

Heterogeneous Dynasties and the Political Economy of Public Debt

Grégory de Walque*and Louis Gevers†

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Abstract

We consider a small open overlapping generation economy with descending altruism. We introduce heterogeneity within dynasties by assuming that each parent procreates a fixed proportion of selfish children at each generation. Altruistic parents can recognize the type of each child and take it into account when bequeathing. There is no Ricardian equivalence and an active public intergenerational transfer policy is attractive to altruistic dynasty members, although there may be no unanimity among them. We display reasonable conditions for indirect preferences to be single-peaked and we apply the median voter theorem. We describe political equilibrium paths and discuss their compatibility with the steady path of an underlying closed economy with autonomous labor productivity growth. *Journal of Economic Literature* Classification Numbers: D31, D64, D72, D91, H63

*University of Namur

†University of Namur and CORE (UCL)

1 Introduction

We consider a small overlapping generations economy facing stable competitive world markets with exogenously growing individual wages and we attempt to explain political decisions regarding public debt, pensions and taxation. Meaningful political debates usually involve some heterogeneity and we tailor our growth model to accommodate this feature. We follow Cukierman and Meltzer (1989) who wrote a pioneering paper with the same objective by adding a twist to the modified Ramsey model and by restricting ourselves to lump-sum taxes and pensions that are uniform within a generation but variable across generations. Moreover, as they do, we consider only policy changes leaving unscathed the vested rights of retirees while they may affect the standard of living of both the currently active generation and their descendants. However, instead of assuming that innate relative labor productivity varies across families, our paper innovates by introducing *heterogeneous preferences within* otherwise identical dynasties.

In the model we shall study, each adult is assumed to procreate m children out of which a constant number pm display the same pattern of loglinear descending altruism, whereas the rest are as selfish as any standard homo economicus. Negative bequests are prohibited. Obviously, zero bequest is a dominating strategy for the selfish children if their own parents' bequest has to be unconditional, as we assume. We further assume that parents can recognize each child's type and tailor their bequest policy so that the anticipated consumption level of an altruistic child be equal to that of a selfish child, at least if nonnegative bequests are optimal.

Although altruistic decision-makers are concerned with the consumption level of every single descendant, their bequest policy can influence the consumption of only a subset of them. This is due to the no-bequest strategy of

the selfish children. Despite this feature, an altruistic decision-maker would like to choose for all his/her descendants the same consumption growth factor, viz. the product of the utilitarian weight times the market interest factor. If altruistic parents consider optimal for their dynasty to accumulate, public saving coupled with a uniform transfer policy that may be time-dependent can solve the problem raised by selfish descendants.

If altruistic parents consider optimal to consume themselves a fraction of their children's autonomous productivity gain, nonnegative bequest constraints are a source of worry for them. However, if dynastic members cannot borrow privately against their descendants' income, society as a whole is endowed with such a possibility. By assumption, it can set up taxes up to the value of the gross wage bill at each future period and cash its present counterpart on the world capital market. Any expenditure plan having the same total present value can be adopted by the budget authorities. By lowering the growth factor of the *disposable* wage stream, the budget authorities are able to free the bequest constrained dynasties from their embarrassment.

If we were to reject our heterogeneity assumption and to suppose instead that each dynasty is homogeneous in composition, whereas the economy consists of two contrasting dynasties, one of which is selfish while the other has altruistic preferences, extreme public dissaving could be achieved at no cost to the latter, assuming that it is not bequest constrained. Indeed, altruistic parents would fully compensate their offspring for the loss they incur in disposable wage. This observation is of course known as Ricardian equivalence, the subject of a classical paper by Barro (1974). Thus, in our example¹, extreme public dissaving would be backed by a Pareto argument limited to two overlapping generations².

¹We are indebted to Philippe Weil for this forceful remark

²If we add the small economy assumption to the Cukierman-Meltzer (1989) model, we

However, Ricardian equivalence vanishes once we introduce heterogeneity within dynasties, since altruistic decision-makers can compensate only a proper subset of their descendants either directly or indirectly. We exhibit reasonable sufficient conditions for single-peakedness of preferences with respect to the disposable wage growth factor. In particular, if the bequest received is equal to zero, an agent's desired wage growth factor is equal to his/her ideal consumption growth factor. In the long run, with at most two elections based on the majoritarian rule, the disposable wage growth factor is set equal to the median voter's ideal consumption growth factor. This is the only stable political equilibrium. In contrast with the example we just described, brutal enslaving of future generations occurs only if less than half the voters are altruistic.

The formal proofs of all the propositions are available upon request from the authors.

2 The model

We consider an economy consisting primarily of private family dynasties deciding about consumption and bequest plans. By assumption, they face a world financial market with stable interest rates and they forecast with certainty the disposable wage sequence that will accrue to them. Therefore the economic description of our model is consistent with two interpretations : either agents live only one period and children simply show up when their parent dies, or agents live two periods and they overlap in standard fashion.

 may generate analogous drastic policy changes by considering dynasties endowed with the same total amount of human wealth while they may differ with respect to the time pattern of wage earnings.

ion but they commit themselves with respect to their lifetime consumption and bequest policy while active. We start with the former interpretation and sketch the other one towards the end. It goes without saying that the structure of the political debate could depend heavily on the choice of interpretation, were it not for our modelling the retirees' vested rights.

We shall suppose that each adult procreates a fixed number m of children where $m \geq 1$. We depart from the traditional dynastic setting by splitting each family in two types respectively called "altruistic" and "selfish". We denote by p a fixed proportion ($p \in]0, 1[$) and we assume that each family consists of pm altruistic children and $(1 - p)m$ selfish ones. Every selfish child is a typical homo economicus, whereas an altruistic child shares common descending utilitarian preferences with every other altruistic member of his/her dynasty.

By assumption, bequests may not be made conditional on the future bequeathing behavior of the recipients. In contrast with Gevers and Michel (1998), we assume that parents can recognize their children types and they are free to bequeath unequal amounts to each child if they please.

