mathbf{v}_t \sim i.i.d.\left(\mathbf{0}, \mathbf{V}\right)$$

Kalman filter

- In general: iterative process to estimate unknown parameters based on consecutive data inputs

Kalman filter

- In general: iterative process to estimate unknown parameters based on consecutive data inputs
- Iterative estimation of beliefs β_t

Kalman filter

- In general: iterative process to estimate unknown parameters based on consecutive data inputs
- Iterative estimation of beliefs β_t

$$\begin{aligned}\beta_{t+1|t} &= \beta_{t|t-1} + \mathbf{K}_t \left[\mathbf{y}_t^f - \mathbf{X}_{t-1}^T \beta_{t|\text{t-1}}\right] \\ \mathbf{P}_{t+1|t} &= \left(\mathbf{I} - \mathbf{K}_t \mathbf{X}_{t-1}^T\right) \mathbf{P}_{t|t-1} + \mathbf{V}\end{aligned}$$

with $\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{X}_{t-1} \left[\mathbf{X}_{t-1}^T \mathbf{P}_{t|t-1} \mathbf{X}_{t-1} + \mathbf{\Sigma}\right]^{-1}$.

Additional slides

Estimation

Estimation

Bayesian estimation using the DYNARE 4.2.4. MATLAB toolbox, modified by Slobodyan and Wouters to allow for adaptive learning.

Vector of estimated parameters consists of

- structural parameters:

$$\left(\gamma_p, \gamma_w, \theta_p, \theta_w, 100(\bar{\Pi} - 1), \rho_R, \phi, \phi_\pi, \phi_{\Delta y}, \sigma, s''\right)$$

- shock processes parameters:

$$(\rho_b, \rho_g, \rho_\pi, \rho_r, \rho_i, \rho_w, \mu_w, \mu_\pi, \sigma_b, \sigma_g, \sigma_i, \sigma_\pi, \sigma_{\pi^*}, \sigma_r, \sigma_w, \sigma_z)$$

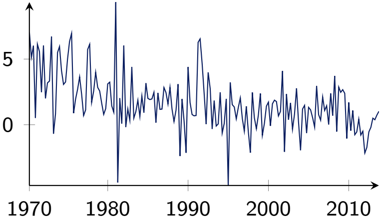
- learning parameters: σ_0 and σ_v

I fix $\alpha = 1/3$, $100(\beta^{-1} - 1) = 0.25$, $\delta = 2.5\%$, $\bar{G}/\bar{Y} = 20\%$, $\bar{\epsilon}_p = 0.2$, $\bar{\epsilon}_w = 0.1$, $\rho_{\pi^*} = 0.985$, $100(\gamma - 1) = 0.334$, and Φ.

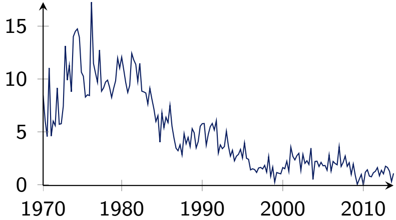
Data

Data

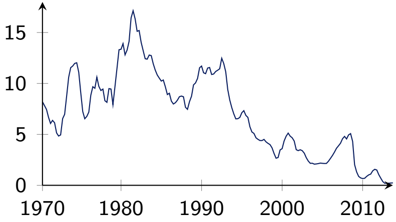
Real government consumption growth



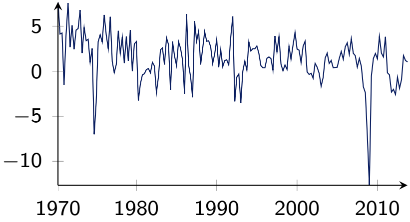
Inflation rate



Nominal interest rate

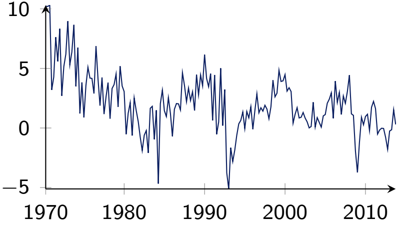


Real GDP growth

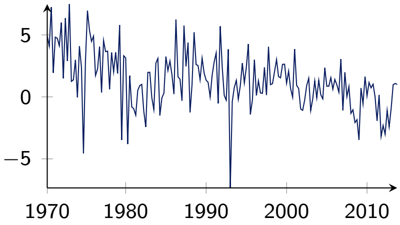


Data (cont.)

