The Geography of Path Dependence*

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First Version: June 2017
This Version: March 2018
Preliminary and Incomplete

Abstract

How much of the distribution of economic activity today is determined by history rather than by geographic fundamentals? And if history matters, does it matter much? We develop an empirical framework that enables answers to these questions. Our model combines a workhorse model of trade subject to geographic frictions with features of local agglomeration externalities as well as an overlapping generations model of labor mobility also subject to spatial fractions. We derive parameter conditions, for arbitrary geographic scenarios, under which equilibrium transition paths are unique and yet steady states may nevertheless be non-unique — that is, where initial conditions ("history") determine long-run steady-state outcomes ("path dependence"). We then estimate the model's parameters (which govern the strength of agglomeration externalities and trade and migration frictions), by focusing on moment conditions that are robust to potential equilibrium multiplicity, using spatial variation across US counties from 1800 to the present. We then simulate a range of counterfactual scenarios that vary the initial conditions of US economic geography in order to shed light on the extent to which path dependence is costly — or equivalently, the extent to which the modern U.S. distribution of economic activity is inefficient because of the long arm of history.

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1 Introduction

Economic activity in modern economies is staggeringly concentrated. For example, almost 20% of value added in the United States is currently produced in just three cities (MSAs) that take up a mere 1.5% of its land area. But perhaps even more remarkable are the historical accidents that purportedly determined the location of those same three cities—one was a Dutch fur trading post, one a *pueblo* designated by a Spanish governor for an original 22 adult and 22 children settlers, and one a river mouth known to Algonquin natives for its distinct *chicagoua* (a wild garlic).

There is no shortage of anecdotes about how the quirks of history have shaped the location of economic activity. But how widespread should we expect these and similar examples of path dependence to be in the real-world economies around us? If the potential for path dependence is widespread, how often did historical shocks actually matter? If history matters, did it merely reshuffle the current location of economic activity? Or does the long arm of history also impact the total amount of economic activity (and, hence, notions of aggregate welfare) by effectively concentrating modern agglomerations into fundamentally inferior locations?

In this paper we develop an empirical framework for answering these questions. We build on a rich vein of theoretical modeling (as developed in, for example, Fujita, Krugman, and Venables, 1999) that outlines stylized environments—models with two or three locations and very little heterogeneity, for example—in which strong agglomeration spillover effects can give rise to a potential multiplicity of equilibria. From there we set up a dynamic, overlapping generations model of economic geography with an arbitrary number of regions separated by arbitrary trading and migration frictions, as well as arbitrary time-varying locational fundamentals; these features allow us to map the model to empirical settings in which unobserved heterogeneity is typically substantial. To this basic setup we add agglomeration spillovers in production and consumption that, if they are sufficiently strong, can create the possibility for history to matter in determining modern outcomes.

Our main set of theoretical results aim to clarify when path dependence could potentially occur. We first characterize a condition for dynamic equilibria—that is, the transition paths that would take this economy from any starting point to any presumptive steady-state—to be unique. This condition depends on two elasticities that promote dispersion (cross-locational elasticities of substitution in consumption and in migration decisions). It also hinges on the strength of two elasticities that govern contemporaneous agglomeration because the local attractiveness of a location at any point in time can potentially rise (due to local amenity and productivity spillovers), if these elasticities are strong, because of the presence of other workers in that location at that same point in time. As long as the agglomeration elasticities are not especially large, the dynamic paths of our economy will be unique for any path of geographic fundamentals.

However, even if these dynamic equilibria are unique, there may still exist multiple steady-state equilibria. Our second result clarifies that this will occur in a model such as ours only for sufficiently large values of a different source of agglomeration externalities—ones that work historically, whereby a location's productivity and amenity values might be functions of that location's *lagged* population level. These effects may capture both the accumulation of local knowledge (either productive or cultural), as well as the enduring payoffs from investments that previous generations may have made in a location's productivity (e.g. through improved roads, or the demarcation of property rights) or in a location's amenity appeal (e.g. through earmarking land for parks, or the discovery of an enjoyable climate).

When contemporaneous spillovers are relatively low, and yet historical spillovers are relatively high, dynamic paths will be unique but steady-states will exhibit multiplicity. And this will be true for any arbitrary (time path of) geographic fundamentals. In this parameter range, we say that our economy exhibits *path dependence,* because the economy's initial conditions—such as the distribution of economic activity in some early starting period, or long-irrelevant productivity shocks—will govern the long-run steady-state in which the economy will end up. In such a setting, steady-states are typically rankable in terms of aggregate welfare (so it is possible that unfortunate initial conditions could lead the economy to a particularly inferior steady-state). And every steady-state has associated with it a basin of attraction (a set of initial population distributions from which the economy will converge to that particular steady-state) whose boundaries are governed by the current and future path of geographic fundamentals.

In summary, this model exhibits the potential for rich and yet also well-behaved path-dependent dynamics. Whether such richness can obtain hinges on six elasticities: two dispersion parameters (the elasticities of trade and migration responses to payoffs), two contemporaneous spillover parameters (for production and amenities), and two historical spillover parameters (again, one for each of production and amenities).

We then set out in Section 3 to estimate these six parameters from a unique dataset on the long-run spatial history of the United States which allows us to trace local (county-level) incomes, populations, and migration flows back several centuries (to 1800) when the US Census began in full force. This estimation strategy uses trade and migration equations to infer, via market clearing conditions, the apparent attractiveness of each location-year as an origin and a destination for both trade and migration. Further, the logic of our contemporaneous and historical spillover effects suggests that these terms should be related to local contemporaneous and historical scale—and it is these expressions that provide simple 2SLS identifying moments for our estimation strategy.

Because of inevitable endogeneity in these equations we draw on an instrumental variables strategy that is based on the model's insight that other locations' geographic shifters of productivity (such as soil and elevation) or amenities (such as January temperature) should not affect any given location's own productivity or amenity values directly, after controlling for the location's own value of these shifters. We use this idea, together with the time variation driven by initial conditions and the spread of population predicted by the model, to estimate the six parameters referred to above. Importantly, this logic, and all of our moment conditions, are valid regardless of the potential for multiplicity (in dynamic paths, or steady-states) in our economy—so the usual concerns about models with non-uniqueness lacking invertibility of the mapping from data to model parameters do not arise.

Our elasticity estimates imply that the conditions for path dependence described above are indeed satisfied (though they are very close to the boundary identified in Propositions 1 and 2). So path dependence exists in the US economy from 1800-2000, in a purely qualitative sense, according to our empirical estimates.

The remaining simulation exercises in Section 4 then go beyond this qualitative result in order to assess the quantitative significance of potential path dependence. We do this by randomly reassigning the geographical incidence of various shocks to different locations—essentially, by swapping pairs of historical accidents from one location to another among pairs of locations with clustered (that is, similar in a multivariate sense) geographic characteristics. In our first set of simulations the historical shocks that are reassigned are populations in the initial period (that is, for 1800, the earliest year in the Census data). Our findings imply that, across hundreds of simulations, the distribution of population (and hence also aggregate welfare) in 2000 is remarkably invariant to the draws of initial historical conditions. In this sense, history can matter, but it matters very little.

Our second set of simulations instead shocks the transition path (in terms of productivity and amenity fundamentals) between 1850 and 2000 by reassigning shocks in 1900 and 1950. Throughout, we hold initial conditions (population levels in 1800 and fundamentals in 1850) constant at their values seen in the actual data. Shocks in 2000 are held similarly constant. This implies that any differences seen in 2000 are only due to the persistent effects of long-redundant shocks, but those differences can be substantial. In particular, not only is the distribution of population in 2000 highly variable across our simulations (implying an important role for path dependence in selecting the steady-state that is prevailing by 2000) but also the spread of aggregate welfare levels in 2000 across our simulations is wide. So in this sense path dependence can be highly consequential for not only the location but also the total amount of economic activity in a spatial economy.

These results shed new light on both theoretical and empirical studies of economic geography that have aimed to speak to the phenomenon of path dependence. An important empirical literature has sought to estimate some of the ingredients of path dependence that our Propositions 1 and 2 identify. In particular, Ellison and Glaeser (1997), Greenstone, Hornbeck, and Moretti (2010), Kline and Moretti (2014) and **?** all estimate the local agglomeration spillover effects of contemporaneous populations onto productivity and amenities. We are not aware, by contrast, of any papers that estimate both the historical and contemporaneous elasticities (effects which are highly correlated and hence difficult to disentangle) as we do here, and as our theoretical results highlight are independently important for the study of path dependence in these environments.

A separate empirical literature has focused on the search for direct evidence of path dependence itself. For example, Davis and Weinstein (2002) document the persistence of economic geography across locations in Japan over several millennia, including in response to the displacement and destruction of the second World War. However, evidence from wartime destruction elsewhere suggests the empirical context may matter for whether or not path dependence can occur, as Bosker, Brakman, Garretsen, and Schramm (2007) find evidence of multiple spatial equilibria in Germany, while persistence was confirmed in Vietnam by Miguel and Roland (2011). More recently, Bleakley and Lin (2012) describe the propensity for US cities today to be located at portage sites, locations with temporarily high demand for labor (due to waterway transshipment and other services) in about 1800—and this seems to be strong evidence for path dependence. Our theory characterizes the conditions on parameter values and the distribution of geographical fundamentals under which such divergent experiences with path dependence could both arise.

On the theory side, we draw on the insights of a theoretical literature that pioneered the understanding of the full dynamics of path-dependent geographic settings. Krugman (1991) and Matsuyama (1991), for example, developed models with two locations and infinitely-lived agents (with timing assumptions that meant that each agent made one locational decision at the start of her life). As fully elucidated in Ottaviano (1999), the dynamics of equilibrium paths (which includes those "sunspot"-like equilibria in which multiplicity derives purely from self-fulfilling expectations) in such settings are dauntingly complex, and would leave effectively no empirical predictability (or scope for parameter estimation) since at any point in time equilibria exhibit true multiplicity, with no mapping from parameters to data or vice versa. Counterfactual simulations in such models are similarly challenging due to indeterminacy. Herrendorf, Valentinyi, and Waldmann (2000) add agent-specific heterogeneity to such a model and reduce the range of parameter values under which extreme multiplicity does not arise, but the small number of regions and the symmetric conditions placed on those regions' fundamental conditions, in the cross-section and over time, make them unsuited to direct empirical analysis so we have endeavored to extract the core lessons of these setups and adapt them to our more empirical framework.

Finally, we build on recent work on quantitative economic geography models such as the static environments of Glaeser (2008), Allen and Arkolakis (2014), Ahlfeldt, Redding, Sturm, and Wolf (2015) – summarized in the excellent review article Redding and Rossi-Hansberg (2017) – as well as the recent dynamic models of Desmet, Nagy, and Rossi-Hansberg (2015) and Caliendo, Dvorkin, and Parro (2015). Our advance is to extend these tools in order to facilitate the explicit study of geographic path dependence, to estimate, in the case of 200 years of US economic geography, the six key elasticities that our extended framework highlights as essential for such a theme, and then to apply the resulting estimates to counterfactual simulations about the consequentiality of path dependence for the location and aggregate efficiency of economic activity in the US today.

2 Theoretical framework

In this section we develop a dynamic economic geography model that is amenable to the empirical study of geographic path dependence throughout US history. A large set of regions possess arbitrary, time-varying fundamentals in terms of productivity and amenities. They interact in product markets that interact with one another via (costly) trade in goods, and in labor markets that interact with one another via (costly) migration. Crucially, production and consumption both potentially involve contemporary and historical non-pecuniary spillovers—the force for potential local agglomeration externalities, and hence path dependence. We now describe each of these ingredients in turn.

2.1 Setup

There are $i \in \{1, ..., N\}$ locations and time is discrete and indexed by $t \in \{0, 1, ...\}$. Each individual lives for two periods. In the first period ("childhood"), an individual is born where her parent lives and chooses where to live as an adult. In the second period ("adulthood"), an individual supplies a unit of labor inelastically to produce the differentiated variety in the location she lives, consumes, and then gives birth to a child. Let L_{it} denote the number of workers (adults) residing in location i at time t, where the total number of workers

$\sum_{i=1}^{N} L_{it} = \bar{L}$, is normalized to a constant in each period t.[1] The population in time $t = 0$, $\{L_{i0}\}$, is given exogenously.

2.1.1 Production

Each location i is capable of producing a unique good—the Armington (1969) assumption. Firms (indexed by ω) in location i produce this homogeneous good under perfectly competitive conditions with the following constant returns-to-scale production function

$$q_{it}(\omega) = A_{it} l_{it}(\omega)$$

where labor $l_{it}(\omega)$ is the only production input. The productivity level for the location, A_{it}, is given by

$$A_{it} = \bar{A}_{it} L_{it}^{\alpha_1} L_{it-1}^{\alpha_2} \tag{1}$$

where $\bar{A}_{it}$ is the exogenous (but unrestricted) component of this location's productivity in year t. Importantly, the two additional components of a location's productivity depend on the number of workers in that location in the current period, L_{it}, and in the previous period, L_{it-1}. We assume that firms are small and hence take these aggregate labor quantities as given. Hence the parameter α_1 governs the strength of any potential (positive or negative) contemporaneous agglomeration externalities working through the size of local production. This is a simple way of capturing Marshallian externalities, external economies of scale, knowledge transfers, thick market effects in output or input markets, and the like. The presence of the term $L_{it}^{\alpha_1}$ is standard in many approaches to modeling spatial economies, albeit typically in static models that would combine effects of L_{it} and L_{it-1}.

The parameter α_2, on the other hand, governs the strength of potential *historical* agglomeration externalities. This allows for the possibility that two cities with equal fundamentals $\bar{A}_{it}$ and sizes L_{it} today might feature different productivity levels A_{it} today because they had differing sizes L_{it-1} in the past. One possible cause of $\alpha_2 > 0$ could be the persistence of local knowledge about production that was generated and accumulated in the past. Alternatively, the term $L_{it-1}^{\alpha_2}$ could act as a stand-in for (un-modeled) durable capital or infrastructure that was installed as a result of historical (i.e. $t-1$) production and that producers in period t are able to benefit freely from.

Finally, total output in location is given by $Q_{it} \equiv \sum_{\omega} q_{it}(\omega) = A_{it} L_{it}$. As is standard, due to constant returns at the firm-level, firm sizes are indeterminate and so in what follows we drop the firm identifiers ω.

[1] Our model economy exhibits a form of scale-invariance that means that, for the purposes of our analysis here, the total number of workers in any time period is irrelevant.

2.1.2 Consumption

Adults are the only consumers.[2] They have constant elasticity of substitution (CES) preferences, with elasticity σ, across the differentiated goods that each location can produce. Letting w_{it} denote the equilibrium nominal wage, and letting P_{it} be the price index (solved for below), the deterministic component of welfare—that is, welfare up to an idiosyncratic shock that we introduce below—of any adult residing in location i at time t is given by

$$W_{it} \equiv u_{it} \cdot \left(\frac{w_{it}}{P_{it}}\right),$$

where the term u_{it} refers to a location-specific amenity shifter that is given by

$$u_{it} = \bar{u}_{it} L_{it}^{\beta_1} L_{it-1}^{\beta_2}. \tag{2}$$

The term $\bar{u}_{it}$ allows for flexible exogenous amenity offerings in any location and time period. Endogenous amenities work analogously to the production externality terms (governed by the elasticities α_1 and α_2) introduced above, with the parameters β_1 and β_2 here capturing the potential for the presence of other adults in a location to directly affect (either positively or negatively, depending on the sign of β_1 and β_2) the utility of any given resident. We assume that consumers take these terms as given, just as they take factor and goods prices as given, when making decisions.

As is well understood, a natural source of a negative value for β_1 in a model such as this one is the possibility of local congestion forces that are not directly modeled here; for example, if non-tradable goods (such as housing and land) are in fixed supply locally and are demanded in fixed (that is, Cobb-Douglas) proportions then $-\beta_1$ would equal the share of expenditure spent on such goods. These effects would work contemporaneously.