An altruistic parent can safely predict that a selfish child will transmit nothing to his/her own children, by a dominating strategy argument, whereas any altruistic child will carry out the plans of his/her decision-making parent exactly as in the traditional modified Ramsey model (where $p = 1$). A typical agent of type i ($i = a, s$ where a stands for "altruistic" and s for "selfish") living at period t ($t \geq 0$) receives from his/her parent a bequest x_t^i . He or she receives also a lump sum payment w_t which we interpret as disposable wage income. These resources may be used for their own consumption, denoted c_t^i , and the balance can be lent on the world capital market. When the one period riskless bonds mature, each heir receives x_{t+1}^i . If the net interest

factor, assumed fixed and perfectly stable over time, is denoted R , our agent's savings at t amount to $\frac{m}{R}[px_{t+1}^a + (1-p)x_{t+1}^s]$. For simplicity, we write

$$q \stackrel{\text{def}}{=} \frac{m}{R}$$

and we summarize the above accounting description as follows :

$$x_t^a + w_t = c_t^a + pqx_{t+1}^a + (1-p)qx_{t+1}^s, \quad \forall t \geq 0 \quad (1a)$$

$$x_t^s + w_t = c_t^s, \quad \forall t \geq 0 \quad (1b)$$

Negative bequest are prohibited in our small open economy :

$$x_t^a \geq 0, \quad x_t^s \geq 0 \quad (2)$$

As stated in proposition 2 of de Crombrughe and Gevers (1998), a direct consequence of the recognizable type assumption is that altruistic parents make sure that each child gets the same consumption level by having $x_t^a > x_t^s$ whenever $x_t^s > 0$ ³:

$$c_t^a = c_t^s = c_t$$

We assume furthermore that individual disposable wages are expected to follow an exponential path

$$w_{t+\tau} = w_t \cdot g_t^\tau \quad \text{where} \quad 0 \leq g_t < \frac{R}{m} = \frac{1}{q}, \text{ and } \tau, t \geq 0 \quad (3)$$

where g_t is the net wage growth factor expected to remain forever constant by agents alive at period t .

³As explained further on, this assumption restricts somewhat the domain of parameter values for which our conclusions hold, but it simplifies greatly algebra. For a discussion about the discrimination versus the non-discrimination case, see de Crombrughe and Gevers (1998).

In this setting, an altruistic decision-maker is aware that his/her bequeathing policy cannot affect any descendants of one of his/her selfish descendants. We shall call the decision-maker's *sub-dynasty* the proper subset of his/her descendants whose consumption level is influenced by his/her bequeathing decision. Thus, for an altruistic decision-maker at t , the relevant subdynasty consists of m descendants at $t+1$, pm^2 descendants at $t+2$ and $\frac{1}{p}(pm)^h$ descendants at period $t+h$. Since there is not finite horizon, any subdynasty from any t onwards can be considered as the union of infinitely many subdynasties that start at a later date. Note that each altruistic child of a selfish parent initiates a new subdynasty that is not the continuation of a preexisting subdynasty. Selfish children of selfish parents do not belong to any subdynasty, whereas selfish children of an altruistic parent belong to the latter's subdynasty.

We denote $H_t(t+T)$ the present value at t of the wage resources for an altruistic decision-maker at time t and his/her subdynastic descendants up to some later date $t+T$ (for $T \geq 0$):

$$\begin{aligned} H_t(t+T) &= w_t \left(1 + \frac{1}{p} \sum_{h=1}^T (pqg_t)^h \right) \\ &= w_t \left[1 + \frac{qg_t(1 - (pqg_t)^T)}{(1 - pqg_t)} \right], \quad \forall t, T \in \mathbb{N}_+ \end{aligned} \quad (4a)$$

Under assumption (3), this expression converges when T grows to infinity.

We denote it H_t :

$$H_t = \lim_{T \rightarrow \infty} H_t(t+T) = w_t \left[1 + \frac{qg_t}{(1 - pqg_t)} \right] \quad (4b)$$

For future reference, we define next a related present value concept for a dynasty starting at period t ; we denote it H_t^* , and we proceed by elucidating its relation with subdynastic human wealth. At each future period, a fraction $(1-p)$ of dynastic members are selfish. Those born at period $t+1$ belong to

the initial parent's subdynasty. Starting at period $t+2$, a subfraction $p(1-p)$ of dynastic members are altruistic children of a selfish parent and they start a new subdynasty whereas a subfraction $(1-p)^2$ are selfish children of a selfish parent and they bequeath nothing. This observation explains the following decomposition of dynastic human wealth :

$$\begin{aligned} H_t^* &= H_t + p(1-p) \sum_{h=2}^{\infty} q^h H_{t+h} + (1-p)^2 \sum_{h=2}^{\infty} q^h w_{t+h} \\ &= w_t \sum_{h=0}^{\infty} (qg_t)^h = \frac{w_t}{1 - qg_t} \end{aligned} \quad (5)$$

We turn now to altruistic preferences. Because they admit of an additive representation, private plans pertaining to the subdynasty are independent of what happens to dynastic members outside it. By assumption, each decision-maker's objective function is a weighted sum of logarithms of individual consumption level with exponentially decreasing weights. As there is no ascending altruism, each altruistic child's preference representation can be obtained simply by omitting the first element in his/her altruistic parent's utilitarian sum.

Let γ stand for the altruism factor of a particular dynasty. By assumption, $0 < \gamma < 1$. Let us denote by W_t the welfare of a subdynasty initiated at generation t . Assuming away average utilitarianism, we can write

$$W_t = \log c_t + \frac{1}{p} \sum_{h=1}^{\infty} (p\gamma)^h \cdot \log c_{t+h} \quad (6)$$

The altruistic parent cares equally about every descendant of any given generation, independently of his/her subdynasty. Much as total dynastic human wealth (5), the welfare of the whole dynasty at period t may be expressed as the weighted sum of its subdynastic components plus the welfare

of the individuals who do not belong to an altruistic chain:

$$W_t^* = W_t + p(1-p) \sum_{h=2}^{\infty} \gamma^h W_{t+h} + (1-p)^2 \sum_{h=2}^{\infty} \gamma^h \log w_{t+h} \quad (7)$$

3 Consumption and bequest path

The altruistic decision-maker's objective is to maximize the welfare of his/her subdynasty while respecting the following subdynastic budget constraint

$$x_t^a + H_t = c_t + \frac{1}{p} \sum_{h=1}^{\infty} (pq)^h c_{t+h} \quad (8)$$

as well as the nonnegative bequest constraints.