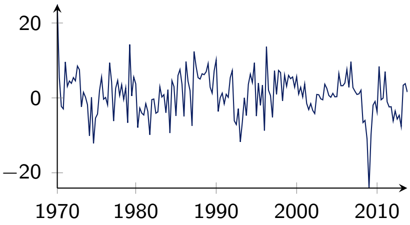
Real wage growth



Real consumption growth



Real investment growth



Results

Results

- Substantial evidence in favour of the learning model

Results

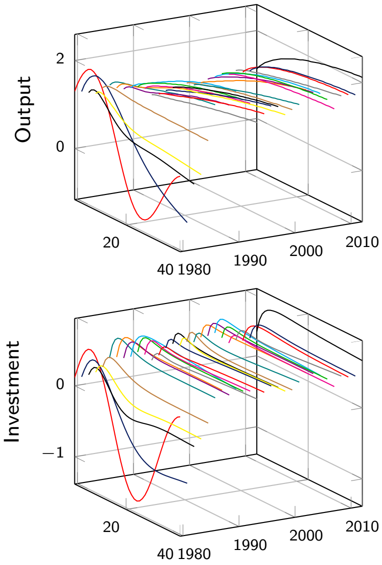
- Substantial evidence in favour of the learning model
- Posterior estimates: assumption on expectations affects some of the parameter values but majority of 90% credible intervals overlap substantially

Results

- Substantial evidence in favour of the learning model
- Posterior estimates: assumption on expectations affects some of the parameter values but majority of 90% credible intervals overlap substantially
- Time-varying impulse responses after a government spending shock

Transmission of government spending shocks

Transmission of government spending shocks



Government spending multipliers

Government spending multipliers

- Measures the effect of government spending on
  - Output
  - Private consumption
  - Private investment

Government spending multipliers

- Measures the effect of government spending on
  - Output
  - Private consumption
  - Private investment
- At different horizons

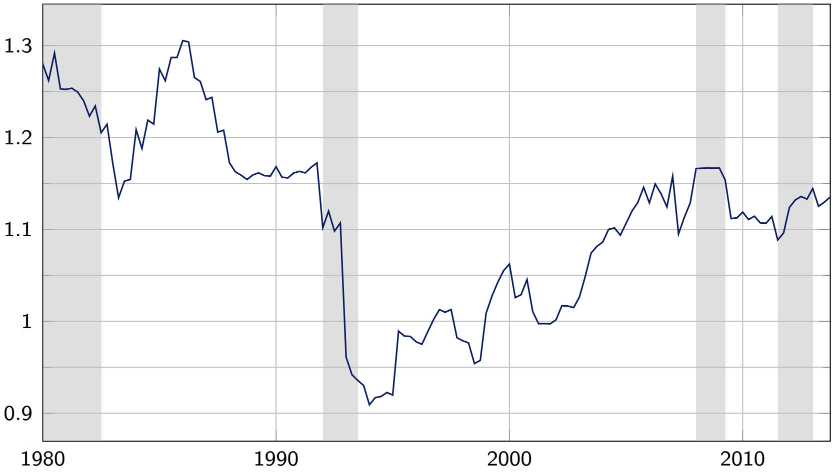
Government spending multipliers

- Measures the effect of government spending on
  - Output
  - Private consumption
  - Private investment
- At different horizons
- Present-value multipliers

$$\left.\frac{PV(\Delta X)}{PV(\Delta G)}\right|_t = \frac{\sum_{s=0}^{k} \bar{R}^{-s} X_{t+s}}{\sum_{s=0}^{k} \bar{R}^{-s} G_{t+s}} \frac{1}{\bar{G}/\bar{X}},$$

