Behaviour Based Price Discrimination with Cross-Group Externalities

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Abstract

This paper analyses the practice of firms to offer different prices to consumers according to the past purchase behaviour (BBPD) in the context of two-sided markets. In a two-period model, two platforms compete for heterogeneous firms and consumers. Platforms are allowed to discriminate prices on the consumers' side according to their past purchase behaviour. When first-period market shares are taken as given, the presence of externalities makes two-direction switching less likely compared to the case of a one-sided market. Second-period competition is strengthened compared to the case in which a uniform price is charged in both sides, whereas in the first period it is relaxed if firms exhibit weaker externalities than consumers, intensified otherwise. The overall effect on inter-temporal profits of platforms is negative, confirming the previous results of BBPD literature.

JEL codes: L1, D4.

Keywords: Behaviour-Based Price Discrimination, Two-Sided Markets.

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1 Introduction

When a firm knows the identity of its customers, it often decides to charge new customers with a lower price in order to conquer new demand. As pointed out by Taylor (2003), price discrimination based on past purchases, called behaviour based price discrimination (BBPD), is very common in subscription markets. In these markets, since transactions are never anonymous, a firm knows the identity of current consumers and can thus propose low introductory prices to whom did not buy its product in the past.

Discounts take different forms such as low introductory prices, trial memberships and free installations. As mentioned in Caillaud and De Nijs (2014), a new subscriber for 3 months to the French newspaper “Le Monde”, pays 50 euros whereas a previous customer is charged 131.30 euros. Similar offers are the free trial memberships to software applications as well as online contents platforms such as Spotify and Amazon.[1] Moreover, first subscriptions to credit cards[2] and TVs/internet services are often offered for free.

All these services have the common feature that subscribers are not the only customers, as business is also made on merchants (credit cards), advertisers (media) and content providers (online platforms). In economic jargon, these markets are run by two-sided platforms allowing the interaction between different groups of customers linked to each other by cross-group externalities. Think for example to credit cards. A cardholder’s utility is increasing in the number of shops where she can use it and merchants are in turn more willing to pay for a card reader as the number of card users increases.

Because of the presence of the externalities, one of the distinctive features of these markets is the pricing rule, which is different from the general rule that applies in a one-sided framework (i.e., market without externalities), both for a monopolistic and a competitive environment. Back to the example of credit

[1]From Amazon website “Amazon Prime members in the U.S. can enjoy instant videos: unlimited, commercial-free, instant streaming of thousands of movies and TV shows through Amazon Instant Video at no additional cost. Members who own Kindle devices can also choose from thousands of books – including more than 100 current and former New York Times Bestsellers – to borrow and read for free, as frequently as a book a month with no due dates, from the Kindle Owners’ Lending Library. Eligible customers can try out a membership by starting a free trial”.

[2]Taylor (2003) also mentions a 1998 *Wall Street Journal*’s article by Bailey and Kilman reported that “the 60% of all Visa and MasterCard solicitations include a “teaser” (low introductory rate) on balances transferred from a card issued by another bank”.

cards, the subscription fee charged to the cardholders affects not only the demand in this group, but also the willingness to pay of merchants to hold an EPOS. This is the basic reason for the observation of a cross-group price discrimination, as the price charged to each group of agents depends on the cross externalities, so that a group whose participation entails a large participation of the other group will be charged less. According to this discussion, in many subscription markets two kinds of strategies are used by competing platforms: the mentioned *cross-group* price discrimination typical of a two-sided market and the *within-group* BBPD in subscribers' side.

These strategies have a common feature: platforms have some information about the characteristics of various groups of customers and exploit this information setting targeted prices to each group. However, the type of information required to implement these strategies is fundamentally different. On the one hand, to engage in *cross-group* price discrimination, platforms simply sort customers according to their externalities. On the other hand, *within-group* BBPD requires platforms to know the identity and the behaviour of customers.

This paper provides a two-sided market analysis investigating the effects of within-group BBPD on switching behaviour, prices and platforms' profits. In particular, in a two-period model, after a first round of purchases, platforms are allowed to price-discriminate on the subscribers' side according to their past purchase behaviour. The model is solved by backward induction and the analysis is two-fold. In the sub-game analysis, the market shares of first period are taken as given and the paper provides an analysis of all possible equilibria. In particular, different switching behaviours arise depending on the first period equilibrium and, in turn, on the strength of externalities. Namely, the stronger the externalities, the less likely to observe two-direction switching (TDS) and vice-versa. The inter-temporal equilibrium is thus provided and the resulting prices and profits are compared with a benchmark case in which price discrimination is not allowed. The main findings are two. Second-period competition is strengthened compared to the case in which a uniform price is charged in both sides of the market, whereas in the first period it is relaxed if the subscribers exhibit stronger externalities than firms, intensified otherwise. The overall effect of BBPD on inter-temporal profits of platforms is unambiguously negative, confirming the previous results of the one-sided literature.