Proposition 1 : *Let us consider an altruistic decision-maker at $t \geq 0$. For any $x_t^a, H_t \in \mathbb{R}_+, p \in]0, 1[, g_t \in [0, \frac{1}{q}[$ and for any γ such that $\min \left\{ 1; \frac{p^{-1/2}-1}{1-p} \right\} > \gamma \geq qg_t$, the consumption and bequest path which maximizes (6) subject to (2) and the subdynastic budget constraint (8) is characterized by*

$$c_{t+h} = \frac{1-p\gamma}{1+(1-p)\gamma} \cdot (x_t^a + H_t) \cdot \left(\frac{\gamma}{q} \right)^h \quad \forall h \geq 0 \quad (9)$$

$$x_{t+h}^a = \left(\frac{\gamma}{q} \right)^h \cdot (x_t^a + H_t) - g_t^h \cdot H_t \quad \forall h \geq 0 \quad (10)$$

The upper bound set on γ in proposition 1 is actually a sufficient condition for having $x_{t+h}^s \geq 0$ at any period $t+h$ even though the latter constraint is disregarded. The condition says that descending altruism ought to remain limited in any dynasty. Indeed, the range of admissible γ values can be split as follows

$$\begin{aligned} qg_t \leq \gamma < 1 & \quad \text{if } p < \sim .382 \\ qg_t \leq \gamma < \frac{p^{-1/2}-1}{1-p} & \quad \text{if } p \geq \sim .382 \end{aligned}$$

and, making use of the l'Hospital rule, we observe that the *RHS* of the latter expression tends to $\frac{1}{2}$ as p approaches unity whereas it is equal to one if $p \simeq .382$.

In view of (10), we conclude that total subdynastic wealth per capita has growth factor $\left(\frac{\gamma}{q}\right)$ along the equilibrium path, while its human wealth component has growth factor g_t . If $g_t \leq \frac{\gamma}{q}$, the nonnegative bequest constraint can never be binding along the equilibrium path. However, if $g_t > \frac{\gamma}{q}$, the constraint must eventually bite, and we observe that, once it does, it never pays the decision-maker to start again bequeathing positively. The beginning of the optimal path can be characterized as follows: there exists some finite T such that total subdynastic resources up to period $t+T$, i.e. $[x_t^a + H_t(t+T)]$, are sufficient for maximizing the objective function (6) truncated at $t+T$, while bequests stay nonnegative until $t+T$ even though this constraint is disregarded, whereas any attempt to do the same up to $t+T+1$ with total resources $[x_t^a + H_t(t+T+1)]$ would violate the constraint if it is ignored a priori. Once T is determined, the decision problem is the same as if the subdynasty would get extinct at $t+T+1$. In this context, considering decision-making at period t , we denote by $c_t(t+T)$ the optimal consumption of a subdynasty member at generation t . This notation reminds us that the choice of a consumption level at $t+h \leq t+T$ depends on T (with $0 \leq h \leq T$), the distance between the period of decision and the endogenous finite horizon, and moreover that, for $T > 0$, bequests are positive even though the constraint (2) is not taken into account. The optimal consumption and bequest paths are described in the following proposition:

Proposition 2 : *Let us consider an altruistic decision-maker at $t \geq 0$. For any $x_t^a, H_t \in \mathbb{R}_+$, $p \in]0, 1[$, $g_t \in [0, \frac{1}{q}[$ and for any γ such that $\gamma < qg_t$, the consumption and bequest path which maximizes subdynastic welfare (6)*

subject to constraints (2) and (8) is

$$c_{t+h}(t+T) = \frac{1-p\gamma}{1-p\gamma+\gamma[1-(p\gamma)^T]} [x_t^a + H_t(t+T)] \cdot \left(\frac{\gamma}{q}\right)^h, \forall 0 \leq h \leq T \quad (11)$$

$$x_{t+h}^a = \frac{1-p\gamma+\gamma[1-(p\gamma)^{T-h}]}{1-p\gamma+\gamma[1-(p\gamma)^T]} [x_t^a + H_t(t+T)] \cdot \left(\frac{\gamma}{q}\right)^h - H_{t+h}(t+T) \geq 0, \quad \forall 0 \leq h \leq T \quad (12)$$

$$c_{t+h}(t+T) = w_{t+h}, \quad \forall h > T \quad (13)$$

$$x_{t+h}^a = 0, \quad \forall h > T \quad (14)$$

where T is the unique positive integer such that the program

$$\text{Max } \log c_t(t+T) + \frac{1}{p} \sum_{h=1}^T (p\gamma)^h \cdot \log c_{t+h}(t+T)$$

$$\text{s.t. } x_t^a + H_t(t+T) = c_t(t+T) + \frac{1}{p} \sum_{h=1}^T (pq)^h \cdot c_{t+h}(t+T)$$

is solved by (11) and (12) whereas the program

$$\text{Max } \log c_t(t+T+1) + \frac{1}{p} \sum_{h=1}^{T+1} (p\gamma)^h \cdot \log c_{t+h}(t+T+1)$$

$$\text{s.t. } x_t^a + H_t(t+T+1) = c_t(t+T+1) + \frac{1}{p} \sum_{h=1}^{T+1} (pq)^h \cdot c_{t+h}(t+T+1)$$

implies $x_{t+T+1}^a < 0$ in the solution.

Expression (9) must be compared to (11). In the latter case, a larger share of the relevant expression of total wealth, i.e. $[x_t^a + H_t(t+T)]$, is consumed at each period $t+h \leq t+T$. Comparing (12) to (10), we observe that the growth factor of total wealth is corrected by a time decreasing factor, viz. $\frac{1-p\gamma+\gamma[1-(p\gamma)^{T-h}]}{1-p\gamma+\gamma[1-(p\gamma)^T]}$. The latter reflects the decrease of human wealth as one gets closer to the finite horizon and the gradual consumption of material wealth.