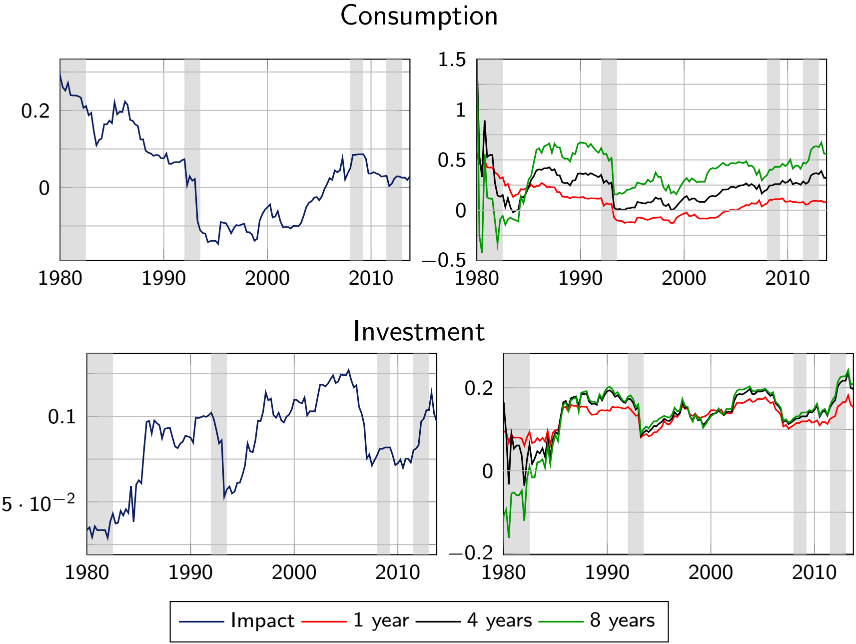
  - X_{t+s} is the response of variable X at period $t+s$,
  - G_{t+s} is government spending at period $t+s$,
  - $\bar{R}$ is the steady state gross nominal interest rate
  - $\bar{G}/\bar{X}$ is the steady state government expenditure to X ratio.

Impact multiplier for output



Medium- and long-term multiplier for output

Consumption and investment multipliers



Time-varying government spending multipliers

- Kalman filter learning generates time variation in the effects of a government spending shock.

Time-varying government spending multipliers

- Kalman filter learning generates time variation in the effects of a government spending shock.
- Findings are similar to time-varying parameter Vector Autoregression (VAR) of Kirchner et al. (2010)

Time-varying government spending multipliers

- Kalman filter learning generates time variation in the effects of a government spending shock.
- Findings are similar to time-varying parameter Vector Autoregression (VAR) of Kirchner et al. (2010)
- Alternative explanations
  - Private debt overhang (Bernardini and Peersman, 2015)
  - Asset market participation, stance of monetary policy (Bilbiie et al., 2008)
  - Government debt-to-GDP ratio, composition of government spending (Kirchner et al., 2010)

Conclusion

Conclusion

- Substantial evidence in favour of the learning mechanism relative to rational expectations.

Conclusion

- Substantial evidence in favour of the learning mechanism relative to rational expectations.
- Responses after a government spending shock are significantly different from those under rational expectations.

Conclusion

- Substantial evidence in favour of the learning mechanism relative to rational expectations.
- Responses after a government spending shock are significantly different from those under rational expectations.
  - Impact multiplier for output on average 1.06 ($\leftrightarrow$ RE: 0.43).

Conclusion

- Substantial evidence in favour of the learning mechanism relative to rational expectations.
- Responses after a government spending shock are significantly different from those under rational expectations.
  - Impact multiplier for output on average 1.06 ($\leftrightarrow$ RE: 0.43).
  - Crowding-in of private consumption for most of the periods

Conclusion

- Substantial evidence in favour of the learning mechanism relative to rational expectations.
- Responses after a government spending shock are significantly different from those under rational expectations.
  - Impact multiplier for output on average 1.06 ($\leftrightarrow$ RE: 0.43).
  - Crowding-in of private consumption for most of the periods
- Expectations channel provides an endogenous explanation for time-varying government spending multipliers.

Household

- Utility-maximising household chooses consumption, bond purchases and investment.
- King et al. (1988) utility specification

$$U(C_t, 1 - N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \exp\left(\frac{\sigma - 1}{1+\phi} N_t^{1+\phi}\right)$$

  - Compatible with balanced growth along the steady state
  - Complementarity between consumption and labour supply if $\sigma > 1$.
- Christiano et al. (2005) capital adjustment costs

Labour unions and employment agencies

Cf. Schmitt-Grohé and Uribe (2006).

Labour union

- Sells differentiated labour inputs on monopolistically competitive labour markets.
- Wage setting à la Calvo (1983)
- In those markets, nominal wages are indexed according to

$$\tilde{W}_t(j) = z_t \left(\Pi_t^*\right)^{1-\gamma_w} \left(\Pi_{t-1}\right)^{\gamma_w} \tilde{W}_{t-1}(j)$$

Labour unions and employment agencies

Cf. Schmitt-Grohé and Uribe (2006).