Related literature. This paper is naturally linked to the two-sided market literature, initially formalized by Rochet and Tirole (2003), Armstrong (2006) and Caillaud and Jullien (2003). The main result around which this literature is built on is the *cross-group* price discrimination, which follows the concept of *Divide and Conquer* firstly proposed by Caillaud and Jullien (2003). In order to develop a business, a platform offers a low (often below-cost) price to one side of the market and thus restores its losses by charging a relatively high price to the other side. As in Rochet and Tirole (2003) and Armstrong (2006), the present paper proposes a Hotelling model, to capture the idea that customers exhibit heterogeneous preferences over rival platforms. The model focuses on the simplest case in which platforms charge only a fee independent of the number of interactions with the other side[3] and customers can join at most one platform.[4]

On the other side, the paper is strongly related with BBPD literature, which main finding is that discrimination is beneficial for consumers, as firms compete more strongly in prices and poach each other's consumers. In particular, the model is built on Fudenberg and Tirole (2000), who provide a Hotelling model played twice, allowing firms to know whether a customer in the second period is new or old. Villas-Boas (1999) provide an infinite time model with overlapping generations of consumers while Esteves (2010) presents different distributions of consumers types. In different setups, they all agree on the result that customer's recognition and consequent price discrimination hurt firms compared to a situation in which the targeted pricing is not possible. Even if a firm alone prefers to obtain the information and so use it to benefit from a surplus extraction, if both get it, then a market stealing effect tends to prevail.

In recent years, first investigations of within-group price discrimination have been presented, both from an empirical and a theoretical viewpoint. Gil and Riera-Crichton (2011) and Angelucci et al. (2013) provide empirical analysis respectively on Spanish TV and French newspaper industries. The first paper is mainly focused on the relationship between competition and price discrimination, while the second one studies how advertisement revenues affect price discrimination on the readers' side. Both competition and advertisement revenues are found to have a negative impact on the likelihood of medias to use price discrimination. From a theoretical point of view, Liu and Serfes (2013) is close to the present

[3]The literature distinguishes between subscription fee and usage fee. In the analysis of the media market of Ferrando et al. (2008) is pointed out how, while readers are charged with the price of the newspaper, advertiser are charged on per-reader basis.

[4]As a matter of fact, literature points out how often at least one side decides to multi-home, i.e. to join more than one platform. Armstrong (2006) and Armstrong and Wright (2007) provide an analysis on the reasons and on the effects of multi-homing on platforms competition.

paper in that both analyse within-group price discrimination. In particular, they allow platforms to engage in perfect price discrimination within both sides of the market. Their main finding is that discrimination might be a tool to neutralise cross-group externalities with a positive effect on prices and platforms' profits. There are two main differences with the present work. First, they only consider one period, keeping the past behaviour of consumers and market's shares as given. Second, they analyse the case of perfect price discrimination rather than discrimination based on past purchase behaviour.

The remainder of the paper is organized as follows. Next section introduces the main features of the model. After, section 3 is devoted to the analysis of the model. Section 4 concludes the paper.

2 The model

Two competing platforms $j = A, B$ aim at selling a service to two different groups of customers, subscribers and firms.[5] Both subscribers and firms are assumed to be uniformly distributed along a unit segment. In turn, platforms' locations are kept fixed at the end-points of this segment, i.e., platform A's location is $l^A = 0$, while platform B is located at $l^B = 1$.

A side-i agent enjoys some utility u from joining a platform, faces a transportation cost normalised to 1 per unit of distance covered[6] and receives a benefit measured by the parameter $\alpha_i \in (0, 1)$ for each side-i agent joining the same platform. According to these assumptions, the per-period utility of a side i agent located at x who joins platform j will be:

$$U_i^j(x) = u + \alpha_i n_{i'}^j - p_i^j - \left|x - l^j\right| \text{ where } i \in \{S, F\},\ i' \neq i, \tag{1}$$

and $n_{i'}^j$ is the total number of the other side's agents joining platform j. Platforms seek to maximise inter-temporal profits, bearing a unitary cost c_i to put a side i's customer "on board" and not discounting the future. The time-profit is simply given by the sum of the products between the price charged to each group and the

[5]Hereafter, the paper uses indifferently the words subscribers, consumers, group S or side S. Similarly, firms are also called side or group F throughout the paper.

[6]The assumption of a common transportation cost equal to 1 is made in order to keep notation as simple as possible, but the intuition behind the results provided in the paper remains the same even assuming side-dependent transportation costs.

number of joiners belonging to the same group. Thus, the time profit of platform j when charging prices p_i^j to each side i is indicated in equation by the following:

$$\pi^j = \sum_{i=S,F} (p_i^j - c_i) n_i^j. \tag{2}$$

Each time period is composed of two stages. In stage (1.1) platforms simultaneously set first-period fees to subscribers (p_{S1}^A, p_{S1}^B) and firms (p_{F1}^A,p_{F1}^B) and in stage (1.2) customers decide which platform to join. In time 2, platforms simultaneously set prices knowing who subscribed to which platform. p_{S2}^{jA} represents the price set by firm j for an A-subscriber in period 1, while p_{S2}^{jB} is charged to B's inherited clients. Firms are instead charged with a uniform price in the second period as well as in the first one (p_{F2}^A,p_{F2}^B). In the very last stage (2.2), after having observed the new fees, firms and subscribers join the preferred platform.