As with α_2, the parameter β_2 captures forces by which the historical population L_{it-1} affects the utility of residents in year t directly (that is, other than through productivity, wages, prices, or current population levels). Again it seems potentially important to allow for such effects given the likelihood that previous generations of residents may leave a durable impact, positive or negative, on their former locations of residence. Positive impacts could include the construction of parks or sewers, and negative impacts could include environmental damage or resource depletion.

[2] If children consumed a fixed fraction of their parents' consumption amounts then allowance for consumption in childhood would simply scale up all consumption amounts in our analysis proportionally.

2.1.3 Trade

Bilateral trade from location i to location j incurs an exogenous iceberg trade cost, $\tau_{ijt} \geq 1$ (where $\tau_{ijt} = 1$ corresponds to frictionless trade). Given this, bilateral trade flows take on the well-known gravity form given by

$$X_{ijt} = \tau_{ijt}^{1-\sigma} \left(\frac{w_{it}}{A_{it}} \right)^{1-\sigma} P_{jt}^{\sigma-1} w_{jt} L_{jt}, \tag{3}$$

where $P_{it} \equiv \left(\sum_{k=1}^{N} \left(\tau_{ki} \frac{w_{kt}}{A_{kt}} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is the CES price index referred to above.

2.1.4 Migration

Recall from the discussion of timing above that L_{jt-1} adults reside in location j at time $t-1$, and they have one child each. Those children choose at the beginning of period t – as they pass into adulthood – where they want to live as adults in order to maximize their welfare.

As described above, adults who reside in a location j will enjoy a deterministic component of utility given by W_{jt} in equilibrium. Similarly to costs of trading, we allow for bilateral impediments to migration $\mu_{ijt} \geq 1$ (with frictionless migration denoted by $\mu_{ijt} = 1$), which act like utility-shifters conditional on migrating from i to j. This means that the deterministic utility for a migrant who moves from location i to location j is $\frac{W_{jt}}{\mu_{ijt}}$. However, we also allow for idiosyncratic unobserved heterogeneity in how each child will value living in each location j in adulthood. Letting the vector of such idiosyncratic taste differences be denoted by $\vec{\varepsilon}$, the actual welfare of a child who receives the draw $\vec{\varepsilon}$ while living in location i in time $t-1$ who chooses to move to location j as an adult is:

$$W_{ijt}\left(\vec{\varepsilon}\right) = \frac{W_{jt}}{\mu_{ijt}} \varepsilon_j, \tag{4}$$

so the particular shock for location j, denoted by ε_j, simply scales up or down the deterministic component of utility, $\frac{W_{jt}}{\mu_{ijt}}$. Hence, a child chooses:

$$\max_j W_{ijt}\left(\vec{\varepsilon}\right) = \max_j \frac{W_{jt}}{\mu_{ijt}} \varepsilon_j$$

We further assume that $\vec{\varepsilon}$ is extreme-value (Frechet) distributed with shape parameter θ (and a set of location parameters that we normalize to one without loss of generality). As is standard, the number of children in location i in time $t-1$ who choose to move to location

j in time t, L_{ijt}, is then given by:

$$L_{ijt} = \frac{\left(W_{jt}/\mu_{ijt}\right)^{\theta}}{\sum_{k=1}^{N}\left(W_{kt}/\mu_{ikt}\right)^{\theta}} L_{it-1}. \tag{5}$$

For future reference, we note that the expected utility of a child location in location i in time $t-1$ prior to realizing their idiosyncratic shocks $\vec{\varepsilon}$, Π_{it}, is:

$$\Pi_{it} = \left(\sum_{k=1}^{N}\left(W_{kt}/\mu_{ikt}\right)^{\theta}\right)^{\frac{1}{\theta}}. \tag{6}$$

So, summarizing, we can write bilateral migration flows in the gravity equation form as

$$L_{ijt} = \mu_{ijt}^{-\theta}\Pi_{it}^{-\theta}L_{it-1}W_{jt}^{\theta}, \tag{7}$$

where we expect higher migration into destination locations j with high destination welfare W_{jt}, out of origin locations i that either have a lot of residents L_{it-1} or poor expected utility at birth Π_{it} or both, and among pairs for which bilateral migration costs μ_{ijt} are low.

2.2 Dynamic Equilibrium

An equilibrium in this dynamic economy is a sequence of values of prices and allocations such that goods and factor markets clear in all periods. More formally, for any initial population vector $\{L_{i0}\}$ and geography vector $\left\{\bar{A}_{it}, \bar{u}_{it}, \tau_{ijt}, \mu_{ijt}\right\}$, an equilibrium is a vector $\{L_{it}, w_{it}, W_{it}, \Pi_{it}\}$ such that, for all locations i and time periods t, we have:

1. *Total sales is equal to payments to labor:* That is, a location's income is equal to the value of all locations' purchases from it, or $w_{it}L_{it} = \sum_j X_{ijt}$. Using equation (3) this can be written as

$$w_{it}^{\sigma}L_{it}^{1-\alpha(\sigma-1)} = \sum_j K_{ijt}L_{jt}^{\beta(\sigma-1)}W_{jt}^{1-\sigma}w_{jt}^{\sigma}L_{jt}, \tag{8}$$

   with $K_{ijt} \equiv \left(\frac{\tau_{ij}}{\bar{A}_{it}L_{it-1}^{\alpha_2}\bar{u}_{jt}L_{jt-1}^{\beta_2}}\right)^{1-\sigma}$ defined as a collection of terms that are either exogenous or predetermined from the perspective of period t.

2. *Trade is balanced:* That is, a location's income is fully spent on goods from all locations,

or $w_{it}L_{it} = \sum_j X_{jit}$. Using equation (3) this can be written as

$$w_{it}^{1-\sigma} L_{it}^{\beta_1(1-\sigma)} W_{it}^{\sigma-1} = \sum_j K_{jit} L_{jt}^{\alpha_1(\sigma-1)} w_{jt}^{1-\sigma}. \tag{9}$$

3. *The total population is equal to the population arriving in a location:* That is, $L_{it} = \sum_j L_{jit}$. From equation (7) this implies

$$L_{it} W_{it}^{-\theta} = \sum_j \mu_{jit}^{-\theta} \Pi_{jt}^{-\theta} L_{jt-1}. \tag{10}$$

4. *The total population in the previous period is equal to the number of people exiting a location:* That is, $L_{it-1} = \sum_j L_{ijt}$. From equation (7) this can be written as

$$L_{it-1} = \sum_j \mu_{ijt}^{-\theta} \Pi_{it}^{-\theta} L_{it-1} W_{jt}^{\theta},$$

which can then be written more compactly as

$$\Pi_{it}^{\theta} \equiv \sum_j \mu_{ijt}^{-\theta} W_{jt}^{\theta}. \tag{11}$$

Summarizing, the dynamic equilibrium can be represented as the system of $4 \times N \times T$ equations (in equations 8-11) in $4 \times N \times T$ unknowns, $\{L_{it}, w_{it}, W_{it}, \Pi_{it}\}$.

This system of equations (8)-(11) comprises a high-dimensional nonlinear dynamic system whose analysis can prove challenging. But its analysis is facilitated by the fact that it is a system of additive power equations, where each of the endogenous variables $\{L_{it}, w_{it}, W_{it}, \Pi_{it}\}$ appears, on either the left-hand or right-hand side, to a particular fixed power, with weights in the system given by an exogenous kernel that comprises variables that are either exogenous or pre-determined from the perspective of period t (K_{ijt} in equations 8 and 9, and $\mu_{ijt}^{-\theta}$ in equations 10 and 11). This means that the solution of each cross-sectional system for t, given values of K_{ijt} and hence solutions from the previous period $t-1$, can be solved using the methods in Allen, Arkolakis, and Li (2015).

Towards this goal, we define the matrix:

$$\mathbf{A}(\alpha_1, \beta_1) \equiv \begin{pmatrix} \left| \frac{\theta(1+\alpha_1\sigma+\beta_1(\sigma-1))-(\sigma-1)}{\sigma+\theta(1+(1-\sigma)\alpha_1-\beta_1\sigma)} \right| & \left| \frac{(\sigma-1)(\alpha_1+1)}{\sigma+\theta(1+(1-\sigma)\alpha_1-\beta_1\sigma)} \right| \\ \left| \frac{\theta/\tilde{\sigma}}{\sigma+\theta(1+(1-\sigma)\alpha_1-\beta_1\sigma)} \right| & \left| \frac{\theta(1-(\sigma-1)\alpha_1-\beta_1\sigma)}{\sigma+\theta(1+(1-\sigma)\alpha_1-\beta_1\sigma)} \right| \end{pmatrix}, \tag{12}$$

where $\tilde{\sigma} \equiv \frac{\sigma-1}{2\sigma-1}$. Given this definition, the following result characterizes a sufficient condition for existence and uniqueness for environments with symmetric trade costs (and unrestricted migration costs) and arbitrary positive geographic fundamentals.

Proposition 1. For any initial population $\{L_{i0}\}$ and geography $\left\{\bar{A}_{it} > 0, \bar{u}_{it} > 0, \tau_{ijt} = \tau_{jit}, \mu_{ijt} > 0\right\}$, there exists an equilibrium. The equilibrium is unique if $\rho\left(\mathbf{A}\left(\alpha_1, \beta_1\right)\right) \leq 1$, where $\rho(\cdot)$ denotes the spectral radius operator.

Proof. See Section A.1. □

We note that this sufficient condition for uniqueness will be satisfied whenever α_1 and β_1 are sufficiently small. Figure 1 illustrates this condition for two particular values of σ and θ, values at which the sufficient condition of $\rho\left(\mathbf{A}\left(\alpha_1, \beta_1\right)\right) \leq 1$ is well approximated by the simple relation of $\alpha_1 + \beta_1 \leq 0$. Finally, we note that this result concerning uniqueness of the dynamic equilibrium does not depend on the values of α_2 and β_2, since the current generation takes L_{it-1} as given.

2.3 Steady-State

Our discussion of path dependence rests on the consideration of the various potential steady-states of this model economy. Intuitively, if local agglomeration economies are strong enough then there could be multiple allocations at which the economy would be in steady-state—agents who happen to come to reside in a location could find it optimal to stay there thanks to the reinforcing logic of local positive spillovers.

To evaluate this possibility we consider a version of the above economy but for which the potentially time-varying fundamentals $\left\{\bar{A}_{it}\right\}$ and $\{\bar{u}_{it}\}$ and trade $\{\tau_{ijt}\}$ and migration $\{\mu_{ijt}\}$ costs are held constant over time at the values $\left\{\bar{A}_i, \bar{u}_i, \tau_{ij}, \mu_{ij}\right\}$. The steady-states of our economy will therefore be a set of time-invariant endogenous variables that we denote by $\{L_i, w_i, W_i, \Pi_i\}$. (Note that while population levels at each location, L_i, will be constant in steady-state, and hence net migration flows are zero, gross migration flows will still be positive in steady-state equilibrium due to the churn induced by idiosyncratic locational preferences in equation 4.) The following result, analogous to Proposition 1, provides a sufficient condition for existence and uniqueness of the steady-state of this economy (again, for arbitrary geographies with symmetric trade and migration costs).

Proposition 2. For any time-invariant geography $\left\{\bar{A}_i > 0, \bar{u}_i > 0, \tau_{ij} = \tau_{ji}, \mu_{ij} = \mu_{ji}\right\}$, there exists a steady-state equilibrium. The equilibrium is uniqueif $\rho\left(\mathbf{A}\left(\alpha_1 + \alpha_2, \beta_1 + \beta_2\right)\right) \leq 1$.

Proof. See Section A.2. □

The condition for uniqueness of the steady-state in Proposition 2 is similar to that for uniqueness of transition paths in Proposition 1. The only difference is that the latter condition depends on the size of contemporaneous spillovers α_1 and β_1, whereas the latter condition depends on the size of total (that is, contemporaneous plus historical) spillovers $\alpha_1 + \alpha_2$ and $\beta_1 + \beta_2$. Combining Propositions 1 and 2, we see that what matters for the potential multiplicity of steady-states (and hence the potential for initial conditions or temporary shocks to affect the steady-state that obtains – or equivalently, for path dependence to occur) is the values of α_2 and β_2. If these historical spillover parameters are large then it is likely for path dependence to occur. Further, if the values of α_1 and β_1 are low then dynamic equilibrium paths will be unique. In this range of parameters (that is, relatively small α_1 and β_1 and yet relatively large α_2 and β_2) path dependence will both exist and be straightforward to study, since the complications of multiplicity for estimation, computation, and interpretation of counterfactuals do not arise. We think of this as well-behaved path dependence.

Before discussing path dependence further, it is useful to point out some features that will obtain in any steady-state equilibrium. First, as one might suspect, a notion of welfare is equalized across locations in steady-state—otherwise, surely gross migration flows, induced by any spatial welfare arbitrage opportunities, would not be zero. The notion of welfare that is constant across locations is $\mathit{\Omega} \equiv E\left[\max_j \left(W_j \Pi_j \varepsilon_j\right)\right]$, the expectation (across the random draws of idiosyncratic preferences in equation 4) of the maximum welfare that an agent can achieve when she has no particular attachment to any location. Recall that the welfare an agent can expect to achieve, conditional on living in location i, is $E\left[\max_j \frac{W_j}{\mu_{ij}} \varepsilon_j\right]$. The essence of the steady-state version of this is to replace the term $1/\mu_{ij}$ (the relevant penalty that an agent living in i must pay to get to j and enjoy $W_j \varepsilon_j$ there) with the term Π_j (which is the appropriate weighted average of costs of getting from any location to j). A simple calculation shows that

$$\Omega = W_i \Pi_i L_i^{-\frac{1}{\theta}} \ \forall i \in \{1, ..., N\},$$

which includes the term $L_i^{-\frac{1}{\theta}}$ to account for the fact that, in equilibrium, a heavily populated location must have an average number of residents there who had relatively unfavorable idiosyncratic draws, so their average welfare is lower than otherwise.

A second feature of the steady-state is useful for fixing intuition. Algebraic manipulations of equations of the steady-state versions of equations (8)-(11) imply that the equilibrium steady state distribution of population can be written as:

$$\gamma \ln L_i = C + (1 - \tilde{\sigma}) \ln \bar{u}_i + \tilde{\sigma} \ln \bar{A}_i + (1 - \tilde{\sigma}) \ln \Pi_i - \ln P_i, \tag{13}$$

where $\gamma \equiv \frac{1}{\theta}(1 - \tilde{\sigma}) - \frac{\tilde{\sigma}}{\sigma - 1} - (\tilde{\sigma} + 1)(\beta_1 + \beta_2) + \tilde{\sigma}(\alpha_1 + \alpha_2)$. This implies that a greater density of residents can be found, in any steady-state equilibrium, in locations with high productivity $\bar{A}_i$, high amenities $\bar{u}_i$, high access to migration destinations Π_i, and high access to imported goods (low P_i), and the elasticities of these characteristics are governed by the strength of the key spillover elasticities ($\alpha_1, \alpha_{2,}\beta_1$ and β_2). Of course, while the first two of these determinants of population density, $\bar{A}_i$ and $\bar{u}_i$, are exogenous in our model, the latter two determinants, Π_i and P_i, are endogenous and involve endogenous features of all other locations. It is the self-reinforcing potential of those cross-location interactions that leads to the possibility of multiple steady-states and hence multiple vectors of $\{L_i\}$ that would satisfy the system in equation (13).