Labour union

- Sells differentiated labour inputs on monopolistically competitive labour markets.
- Wage setting à la Calvo (1983)
- In those markets, nominal wages are indexed according to

$$\tilde{W}_t(j) = z_t \left(\Pi_t^*\right)^{1-\gamma_w} \left(\Pi_{t-1}\right)^{\gamma_w} \tilde{W}_{t-1}(j)$$

Employment agency

- Bundles the differentiated labour supplies and sells it to the intermediate goods producers

$$N_t(i) = \left[\int_0^1 N_t\left(i,j\right)^{\frac{1}{1+\epsilon_{w,t}}} dj\right]^{1+\epsilon_{w,t}}$$

Firms: final good producer

- Representative, perfectly competitive firm
- Bundles a continuum of intermediate goods
- Production function

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\epsilon_{p,t}}} di \right)^{1+\epsilon_{p,t}}$$

where $1 + \epsilon_{p,t}$ is an ARMA(1,1) price mark-up shock.

Firms: intermediate goods producers

- Monopolistically competitive firms populating $[0, 1]$
- Each firm rents labour and capital so as to minimise costs.

Firms: intermediate goods producers

- Monopolistically competitive firms populating $[0, 1]$
- Each firm rents labour and capital so as to minimise costs.
- Production function

$$Y_t(i) = A_t^{1-\alpha} K_{t-1}(i)^{\alpha} N_t(i)^{1-\alpha} - \Phi A_t$$

where technology A_t evolves according to
$A_t / A_{t-1} = \gamma \exp(\epsilon_t^Z)$.

Firms: intermediate goods producers

- Monopolistically competitive firms populating $[0, 1]$
- Each firm rents labour and capital so as to minimise costs.
- Production function

$$Y_t(i) = A_t^{1-\alpha} K_{t-1}(i)^{\alpha} N_t(i)^{1-\alpha} - \Phi A_t$$

where technology A_t evolves according to $A_t/A_{t-1} = \gamma \exp(\epsilon_t^Z)$.

- Staggered price setting à la Calvo (1983). If a firm cannot re-optimise, its price is indexed according to

$$\tilde{P}_t(i) = (\Pi_t^*)^{1-\gamma_p} (\Pi_{t-1})^{\gamma_p} \tilde{P}_{t-1}(i)$$

Inflation equation

Government policies

Central bank

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\hat{\Pi}_t^* + \rho_R \left(\hat{\Pi}_t^* - \hat{\Pi}_{t-1}^*\right)$$
$$+ (1-\rho_R)\left[\phi_\pi(\hat{\Pi}_t - \hat{\Pi}_t^*)\right] + \phi_{\Delta y}\Delta\hat{y}_t + \hat{u}_t^r$$

Return

Government policies

Central bank

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\hat{\Pi}^*_t + \rho_R \left(\hat{\Pi}^*_t - \hat{\Pi}^*_{t-1}\right)$$
$$+ (1-\rho_R)\left[\phi_\pi(\hat{\Pi}_t - \hat{\Pi}^*_t)\right] + \phi_{\Delta y}\Delta\hat{y}_t + \hat{u}^r_t$$

- Time-varying inflation target Π^*_t: $\hat{\Pi}^*_t = \rho_{\pi^*}\hat{\Pi}^*_{t-1} + \epsilon^{\pi^*}_t$
- Monetary policy shock $\hat{u}^r_t = \rho_r \hat{u}^r_{t-1} + \epsilon^r_t$

Return

Government policies

Central bank

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\hat{\Pi}_t^* + \rho_R \left(\hat{\Pi}_t^* - \hat{\Pi}_{t-1}^*\right) + (1-\rho_R)\left[\phi_\pi(\hat{\Pi}_t - \hat{\Pi}_t^*)\right] + \phi_{\Delta y}\Delta\hat{y}_t + \hat{u}_t^r$$

- Time-varying inflation target Π_t^*: $\hat{\Pi}_t^* = \rho_{\pi^*}\hat{\Pi}_{t-1}^* + \epsilon_t^{\pi^*}$
- Monetary policy shock $\hat{u}_t^r = \rho_r \hat{u}_{t-1}^r + \epsilon_t^r$

Fiscal authority

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_t^g$$

Return

Agents' forecasting model III

We can write the forecasting model in a SURE format

$$y^f_{j,t} = \mathbf{X}^T_{j,t-1}\beta_{j,t-1} + u_{j,t}, \qquad j = 1, 2, \ldots, m$$

$$\Updownarrow$$

$$\begin{bmatrix} y^f_{1,t} \\ y^f_{2,t} \\ \vdots \\ y^f_{m,t} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{1,t-1} & 0 & \cdots & 0 \\ 0 & \mathbf{X}_{2,t-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}_{m,t-1} \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \vdots \\ \beta_{m,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{bmatrix}$$

$$\Updownarrow$$

$$\mathbf{y}^f_t = \mathbf{X}_{t-1}\beta_{t-1} + \mathbf{U}_t, \qquad \mathbf{\Sigma} = E\left[\mathbf{U}_t\mathbf{U}^T_t\right]$$