Three main assumptions are used throughout the paper: *(i)* the utility u is big enough so that every agent prefers to join at least one platform instead of joining none (Full Market Coverage); *(ii)* each agent joins at most one platform (Single-Homing); *(iii)* profit functions are concave. As shown in Armstrong and Wright (2007), single-homing in both sides is the case when $1 > \max\{\alpha_S, \alpha_F\}$, meaning that agents are interested in reaching the other side, but not so much to decide to join both platforms and bear price and transportation cost twice. Moreover, $1 > 2(\alpha_S + \alpha_F)^2$ is the condition needed for the profits to be concave in prices.

3 Analysis

This section provides a complete analysis of the model. In particular, it firstly introduces and explains the benchmark case in which customer's recognition is not allowed in the next subsection. Subsequently, Subsection 3.2 describes the possible equilibria when platforms are allowed to engage in BBPD and compares the results of the two regimes.

3.1 No customer's recognition

Assume there exists a ban on price discrimination or that customers cannot be recognized. In this scenario, platforms cannot distinguish between old and new subscribers, and thus can only engage in cross-side but not within-side price discrimination, i.e., $p_{S2}^{jj} = p_{S2}^{ji} = p_{S2}^{j}$. This would imply that the oligopoly competition in prices takes the form of a two-period repeated game in which nothing changes from the first to the second period. For this reason, the solution of the repeated game is nothing more than the solution of the per period game, with prices $p_S^A, p_F^A, p_S^B, p_F^B$ in both time periods.

According to the utility defined in equation (1), given prices p_S^A, p_F^A and p_S^B, p_F^B the locations $\bar{x}_i$ of the side i consumer indifferent between the two platforms will be:

$$\bar{x}_i = \frac{1}{2} + \frac{p_i^B - p_i^A + \alpha_i(p_{i'}^B - p_{i'}^A)}{2} \text{ where } i' \neq i.$$

Given these expectations on joining decisions, platform j maximizes the following profits choosing the prices p_S^j and p_F^j:

$$\Pi^j = (1+\delta)\left[(p_S^j - c_S)|\bar{x}_S - l^j| + (p_F^j - c_F)|\bar{x}_F - l^j|\right].$$

Using the first-order conditions of these two problems, the equilibrium prices in the two sides are the following:

$$\bar{p}_S^A = \bar{p}_S^B = c_S + 1 - \alpha_F \text{ and } \bar{p}_F^A = \bar{p}_F^B = c_F + 1 - \alpha_S. \tag{3}$$

These prices result in the market splitting locations $\bar{x}_S = \bar{x}_F = 1/2$ and in the following equilibrium profits:

$$\bar{\Pi}^A = \bar{\Pi}^B = \bar{\Pi} = (1+\delta)\left[1 - \frac{\alpha_S + \alpha_F}{2}\right]. \tag{4}$$

3.2 Customer's recognition

In this section, first period prices as well as the identity of first period subscribers are assumed to be observable to both platforms when they choose second period fees. Sub-game perfection is the equilibrium concept. In stage (2.2) both firms and subscribers observe all prices and decide which platform to join.

Subscribers. In what follows, x_2^A represents the location of that first-period A's subscriber who is indifferent between switching to the rival or being loyal for given prices p_{S2}^{AA} and p_{S2}^{BA} offered to him. Following the same reasoning, x_2^B is the location of the indifferent first-period B-joiner. Simply equalizing utilities in both turfs, the two cutoffs will be:

$$x_2^j = \frac{1}{2} + \frac{\alpha_S n_{F2}^A - \alpha_S n_{F2}^B + p_{S2}^{Bj} - p_{S2}^{Aj}}{2} \text{ with } j \in \{A, B\}. \tag{5}$$

Assume the population of subscribers to split in time 1 at location x_{S1}, so that all consumers located below this cutoff joined platform A and all the ones above joined platform B. Therefore, the number of subscribers switching from platform A to platform B is given by $n_{S2}^{BA} = \max\{x_{S1} - x_2^A, 0\}$, while $n_{S2}^{AB} = \max\{x_2^B - x_{S1}, 0\}$ move towards the other direction. The remaining $n_{S2}^{AA} = \min\{x_2^A, x_{S1}\}$ and $n_{S2}^{BB} = \min\{1 - x_2^B, 1 - x_{S1}\}$ are loyal respectively to platform A and platform B.