The length of the finite horizon T depends on two main determinants, viz. qg_t and the ratio of the non-human wealth inherited by the altruistic decision-maker towards the wage $\frac{x_t^a}{w_t}$. Their influence is described in the following

Proposition 3 *Under the assumptions of proposition 2 (in particular for every $qg_t \in]\gamma, 1[$), T increases stepwise from zero to infinity as x_t^a grows from zero to infinity. Moreover, for any $x_t^a > 0$, T is stepwise increasing as qg_t decreases and it grows to infinity as qg_t approaches γ .*

The last sentence in the proposition can be explained as follows. The larger the wage growth factor g_t , the stronger the incentive for the bequest constrained decision-maker to consume out of the inherited wealth and the less distant the finite horizon. As qg_t approaches its upper bound, human wealth grows without bound and the incentive to dissave from x_t^a becomes stronger, so that T goes to its minimum value. This limit value of T is a positive function of $\frac{x_t^a}{w_t}$. In particular, if this ratio is sufficiently small (more precisely if $\frac{x_t^a}{w_t} < \frac{1-\gamma}{\gamma}$), this limit value is equal to zero.

We study next the dynastic distribution of wealth generated by our model if exogenous parameters remain constant; in particular, we assume that the disposable wage growth factor g has remained stable ever since the dynasty started. For simplicity, we restrict ourselves to the altruistic members of the dynasty. As it turns out, the bequest received by an altruistic agent depends on two things: the number of links of the longest subdynasty ending with him/her and the non-human wealth of the initiator of the chain. Consider an altruistic ancestor at period zero with non-human wealth x_0^a ; from $t \geq 2$ onwards, there are $p(1-p)m^t$ altruistic children of selfish parents who live at t . These children are the first link of a new subdynasty. From $t \geq 3$ onwards, there are also $p^2(1-p)m^t$ altruistic grand-children of selfish grand-parents. They constitute the second link in a subdynasty. The reasoning may be generalized and, from $t \geq h+1$ onwards, there are $p^{t-h}(1-p)m^t$ members of generation t who are the $(t-h)$ th link of a subdynasty. The corresponding amounts of wealth can be calculated with help of (10), (12) and (14). Table

1 summarizes our findings for a dynasty initiated at $t = 0$ with $x_0^a \geq 0$. If $\gamma \leq qg$ and $x_0^a = 0$, private bequests never become positive and consumption is uniform within each generation.

Table 1^a : distribution of total subdynastic wealth per altruistic member of the dynasty at generation $t \geq 2$. The dynasty was initiated at $t = 0$ with $x_0^a \geq 0$

No. of uninterrupted links	$x_t^a + H_t$		conditional proportion of altruists in the dynasty
	$\gamma \geq qg$	$\gamma < qg$	
0	H_t	H_t	$(1 - p)$
1	$H_t \left(\frac{\gamma}{qg}\right)$	H_t	$p(1 - p)$
2	$H_t \left(\frac{\gamma}{qg}\right)^2$	H_t	$p^2(1 - p)$
...
$t - h$ (with $h < t$)	$H_t \left(\frac{\gamma}{qg}\right)^{t-h}$	H_t	$p^{t-h}(1 - p)$
...
t	$\left(\frac{\gamma}{q}\right)^t x_0^a + H_t \left(\frac{\gamma}{qg}\right)^t$	$\begin{cases} \xi(t, T) + H_t \Leftrightarrow t \leq T \\ H_t \Leftrightarrow t > T \end{cases}$	p^t

^a where $\xi(t, T)$ is given by expression (12)

If $\gamma > qg$ and $x_0^a = 0$, the conditional distribution of total subdynastic wealth among altruistic members of the dynasty approaches the discrete analogue of the Pareto distribution as $T \rightarrow \infty$. Under the same pair of assumptions, we can easily compute the ratio at period t of expected total

subdynastic wealth with respect to its human wealth component

$$\begin{aligned}
E \left\{ \frac{x_t^a + H_t}{H_t} \right\} &= p^t \left(\frac{\gamma}{qg} \right)^t + (1-p) \sum_{h=0}^{t-1} p^h \left(\frac{\gamma}{qg} \right)^h \\
&= \left(\frac{p\gamma}{qg} \right)^t \frac{p \left(1 - \frac{\gamma}{qg} \right)}{1 - \frac{p\gamma}{qg}} + \frac{(1-p)}{1 - \frac{p\gamma}{qg}}
\end{aligned} \tag{15}$$

Expression (15) tells us about wealth dispersion among altruistic dynasty members in the lower part of the distribution, since $x_t^a = 0$ for the children of a selfish parent. It is also revealing from the point of view of their consumption distribution, since their consumption is proportional to their wealth. If $\frac{\gamma}{qg} \geq \frac{1}{p}$, this ratio grows without bound. If $\frac{\gamma}{qg} < \frac{1}{p}$, it converges from below to $\frac{1-p}{1-\frac{p\gamma}{qg}}$ which is increasing and convex in $\frac{\gamma}{qg}$. We conclude that the following range of interest factors are compatible with a steady path with positive private bequest expectation:

$$\frac{gm}{\gamma} < R < \frac{gm}{p\gamma}$$

where the lower bound is simply the condition for positive private bequest expectation. The *RHS* inequality is actually identical to the stationarity condition obtained by Blanchard (1985) in a related model. Note that $\frac{1}{p}$ corresponds to the inverse proportion of altruistic agents which received a strictly positive bequest in the subset of altruistic agents, or equivalently, to the dilution of altruistic agents with strictly positive non-human wealth within the dynasty⁴.

⁴Indeed, at each period t , p is the proportion of altruistic members of the dynasty, while $p - p(1-p) = p^2$ is the proportion of altruistic members whose parent was altruistic.

4 Public transfer policy

So far, we have described the optimal consumption path chosen by an altruistic decision-maker acting in his/her private capacity at period t . We proceed with a change of perspective: indeed, instead of considering our decision-maker's private capacity, we would now like him or her to evaluate debt and transfer policy as a voter who takes into account the state budget constraint.