2.4 A path dependence example

To see the logic of path dependence in this model more concretely, consider a simple example of three locations. Suppose, to begin, that these locations have identical and time-invariant geographies $\left\{\bar{A}_{it}, \bar{u}_{it}, \tau_{ijt}, \mu_{ijt}\right\}$, and trade and migration costs are symmetric across locations. Figure 2 shows the phase diagram on the two-dimensional phase space of L_{it} shares in this economy. (To interpret these figures, note that each red dot has associated with it a blue ray; the direction of the ray illustrates the direction to which the system dynamics move towards the red dot, and the length of the ray conveys the speed with which those dynamics take place.) We begin in panel (a) with a setting in which the spillover parameters (α_1, β_1, α_2 and β_2) are all zero. Naturally, this symmetric economy with no spillovers has a unique steady-state, and this steady-state is located the center of the simplex because of symmetry. Panels (b) through (f) then increase α_2 but keep all other parameters in the economy constant. At $\alpha_2 = 0.1$ this increase in α_2 has no apparent qualitative impact on the dynamics of the economy. But at $\alpha_2 = 0.2$ we see a dramatic change. The central location, a unique and stable steady-state when $\alpha_2 = 0.1$, is still a steady-state but it is no longer stable (all dynamic rays near that central point lead away from it). And, further, this steady-state is no longer unique—six additional steady-states have emerged, three stable steady states with relatively concentrated population shares (the corners of the simplex) in a single location, and three unstable steady states with equal concentration in two of the three locations (and little population in the third). As we increase α_2 even further this basic picture doesn't change, though speeds of convergence to steady-state do increase. One final thing to note in this example is that each steady-state will be surrounded by points that will map dynamically to it. The locus of such points around any steady-state comprise its *basin of attraction*. In this symmetric case, the three symmetric, stable steady-states have symmetric basins of attraction that partition the space of all possible starting points in the simplex. (The stable steady-state, of course, has no basin of attraction.)

Now consider the same example but with asymmetric fundamentals. Suppose that location 2 has worse fundamental amenity value $\bar{u}_{it}$ than do the other two regions. When $\alpha_2 = 0$, as shown in Figure 3, the steady-state is again unique and relatively central, just as with the previous symmetric example. But the difference now is that the asymmetric fundamentals (the relative unattractiveness of location 2 in terms of fundamentals) have shifted the location of that unique steady-state—intuitively, it is shifted in the direction of the location 2 axis of the simplex, implying less population at location 2 in steady-state. As we increase α_2 from 0 to 0.5 in panels (a) to (f) we see behavior that is similar to that of the symmetric case. The unique steady-state under $\alpha_2 \leq 0.1$ shifts to a multiplicity of steady-states, each displaying relative concentration in the corners of the simplex, for $\alpha_2 \geq 0.2$. In this case, however, the three steady-states have different levels of aggregate welfare (in the sense described above), so we could speak of an economy that might, due to a bad set of initial conditions, end up in a dominated steady-state. Reassuringly, however, the basin of attraction of a relatively good steady-state is larger than that of a dominated one. So, in the space of all possible initial conditions, good steady-states may be more likely to arise.

3 Identification and Estimation

We now describe a procedure for mapping the above model into observable features of the US economy throughout the past two centuries. The goal is to estimate the elasticity parameters (α_1, β_1, α_2, β_2, σ and θ) that are critical for assessing the likelihood and strength of path dependence, as well as the geographic fundamentals $\left\{\bar{A}_{it}, \bar{u}_{it}, \tau_{ijt}, \mu_{ijt}\right\}$ that shift the consequences of path dependence.

3.1 Data

We aim to track subnational regions throughout the period from 1800-2000. In each decennial census, information is available at the county level, but these county border definitions change over the years, so we track the sub-county units that comprise the full set of county intersections across the full set of years and denote those unique intersection units as our geographical units of analysis i in the model above. We then apportion uniformly the county-level information for county c in any year t into each of the sub-county units i that map to county c in year t.

Data limitations mean that obtaining consistent time series on the long sweep of American geographical (county-level) history can be challenging. Thankfully, one variable that is available throughout is a proxy for internal migration, which then corresponds to L_{ijt} in the model above. The decennial US Census tracks, from 1790-present, data on the population by county of current residence and state of birth (and age). These can be extracted from publicly available 5% samples for each year. Given that "adults" in the model are the generation that produces and consumes, we take the number of people aged 20-70 in this dataset, in each county j and year t, and use these adults' location of birth as our proxy for the origin of their adult migration journey, location i. To avoid overlaps of these cohorts of 20-70 year-olds, we then work only with the Census data for every 50 years, i.e. 1800, 1850, 1900, 1950, and 2000. Residents aged 0-19 or over-70 then play no role in our subsequent analysis. Finally, we apportion uniformly the number of people born in state s in year t equally into each of the sub-county units i contained in state s in year t. This procedure delivers our proxy for migration flows L_{ijt} (and hence also total populations $L_{jt} \equiv \sum_i L_{ijt}$).

Our second important variable is that for nominal per-capita incomes, w_{it}. The US Census did not track wage income until 1940, but an available proxy is available for the value of county-level total agricultural and manufacturing output from 1850-present. Under the assumption that local expenditure on (and hence income from) non-tradable services tracks that for agriculture and manufacturing, this data series provides a proxy for $w_{it}L_{it}$. Given data on L_{it} this can be used to proxy for w_{it}. Because this essential ingredient of our estimation procedure is only available from 1850 onwards, we treat 1800 as date 0 (and hence $\{L_{i0}\}$ comes from L_{it} in 1800).

The third data ingredient concerns intra-national trade flows, X_{ijt}. To the best of our knowledge this is only publicly available (within 1850-2000) beginning in the year 1997 from the Commodity Flow Survey (CFS)

Finally, an instrumental variable estimation procedure that we describe (and only that procedure) below requires observable proxies for the geographic productivity and amenity terms, for which we collect contemporary measures of elevation, soil quality, temperature, precipitation, and distance to coast and navigable bodies of water. For the purpose of constructing valid instruments, we treat these observed geographic characteristics as time-invariant properties of a location.

3.2 Identification and Estimation

We now describe a three-step estimation procedure designed to recover estimates of the elasticity parameters (α_1, β_1, α_2, β_2, σ and θ) and geographic fundamentals $\left\{\bar{A}_{it}, \bar{u}_{it}, \tau_{ijt}, \mu_{ijt}\right\}$ for all locations i and years t from 1850-2000 through the use of the above data on L_{ijt} from the years 1800-2000, on w_{it} from the years 1850-2000, and on X_{ijt} from one cross-section (in

1997).

In the *first step* of this procedure we assume that trade and migration costs, τ_{ijt} and μ_{ijt} are functions of observable (potential) shifters of these costs. For now we use only distance and model these costs as $\ln \tau_{ijt} = \kappa_t \ln dist_{ij}$ and $\ln \mu_{ijt} = \lambda_t \ln dist_{ij}$. Substituting these expressions into the gravity equations for trade and migration flows, equations (3) and (7) respectively, we obtain

$$\ln X_{ijt} = (1-\sigma)\, \kappa_t \ln dist_{ij} + \gamma_{it} + \delta_{jt} + \varepsilon_{ijt} \tag{14}$$

$$\ln L_{ijt} = -\theta \lambda_t \ln dist_{ij} + \rho_{it} + \pi_{jt} + \nu_{ijt}, \tag{15}$$

where the terms γ_{it}, δ_{jt}, and represent fixed-effects in these gravity estimation equations, and ε_{ijt} and ν_{ijt} correspond to potential measurement error in trade and migration flows respectively.

In principle, one could estimate κ_t for any year t in which data on trade flows X_{ijt} are available. However, as describe above, we only have access to such data for one year, 1997. So we assume that κ is constant over time. By contrast, data on migration flows L_{ijt} are available for all decades from 1850 onwards so we estimate corresponding λ_t separately for each year. The result of this first step is an estimate of the composite parameters $(1-\sigma)\kappa_t$ and $\theta\lambda_t$.[3]

Turning to our *second step,* we define $T_{ijt} \equiv \hat{\tau}_{ijt}^{1-\sigma} = dist_{ij}^{(1-\sigma)\kappa_t}$, $M_{ijt} \equiv \hat{\mu}_{ijt}^{-\theta} = dist_{ij}^{-\theta\lambda_t}$, which are identified in step one (as they are a function of observables and the identified composite parameters only). For notational ease, further define $p_{it} \equiv \frac{w_{it}}{A_{it}}$, and $Y_{it} \equiv w_{it} L_{it}$. Then the system of equations (8)-(11) defining equilibrium for each period can be written as

$$p_{it}^{\sigma-1} = \sum_j T_{ijt} \left(\frac{Y_{jt}}{Y_{it}} \right) P_{jt}^{\sigma-1} \tag{16}$$

$$P_{it}^{\sigma-1} = \sum_j T_{jit} \left(p_{jt}^{\sigma-1} \right)^{-1} \tag{17}$$

$$\left(W_{it}^{\theta} \right)^{-1} = \sum_j M_{jit} \frac{L_{jt-1}}{L_{it}} \left(\Pi_{jt}^{\theta} \right)^{-1} \tag{18}$$

$$\Pi_{it}^{\theta} = \sum_i M_{ijt} W_{jt}^{\theta}. \tag{19}$$

[3]Given data limitations, we estimate the trade gravity regression on bilateral state to state trade flows, whereas we estimate the migration gravity regression on state (of birth) to county (of residence) population flows.

Noting that we have data on Y_{it} and L_{it} for all locations i and periods t and that the values T_{ijt} and M_{ijt} were identified in step 1, the following proposition shows that the four remaining variables in equations (8)-(11) are identified because this system of equations has a unique solution given (Y_{it}, L_{it}, T_{ijt} and M_{ijt}).

Proposition 3. Given observed data on $\{Y_{it}, L_{it}, L_{it-1}\}$ and identified values of $\{T_{ijt}, M_{ijt}\}$ from step two there exists unique (up to-scale) values of$\{p_{it}^{\sigma-1}, P_{it}^{\sigma-1}, W_{it}^{\theta}, \Pi_{it}^{\theta}\}$ that satisfy equations *(8)-(11)*.

Proof. See Section A.3. □

An important feature of this second step result is that it does not depend on the values of the trade or migration elasticities, σ and θ, only on the composite parameters that were recovered in step 1. The basic intuition of this recovery procedure is that we are recovering the analogs of the exporter and importer fixed-effects of the trade gravity equation (effectively $p_{it}^{\sigma-1}$ and $P_{it}^{\sigma-1}$, respectively) and the origin and destination fixed-effects of the migration gravity equation (W_{it}^{θ} and Π_{it}^{θ}, respectively) from non-bilateral data (unlike the usual approach to recovery of such fixed-effects) by making use of the goods and labor market clearing equations for equilibrium.

Finally, we turn to the *third step* of our estimation procedure. By definition, $p_{it} \equiv \frac{w_{it}}{A_{it}}$; therefore, given definition of $A_{it} = \overline{A}_{it} L_{it}^{\alpha_1} L_{it-1}^{\alpha_2}$ we take logs of $p_{it}^{\sigma-1}$ to obtain:

$$\begin{aligned} \ln\left(p_{it}^{\sigma-1}\right) = {} & (\sigma-1)\ln w_{it} + \alpha_1(1-\sigma)\ln L_{it} + \alpha_2(1-\sigma)\ln L_{it-1} \\ & + (1-\sigma)\ln \bar{A}_{it}. \end{aligned} \tag{20}$$

Recall that the value of $p_{it}^{\sigma-1}$ was identified in step two. Therefore, equation (20) represents an equation that can be used as a simple regression estimating equation given data on the right-hand side variables, w_{it}, L_{it} and L_{it-1}. Suitable estimates of this equation would therefore identify σ, $\alpha_1$and α_2. However, the unobservable term in equation (20), $(1-\sigma)\ln \bar{A}_{it}$, would be the residual in this estimating equation and we would expect it to be correlated with the regressors w_{it} and L_{it}— indeed, the migration behavior in equation (7) suggests that migrants would move to locations with exceptional values of this residual, $\bar{A}_{it}$. We come back to our (instrumental variables) strategy to deal with this endogeneity problem below.

Analogous manipulations on the migration side imply

$$\ln\left(W_{it}^{\theta}\right) = \theta \ln w_{it} + \left(\frac{1}{1-\sigma}\right)\ln\left(P_{it}^{1-\sigma}\right) + \beta_1\theta\ln L_{it} + \beta_2\theta\ln L_{it-1} + \theta\ln \bar{u}_{it}, \tag{21}$$

which is again an equation that relates a variable recovered from step two, the migration origin fixed-effect W_{it}^{θ}, on observables (w_{it}, L_{it} and L_{it-1}) as well as another variable recovered from step two, the trade destination fixed-effect $P_{it}^{1-\sigma}$. Again, this regression specification allows the opportunity to estimate three key elasticities (θ, β_1 and β_2), but the logic of migration suggests that there is an unavoidable endogeneity problem due to the correlation between the unobserved amenity shifter $\bar{u}_{it}$, the regression residual in equation (21), and regressors such as $\ln w_{it}$ and $\ln L_{it}$. Finally, we note that equations (20) and (21) together over-identify the parameter σ, so there are opportunities for testing this restriction.

To construct instrumental variables (IVs) for the endogenous regressors $\{\ln w_{it}, \ln L_{it}, \ln L_{it-1}, \ln P_{it}\}$ in equations (20) and (21) , we construct instruments from model-based simulations of these variables. This proceeds as follows. We begin with a candidate guess of the elasticity parameters, at values motivated by earlier work. We then seek to obtain proxies for $\overline{A}_{it}$ and $\overline{u}_{it}$ (purely for the purpose of constructing our IV) by modeling these terms as a function of the observable (and time-invariant) geographical characteristics mentioned above (soil quality, elevation, climate, and water access). Finally, we start the IV-generating model simulation at an $\{L_{i0}\}$ equal to the observed 1800 population shares. This procedure fully specifies the simulated model, which then can be run forwards in time to generate model predictions for the endogenous regressors. We further control for the geographical characteristics of each location, as well as their initial population value L_{i0}, in the IV estimation of equations (20) and (21) . This implies that the effective exclusion restriction needed here is that a location's unobserved productivity and amenity shifters, $\overline{A}_{it}$ and $\overline{u}_{it}$, are not correlated with the 1800 population values or geographical characteristics of that location's neighbors conditional on the location's own values of these attributes.

Finally, we note that, conditional on obtaining consistent estimates of the elasticity parameters (α_1, β_1, α_2, β_2, σ and θ), equations (20) and (21) allow recovery of the geographic fundamentals $\left\{\bar{A}_{it}, \bar{u}_{it}\right\}$ as well. Combined with the earlier estimates of $\{T_{ijt}, M_{ijt}\}$ from step two all model parameters are thereby identified.

3.3 Estimation Results

Our 2SLS estimates of the coefficients in equations (20) and (21) are reported in Table 1. This table reports, in columns (1)-(3), the first-stage (one for each of the three endogenous variables), as well as, in columns (4) and (5), the second-stage coefficients. We also report, in the bottom sections of columns (4) and (5), the corresponding values of the elasticity parameter estimates. These estimates have some noteworthy features. The productivity spillovers, α_1 and α_2, are both positive, as prior work (on static estimation settings, which lump these two parameters together) might suggest. The amenity spillovers, β_1 and β_2, are both negative.

Because the productivity spillovers are positive and the amenity spillovers negative, it is not clear whether these estimates are in the range for uniqueness of equilibria, and of steady-states, implied by Propositions 1 and 2. Figure 4 plots these estimates in the ranges implied by these propositions (which also depend on the estimates of σ and θ). From the fact that the red star is inside the yellow region we see that the equilibria will be unique at these parameter estimates; similarly, from the fact that the green star is (just) inside the blue region we see that multiple steady-states are a possibility (that is, the sufficient condition for uniqueness of steady-states identified in Proposition 2 is not satisfied by these estimates). These two findings suggest that path dependent outcomes are a possibility in this model economy.