Firms. Firms take their decision following the same reasoning as users. They observe prices offered by both platforms and form expectations about how many consumers will subscribe to each platform. According to the discussion of last paragraph, the total number of subscribers to platform j is the sum of loyalists n_{S2}^{jj} and switchers $n_{S2}^{j'j}$. Firms correctly anticipate the switching behaviour of the other side. By simple comparisons of utilities, the indifferent firm is located at:

$$x_{F2} = \frac{1}{2} + \alpha_F \left(n_{S2}^{AA} + n_{S2}^{AB} - \frac{1}{2} \right) + \frac{1}{2} \left(p_{F2}^B - p_{F2}^A \right). \tag{6}$$

All firms located below this cutoff will join platform A (i.e., $n_{F2}^A = x_{F2}$) and all above will prefer platform B ($n_{F2}^B = 1 - x_{F2}$). We can have two different cases. In the first one, platforms expect some bi-directional movements of consumers in the second period, i.e., they expect Two-Direction Switching (TDS). Formally, it means that the cutoffs in equation (5) are located in such a way that $x_2^A < x_{S1} < x_2^B$, as depicted in Figure 1 below. If instead platforms expect switching to be One-Direction (ODS) towards platform A, they expect the cutoffs in (5) to be located in such a way that $x_{S1} \leq x_2^A$ and $x_{S1} < x_2^B$ as depicted in Figure 2. According to these expectations, the maximisation problems of the platforms change dramatically and give different equilibrium prices, summarised in the following proposition:

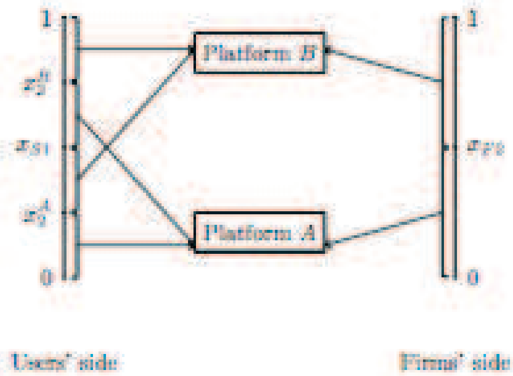

Figure 1: Two-Direction Switching.

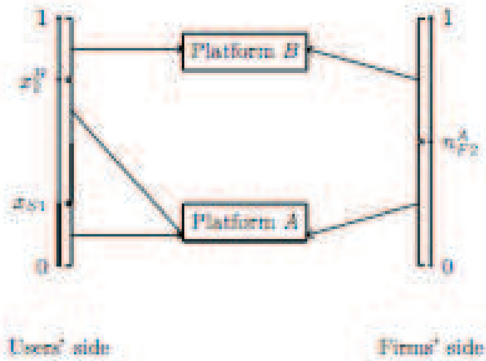

Figure 2: Switching only to A (ODS).

Proposition 1. *Assume that $n_{S1}^A = x_{S1}$ and $n_{S1}^B = 1 - x_{S1}$ side S agents subscribed respectively to platform A and B in the past, then the equilibrium prices will be:*

1. *If TDS is expected:*

$$\begin{aligned} p_{S2}^{ii} &= c_S + \tfrac{5}{12} - \alpha_F + \left(\tfrac{1}{2} + 2\Lambda\right) n_{S1}^i - \Lambda, \\ p_{S2}^{ij} &= c_S + \tfrac{13}{12} - \alpha_F - (\tfrac{3}{2} - 2\Lambda) n_{S1}^i - \Lambda, \qquad \text{with } i \in \{A, B\} \text{ and } i \neq j \\ p_{F2}^{i} &= c_F + 1 - \alpha_S + 2\Omega n_{S1}^i - \Omega, \end{aligned}$$

where $\Lambda \equiv \frac{3(3-2\alpha_S(2\alpha_S+\alpha_F))}{4(9-2(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F))} \in (0, \frac{1}{2})$ *and* $\Omega \equiv \frac{(\alpha_S-\alpha_F)}{4(9-2(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F))}$.

2. *If ODS to platform i is expected:*

$$\begin{aligned} q_{S2}^{ii} &= 1 + \Phi n_{S1}^i, & q_{S2}^{ji} &= 0. \\ q_{S2}^{ij} &= c_S + 1 - (1 + \Psi) n_{S1}^i - \alpha_F, & q_{S2}^{jj} &= c_S + 1 - (1 - \Psi) n_{S1}^i - \alpha_F, \\ q_{F2}^{A} &= c_F + 1 - \alpha_S + (2\alpha_S - \Gamma) n_{S1}^i, & q_{F2}^{B} &= c_F + 1 - \alpha_S + \Gamma n_{S1}^i, \end{aligned}$$

where $\Psi \equiv \frac{3(1-\alpha_F\alpha_S)}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$, $\Gamma \equiv \frac{2((4\alpha_S-\alpha_F)+\alpha_S^2(\alpha_S+2\alpha_F))}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$ *and*

$\Phi \equiv \left(\frac{2\alpha_S(\alpha_F-\alpha_S)}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)} - 2\right)$.