We thus leave the realm of partial equilibrium and proceed to an exercise in general equilibrium analysis relying on the small economy assumption. Let us suppose a state consisting of identical heterogeneous dynasties which is vested with the power of taxing labor income and transferring and which can act on the world financial market both as a lender and as a borrower, by either buying or issuing riskless one-period bonds. Assume no redistribution takes place at period 0 and let g_0 stands for the growth factor of individual gross wage, so that we may define $w_0 g_0^t$ as the gross wage at period t . Let D_t stand for the net amount to be repaid to creditors at the beginning of period t per member of generation t ; this number can be of either sign. The state intertemporal budget has much the same pattern as equation (1a); it can be described as follows:

$$w_0 g_0^t - D_t = w_t - q D_{t+1} \quad (16)$$

By consolidating intertemporal budgets from t to $t + l$, we obtain

$$\sum_{h=0}^l w_0 q^h g_0^{t+h} - D_t = \sum_{h=0}^l q^h w_{t+h} - q^{l+1} D_{t+l+1} \quad (17)$$

For simplicity, we further assume that the state can raise on the world financial market the excess over D_t of the present value of the entire flow of individual *gross* wages, henceforth denoted \tilde{H}_t^* . This is tantamount to

letting $q^{l+1}D_{t+l+1}$ vanish in the limit as l grows without bound; if debt is positive, this is the traditionnal *no Ponzi game* condition. Thus, the state's borrowing capacity must be equal to the present value of *net* wages from t onwards, as defined by equation (5). Formally,

$$\sum_{h=0}^{\infty} w_0 q^h g_0^{t+h} - D_t = \tilde{H}_t^* - D_t = \sum_{h=0}^{\infty} q^h w_{t+h} = H_t^* \quad (18)$$

In order to economize on the number of dimensions of the political debate, we shall further assume that $w_{t+h} = g_t^h w_t$ ($h = 0, 1, \dots$) and we define the set of feasible policies at $t \geq 1$

$$G_t = \left\{ (g, w) \in \mathbb{R}_+^2, g < \frac{1}{q}, w = (1 - qg) \left(\tilde{H}_t^* - D_t \right) \right\} \quad (19)$$

As an illustration, let public authorities choose at period t a debt policy driven by the feasible pair (g_t, w_t) and suppose this policy remains unchanged until period $t + l$ so that $g_h = g_t$ ($\forall t \leq h \leq t + l$). We obtain the following expression for the ratio of net over gross dynastic human wealth at $t + l$:

$$\frac{\tilde{H}_{t+l}^* - D_{t+l}}{\tilde{H}_{t+l}^*} = \frac{H_{t+l}^*}{\tilde{H}_{t+l}^*} = \frac{H_t^*}{\tilde{H}_t^*} \left(\frac{g_t}{g_0} \right)^l \quad (20)$$

What happens to national debt if g_t remains for ever at the same level? We have to distinguish whether g_t is set below or above g_0 . If $g_t < g_0$, public debt accumulates and the ratio of the net value of dynastic human wealth over its gross counterpart approaches gradually 0, whereas the growth factor of D_{t+l} approaches g_0 from below, as can be readily seen by inspecting the limit value of the *RHS* of the last expression. This process does not prevent consumption per head from growing without bound provided $g_t > 1$.

If on the contrary $g_t > g_0$, national material wealth ($-D_{t+l}$) piles up and its growth factor approaches g_t from below. To see this, we multiply through the last pair of equalities by $(\tilde{H}_{t+l}^*)/(g_t)^l$ to obtain after rearrangement

$$\frac{(-D_{t+l})}{(g_t)^l} = H_t^* - \frac{\tilde{H}_{t+l}^*}{(g_t)^l} = H_t^* - \tilde{H}_t^* \left(\frac{g_0}{g_t} \right)^l \quad (21)$$

As $l \rightarrow \infty$, the *LHS* approaches H_t^* , which remains constant and positive. We summarise our observations in the next proposition:

Proposition 4 *Suppose the state intertemporal budget is described by equation (16) and $\lim_{l \rightarrow \infty} q^l D_{t+l} = 0$. Suppose both g_0 , the growth factor of individual gross wage, and g_t , the growth factor of net wage, remain constant from t onwards. Then, whenever $g_t < g_0$ (resp. $g_t > g_0$) the economy will be in net debt (resp. credit) position within finite time. From then on, net national debt (resp. national material wealth) will not stop accumulating and its growth factor will approach g_0 (resp. g_t) from below as time goes by. If $g_t = g_0$, the net indebtedness (resp. credit) position of the state at t does not change sign: it keeps growing and its constant growth factor is $g_t = g_0$.*

5 Preferences with respect to public transfers

In the following propositions, we describe the indirect preferences of an agent living at period $t \geq 1$ with respect to intergenerational public transfers.

Proposition 5 : *For any $t \geq 1$, any $q > 0$, any $0 < \gamma < \min \left\{ 1; \frac{p^{-1/2}-1}{1-p} \right\}$ and any $x_t^a, H_t^* \in \mathbb{R}_+$, for any altruistic decision-maker at t , total indirect dynastic welfare W_t^* as defined by (7) and (9) to (14) is continuous and single-peaked in g_t over the interval $[0, \frac{1}{q}[$ provided w_t is adapted so that $(g_t, w_t) \in G_t$. Defining $g(x_t^a)$, the argument maximizing indirect dynastic wel-*

fare at t in this context, we observe that :

- (i) $g(x_t^a) \in [\frac{\gamma}{q}, \frac{1}{q}[$
- (ii) $g(x_t^a)$ is increasing in both x_t^a and γ
- (iii) $x_t^a = 0 \Rightarrow g(x_t^a) = \frac{\gamma}{q}$
- (iv) $x_t^a \rightarrow \infty \Rightarrow g(x_t^a) \rightarrow \frac{1}{q}$
- (v) $\gamma \rightarrow 0 \Rightarrow g(x_t^a) \rightarrow 0$
- (vi) for $p \lesssim .382$, $\gamma \rightarrow 1 \Rightarrow g(x_t^a) \rightarrow \frac{1}{q}$