Finally, while the estimates in Table 1 pool the data used to estimate equations (20) and (21) across all years for which data is available for these regressions (1850, 1900, 1950 and 2000), Table 2 presents estimates of our elasticity parameters for each year separately. There is a surprising amount of stability of theses estimates over these samples, despite the fact that they draw from such diverse historical periods in American history.

4 Quantifying the Consequences of Path Dependence

Having estimated all of the ingredients of the model introduced above on data from the history of US economic geography from 1800 onwards, we now use the estimated model to obtain a quantitative understanding of its path-dependent features. While this draws exactly on the primitives estimated above, in practice to reduce computational burdens (resulting from the large number of simulations performed below) we work with gridded spatial units that are larger than (i.e. aggregates over) those tracked in the data above. There are 570 such units in our current simulations.

We do so by comparing alternative paths of the distribution of US economic activity under differing historical conditions. We describe those various sorts of differing conditions in detail below. But one common feature is the distribution of alternative conditions from which to draw. Our basic idea is to use an approach that is analogous to bootstrap sampling (without replacement), where the historical conditions of a sample of US locations are allowed to be randomly re-ordered over space. In order to discipline the distribution of such alternative spatial conditions, we develop clusters with 10 locations each and use a k-mean clustering algorithm, based on the geographic variables described above, to partition all regions of the US into non-overlapping clusters. This algorithm effectively finds the construction of partitions into k different groups (where $k = 57$ here, in the presence of 570 locations and

10 locations per cluster) of locations that minimizes differences on geographic observables within groups. A map of the resulting clusters is shown in Figure 5, where each of 57 colors refers to a different cluster, but the absolute color scale is irrelevant.

Any given simulation run then redraws, without replacement, the historical conditions of each location within each cluster. The end result is a set of alternative distributions of historical shocks that should resemble the historical distribution of actual shocks. Whenever we do this re-draw of historical conditions (the exact type of which we vary and discuss below), all other locational features are deliberately held constant so that we can assess the pure contribution of alternative historical conditions alone.

4.1 Alternative initial populations

We begin with the case in which the alternative historical conditions referred to above comprise alternative initial (that is, for the year 1800) population distributions $\{L_{i0}\}$. We conduct 200 simulations in which these initial populations are redrawn (without replacement) from the distribution of such initial populations within the geographically similar clusters of Figure 5 (that is, as described above, those simulations with similar geographic features, not necessarily those that are proximate to one another). The fact that we redraw without replacement means that our simulations effectively randomly reshuffle the allocations of initial populations among geographically similar locations.

Figure 6 illustrates the distribution of equilibrium 1850 population for each of our 570 gridded simulation locations, among the first three of our 200 simulations. Likewise, Figures 7 through 9 do the same for the years 1900-2000. While these are only three random simulations, a clear sense of convergence occurs—that is, while the location of the nation's population is quite different across simulations in the early years of 1850 and 1900, by 1950 and 2000 the population distributions look similar. These maps show only three simulations, but the results from all 200 simulations follow a similar pattern. This is illustrated in Figure 10, where we plot, for each year from 1850-2000, the spread of population in each location across the simulations. For each simulation we calculate the rank of each location in the nationwide population size ranking, and the y-axis reports the tendencies of this ranking across the simulations—the thin blue bar indicates the max and min, the thicker blue bar the interquartile range, and the black dot the mean. The locations are ordered along the x-axis by their median ranking across simulations. As suggested by the maps in Figures 6-9, early on, in 1850 and 1900, there is substantial heterogeneity across simulations in whether a given location is likely to be highly ranked in the national population distribution or not. But by 2000 this volatility across simulations has settled down to a large extent. The fact that there remains substantial cross-simulation differences in the location of production is indicative of likely path dependence, but it is clear that it has only relatively weak impacts on the location of production and population.

Relatedly, Figure 11 reports the histogram of nationwide (averaged across locations) welfare levels for each of the 200 simulations. Consistent with the idea from Figure 10 that random alternatives in initial conditions lead to only small differences in the location of production, the differences in welfare seen in Figure 11 are tiny—the max-min spread is less than 1%. Clearly there is very little scope for initial conditions, as proxied for by the distribution of population in 1800, to matter for welfare.

4.2 Alternative paths of productivity and amenity shocks

We now perform simulations in a similar spirit to those above. Though rather than exploring the role of initial conditions, our goal now is to assess the importance of historical events along the transition path from our initial data point (1800) to our last one (2000). We do this by starting the model economy at the same point (that is, the same 1800 populations, and the same value for the 1850 productivity and amenity shifters $\{\bar{A}_{it}, \bar{u}_{it}\}$), and then redraw (within the geographically similar clusters of Figure 5, without replacement) values for $\bar{A}_{it}$ in years 1900 and 1950. That is, the factual path of productivity shocks $\bar{A}_{it}$ is replaced in our simulations with counterfactual paths of such shocks drawn from random reorderings within geographically similar regions. However, we emphasize that the current (year 2000) productivity and amenity shifters remain unchanged; as a result, the only affect of these shocks on current outcomes is through the effect of productivity and amenity spillovers α_2 and β_2.

Figure 12 presents our findings (in a manner that is analogous to Figure 10). Naturally, the picture from 1850 shows no variance since we are not perturbing any shocks in that year. But for 1900 and 1950 the shocks are evidently extremely disruptive in terms of generating a wide spread of alternative histories, simply from reordering local (that is, geographically similar) shocks. By 2000 the variance is reduced, but it is still substantial, and a great deal larger than that for the shocks to initial population levels of Figure 10. Consistent with this, Figure 13 shows that the spread of possible aggregate welfare outcomes across simulations here is a good order of magnitude larger than it was for the case of Section 4.1—indeed, the spread is now substantial, with a good 45 log point difference across the max-min range in these 200 simulations. Another feature of Figure 10 is the fact that the actual 2000 aggregate welfare in the US (shown with a yellow star) is right in the middle of the distribution of possible welfare levels across our simulations. So the actual path of productivity shocks $\bar{A}_{it}$ that occurred in 1900-1950 was evidently not that out of line with what was likely, on average, across our simulations.

Finally, we repeat for the case of amenity shifters $\bar{u}_{it}$ the above analysis that was done on productivity shifters $\bar{A}_{it}$. Figure 14 reports the variance of population ranks across locations, across the 200 simulations. And Figure 15 illustrates the variance in aggregate welfare across the simulations. These findings are similar to those in Figures 10 and 13 for the case of productivity shocks: again, the locational variance is high, and the welfare spread even higher (now up to 80 log points), but in this case, as it happens, the factual welfare draw was on the high end of the possible distribution.

5 Conclusion

It is not hard to look at the economic patterns around us and believe both that agglomeration forces are important, and that they are strong enough to be the source of self-reinforcing, stable clustering of economic activity. This opens up the likelihood that there are many such steady-states—some good, some bad—and the potential for historical accidents, such as initial conditions or long-defunct technological shocks, to play an outsized role in governing both the location and total amount of economic activity.

This paper has drawn on rich historical data available from the US Census from 1800 onwards in order to improve our understanding of these spatial forces. We have developed a model in which path dependence is both easy to characterize qualitatively, and to scrutinize quantitatively. Six elasticities matter for spatial path dependence: two dispersion elasticities coming from the desire for goods and migrants to seek substitute locations, two elasticities governing the the strength of contemporaneous local productivity and amenity agglomeration externalities, and two elasticities capturing the propensity for lagged agglomeration spillovers to matters. We estimate these elasticities using variation coming from the initial conditions and geographical characteristics of a location's neighboring locations and show that path dependence emerges as a theoretical possibility at those parameter values. Our simulations of randomly chosen, locally spatial permutations in initial conditions and historical shocks suggest that the location of economic activity in the US today is remarkably stable across alternative initial (as of 1800) initial population allocations. Correspondingly, aggregate welfare is stable too. However, our simulations also show that alternative historical shocks (to fundamentals of productivity and amenities between 1900 and 1950) can be far more disruptive.

While we have developed these empirical and theoretical tools in the hopes of an improved understanding of inter-city economic geography, these advances could be applied to other areas of economics in which increasing returns, multiplicity, and path dependence have long appeared as objects of theoretical interest that lack a corresponding amount of empirical estimation, quantification, and simulation. One possible such area would be to intra-city issues such as residential segregation, sorting, and so-called "tipping" dynamics (Card, Mas, and Rothstein, 2008); another would be to the dynamics of industrial clustering across countries (World Bank, 1993; Krugman and Venables, 1995; Matsuyama, Sushko, and Gardini, 2014; and Kucheryavyy, Lyn, and Rodríguez-Clare, 2017); a final application could be to dynamic questions of political economy such as those surveyed in Acemoglu and Robinson (2005).

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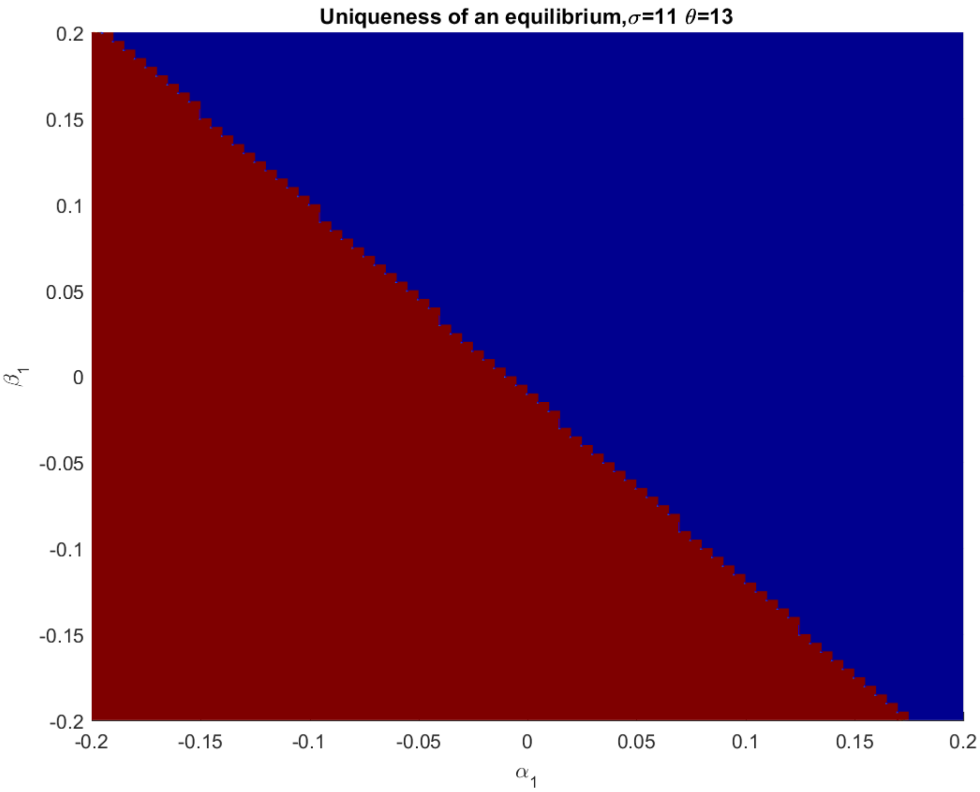
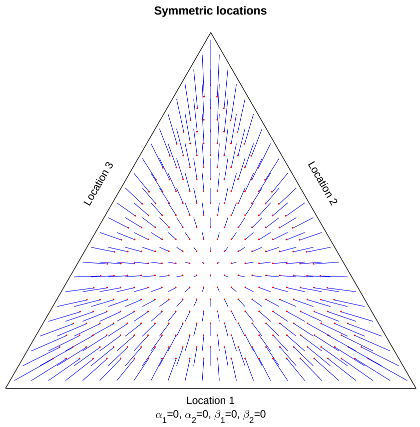
Figure 1: Illustration of Proposition 1

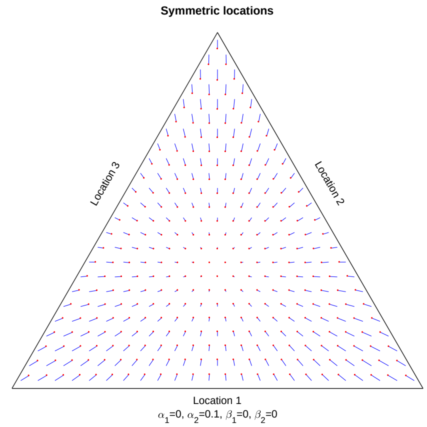


Notes: This figure illustrates the regions of the parameter range (in the space of α_1 and β_2, holding σ and θ constant at the values shown above) that satisfy the condition for uniqueness of equilibrium, as per Proposition 1.

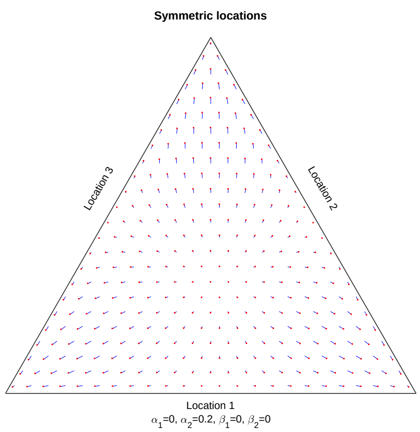
Figure 2: Phase diagrams for 3-region symmetric example



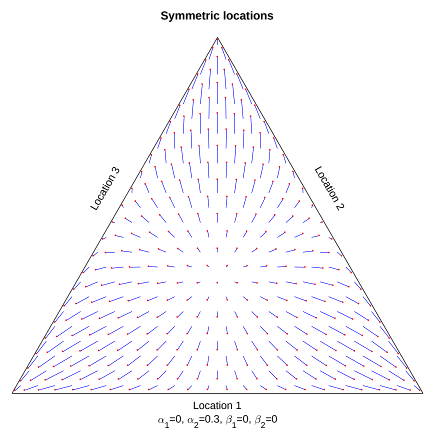
(a) $\alpha_2 = 0$



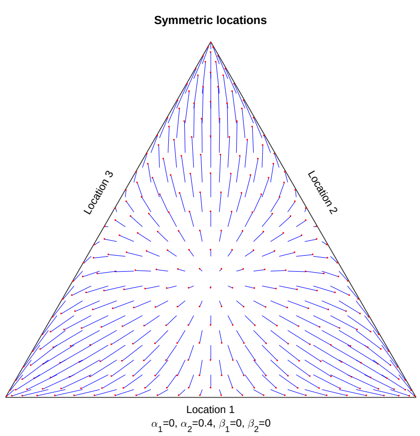
(b) $\alpha_2 = 0.1$



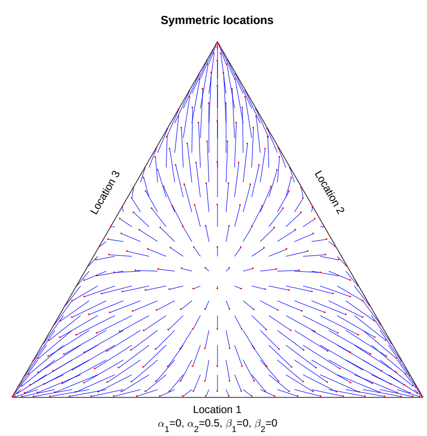
(c) $\alpha_2 = 0.2$



(d) $\alpha_2 = 0.3$



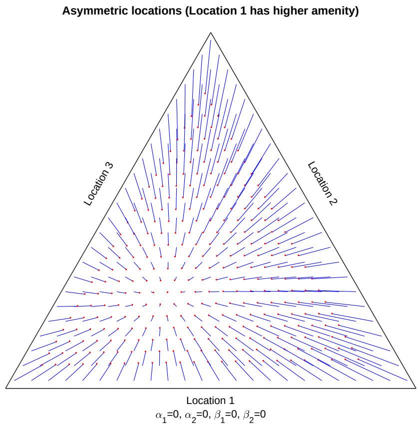
(e) $\alpha_2 = 0.4$



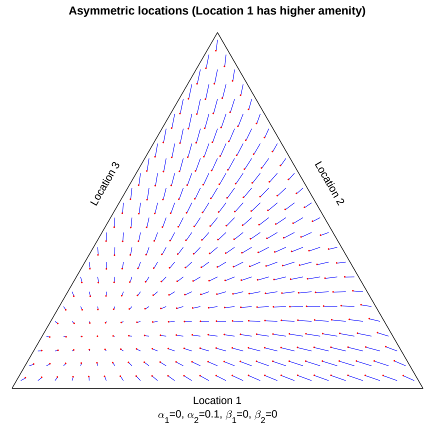
(f) $\alpha_2 = 0.5$

Notes: This figure illustrates phase diagrams for an asymmetric three-region example economy. The parameters $\alpha_1, \beta_1, \beta_2, \sigma$ and θ are held constant as α_2 varies.