Proof. See Appendix. ■

In order to better grasp the intuition behind Proposition 1 let us consider the equilibrium prices in point 1, which describes the symmetric equilibrium in which both platforms steal rival's consumers in the second period.

First, the own inherited market share in the subscribers side affects positively the price a given platform charges to the old loyal consumers and negatively the one offered to the switchers. Intuitively, the relation between prices and market share follows directly from the effective power that the size of the first-period market creates in each turf for the "attacking" (else turf) and the "defending" firm (own turf). Clearly, the attack in the rival turf turns out to be more costly as the size of the market already conquered in the first period becomes higher. In other words, the price offered to the switchers should be lower when a lot of consumers were attracted in the first period, since the non-conquered portion is very far away in the Hotelling line. Therefore, from the point of view of the defending firm, the higher the market share inherited from the past the weaker the price competition in its own turf, as the rival becomes less aggressive. For this reason, the equilibrium price for loyalists is increasing in the inherited market share.

On the other hand, how the inherited number of subscribers affects the equilibrium price chosen in firms' side ultimately depends on the relative strength of externalities between the two sides. If firms are more interested in meeting consumers than the other way around (i.e., $\alpha_F > \alpha_S$), then the equilibrium prices for firms decreases with the number of inherited consumers. In this case, competition for users is very strong and switching is due to offers in the subscribers side. Since firms expect switching movements towards the small-sized platform, they are willing to pay less as the number of inherited subscribers increases. Differently, if users are more interested than firms in the interaction, the latter are charged more as the inherited market increases. In this case, since competition for subscribers is less intense, switching is mainly driven by a decrease in the price offered to firms. This decrease will be weaker as the inherited market number of users increases, since the incentives to attract new subscriptions are lower (smaller potential market to conquer).

When switching is one-direction, it means that for a given platform becomes too costly to attract new subscribers: even offering a price equal to 0 (i.e., lower than the marginal cost) is not sufficient to attract anybody. Therefore, the defending firm can charge a price for inherited subscriptions just sufficient to keep all of them. The other prices (in the other turf and in the other side) keep the same qualitative features of the symmetric case. These equilibrium prices will determine peculiar switching behaviours of consumers. If the inherited number of subscribers is very high, a platform finds it too costly to attract the small residual number of rival's ones. Moreover, having a high inherited number of subscribers makes it more difficult for a platform to retain old customers, as stated in the following proposition.

Corollary 2. *Given the equilibrium prices in Proposition 1:*

1. *If $x_{S1} \in (\hat{x}, 1-\hat{x})$, with $\hat{x} \equiv \frac{1}{6} + \frac{1}{12(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))}$, then TDS occurs.*
2. *If $x_{S1} \leq \hat{x}$ (respectively $x_{S1} \geq 1-\hat{x}$), then ODS occurs towards platform A (resp. B).*
3. *The presence of externalities reduces the length of the interval of inherited market splitting location compatible with TDS compared with the case of a one-sided market.*

Proof. See Appendix. ■

Due to these reasonings, the inherited market splitting location should be symmetric enough for TDS to occur,[7] while an unbalanced market implies that switching occurs from the "strong" to the "weak" platform. The likelihood of the TDS equilibrium to arise depends on the strength of externalities, through the positive effect that α_S and α_F have on the cutoff $\hat{x}$. Moreover, since the externality parameters are bound by 0 from below $\hat{x}$ is always higher than $\frac{1}{4}$.[8]

Two main considerations can be made. On the one hand, the higher the externalities, the narrower the interval allowing TDS. On the other hand, compared to a one-sided market, TDS is less likely to occur and, if externalities are particularly strong, even a slight inherited asymmetry might determine the impossibility of TDS to occur. This is due to the fact that, in the presence of externalities, platforms face a cross-side coordination problem that emphasises the effects of inherited asymmetries on the second-period switching behaviour of customers. Namely, the incentives for the "weak" platform to be aggressive in the rival's turf are stronger than in a one-sided market, as the gain coming from new subscriptions is also associated with the attraction of new firms. For the same reason,

[7]In the analysis of their two periods model of BBPD in a one-sided market, Fudenberg and Tirole (2000) use exactly this assumption to solve backward the model.

[8]In an unpublished paper Gehrig et al. (2007) provide an analysis of the BBPD with inherited market shares and finding how $\hat{x}$ is equal to $\frac{1}{4}$ in a one-sided market, which corresponds to the case with $\alpha_S, \alpha_F = 0$ in the present model.

the cost for the "strong" platform to attract some of the residual customers is higher in a two-sided market, as both sides have to be carried "on-board".

To conclude this paragraph, it is worth noticing what happens when the inherited market is perfectly symmetric. The results will be summarised in the following corollary.