It is important to note that the single-peakedness of total indirect dynastic welfare W_t^* in g_t is due to the presence of selfish descendants who interrupt the bequest chain. To show this, let us turn temporarily to the traditional model of homogeneous dynasties by studying the limiting case where $p = 1$ ⁵. Under this assumption, $H_t = H_t^*$ and $W_t = W_t^*$ and we may state

Proposition 6 : *If $p = 1$, for any $t \geq 1$, any $q > 0$, any $g_t \in [0, \frac{1}{q}[$, any $0 < \gamma < 1$ and any $x_t, H_t^* \in \mathbb{R}_+$, for any altruistic decision-maker at t , total indirect dynastic welfare W_t^* as defined by (7) and (9) to (14) is continuous and single-plateau in g_t provided w_t is adapted so that $(g_t, w_t) \in G_t$. Indirect dynastic welfare is maximum as long as $g_t \leq \frac{\gamma}{q}$.*

The plateau is just a reflection of Ricardian equivalence between debt and lump-sum taxation to finance public expenditure: in our model, public transfer policy can be nullified by counteracting private bequests. As a direct corollary to this proposition, we obtain the following

Proposition 7 : *Suppose that $p = 1$. Consider a successive generations economy consisting of individuals who differ only with respect to their material wealth endowment as well as their altruistic factor $\gamma \in]\underline{\gamma}, \bar{\gamma}[$ with $\underline{\gamma} \geq 0$ and $\bar{\gamma} < 1$. Then if some dynasties are bequest constrained from a given*

⁵Or equivalently we could prevent selfish agents from having children. In this case the dynasty reduces to its first subdynasty.

generation on, it is a strict Pareto improvement from the viewpoint of the living generations to set an intergenerational redistribution policy driven by the feasible pair $\{g_t = \underline{\gamma}/q, w_t = (1 - \underline{\gamma})g_{t-1}H_{t-1}^\}$*

However, it should be stressed that the descendants of all the decision-makers belonging to a dynasty initially bequest constrained suffer from such policy changes. As is well known, a Pareto argument restricted to the coalition of living agents may be both cogent from the positive viewpoint and dubious as an ethical norm.

This quick glance at the model of homogeneous dynasties and the unsatisfactory conclusions to which it leads, shows the interest of the heterogeneous dynasties approach. As stated in proposition 5, W_t^* is single-peaked in g_t in this setting. This property proves quite useful when we analyse the political economy of public debt. Why is W_t^* increasing in g_t for $qg_t < qg(x_t^a)$ for $p < 1$ and constant in g_t for $p = 1$ over the same interval? This is due to the mitigation of the adverse effect of selfishness in the former case. Since initial subdynastic human wealth gets reduced when g_t increases and $p < 1$, this adverse effect is more than compensated by what happens to the rest of the decision-maker's descendants. In order to grasp more fully the intuition behind proposition 5, let us consider the particular case where $x_t^a = 0$. Under this condition, the welfare of the altruistic decision-maker at t is maximized if the disposable wage growth factor is equal to the ideal consumption growth factor as stated under (iii). Indeed, the disposable wage growing at this pace induces both selfish and altruistic members of the dynasty to enjoy the same consumption level, a level considered first best by their altruistic ascendant. Since there is no private bequeathing, consumption per capita is equal within each generation. In other words, at $x_t^a = 0$ and $g_t = \frac{\gamma}{q}$, the consumption path of an heterogeneous dynasty is the same as the path of an homogenous one.

An altruistic decision-maker at t can indeed ignore the consequence of having selfish descendants. Furthermore, all his/her altruistic descendants would in turn consider ideal the same g_t value. Public intergenerational transfers allow to circumvent perfectly the impossibility of bequeathing anything to the children of one's selfish descendants.

In the presence of a positive initial wealth x_t^a , the decision-maker who is to set g_t at period t is no longer able to ensure consumption equality within every ensuing generation. Indeed, his/her selfish children will not transmit the slightest part of their inheritance to their own descendants. To limit this squandering, it is optimal to set $g_t > \frac{\gamma}{q}$. Then, from some period $t + h$ onwards⁶, every descendant that is not a member of the decision-maker subdynasty gets a higher standard of living than if $g_t = \frac{\gamma}{q}$, whereas those born earlier undergo a relative loss. On the other hand, the members of the decision-maker's subdynasty get at least partly compensated for this loss until $t + T$ is reached.

The consequence of this choice of g_t is twofold. First, all the altruistic descendants of the decision-maker at t who are not members of his/her subdynasty receive no bequest. They would be better off if g_t could be set equal to $\frac{\gamma}{q}$ from their generation onwards. Second, the positive bequest received by the altruistic members of the decision-maker's subdynasty are *decreasing over time* since $g_t > \frac{\gamma}{q}$. After a finite number T of periods, bequeathing stops in the dynasty. It follows from proposition 5 that the $g(x_{t+h})$ value preferred by the altruistic decision-makers of the subdynasty at any period $t + h$ ($0 \leq h \leq T$) is decreasing until $t + T$ at which it settles at $g(x_{t+T}) = \frac{\gamma}{q}$. In conclusion, if a dynasty starts with a positive material wealth, its altruistic members will disagree with respect to the future ideal g -value as long as they

⁶It is easy to show that h is an integer close to $\frac{\log(1-\gamma) - \log(1-qg_t)}{\log g_t - \log(\frac{\gamma}{q})}$.

receive unequal bequests. If the dynasty starts at period zero, this is bound to occur from period 2 to T . We conclude that it is worthwhile to study the political determination of g_t within an economy consisting of identical heterogeneous dynasties.

For this purpose, we consider a given initial pair (g_0, w_0) . Let us count at every generation $t \geq 2$ the proportions of individuals who share the same preferences with respect to the set of feasible pairs G_t . Within this economy, parameter γ is common to all altruistic individuals; their preferred pair can thus only differ in relation with their type (selfish or altruistic) and the bequest received. We already know that a proportion $(1 - p)$ of agents is selfish; they all favor $g_t = 0$. If we turn to the altruistic subpopulation, we may infer from Table 1 the distribution of their preferences with respect to redistribution.