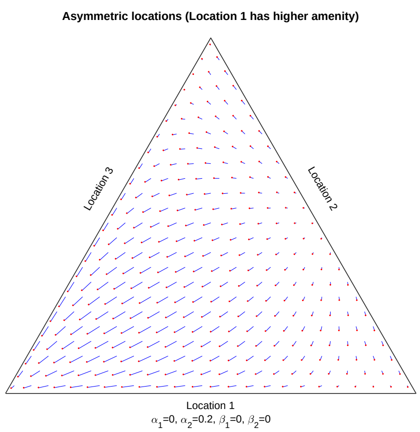
Figure 3: Phase diagrams for 3-region asymmetric example



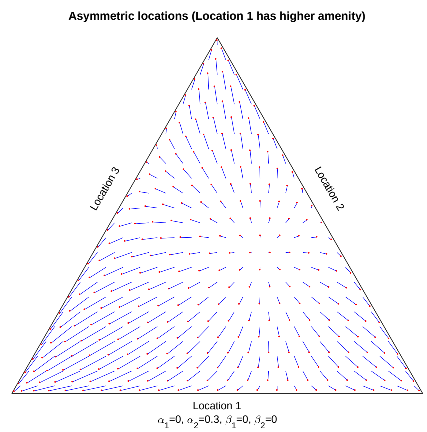
(a) $\alpha_2 = 0$



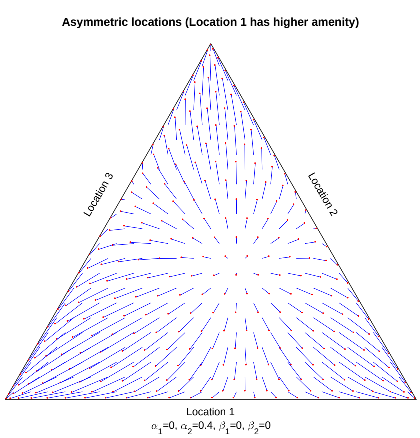
(b) $\alpha_2 = 0.1$



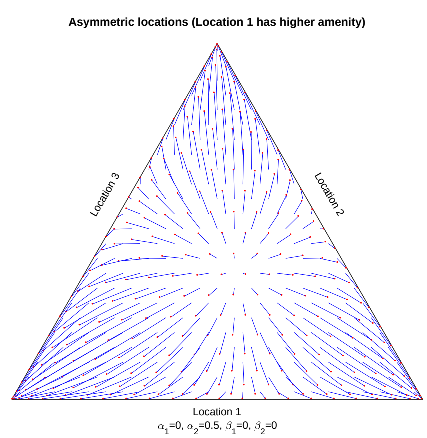
(c) $\alpha_2 = 0.2$



(d) $\alpha_2 = 0.3$



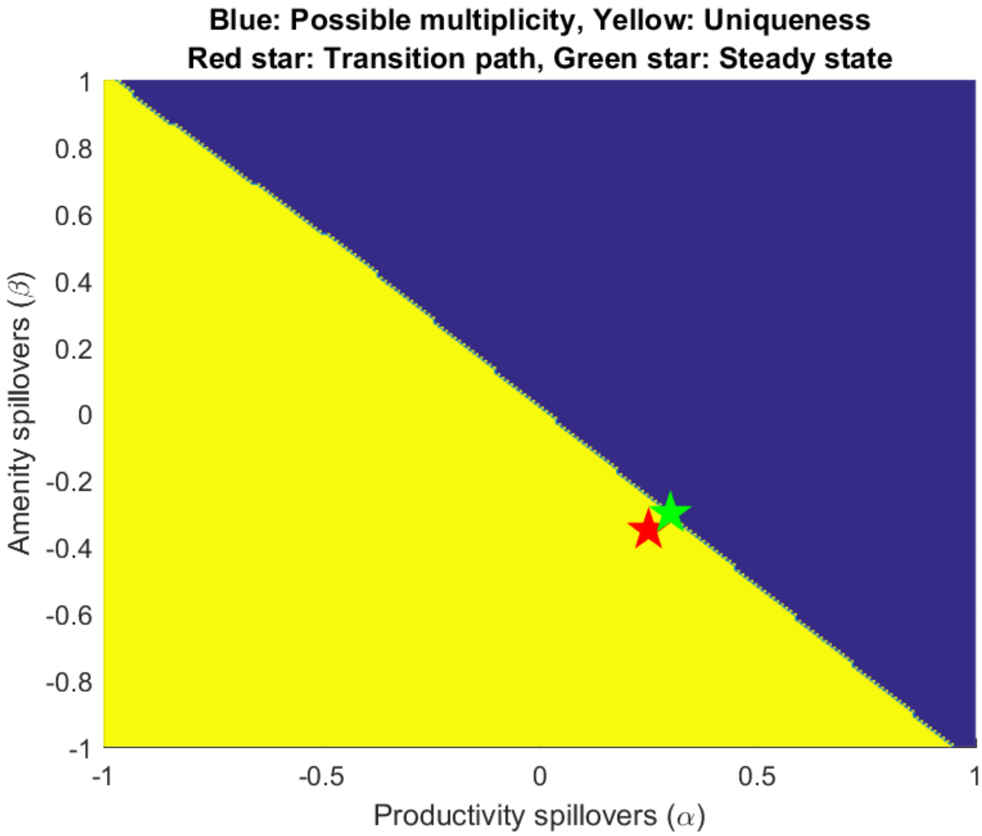
(e) $\alpha_2 = 0.4$



(f) $\alpha_2 = 0.5$

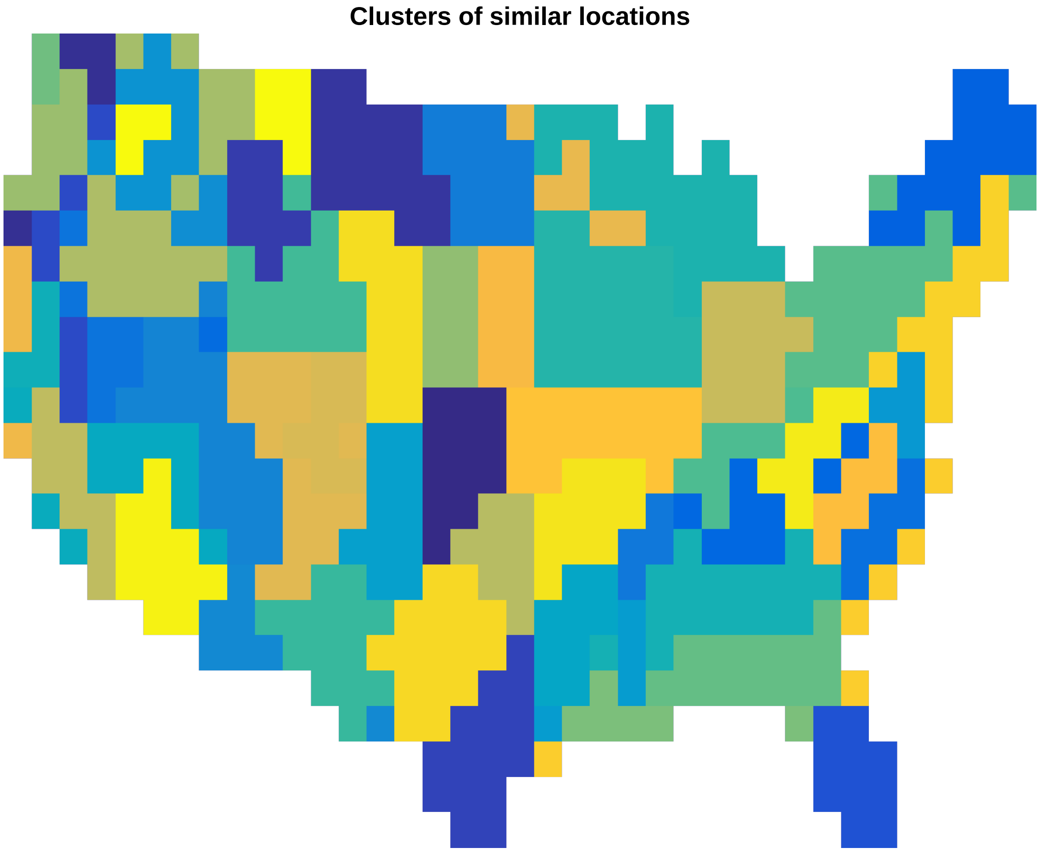
Notes: This figure illustrates phase diagrams for an asymmetric three-region example economy. The parameters $\alpha_1, \beta_1, \beta_2, \sigma$ and θ are held constant as α_2 varies.

Figure 4: Parameter Estimates



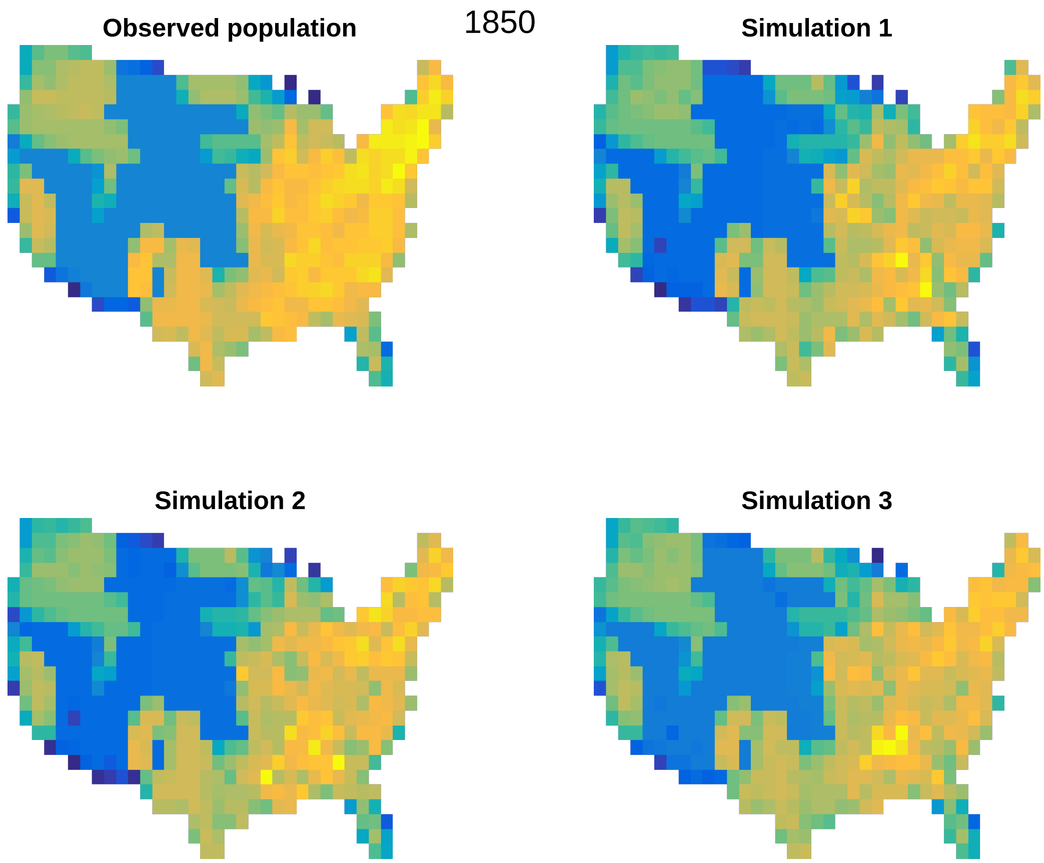
Notes: This figure illustrates the regions of the parameter range (in the space of $\alpha_1 + \alpha_2$ and $\beta_1 + \beta_2$, holding σ and θ constant at the values estimated) that satisfy the condition for uniqueness of equilibrium, as per Proposition 1, and uniqueness of steady-states, as per Proposition 2.

Figure 5: Map of geographic clusters



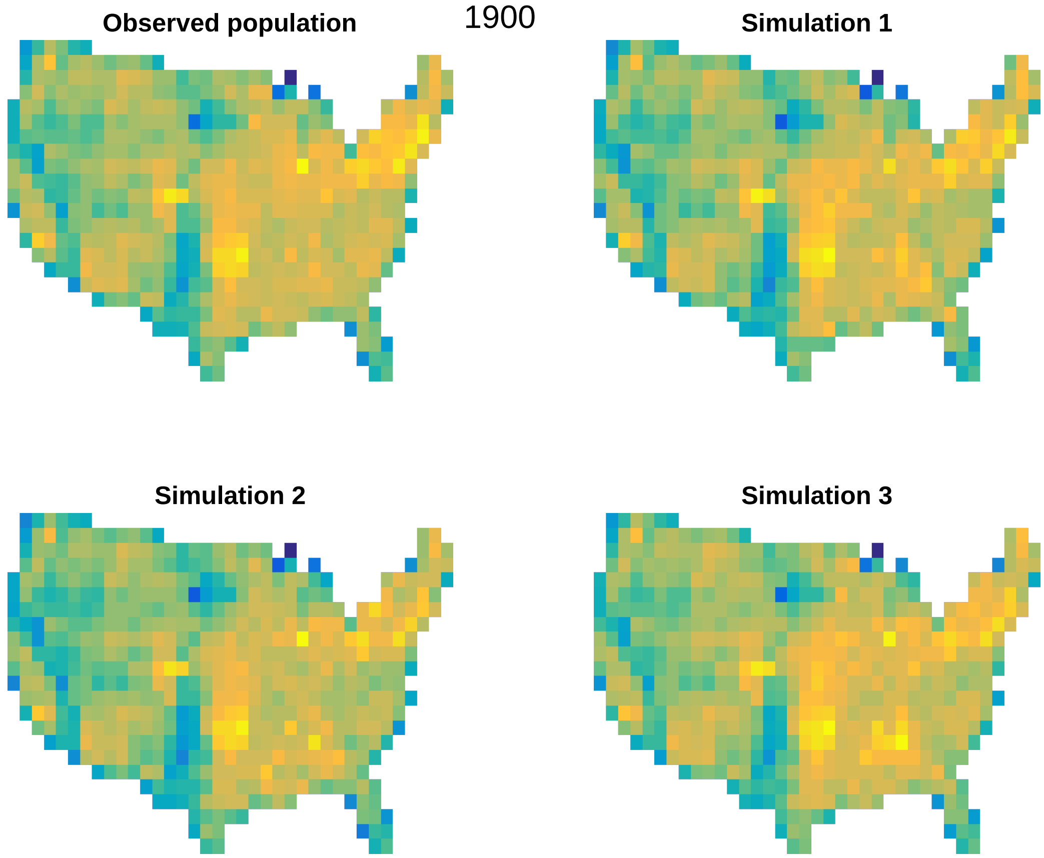
Notes: This figure illustrates a map of the 570 locations in our simulations, as well as how they are grouped into 57 clusters designed to minimize the within-cluster variation in geographic characteristics.