Real wage equation

$$\hat{w}_t = w_1 \left(\phi \hat{N}_t + \hat{c}_t - \hat{w}_t\right) + w_2 \hat{w}_{t-1} + w_3 E_t^* \hat{w}_{t+1} + w_4 \hat{\Pi}_t + w_5 \hat{\Pi}_{t-1} + w_6 E_t^* \hat{\Pi}$$

with

$$
\begin{aligned}
w_1 &= \frac{(1-\theta_w)\left(1-\beta\theta_w\gamma^{1-\sigma}\right)}{\theta_w\left(1+\beta\gamma^{1-\sigma}\right)}, \\
w_2 &= \frac{1}{1+\beta\gamma^{1-\sigma}}, \\
w_3 &= \frac{\beta\gamma^{1-\sigma}}{1+\beta\gamma^{1-\sigma}}, \\
w_4 &= -\frac{1+\beta\gamma^{1-\sigma}\gamma_w}{1+\beta\gamma^{1-\sigma}}, \\
w_5 &= \frac{\gamma_w}{1+\beta\gamma^{1-\sigma}}, \\
w_6 &= \frac{\beta\gamma^{1-\sigma}}{1+\beta\gamma^{1-\sigma}},
\end{aligned}
$$

New Keynesian Phillips curve

$$\hat{\Pi}_t = \pi_1 \hat{MC}_t + \pi_2 \hat{\Pi}_{t-1} + \pi_3 E_t^* \hat{\Pi}_{t+1} + \pi_4 \hat{\Pi}_t^* + \hat{u}_t^{\pi}$$

with

$$\begin{aligned}
\pi_1 &= \frac{(1-\theta_p)\left(1-\beta\theta_p\gamma^{1-\sigma}\right)}{\theta_p\left(1+\beta\gamma^{1-\sigma}\gamma_p\right)}, \\
\pi_2 &= \frac{\gamma_p}{1+\beta\gamma^{1-\sigma}\gamma_p}, \\
\pi_3 &= \frac{\beta\gamma^{1-\sigma}}{1+\beta\gamma^{1-\sigma}\gamma_p}, \\
\pi_4 &= \frac{(1-\gamma_p)\left(1-\rho_{\pi^*}\beta\gamma^{1-\sigma}\right)}{1+\beta\gamma^{1-\sigma}\gamma_p}.
\end{aligned}$$

Return

Log-linearised equations

$$\hat{y}_t = \left(1 - \frac{\bar{i}}{\bar{y}} - \frac{\bar{g}}{\bar{y}}\right)\hat{c}_t + \frac{\bar{i}}{\bar{y}}\hat{i}_t + \frac{\bar{g}}{\bar{y}}\hat{g}_t$$

$$\hat{y}_t = \frac{\bar{y} + \Phi}{\bar{y}}\left[\alpha \hat{k}_{t-1} + \hat{z}_t + (1-\alpha)\hat{N}_t\right]$$

$$\hat{c}_t = E_t^{*}\hat{c}_{t+1} + c_1\left(\hat{N}_t - E_t^{*}\hat{N}_{t+1}\right) - c_2\left(\hat{R}_t - E_t^{*}\hat{\Pi}_{t+1}\right) + \hat{u}_t^{b}$$

$$\hat{w}_t = w_1\left(\phi \hat{N}_t + \hat{c}_t - \hat{w}_t\right) + w_2\hat{w}_{t-1} + w_3 E_t^{*}\hat{w}_{t+1} + w_4\hat{\Pi}_t + w_5\hat{\Pi}_{t-1} + w_6 E_t^{*}\hat{\Pi}_{t}$$

$$\hat{i}_t = i_1\left(\hat{i}_{t-1} - \hat{z}_t\right) + (1 - i_1)\,E_t^{*}\hat{i}_{t+1} + i_2\hat{Q}_t + \hat{u}_t^{i}$$

$$\hat{Q}_t = -\left(\hat{R}_t - E_t^{*}\hat{\Pi}_{t+1} - \sigma\hat{u}_t^{b}\right) + \beta\gamma^{-\sigma}\left[\bar{r}^{k} E_t^{*}\hat{r}_{t+1}^{k} + (1-\delta)\,E_t^{*}\hat{Q}_{t+1}\right]$$

$$\hat{k}_t = k_1(\hat{k}_{t-1} - \hat{z}_t) + (1 - k_1)\,\hat{i}_t + k_1\hat{u}_t^{i}$$