Let $\overline{\mathcal{E}}_{\mathcal{S}}$ (resp. $\overline{\mathcal{E}}_{\mathcal{O}}$) be a successive (resp. overlapping) generation economy consisting of identical heterogeneous dynasties initiated at $t = 0$ and considered at the beginning of period $t \geq 2$ with parameters $p, \gamma, q, g_{t-1}, w_{t-1} \in IR_{++}^5$ with $\frac{1}{2} < p < 1, \gamma \leq \frac{p^{-1/2}-1}{1-p}, qg_{t-1} < 1$ and such that $H_t^* = \frac{w_{t-1}g_{t-1}}{1-qg_{t-1}}, x_0^a \geq 0$.

The economy $\overline{\mathcal{E}}_{\mathcal{S}}$ (resp. $\overline{\mathcal{E}}_{\mathcal{O}}$) is moreover said to fulfill condition $\mathcal{C}_{\mathcal{S}}$ (resp. $\mathcal{C}_{\mathcal{O}}$) if $x_t^a = 0$ for altruistic agents making up at least a fraction $\frac{1}{2} - (1 - p) = p - \frac{1}{2} \geq 0$ (resp. $\frac{1}{2} (1 - \frac{1-p}{1+m}) - (1-p)\frac{m}{1+m} = \frac{p(1+2m)-m}{2(1+m)} \geq 0$) of every dynasty⁷.

Proposition 8 *In $\overline{\mathcal{E}}_{\mathcal{S}}$ (resp. $\overline{\mathcal{E}}_{\mathcal{O}}$), if condition $\mathcal{C}_{\mathcal{S}}$ (resp. $\mathcal{C}_{\mathcal{O}}$) is fulfilled,*

⁷Condition $\mathcal{C}_{\mathcal{O}}$ is obtained by taking into account the fact that, under the assumption of precommitment at the time of private decision-making and vested interests of retirees, selfish retirees are indifferent with respect to the choice of a feasible pair (g_t, w_t) whereas altruistic retirees display the same preference as their altruistic children

a proposal to establish from $t \geq 1$ onwards the feasible pair $\left(\frac{\gamma}{q}, (1 - \gamma)H_t^*\right)$ is a Condorcet winner against any other alternative $(g, w) \in G_t$. Moreover, if there is a policy reappraisal at $t + \tau$, $\tau > 0$, the feasible pair $\left(\frac{\gamma}{q}, (1 - \gamma)\left(\frac{\gamma}{q}\right)^\tau H_t^*\right)$ is a Condorcet winner over any other $(g, w) \in G_{t+\tau}$.

Our next two propositions are rather technical: they distill the information contained in Table 1 and display sufficient conditions for \mathcal{C}_S to hold in $\overline{\mathcal{E}}_S$. Our general conclusion is that \mathcal{C}_S is no difficult to satisfy.

Proposition 9 *Condition \mathcal{C}_S is fulfilled in $\overline{\mathcal{E}}_S$ if*

$$\begin{aligned} \text{either } & \frac{\gamma}{q} \leq g_{t-1} = g_{t-2} = \dots = g_{t-h} \text{ for } h \geq \frac{\log 1/2}{\log p} \\ \text{or } & \frac{p^{-1/2}-1}{q(1-p)} > \frac{\gamma}{q} > g_{t-1} \text{ and } \frac{1}{2} < p < \sqrt{\frac{1}{2}} \simeq .707 \end{aligned}$$

Proposition 9 gives values of the parameters that are sufficient for making sure that the median voter at period $t \geq h \geq 0$ be a member of the coalition of altruistic agents who received zero bequest. If $\frac{\gamma}{q} > g_{t-1}$, the size of this coalition is fixed⁸. If $\frac{\gamma}{q} \leq g_{t-h}$, $p^h \leq \frac{1}{2}$ is the condition for the median voter to belong to the same group of voters⁹. Indeed, the new subdynasties born between period $t - h$ and t transmit no private wealth in this case; their altruistic members join the coalition of altruistic agents who received no bequest, so that, at period t , this coalition includes a majority of voters. Adapting proposition 9 to fit an overlapping generation economy can be done by relying on the same technique as for condition \mathcal{C}_O (cf. footnote 7)¹⁰.

Our last proposition is actually a corollary of proposition 5 and it deals again with condition \mathcal{C}_S . If the latter is not fulfilled when voting takes place

⁸In this case, in view of proposition 1, the extreme values of the upper bound of γ are given by $\lim_{p \rightarrow \frac{1}{2}} \frac{p^{-1/2}-1}{1-p} \simeq .828$ and $\lim_{p \rightarrow \sqrt{\frac{1}{2}}} \frac{p^{-1/2}-1}{1-p} \simeq .646$.

⁹With $\lim_{p \rightarrow 1} \frac{\log 1/2}{\log p} = +\infty$ and $\lim_{p \rightarrow 1/2} \frac{\log 1/2}{\log p} = 1$.

¹⁰ $h \geq \frac{\log 1/2}{\log p}$ must then be replaced by $\frac{\log(\frac{1}{2}) + \log(1 + \frac{p}{m})}{\log p}$ while in the $\frac{\gamma}{q} > g_{t-1}$ case, the upper bound on p should be replaced by $\frac{1}{2} \frac{1 + \sqrt{1 + 2m + 2m^2}}{m + 1}$

at period $t - h$ ($h < t$), it may be satisfied within at most h generations, with $h \leq \frac{\log 1/2}{\log p}$. Let us denote $g(x_t^a(j))$ the g value belonging to the pair $(g, w) \in G_t$ that is best from the viewpoint of any agent j living at t . Let $g_t^{med.}$ be the median value of the $g(x_t^a(j))$ distribution at period t .

Proposition 10 *In $\overline{\mathcal{E}}_S$, a proposal made at period $t - h$ ($h < t$) to establish from then onwards $(g_{t-h}, w_{t-h}) \in G_{t-h}$ where $g_{t-h} = g_{t-h}^{med.} \geq \frac{\gamma}{q}$ and $w_{t-h} = [1 - qg_{t-h}^{med.}] \cdot H_{t-h}^*$ is a Condorcet winner against any other $(g, w) \in G_{t-h}$ if $\frac{\gamma}{q} > g_{t-h-1}$. Therefore, from period $t-h$ onwards, $\frac{\gamma}{q} \leq g_{t-h}$. As a consequence, the first part of proposition 9 applies.*

The last proposition is also valid in the $\overline{\mathcal{E}}_O$ economy if proposition 9 is adapted as described above.