Figure 6: Map of 3 example simulations of random (within-cluster) initial populations



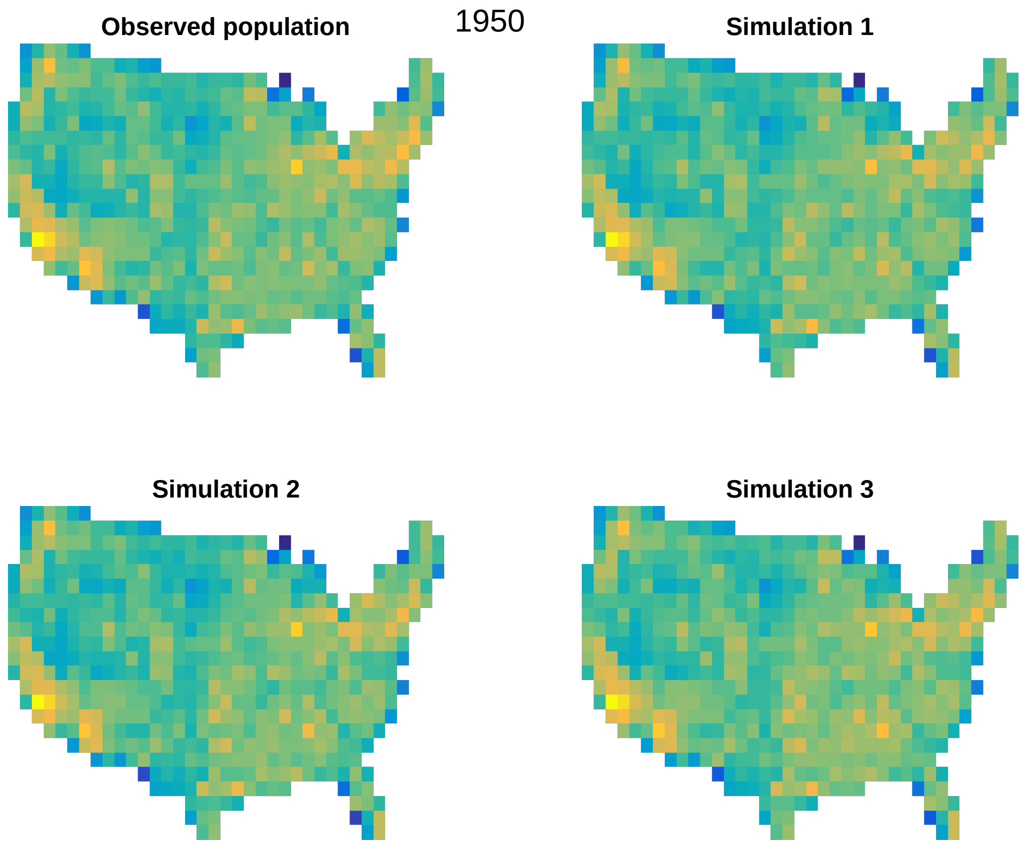
Notes: This figure illustrates (for 1850) the results of the first three of our 200 simulations of randomized (within the geographically similar cluster regions of Figure 5, drawn without replacement) initial year 1800 population levels. Also show, at the top left, is the map of the actual population distribution.

Figure 7: Map of 3 example simulations of random (within-cluster) initial populations



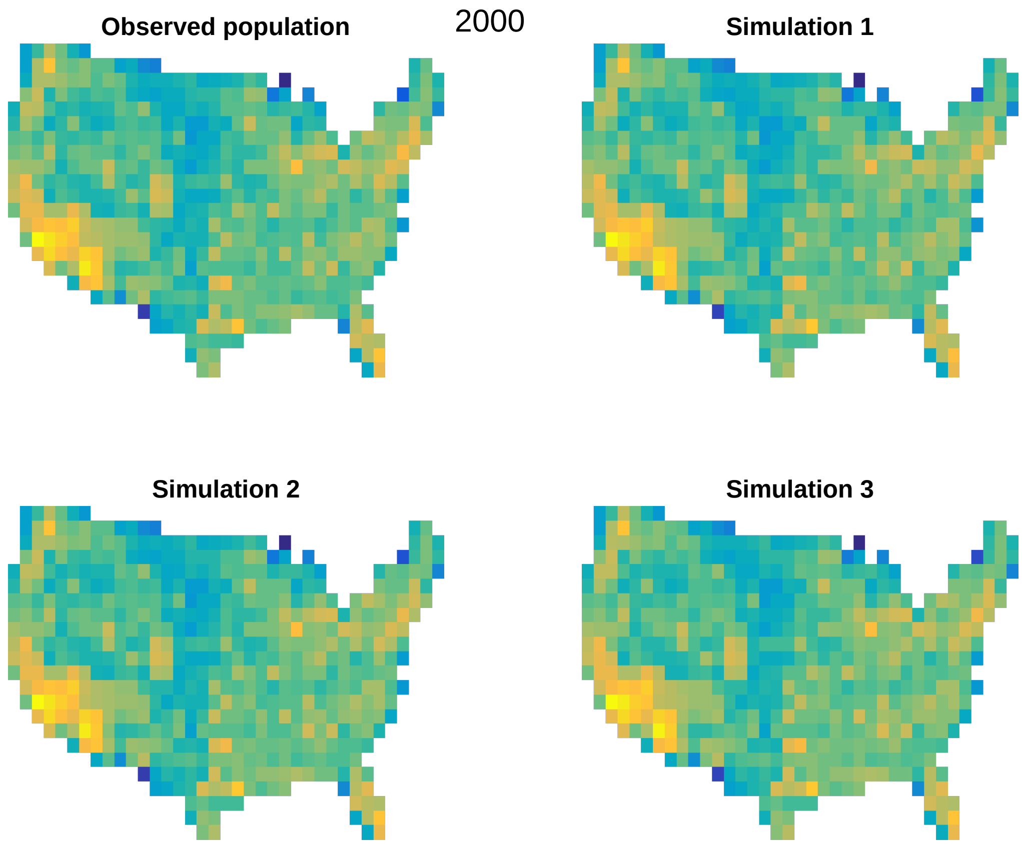
Notes: This figure illustrates (for 1900) the results of the first three of our 200 simulations of randomized (within the geographically similar cluster regions of Figure 5, drawn without replacement) initial year 1800 population levels. Also show, at the top left, is the map of the actual population distribution.

Figure 8: Map of 3 example simulations of random (within-cluster) initial populations



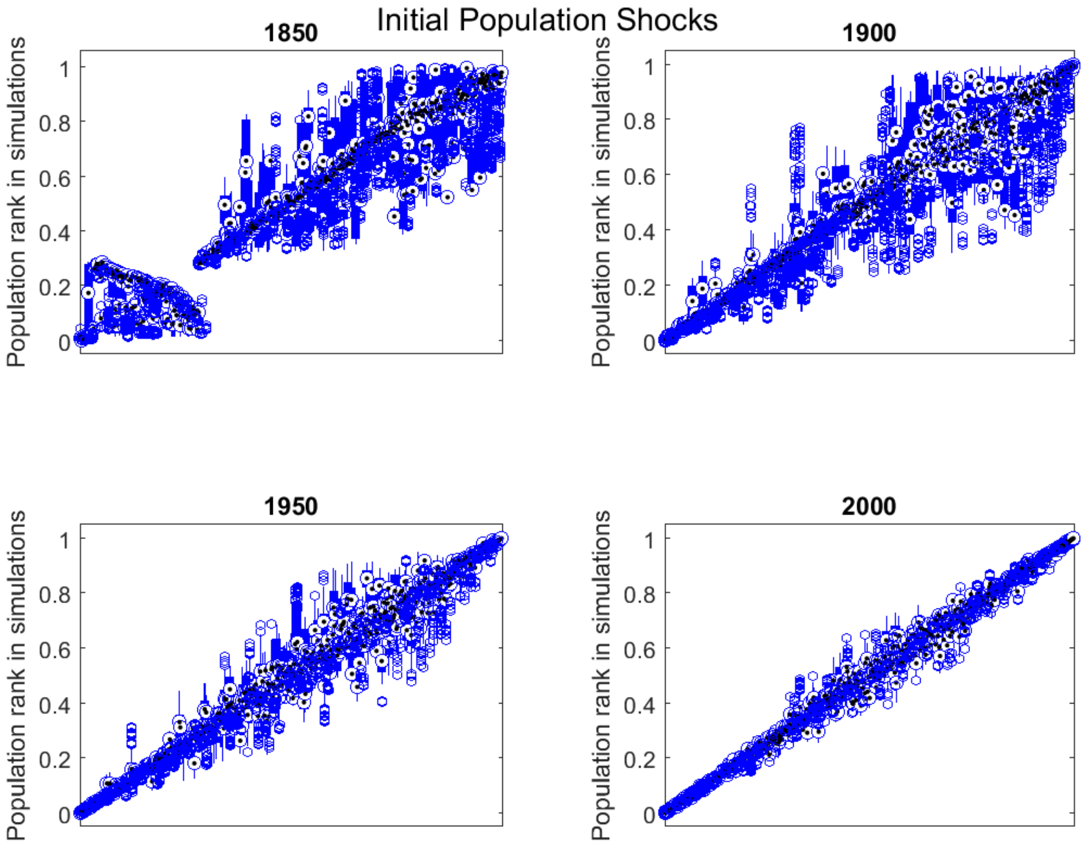
Notes: This figure illustrates (for 1950) the results of the first three of our 200 simulations of randomized (within the geographically similar cluster regions of Figure 5, drawn without replacement) initial year 1800 population levels. Also show, at the top left, is the map of the actual population distribution.

Figure 9: Map of 3 example simulations of random (within-cluster) initial populations



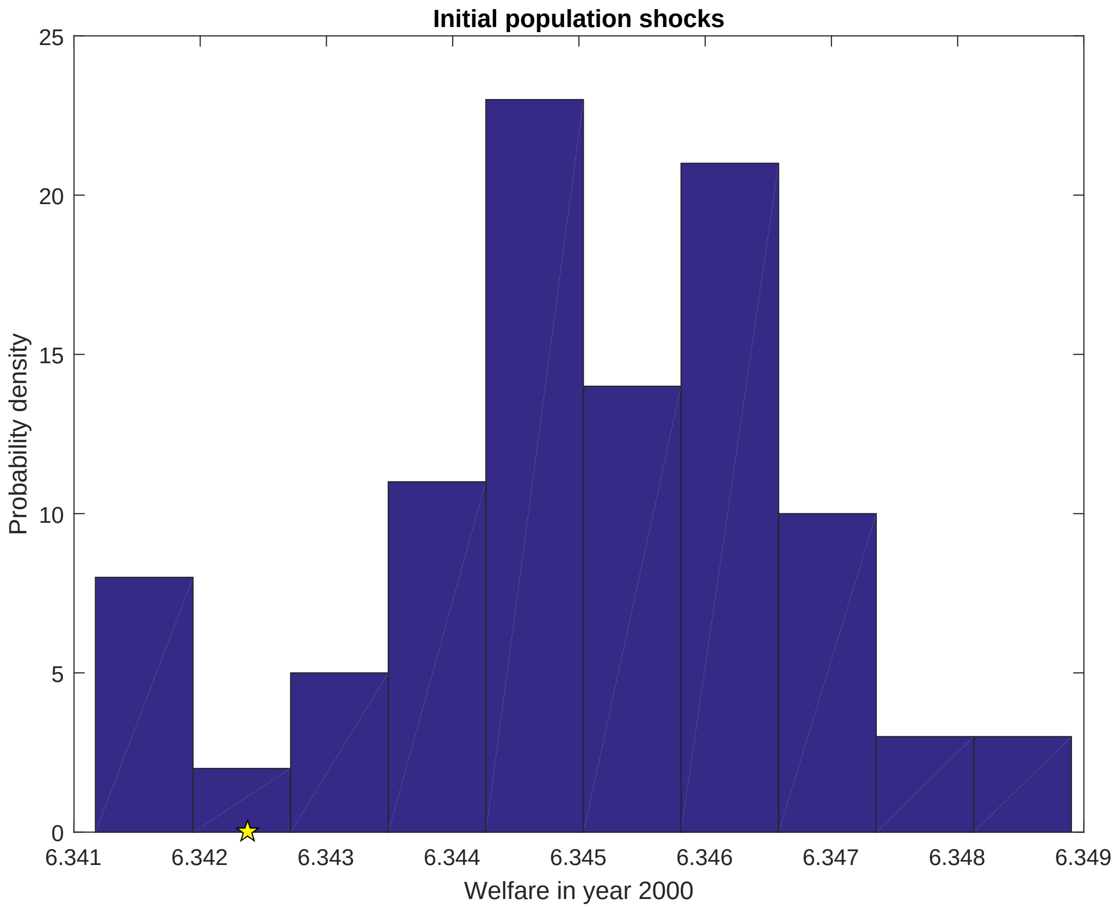
Notes: This figure illustrates (for 2000) the results of the first three of our 200 simulations of randomized (within the geographically similar cluster regions of Figure 5, drawn without replacement) initial year 1800 population levels. Also show, at the top left, is the map of the actual population distribution.

Figure 10: The distribution of population, by year, across 200 simulations of random initial populations



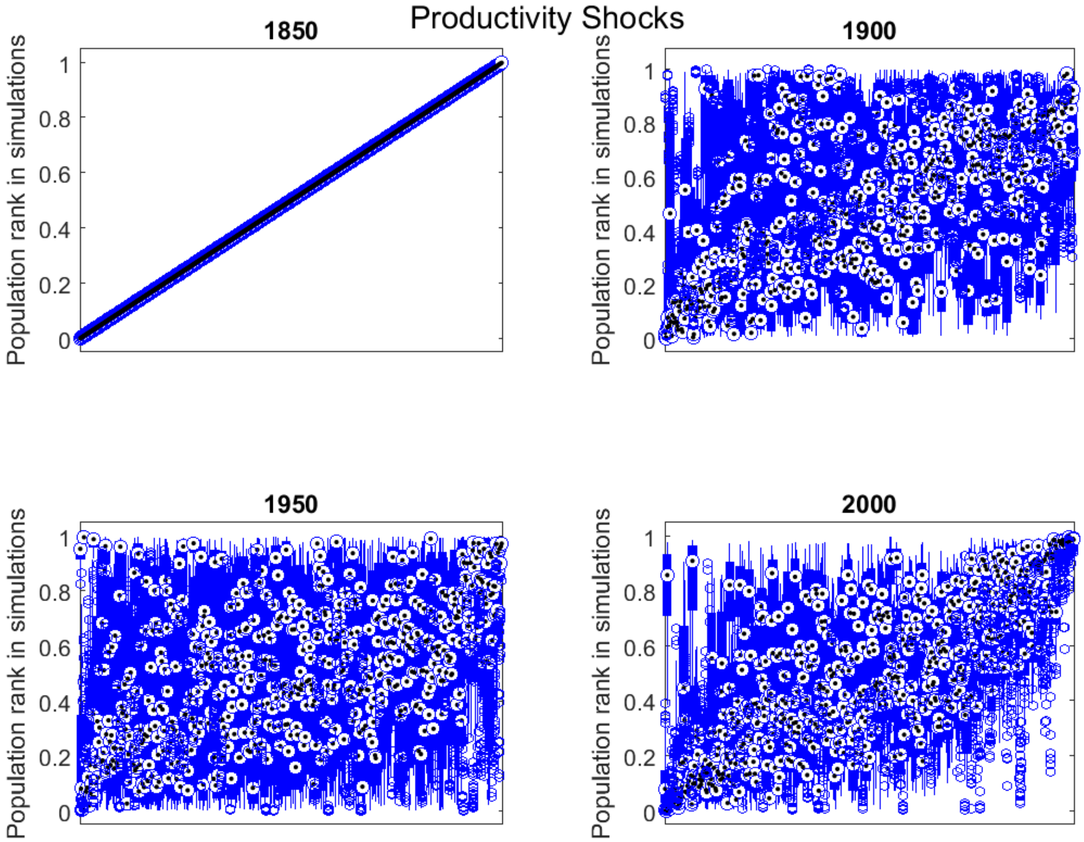
Notes: This figure illustrates how a location's rank in the nationwide population distribution (within any given year, as shown) can vary across 200 simulations of random (drawn, within-the geographically similar clusters of Figure 5, without replacement) population levels from year 1800. The x-axis refers to each location, ordered according to their across-200 simulations median nationwide population rank within the year shown. Then, for each location, the figure contains box plots (with the max-min range in narrow blue, the interquartile range in wide blue, and the mean shown with a black circle) if that location's cross-simulations distribution of nationwide population rank.