Corollary 3. *Assume that* $n^A_{S1} = n^B_{S1} = 1/2$*, then:*

1. *the second-period equilibrium prices will be:*

$$p^{ii}_{S2} = c_S + \tfrac{2}{3} - \alpha_F, \quad p^{ij}_{S2} = c_S + \tfrac{1}{3} - \alpha_F, \quad p^i_{F2} = c_F + 1 - \alpha_S$$

2. *TDS will occur.*

In this case, the market is symmetric enough to have TDS and prices take into account the externality they create on the other side of the market. In particular, since the attraction of an additional subscriber makes firms more willing to pay for a factor α_F, each subscriber is rewarded in that measure. More comments on prices will be done when they are compared with the benchmark case of the intra-side uniform price.

First period. This paragraph is devoted to the analysis of first-period decisions. Two main assumptions are made in the following analysis. First, what follows relies on the fact both platforms expect TDS to occur tomorrow. This is mainly required for the results to be "readable" and interpretable and follows the idea of Fudenberg and Tirole (2000) of symmetric (enough) market shares in the first period. Secondly, consumers are assumed to be myopic, i.e., they only care about the utility they get at stage (1.2), without anticipating the second period (possible) switching. Myopia here is assumed just in order to keep the analysis as simple as possible. Accordingly, by simple comparison of utilities, the indifferent side-i agent will be located at:

$$x_{i1} = \frac{1}{2} + \frac{\alpha_i}{2t}\left(n^A_{i'1} - n^B_{i'1}\right) + \frac{1}{2t}(p^B_{i1} - p^A_{i1}), \text{ with } i \in \{S, F\}, i \neq i',$$

and under full market coverage, it holds that the total numbers of customers joining respectively platform A and platform B will be $n^A_{i1} = x_{i1}$ and $n^B_{i1} = 1 - x_{i1}$. Taking into account that both sides correctly anticipate the other side participation, the indifferent indifferent agent is side i will be located at:

$$x_{i1} = \frac{1}{2} + \frac{\alpha_i\left(p^B_{i'1} - p^A_{i'1}\right) + t(p^B_{i1} - p^A_{i1})}{t^2 - \alpha_i\alpha_{i'}}, \text{ with } i \in \{S, F\}, i \neq i'.$$

First-period prices chosen by platforms have an effect not only on current profits but also on second period profits, as the market share of period 1 determines second-period. Indeed, having a high number of previous subscribers today reduces the possibilities both to steal customers from the rival and to retain old customers overcoming the poaching attempted by the rival. As demonstrated in the appendix, we will have the following equilibrium:

Proposition 4. *When platforms expect symmetry, the equilibrium is characterised by:*

1. *subscription fees equal to* $p_{S1}^A = p_{S1}^B = c_S + 1 - \alpha_F + \frac{\delta(3-2\alpha_S-\alpha_F)(\alpha_S-\alpha_F)}{3(9-2(2\alpha_S+\alpha_F)(2\alpha_F+\alpha_S)}$,
2. *firms' prices equal to* $p_{F1}^A = p_{F1}^B = c_F + 1 - \alpha_S$,
3. $x_{S1} = x_{F1} = x_{F2} = 1/2$ *and* $x_2^A = 1 - x_2^B = 1/3$.

Proof. See Appendix. ■

Proposition 4 summarises the main characteristics of the equilibrium first-period prices and their effects on the first-period price competition and the second-period switching behaviour. Unsurprisingly, the market splits in the first period at locations 1/2 in both sides of the markets. Moreover, 1/6 of subscribers switch platform in the second period and 1/3 of them remain loyal,[9] whereas no switching occurs in the firms side going from the first to the second period. Concerning the prices in both period and their comparison with the intra-side uniform pricing, the results are summarised in the following proposition.

Proposition 5. *Allowing platforms to price subscribers according to their past purchase behaviour will entail:*

(i) same first- and second-period prices in firms' side,

(ii) lower second-period prices for subscribers,

(iii) lower (higher) first-period prices for subscribers if they have stronger (weaker) externalities,

(iv) lower inter-temporal profits.

[9] The switchers towards platform A are located between 2/3-1/2, whereas 1/2-1/3 go towards platform B. The remaining agents closer to the extremes remain loyal.

Proof. See Appendix. ■

Two main effects are playing a role in the determination of optimal prices when platforms engage in within-group price discrimination.

On the one hand, knowing the identity of subscribers pushes firms to compete fiercely in the second period in order to steal each other's consumers (poaching effect) and to "defend" their inherited market from rival's attack. This poaching effect has clear-cut negative effects on second period prices charged to subscribers (point (*ii*) of Proposition 5). On the other hand, this effect goes towards a reduction of first-period competition, as being aggressive in the first period entails a relative disadvantage in terms of tomorrows' conquest of new subscriptions.