In conclusion, if our small economy consists of identical heterogeneous dynasties having a majority of altruistic agents, a unique long run political equilibrium exists under fairly general circumstances. Furthermore, once established, this unique political equilibrium is robust with respect to unexpected and exogenous changes of parameter m taking place at t once and for all. Indeed, dynastic welfare as defined in (7) belongs to the tradition of total (as opposed to average) utilitarianism. This tradition is defended and illustrated by Arrow and Kurz (1970). It involves reinterpreting the utilitarian weight as $\gamma = \delta m$ where δ does not depend on m . As a result, the stable political equilibrium at period $t \geq 0$ does not depend on m since it is characterized by the pair $(g_t, w_t) \in G_t$ with

$$g_t = \frac{\gamma}{q} = \frac{\gamma R}{m} = \delta R$$

As a direct consequence of equilibrium, expected private bequest per head remains at zero level if it had been so in the past, and it vanishes gradually even though it had been positive at some time in the past. However,

from the viewpoint of altruistic parents, uniform public intergenerational transfers are an improvement over imperfect private transfers. Public saving (resp. dissaving) occurs in equilibrium on a per capita basis if $g_t = \frac{\gamma}{q} > g_0$ (resp. $g_t = \frac{\gamma}{q} < g_0$). In the former case, public saving is an almost perfect tool for circumventing the impossibility to bequeath to one's grandchildren if their parent is selfish. In the latter case, bequest-constrained parents manage to extract resources from their offspring through public transfers; in other words, public dissaving circumvents the prohibition of negative private bequests. As the discussion leading to Proposition 4 indicates, the ratio of national debt to gross human wealth is then bound to approach one, and one can speak of gradual enslaving of future generations. Note however that the above tendency may be consistent with $1 < g_t = \frac{\gamma}{q} < g_0$, in which case consumption per head grows over time without bound. It is worth pointing out that equilibrium public (dis)saving is exactly the same as private (dis)saving in a modified Ramsey dynasty which would consist only of altruistic agents ready to accept negative bequests.

In either case of strict inequality between g_t and g_0 , political equilibrium of our economy is not consistent with a steady path of a closed economy, which would require productive capital per head to exhibit a growth factor equal to the autonomous growth factor of labor productivity, viz. g_0 . Indeed, if $g_t = \frac{\gamma}{q} < g_0$, the growth factor of public debt per head approaches g_0 , as we state in Proposition 4, and life-cycle savers cannot possibly absorb it, so that one must predict a demand of capital above the required level. If $g_t = \frac{\gamma}{q} > g_0$, the growth factor of public savings approaches gradually g_t so that the supply of loanable funds is bound to exceed the productive sector required demand. However, if $g_t = \frac{\gamma}{q} = g_0$, political equilibrium is consistent with the steady path of a closed economy provided the net credit position of the

state, once invested in the productive sector at the time when voting occurs has a marginal productivity just equal to $(g_0 m)/\gamma$. Although this condition is identical to the one obtained in the limiting modified Ramsey model of homogeneous dynasties ($p = 1$), at first glance, this condition seems to be anything but robust. Yet, if the net credit position of the state lies above or below the required level when voting occurs, one cannot have $g_t = \frac{\gamma}{q} = g_0$ in political equilibrium and the economy will be off the steady path as described at the beginning of this paragraph.

It seems awkward to us to maintain our picture of agents entertaining the kind of inflexible expectations we have been postulating if the underlying closed economy is off its steady path, since past expectations would prove systematically wrong. However, if we pursue this approach, we believe that our seemingly tenuous steady path might well prove stable if the long run stability condition of the underlying Diamond (1965) economy is satisfied, because it implies that excess demand (resp. supply) of productive capital be self corrective in the long run.

Finally, it is interesting to consider an economy made up of identical dynasties sharing all the same descending altruism parameter γ and sharing all but one the same proportion of altruistic children $p < 1$, the last dynasty being characterized by $p = 1$. Reproducing the reasoning of Pestieau and Michel (1998) and Vidal (1996), we obtain, in the closed economy framework, that the steady state interest factor is dictated by the homogeneous dynasty at the traditional modified golden rule level: $R = \frac{m g_0}{\gamma}$. Along this steady path, we verify the double equality $g_t = \frac{\gamma}{q} = g_0$.

6 Conclusion

The small open economy model we developed displays the limits of the assumption of homogeneous dynasties differing only in the intensity of altruism. In such an economy, bequest constrained agents favor indeed public debt accumulation whereas altruistic members of unconstrained dynasties do not oppose it. The absence of political opposition is due to Ricardian equivalence. If one adds to it a Paretian argument restricted to the coalition of agents living at the moment of the vote, public debt policy is actually decided by the most constrained dynasty.

Assuming selfish intermissions within economic dynasties allows for a more meaningful political debate, even though each member of any given generation is assumed to receive the same gross wage. In our model of heterogeneous dynasties, unanimity disappears because of a basic asymmetry: on the one hand, the budget authorities are enabled to tap resources from unborn generations, but on the other hand, private agents are not able to give these resources back to their original owners. This argument was already used by Weil (1989). Following our model predictions, the more wealthy the altruistic agents, the more they oppose too large a level of public debt. In particular, we point out that altruistic agents with a strictly positive non-human wealth prefer a transfer policy which would leave them bequest constrained. They would actually be better off could they transform their present material wealth into future human wealth. We also establish that the political outcome is stable in the long run. Moreover the net wage growth factor as dictated by the median voter must be set equal to his/her desired consumption growth factor, which happens to be independent from the dynastic population growth rate. We conjecture also that our results concerning the political equilibrium would extend to the case of heterogeneous dynasties

where parents can not discriminate among their children types (cf. Gevers and Michel, 1998). In the latter case, altruistic agents would even have another motive for choosing the same kind of public policy with respect to intergenerational transfers as in the model under study: since positive private bequests would become redundant, they would not feel that they waste resources by bequeathing too much to their selfish children.

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