Log-linearised equations (continued)

$$\hat{\Pi}_t = \pi_1 \widehat{MC}_t + \pi_2 \hat{\Pi}_{t-1} + \pi_3 E_t^* \hat{\Pi}_{t+1} + \pi_4 \hat{\Pi}_t^* + \hat{u}_t^\pi$$

$$\hat{w}_t = \hat{M}C_t + \alpha(\hat{k}_{t-1} - \hat{N}_t) + \hat{z}_t$$

$$\hat{r}_t^k = \hat{M}C_t + (\alpha - 1)(\hat{k}_{t-1} - \hat{N}_t) + \hat{z}_t$$

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1-\rho)\hat{\Pi}_t^* + \rho \left(\hat{\Pi}_t^* - \hat{\Pi}_{t-1}^* \right) + (1-\rho) \left[\phi_\pi (\hat{\Pi}_t - \hat{\Pi}_t^*) + \phi_y \hat{y}_t \right] + \phi_{\Delta y} \Delta y_t + \hat{u}_t^r$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_t^g$$

Return

Model dynamics

Recall the linear approximation of the model

$$\mathbf{A}_0 \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{A}_1 \begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} + \mathbf{A}_2 E_t^* \mathbf{y}_{t+1} + B_0 \epsilon_t = cst$$

Model dynamics

Recall the linear approximation of the model

$$\mathbf{A}_0 \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{A}_1 \begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} + \mathbf{A}_2 E_t^* \mathbf{y}_{t+1} + B_0 \epsilon_t = cst$$

- The estimate $\beta_{t|t-1}$ from the Kalman filter is substituted in the agents' forecasting model to generate $E_t^* \mathbf{y}_{t+1}$.

Model dynamics

Recall the linear approximation of the model

$$\mathbf{A}_0 \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{A}_1 \begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} + \mathbf{A}_2 E_t^* \mathbf{y}_{t+1} + B_0 \epsilon_t = cst$$

- The estimate $\beta_{t|t-1}$ from the Kalman filter is substituted in the agents' forecasting model to generate $E_t^* \mathbf{y}_{t+1}$.
- Actual law of motion under learning

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} = \boldsymbol{\mu}_t + \mathbf{T}_t \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{R}_t \epsilon_t.$$

Kalman filter

- In general: iterative process to estimate unknown parameters based on consecutive data inputs

Kalman filter

- In general: iterative process to estimate unknown parameters based on consecutive data inputs
- Iterative estimation of beliefs β_t

Kalman filter

- In general: iterative process to estimate unknown parameters based on consecutive data inputs
- Iterative estimation of beliefs β_t

$$\begin{aligned}
\beta_{t+1|t} &= \beta_{t|t-1} + \mathbf{K}_t \left[\mathbf{y}_t^f - \mathbf{X}_{t-1}^T \beta_{t|\text{t-1}}\right] \\
\mathbf{P}_{t+1|t} &= \left(\mathbf{I} - \mathbf{K}_t \mathbf{X}_{t-1}^T\right) \mathbf{P}_{t|t-1} + \mathbf{V}
\end{aligned}$$

with $\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{X}_{t-1} \left[\mathbf{X}_{t-1}^T \mathbf{P}_{t|t-1} \mathbf{X}_{t-1} + \mathbf{\Sigma}\right]^{-1}$.

Additional slides

Initialisation

We need to specify

- $\beta_{1|0} = \bar{\beta} =$ initial belief coefficients
- $\mathbf{P}_{1|0} =$ covariance matrix of $\beta_{1|0}$
- $\mathbf{V} =$ covariance matrix of the shocks to the beliefs β_t
- $\mathbf{\Sigma} =$ covariance matrix of the shocks to forward-looking variables $\mathbf{y}_t^f$ (measurement errors)

Initialisation

We need to specify

- $\beta_{1|0} = \bar{\beta} =$ initial belief coefficients
- $\mathbf{P}_{1|0} =$ covariance matrix of $\beta_{1|0}$
- $\mathbf{V} =$ covariance matrix of the shocks to the beliefs β_t
- $\mathbf{\Sigma} =$ covariance matrix of the shocks to forward-looking variables $\mathbf{y}_t^f$ (measurement errors)