On the other hand, any price cut in one side of the market involves a positive effect on other side's participation (externality effect). This is captured by the terms $-\alpha_j$ found in all prices, which are nothing more than the "rewards" that a side i agent receives for the externality that his presence in the platform creates in side j. For this reason, the group exhibiting the lower externality becomes a loss leader and is basically subsidised by the other group, on which platforms mostly make profits: this is the so-called "Divide and Conquer" strategy typical of two-sided markets. The intuitive result coming from the externality effect is that the symmetry of the model brings to a situation in which firms are charged precisely in the same way both under within-group uniform pricing and price discrimination (point (*i*) of Proposition 5). This depends on the occurrence that subscriptions are equally split between the two platforms, and thus firms willingness to pay is the same in both regimes. In the second period, switching determines a change in "who" joins each network, but platforms steal each other the same number of subscribers, keeping the aggregate market shares unchanged.

The interplay between externality and poaching effect determines point (*iii*) of Proposition 5. With the two effects discussed above in mind, the result is rather intuitive. In the first period, the main trade-off faced by platforms is an inter-temporal one. Indeed, they can either compete fiercely in order to conquer a large first-period market or make high margins postponing the attack to the rival's territory. The balance between these two opposite forces ultimately depends on the relative strength of externalities.

Assume subscribers to exhibit stronger externalities than firms, i.e., $\alpha_S > \alpha_F$. Therefore, the optimal "Divide and Conquer" strategy in the early competition will be to charge firms with a very low price and then make profits on the subscribers. In this case, the temptation of making high margins on few subscriptions prevails as BBPD offers the platforms a new opportunity to conquer a large market in the second period. On the other hand, if $\alpha_F > \alpha_S$ platforms "divide" on subscribers' side and "conquer" firms' side, offering a very low price to the former and making profits on the latter. In this situation, subscribers are the loss-leader segment and there is no advantage in implementing a strategy of high margins on a small market.

To conclude, the model confirms the idea that profits are lower when competitors discriminate prices, as platforms are very aggressive in the rival's turf, a well-known result of one-sided markets BBPD literature. The only difference going from a one-sided to a two-sided market is on the magnitude of the loss suffered by sellers in the discriminatory regime. According to the discussion about first period prices, the presence of cross-group network effects emphasises the negative effect of BBPD when this is used in the group exhibiting lower externalities, whereas it mitigates it when subscribers are strongly interested in meeting firms.

4 Conclusion

The paper provides a two-period model of platform competition, in which the demand is composed by two sides, firms and subscribers. Platforms are allowed to discriminate prices among subscribers, according to the fact that BBPD is often used in subscription markets. Cross-group externalities do involve some effects on prices and competition when platforms charge past and new subscribers with different fees.

First, when the first-period market shares are taken as given, externalities have a negative impact on the concrete possibility for two-direction switching to occur. Indeed, the stronger the externalities, the narrower the interval of inherited market splitting locations compatible with a TDS scenario, since the maximal market share a platform can inherit compatible with enjoying the attraction of new customers depends positively on the externalities. When externalities are set to 0 (i.e., one-sided market), the model replicates the results of the analysis provided by Gehrig et al. (2007).

Secondly, when the first-period decisions are taken into account, platforms face a strategic situation similar to a prisoner's dilemma. Each one of them alone has the incentive to offer discounted prices for new subscriptions but, if both of them do it, the level of profits turns out to be lower than the one that would be reached if they committed not to poach rival's consumers. This result follows from two reasons.

On the one hand, going from the non-discriminatory to the discriminatory regime entails a loss in the subscribers side. This loss mainly follows from a decrease in the level of second-period subscription fees, as platform compete fiercely in order to poach each other's clients. Moreover, the presence of cross-group externalities strengthens this loss because of a decrease in first-period subscription fees when firms are more interested than subscribers in meeting the other side of the market. This is due to the fact that the latter group is pivotal to attract the former and BBPD make platforms worried about the second-period attack of the competitor. Oppositely, when subscribers exhibit the highest network externalities, first period prices are higher under the discriminatory regime, which gives platforms a further possibility to attract subscribers in the second period. However, even in this second case, the negative effect of BBPD on second period prices more than compensate the softening of first-period competition, making platforms worse off.

On the other hand, the losses made in the subscribers' side are not recouped on the firms' side. Indeed, the symmetry of the model makes firms indifferent between the scenario in which platforms use within-group price discrimination and uniform subscription fees. This is due to the fact that subscriptions are equally distributed between the two platforms in the two periods, and thus firms participation does not vary. In the second period, switching determines a change in the identity of some subscribers, but their total number (what matters to firms) remains constant over time.

The irrelevance of discounted new subscriptions on the firms' side is doubtless an important weakness of the present paper. In the context of two-sided markets models à la Armstrong with heterogeneity of consumers in terms of locations and symmetric platforms, this irrelevance result will be always there. This is because firms expect and anticipate the future bi-directional movements of subscribers from one platform to the other. Furthermore, even the consideration of multi-homing firms would not modify the picture. When agents are mostly interested in the interaction with the other group rather than the product offered by the platforms themselves, they may take the decision to join both platforms in order to meet the other side. As pointed out by Armstrong (2006), the main implication of this assumption is that price competition is relaxed in the firms' side, since it exhibits a lower elasticity to price. In the present setting, only the sharing of the surplus between platforms and subscribers would be concerned by BBPD, but again no effect would arise in the firms' side. In particular, platforms would charge exactly the same subscription fees found in the analysis above and would behave as monopolists in the firms' side, both in the within-group discriminatory and uniform price regime.