Figure 11: The distribution of aggregate welfare in 2000 across 200 simulations of random initial populations



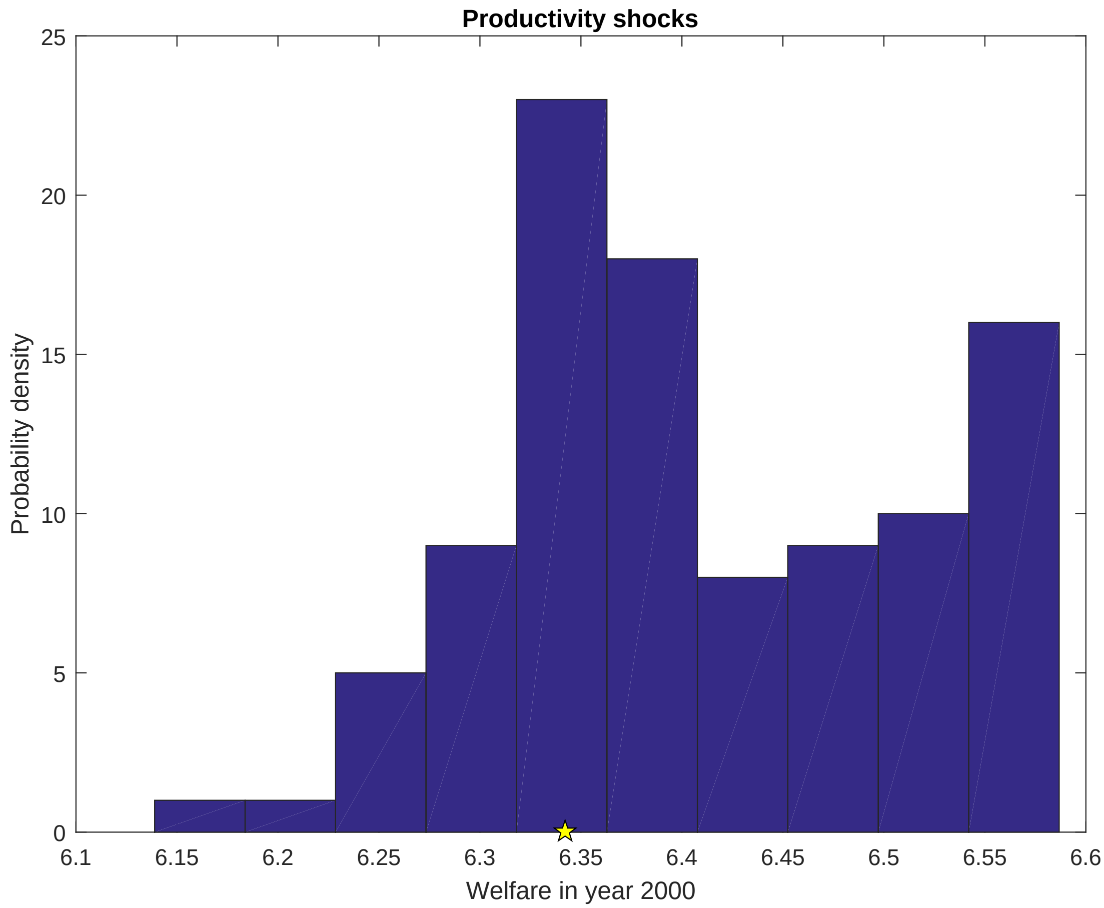
Notes: This figure illustrates how aggregate (population-weighted average) welfare, in logs, in 2000 varies across all 200 simulations of random (drawn, within-the geographically similar clusters of Figure 5, without replacement) population levels from year 1800. The yellow star indicates the factual value from 2000.

Figure 12: The distribution of population, by year, across 200 simulations of random productivity values along transition path



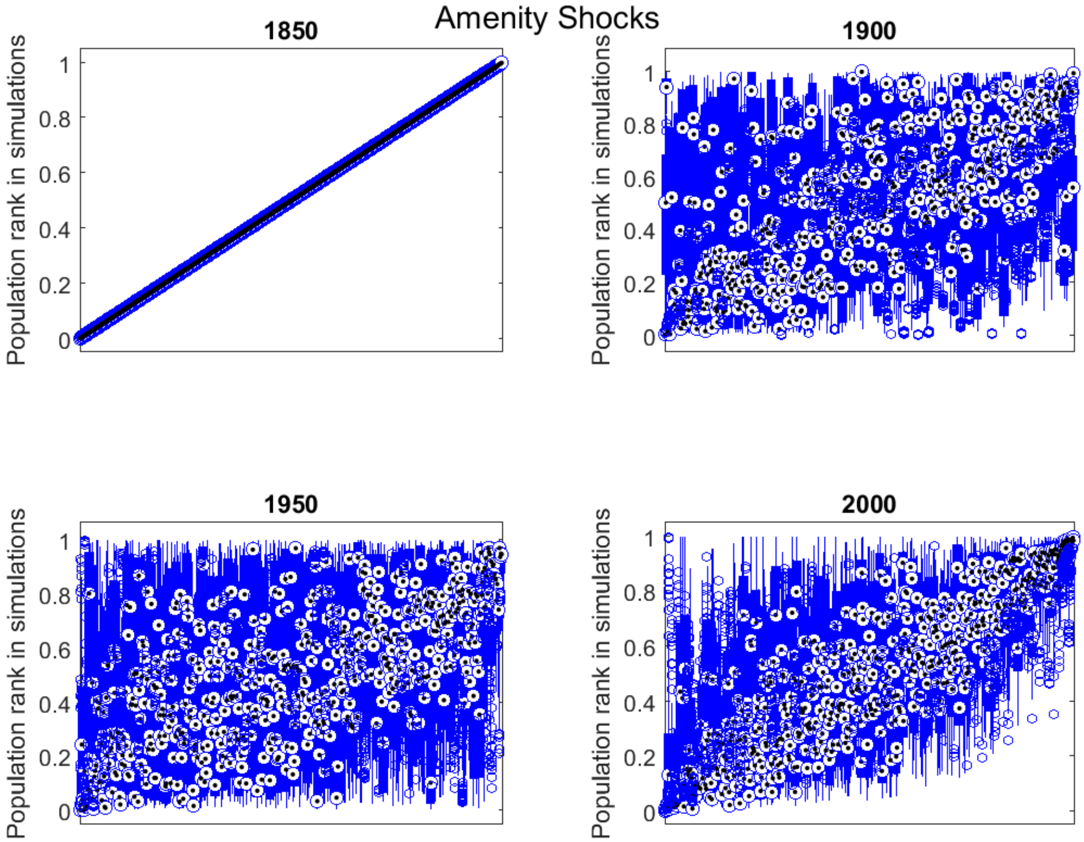
Notes: This figure illustrates how a location's rank in the nationwide population distribution (within any given year, as shown) can vary across 200 simulations of random (drawn, within-the geographically similar clusters of Figure 5, without replacement) productivity shocks $\overline{A}_{it}$ in years 1900 and 1950 (but not 1850 or 2000). The x-axis refers to each location, ordered according to their across-200 simulations median nationwide population rank within the year shown. Then, for each location, the figure contains box plots (with the max-min range in narrow blue, the interquartile range in wide blue, and the mean shown with a black circle) if that location's cross-simulations distribution of nationwide population rank.

Figure 13: The distribution of aggregate welfare in 2000 across 200 simulations of random productivity values along transition path



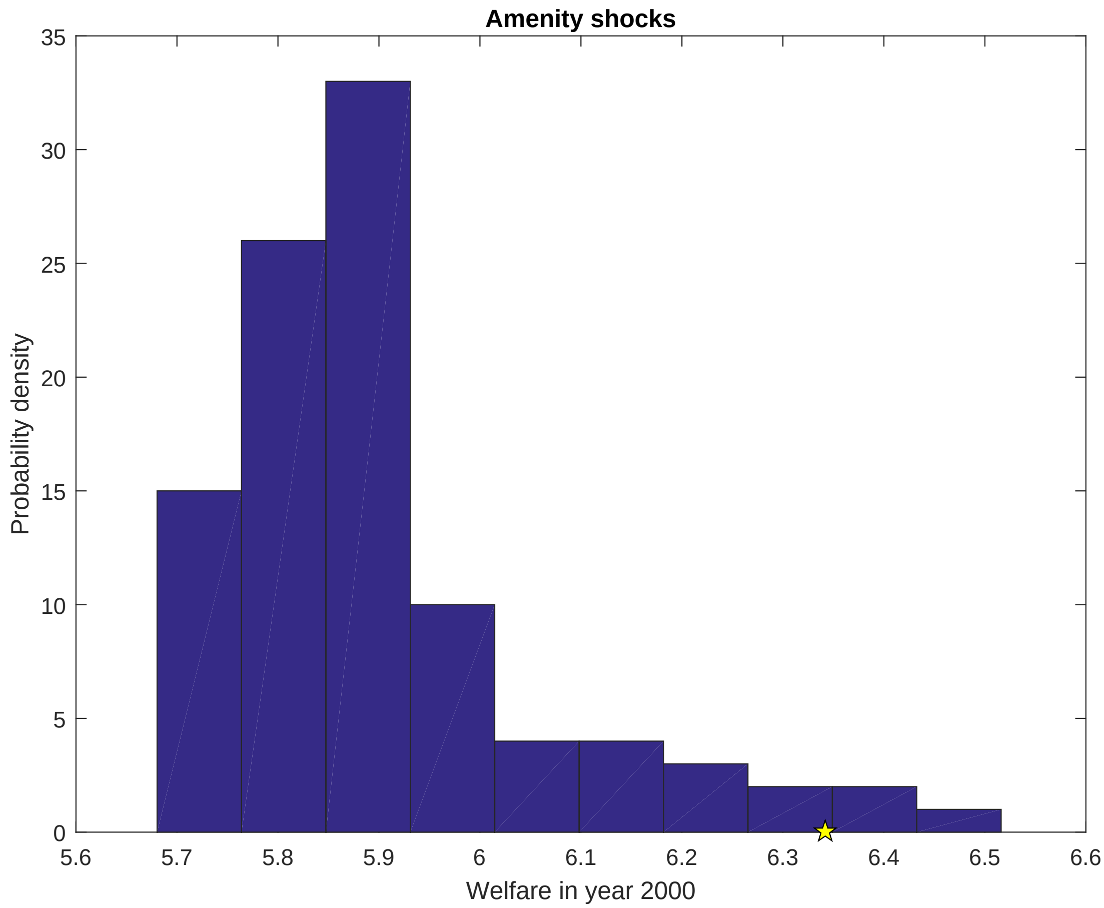
Notes: This figure illustrates how aggregate (population-weighted average) welfare, in logs, in 2000 varies across all 200 simulations of random (drawn, within-the geographically similar clusters of Figure 5, without replacement) productivity shocks $\overline{A}_{it}$ in years 1900 and 1950 (but not 1850 or 2000). The yellow star indicates the factual value from 2000.

Figure 14: The distribution of population, by year, across 200 simulations of random amenity values along transition path



Notes: This figure illustrates how a location's rank in the nationwide population distribution (within any given year, as shown) can vary across 200 simulations of random (drawn, within-the geographically similar clusters of Figure 5, without replacement) amenity shocks $\overline{u}_{it}$ in years 1900 and 1950 (but not 1850 or 2000). The x-axis refers to each location, ordered according to their across-200 simulations median nationwide population rank within the year shown. Then, for each location, the figure contains box plots (with the max-min range in narrow blue, the interquartile range in wide blue, and the mean shown with a black circle) if that location's cross-simulations distribution of nationwide population rank.

Figure 15: The distribution of aggregate welfare in 2000 across 200 simulations of random amenity values along transition path



Notes: This figure illustrates how aggregate (population-weighted average) welfare, in logs, in 2000 varies across all 200 simulations of random (drawn, within-the geographically similar clusters of Figure 5, without replacement) amenity shocks $\overline{u}_{it}$ in years 1900 and 1950 (but not 1850 or 2000). The yellow star indicates the factual value from 2000.

Table 1: Estimating elasticities and spillovers

	First stage			Second stage	
	(1)	(2)	(3)	(4)	(5)
	Wage	Pop.	Trade dest. FE	Trade orig. FE	Migr. dest. FE
Model log wage	0.612***				
	(0.110)				
Predicted log wage				-12.676***	11.736***
				(1.913)	(1.621)
Model log population		0.315***			
		(0.024)			
Predicted log population				3.034***	-4.000***
				(0.512)	(0.515)
Predicted log population 50 years ago				0.351***	-0.045*
				(0.028)	(0.025)
Model log price index			-4.141***		
			(0.052)		
Predicted log trade destination FE					0.240
					(0.187)
Elasticity of substitution σ				13.676***	49.821
				(1.913)	(36.513)
Migration elasticity θ					11.736***
					(1.621)
Contemporaneous productivity spillover α_1				0.239***	
				(0.010)	
Lagged productivity spillover α_2				0.028***	
				(0.003)	
Contemporaneous amenity spillover β_1					-0.341***
					(0.018)
Lagged amenity spillover β_2					-0.004*
					(0.002)
1800 Population	Yes	Yes	Yes	Yes	Yes
Geographic controls	Yes	Yes	Yes	Yes	Yes
Box-year FE	Yes	Yes	Yes	Yes	Yes
F-statistic	165.7	384.5	1298.0	299.0	255.2
R-squared	0.504	0.523	0.846	0.432	0.628
Observations	44408	44408	44408	44408	44408

Table 2: Estimating elasticities and spillovers over time

	Trade origin FE				Migration destination FE			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	1850	1900	1950	2000	1850	1900	1950	2000
Elasticity of substitution σ	13.975***	14.421***	13.495***	16.216***	4.877***	4.247***	7.834**	4.505***
	(2.051)	(2.079)	(2.122)	(2.138)	(0.807)	(0.925)	(3.288)	(1.607)
Migration elasticity θ					8.449***	6.756***	5.582***	4.504***
					(1.678)	(1.699)	(1.733)	(1.744)
Contemporaneous productivity spillover α_1	0.144***	0.186***	0.266***	0.268***				
	(0.021)	(0.016)	(0.013)	(0.010)				
Lagged productivity spillover α_2	0.033***	0.032***	0.010	-0.010				
	(0.004)	(0.009)	(0.010)	(0.008)				
Contemporaneous amenity spillover β_1					-0.763***	-0.797***	-0.670***	-0.610***
					(0.104)	(0.142)	(0.136)	(0.148)
Lagged amenity spillover β_2					0.013**	0.048***	-0.071**	-0.232**
					(0.005)	(0.018)	(0.029)	(0.095)
1800 Population	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Geographic controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Box-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	196.8	196.8	196.8	196.8	179.0	179.0	179.0	179.0
Observations	0.441	0.441	0.441	0.441	0.638	0.638	0.638	0.638
N	44408	44408	44408	44408	44408	44408	44408	44408

A Proofs

The following three proofs are special cases of Theorem 3 (parts (i) and (ii)) of Allen, Arkolakis, and Li (2015), which we restate here for convenience:

Consider the following system of $N \times K$ system of equations

$$\prod_{h=1}^{K} \left(x_i^h\right)^{\beta_{kh}} = \sum_{j=1}^{K} K_{ij}^k \left[\prod_{h=1}^{H} \left(x_j^h\right)^{\gamma_{kh}}\right],$$

where $\{\beta_{kh,}\gamma_{kh}\}$ are known elasticities and $\left\{K_{ij}^k > 0\right\}$ are known bilateral frictions. Let $\mathbf{B} \equiv [\beta_{kh}]$ and $\mathbf{\Gamma} \equiv [\gamma_{kh}]$ be the $K \times K$ matrices of the known elasticities. Define $\mathbf{A} \equiv \mathbf{\Gamma}\mathbf{B}^{-1}$ and the absolute value (element by element) of $\mathbf{A}$ as $\mathbf{A}^p$. Then there exists a strictly positive set of $\left\{x_i^h > 0\right\}_{i\in\{1,\dots,N\}}^{h\in\{1,\dots,K\}}$ and that solution is unique if the spectral radius (i.e. the absolute value of the largest eigenvalue, denote $\rho(\cdot)$) of $\mathbf{A}^p$ is weakly less than one, i.e. $\rho(\mathbf{A}^p) \leq 1$.