We follow Slobodyan and Wouters (2012a)

- $\beta_{1|0} = \hat{\beta}_{OLS} = E\left(\mathbf{X}^T\mathbf{X}\right)^{-1} E\left(\mathbf{X}^T\mathbf{y}^f\right)$
- $\mathbf{\Sigma} = E\left[\mathbf{U}\mathbf{U}^T\right] = E\left[\left(\mathbf{y}^f - \mathbf{X}\beta\right)\left(\mathbf{y}^f - \mathbf{X}\beta\right)^T\right]$
- $\mathbf{P}_{1|0} = \sigma_0\left(\mathbf{X}^T\mathbf{\Sigma}^{-1}\mathbf{X}\right)^{-1}$
- $\mathbf{V} = \sigma_v\left(\mathbf{X}^T\mathbf{\Sigma}^{-1}\mathbf{X}\right)^{-1}$

Initialisation: $\beta_{1|0}$ and $\mathbf{\Sigma}$

Initialisation of $\beta_{1|0}$ and $\mathbf{\Sigma}$ is based on the OLS estimator

Given that the OLS estimate

$$\hat{\beta}_{OLS} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}^f$$

is unbiased, we let

$$\beta_{1|0} = E\left(\mathbf{X}^T\mathbf{X}\right)^{-1}E\left(\mathbf{X}^T\mathbf{y}^f\right)$$

where we use the theoretical moments matrices of the Rational Expectations Equilibrium.

It follows that the covariance matrix

$$\mathbf{\Sigma} = E\left[\mathbf{U}\mathbf{U}^T\right] = E\left[(\mathbf{y}-\mathbf{X}\beta)(\mathbf{y}-\mathbf{X}\beta)^T\right]$$

Initialisation: $\mathbf{P}_{1|0}$ and $\mathbf{V}$

Recall the formulas of the GLS estimator

$$\hat{\beta}_{GLS} = \left(\mathbf{X}^T\mathbf{\Sigma}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{\Sigma}^{-1}\mathbf{y}^f$$

$$Var\left(\hat{\beta}_{GLS}\right) = \left(\mathbf{X}^T\mathbf{\Sigma}^{-1}\mathbf{X}\right)^{-1}$$

$\mathbf{P}_{1|0}$ and $\mathbf{V}$ are both taken to be proportional to $Var\left(\hat{\beta}_{GLS}\right)$:

$$\begin{aligned} \mathbf{P}_{1|0} &= \sigma_0\left(\mathbf{X}^T\mathbf{\Sigma}^{-1}\mathbf{X}\right)^{-1} \\ \mathbf{V} &= \sigma_v\left(\mathbf{X}^T\mathbf{\Sigma}^{-1}\mathbf{X}\right)^{-1} \end{aligned}$$

Motivation: with these initial values and if $\rho = 1$ and $\sigma_v = \sigma_0^2$ the mean dynamics of Kalman filter learning are approximately equal to the dynamics of constant gain RLS learning (see Sargent and Williams, 2005).