A.1 Proof of Proposition 1

When trade costs are symmetric, Allen and Arkolakis (2014) show that the origin and destination fixed effects of the gravity trade equation are equal up to scale. That is if $X_{ij} = K_{ij}\gamma_i\delta_j$, $K_{ij} = K_{ji}$, and $\sum_j X_{ij} = \sum_j X_{ji}$, then we have:

$$\gamma_i \propto \delta_i.$$

[4]From equation (3), this implies:

$$\begin{aligned}
w_i^{1-\sigma} A_i^{\sigma-1} &\propto P_i^{\sigma-1} w_i L_i \iff \\
w_i^{1-\sigma} A_i^{\sigma-1} &\propto \left(\frac{w_i u_i}{W_i}\right)^{\sigma-1} w_i L_i \iff \\
w_i &\propto W_i^{\tilde{\sigma}} u_i^{-\tilde{\sigma}} A_i^{\tilde{\sigma}} L_i^{\frac{1}{1-2\sigma}} \iff \\
w_i &\propto W_i^{\tilde{\sigma}} \bar{u}_i^{-\tilde{\sigma}} \bar{A}_i^{\tilde{\sigma}} L_i^{\left(\alpha_1-\beta_1+\frac{1}{1-\sigma}\right)\tilde{\sigma}} \left(L_i^{lag}\right)^{(\alpha_2-\beta_2)\tilde{\sigma}}
\end{aligned}$$

where $\tilde{\sigma} \equiv \frac{\sigma-1}{2\sigma-1}$.

We can use this to simplify our equilibrium equations:

$$\left(W_i^{\tilde{\sigma}} u_i^{-\tilde{\sigma}} A_i^{\tilde{\sigma}} L_i^{\frac{\tilde{\sigma}}{1-\sigma}}\right)^{\sigma} L_i = \sum_j \tau_{ij}^{1-\sigma} A_i^{\sigma-1} u_j^{\sigma-1} W_j^{1-\sigma} \left(W_j^{\tilde{\sigma}} u_j^{-\tilde{\sigma}} A_j^{\tilde{\sigma}} L_j^{\frac{\tilde{\sigma}}{1-\sigma}}\right)^{\sigma} L_j$$

$$\Pi_i^{\theta} = \sum_j \mu_{ij}^{-\theta} W_j^{\theta}$$

[4]The exact scale will be determined by the aggregate labor market clearing condition. However, the scale can be ignored by first solving for the "scaled" labor (i.e. imposing the scalar is equal to one) and then recovering the scale by imposing the labor market clearing condition. Note that this does not affect any of the other equilibrium equations below, as they are all homogeneous of degree 0 with respect to labor.

$$L_i = \sum_j \mu_{ji}^{-\theta} W_i^{\theta} \Pi_j^{-\theta} L_j^{lag}$$

or equivalently:

$$W_i^{\tilde{\sigma}\sigma} L_i^{1+\frac{\sigma}{1-\sigma}\tilde{\sigma}} = \sum_j \tau_{ij}^{1-\sigma} A_i^{\sigma-1-\sigma\tilde{\sigma}} u_i^{\tilde{\sigma}\sigma} u_j^{\sigma-1-\tilde{\sigma}\sigma} A_j^{\tilde{\sigma}\sigma} W_j^{1-\sigma+\sigma\tilde{\sigma}} L_j^{1+\frac{\sigma}{1-\sigma}\tilde{\sigma}}$$

$$\Pi_i^{\theta} = \sum_j \mu_{ij}^{-\theta} W_j^{\theta}$$

$$L_i = \sum_j \mu_{ji}^{-\theta} W_i^{\theta} \Pi_j^{-\theta} L_j^{lag}$$

Let us then use our spillover equations:

$$A_i = \bar{A}_i L_i^{\alpha_1} \left(L_i^{lag}\right)^{\alpha_2}$$

$$u_i = \bar{u}_i L_i^{\beta_1} \left(L_i^{lag}\right)^{\beta_2}$$

to get:

$$W_i^{\tilde{\sigma}\sigma} L_i^{1+\frac{\sigma}{\sigma-1}\tilde{\sigma}-\alpha_1(\sigma-1-\sigma\tilde{\sigma})-\beta_1\sigma\tilde{\sigma}} = \sum_j \tau_{ij}^{1-\sigma} \bar{A}_i^{\sigma-1-\sigma\tilde{\sigma}} \bar{u}_i^{\sigma\tilde{\sigma}} \beta_j^{\sigma-1-\tilde{\sigma}\sigma} \bar{A}_j^{\tilde{\sigma}\sigma} \left(L_i^{lag}\right)^{\alpha_2(\sigma-1-\sigma\tilde{\sigma})+\beta_2\tilde{\sigma}\sigma} \left(L_j^{lag}\right)^{\beta_2(\sigma-1-\tilde{\sigma}\sigma)+\alpha_2(\ldots}$$

$$\Pi_i^{\theta} = \sum_j \mu_{ij}^{-\theta} W_j^{\theta}$$

$$L_i = \sum_j \mu_{ji}^{-\theta} W_i^{\theta} \Pi_j^{-\theta} L_j^{lag}$$

With a little algebra, this simplifies to:

$$W_i^{\tilde{\sigma}\sigma} L_i^{\tilde{\sigma}(1-\alpha_1(\sigma-1)-\beta_1\sigma)} = \sum_j \tau_{ij}^{1-\sigma} \bar{A}_i^{(\sigma-1)\tilde{\sigma}} \bar{u}_i^{\tilde{\sigma}} \beta_j^{(\sigma-1)\tilde{\sigma}} \bar{A}_j^{\tilde{\sigma}\sigma} \left(L_i^{lag}\right)^{\tilde{\sigma}(\alpha_2(\sigma-1)+\beta_2\sigma)} \left(L_j^{lag}\right)^{\tilde{\sigma}(\alpha_2\sigma+\beta_2(\sigma-1))} W_j^{-(\sigma-1)\tilde{\sigma}} L_j^{\tilde{\sigma}(1 \ldots}$$

$$\Pi_i^{\theta} = \sum_j \mu_{ij}^{-\theta} W_j^{\theta}$$

$$L_i W_i^{-\theta} = \sum_j \mu_{ji}^{-\theta} \Pi_j^{-\theta} L_j^{lag}$$

Let me order the endogenous variables as L, W, Π Then the matrix of LHS coefficients becomes:

$$\boldsymbol{B} \equiv \begin{pmatrix} \tilde{\sigma}\left(1-\alpha_1\left(\sigma-1\right)-\beta_1\sigma\right) & \tilde{\sigma}\sigma & 0 \\ 0 & 0 & \theta \\ 1 & -\theta & 0 \end{pmatrix}$$

and the matrix on the RHS coefficients becomes:

$$\mathbf{\Gamma} \equiv \begin{pmatrix} \tilde{\sigma}\left(1+\alpha_1\sigma+\beta_1\left(\sigma-1\right)\right) & -\left(\sigma-1\right)\tilde{\sigma} & 0 \\ 0 & \theta & 0 \\ 0 & 0 & -\theta \end{pmatrix}$$

Hence, we have:

$$\mathbf{A} \equiv \mathbf{\Gamma}\mathbf{B}^{-1} = \begin{pmatrix} \frac{\theta-\sigma-\beta_1\theta+\alpha_1\sigma\theta+\beta_1\sigma\theta+1}{\sigma+\theta+\alpha_1\theta-\alpha_1\sigma\theta-\beta_1\sigma\theta} & 0 & \frac{\tilde{\sigma}(2\sigma-1)(\alpha_1+1)}{\sigma+\theta+\alpha_1\theta-\alpha_1\sigma\theta-\beta_1\sigma\theta} \\ \frac{\theta/\tilde{\sigma}}{\sigma+\theta+\alpha_1\theta-\alpha_1\sigma\theta-\beta_1\sigma\theta} & 0 & \frac{-\theta(\alpha_1-\alpha_1\sigma)-\beta_1\sigma+1}{\sigma+\theta+\alpha_1\theta-\alpha_1\sigma\theta-\beta_1\sigma\theta} \\ 0 & -1 & 0 \end{pmatrix}$$

Note that the spectral radius of the absolute value will be equal to no less than one given the -1 in the third row and second column. Hence the uniqueness condition requires the absolute remainder of the matrix (removing the third row and second column) has a spectral radius no greater than one, i.e.:

$$\rho\begin{pmatrix} \left|\frac{\theta(1+\alpha_1\sigma+\beta_1(\sigma-1))-(\sigma-1)}{\sigma+\theta(1+(1-\sigma)\alpha_1-\beta_1\sigma)}\right| & \left|\frac{(\sigma-1)(\alpha_1+1)}{\sigma+\theta(1+(1-\sigma)\alpha_1-\beta_1\sigma)}\right| \\ \left|\frac{\theta/\tilde{\sigma}}{\sigma+\theta(1+(1-\sigma)\alpha_1-\beta_1\sigma)}\right| & \left|\frac{\theta(1-(\sigma-1)\alpha_1-\beta_1\sigma)}{\sigma+\theta(1+(1-\sigma)\alpha_1-\beta_1\sigma)}\right| \end{pmatrix} \leq 1,$$

as required.

A.2 Proof of Proposition 2

The proof proceeds similarly to the proof of Proposition 1. If migration costs are symmetric and we are in the steady state, we have: $\sum_i L_{ij} = \sum_j L_{ji}$, $L_{ij} = M_{ij}g_i d_j$, and $M_{ij} = M_{ji}$, then it will be the case that:

$$g_i \propto d_i$$

In our case, this implies:

$$W_i^{\theta} \propto \Pi_i^{-\theta} L_i$$

which simplifies our system of equations as follows:

$$W_i^{\tilde{\sigma}\sigma} L_i^{\tilde{\sigma}(1-(\alpha_1+\alpha_2)(\sigma-1)-\sigma(\beta_1+\beta_2))} = \sum_j \tau_{ij}^{1-\sigma} \bar{A}_i^{(\sigma-1)\tilde{\sigma}} \bar{u}_i^{\tilde{\sigma}} u_j^{(\sigma-1)\tilde{\sigma}} \bar{A}_j^{\tilde{\sigma}\sigma} W_j^{-(\sigma-1)\tilde{\sigma}} L_j^{\tilde{\sigma}(1+(\alpha_1+\alpha_2)\sigma+(\beta_1+\beta_2)(\sigma-1))}$$

$$L_i W_i^{-\theta} = \sum_j \mu_{ij}^{-\theta} W_j^{\theta}.$$

Let me order the endogenous variables as L, W. Define $\tilde{\alpha} \equiv \alpha_1 + \alpha_2$ and $\tilde{\beta} \equiv \beta_1 + \beta_2$ Then the matrix of LHS coefficients becomes:

$$\boldsymbol{B} \equiv \begin{pmatrix} \tilde{\sigma}\left(1-\tilde{\alpha}\left(\sigma-1\right)-\tilde{\beta}\sigma\right) & \tilde{\sigma}\sigma \\ 1 & -\theta \end{pmatrix}$$

and the matrix on the RHS coefficients becomes:

$$\mathbf{\Gamma} \equiv \begin{pmatrix} \tilde{\sigma}\left(1+\tilde{\alpha}\sigma+\tilde{\beta}\left(\sigma-1\right)\right) & -\left(\sigma-1\right)\tilde{\sigma} \\ 0 & \theta \end{pmatrix}$$

Hence, we have:

$$\mathbf{A} \equiv \mathbf{\Gamma}\mathbf{B}^{-1} = \begin{pmatrix} \frac{\theta-\sigma-\tilde{\beta}\theta+\tilde{\alpha}\sigma\theta+1}{\sigma+\theta\left(1+(1-\sigma)\tilde{\alpha}-\tilde{\beta}\sigma\right)} & \frac{-(\sigma-1)(\tilde{\alpha}+1)}{\sigma+\theta\left(1+(1-\sigma)\tilde{\alpha}-\tilde{\beta}\sigma\right)} \\ \frac{\theta/\tilde{\sigma}}{\sigma+\theta\left(1+(1-\sigma)\tilde{\alpha}-\tilde{\beta}\sigma\right)} & \frac{-\theta\left(\tilde{\alpha}(1-\sigma)-\tilde{\beta}\sigma+1\right)}{\sigma+\theta\left(1+(1-\sigma)\tilde{\alpha}-\tilde{\beta}\sigma\right)} \end{pmatrix}$$

As a result, the condition for uniqueness is identical above, where we simply replace α_1 and β_1 with $\tilde{\alpha} \equiv (\alpha_1+\alpha_2)$ and $\tilde{\beta} \equiv (\beta_1+\beta_2)$, as required:

$$\rho\left(\begin{vmatrix} \frac{\theta\left(1+\tilde{\alpha}\sigma+\tilde{\beta}(\sigma-1)\right)-(\sigma-1)}{\sigma+\theta\left(1+(1-\sigma)\tilde{\alpha}-\tilde{\beta}\sigma\right)} \end{vmatrix} \quad \begin{vmatrix} \frac{(\sigma-1)(\tilde{\alpha}+1)}{\sigma+\theta\left(1+(1-\sigma)\tilde{\alpha}-\tilde{\beta}\sigma\right)} \\ \frac{\theta\left(1-(\sigma-1)\tilde{\alpha}-\tilde{\beta}\sigma\right)}{\sigma+\theta\left(1+(1-\sigma)\tilde{\alpha}-\tilde{\beta}\sigma\right)} \end{vmatrix} \right. \\ \left. \begin{vmatrix} \frac{\theta/\tilde{\sigma}}{\sigma+\theta\left(1+(1-\sigma)\tilde{\alpha}-\tilde{\beta}\sigma\right)} \end{vmatrix} \right) \leq 1.$$

A.3 Proof of Proposition 3

The two systems of equations are:

$$p_{it}^{\sigma-1} = \sum_j T_{ijt}\left(\frac{Y_{jt}}{Y_{it}}\right)P_{jt}^{\sigma-1}$$
$$P_{it}^{\sigma-1} = \sum_j T_{jit}\left(p_{jt}^{\sigma-1}\right)^{-1}$$

and:

$$\left(W_{it}^{\theta}\right)^{-1} = \sum_j M_{jit}\frac{L_{jt-1}}{L_{it}}\left(\Pi_{jt}^{\theta}\right)^{-1}$$
$$\Pi_{it}^{\theta} = \sum_i M_{ijt}W_{jt}^{\theta}.$$

Both systems of equations can be written as:

$$x_i = \sum_j K_{ij}^{A}y_j$$
$$y_i = \sum_j K_{ij}^{B}x_j^{-1}$$

which has a corresponding LHS matrix of coefficients:

$$\boldsymbol{B} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and the matrix on the RHS coefficients becomes:

$$\mathbf{\Gamma} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Hence, we have:

$$\mathbf{A} \equiv \mathbf{\Gamma}\mathbf{B}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

so that $\mathbf{A}^p = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. It is straightforward to check that $\rho\left(\mathbf{A}^p\right) = 1$, as required.