Return

Posterior estimates

Parameter	Description	Prior distribution			Posterior distribution		
		Type	Mean	Std.	Mean	Mode	Interval
γ_p	Price indexation to past inflation	B	0.5	0.1	0.4268	0.4298	[0.414,0.436]
γ_w	Wage indexation to past inflation	B	0.5	0.1	0.5135	0.5211	[0.505,0.525]
θ_p	Degree of nominal price rigidity	B	0.75	0.05	0.7612	0.761	[0.756,0.764]
θ_w	Degree of nominal wage rigidity	B	0.75	0.05	0.648	0.6502	[0.645,0.653]
$100(\bar{\Pi} - 1)$	Quarterly steady-state inflation rate	G	0.5	0.1	0.696	0.6928	[0.687,0.706]
ρ_R	Degree of interest rate smoothing	B	0.75	0.1	0.836	0.8388	[0.831,0.84]
ϕ	Inverse Frisch elasticity of labour supply	N	2	0.25	1.954	1.9482	[1.945,1.967]
ϕ_π	Taylor rule inflation rate coefficient	N	1.5	0.1	1.519	1.5175	[1.514,1.528]
$\phi_{\Delta y}$	Taylor rule output growth coefficient	N	0.125	0.05	0.0728	0.0692	[0.0658,0.0789]
σ	Degree of risk aversion	G	1.5	0.37	1.0744	1.0847	[1.06,1.086]
s''	Investment adjustment cost parameter	N	4	1.5	5.3082	5.3135	[5.293,5.318]
ρ_b	Risk premium shock AR coefficient	B	0.5	0.2	0.7289	0.7262	[0.725,0.734]
ρ_g	Government expenditure AR coefficient	B	0.5	0.2	0.9943	0.995	[0.993,0.995]
ρ_π	Price mark-up shock AR coefficient	B	0.5	0.2	0.6379	0.6322	[0.629,0.647]
ρ_r	Monetary policy shock AR coefficient	B	0.25	0.1	0.4802	0.4816	[0.474,0.488]
ρ_i	Investment shock AR coefficient	B	0.5	0.2	0.1025	0.1071	[0.0894,0.111]
ρ_w	Wage mark-up AR coefficient	B	0.5	0.2	0.967	0.9606	[0.961,0.972]
μ_w	Wage mark-up shock MA coefficient	B	0.5	0.2	0.7073	0.7048	[0.701,0.716]
μ_π	Price mark-up shock MA coefficient	B	0.5	0.2	0.6368	0.635	[0.631,0.643]
σ_0	Scale of $\beta_{1\|0}$ cov. matrix matrix $\mathbf{P}_{1\|0}$	G	0.04	0.03	0.0124	0.012	[0.0093,0.0152]
σ_v	Scale of belief cov. matrix matrix $\mathbf{V}$	G	0.004	0.003	0.0109	0.0106	[0.0106,0.0114]

Note: B represents beta, G gamma, IG inverse gamma, and N normal.

Table: Parameter estimates.

Posterior estimates

Parameter	Prior distribution			Posterior distribution		
	Type	Mean	Std.	Mean	Mode	Interval
σ_b	IG	0.1	2	0.7	0.7	[0.7,0.71]
σ_g	IG	0.1	2	0.19	0.19	[0.19,0.2]
σ_i	IG	0.1	2	1.15	1.15	[1.14,1.16]
$\sigma_{\pi*}$	IG	0.02	2	0.061	0.061	[0.059,0.064]
σ_π	IG	0.1	2	0.2	0.2	[0.19,0.2]
σ_r	IG	0.1	2	0.1	0.1	[0.1,0.11]
σ_w	IG	0.1	2	0.42	0.42	[0.4,0.43]
σ_z	IG	0.1	2	0.84	0.84	[0.83,0.84]

Note: B represents beta, G gamma, IG inverse gamma, and N normal.

Table: Parameter estimates.

References I

Bernardini, M. and Peersman, G. (2015). Private Debt Overhang And the Government Spending Multiplier: Evidence for the United States. Working Papers of Faculty of Economics and Business Administration, Ghent University, Belgium 15/901, Ghent University, Faculty of Economics and Business Administration.

Bilbiie, F. O., Meier, A., and Müller, G. J. (2008). What Accounts for the Changes in U.S. Fiscal Policy Transmission? *Journal of Money, Credit and Banking*, 40(7):1439–1470.

Branch, W. A. and Evans, G. W. (2006). A simple recursive forecasting model. *Economics Letters*, 91(2):158–166.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45.

King, R. G., Plosser, C. I., and Rebelo, S. T. (1988). Production, growth and business cycles : I. the basic neoclassical model. *Journal of Monetary Economics*, 21(2-3):195–232.

References II

Kirchner, M., Cimadomo, J., and Hauptmeier, S. (2010). Transmission of government spending shocks in the euro area: Time variation and driving forces. Working Paper Series 1219, European Central Bank.

Sargent, T. J. (1999). *The Conquest of American Inflation*. Princeton University Press.

Sargent, T. J. and Williams, N. (2005). Impacts of Priors on Convergence and Escapes from Nash Inflation. *Review of Economic Dynamics*, 8(2):360–391.

Schmitt-Grohé, S. and Uribe, M. (2006). Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model. In *NBER Macroeconomics Annual 2005, Volume 20*, NBER Chapters, pages 383–462. National Bureau of Economic Research, Inc.

Slobodyan, S. and Wouters, R. (2012a). Learning in a medium-scale DSGE model with expectations based on small forecasting models. *American Economic Journal: Macroeconomics*, 4(2):65–101.

Slobodyan, S. and Wouters, R. (2012b). Learning in an estimated medium-scale DSGE model. *Journal of Economic Dynamics and Control*, 36(1):26–46.