In order to enrich the results of the present paper and, more in general, of the two-sided market literature, asymmetries in networks sizes can play a crucial role in the effects of price discrimination on the welfare of platforms and customers. As pointed out by Chen (2008), Shaffer and Zhang (2002) and Liu and Serfes (2005) for one-sided markets, price discrimination may lead to very different scenarios going from symmetric and asymmetric markets. Without assuming any ex-ante asymmetry, Ambrus and Argenziano (2009) and Gabszewicz and Wauthy (2014) find that if consumers are heterogeneous in terms of the strength of the externalities rather in terms of locations, then asymmetric networks arise at equilibrium. In particular, Gabszewicz and Wauthy (2014) demonstrate how platform competition brings to two vertically differentiated markets (one per side) in which the "quality" of the product sold is simply given by the size of the network. Their setup allows for the co-existence of asymmetric platforms in equilibrium, so that if platforms discriminate price to induce switching of one side, this would well entail some effect in the other side in terms of switching behaviour.

The last point to notice is that the present model assumes myopic customers. Fudenberg and Tirole (2000) and Villas-Boas (1999) show that if customers are forward looking early competition is relaxed, as knowing that tomorrow it will be possible to switch paying a discounted price reduces price elasticity today. In the setup proposed by this paper, which already assumes deep rationality, it can be interesting to see how competition is affected by the fact that both firms and end-users expect platforms to use within-group price discrimination and take it into account when taking their ex-ante decisions.

5 Appendix

5.1 Proof of Proposition 1.

TDS. Expecting to lose some subscribers, platform j expects to keep $n^{jj}_{S2} = |x^j_2 - l^j|$ of them. These agents are going to pay the fee that platform j charges to its loyalists, i.e., p^{jj}_{S2}. On the other hand, $|x^i_2 - x_{S1}|$ are expected to switch from the rival platform i and these switchers are going to pay price p^{ji}_2. Plugging these results into equation (6) and putting together with (5), all the cutoffs depend on all prices as follows:

$$x^A_2 = \tfrac{1}{2} + \tfrac{\alpha_S\left(p^B_{F2}-p^A_{F2}\right)}{2-4\alpha_F\alpha_S} + \tfrac{(1-\alpha_F\alpha_S)\left(p^{BA}_{S2}-p^{AA}_{S2}\right)}{2-4\alpha_F\alpha_S} + \tfrac{\alpha_F\alpha_S\left(p^{BB}_{S2}-p^{AB}_{S2}\right)}{2-4\alpha_F\alpha_S} + \tfrac{\alpha_F\alpha_S(1-2x_{S1})}{2-4\alpha_F\alpha_S};$$

$$x^B_2 = \tfrac{1}{2} + \tfrac{\alpha_S\left(p^B_{F2}-p^A_{F2}\right)}{2-4\alpha_F\alpha_S} + \tfrac{(1-\alpha_F\alpha_S)\left(p^{BB}_{S2}-p^{AB}_{S2}\right)}{2-4\alpha_F\alpha_S} + \tfrac{\alpha_F\alpha_S\left(p^{BA}_{S2}-p^{AA}_{S2}\right)}{2-4\alpha_F\alpha_S} + \tfrac{\alpha_F\alpha_S(1-2x_{S1})}{2-4\alpha_F\alpha_S};$$

$$x_{F2} = \tfrac{1}{2} + \tfrac{p^B_{F2}-p^A_{F2}}{2-4\alpha_F\alpha_S} + \tfrac{\alpha_F\left(1-2x_{S1}+p^{BA}_{S2}-p^{AA}_{S2}+p^{BB}_{S2}-p^{AB}_{S2}\right)}{2-4\alpha_F\alpha_S}.$$

In stage (2.1), anticipating the joining behaviour of both sides of the market, platform j solves the following maximization problem:

$$\max_{p^{jj}_{S2},p^{ji}_{S2},p^j_{F2}} (p^{jj}_{S2} - c_S)|x^j_2 - l^j| + (p^{ji}_{S2} - c_S)|x^i_2 - x_{S1}| + (p^j_{F2} - c_F)|x^j_{F2} - l^j|.$$

Using the first-order conditions of this problem and solving the system of best responses, the equilibrium prices are the following:

$$p^{AA}_{S2} = c_S + \tfrac{5}{12} - \alpha_F + \tfrac{1}{2}x_{S1} + \Lambda, \qquad p^{BB}_{S2} = c_S + \tfrac{5}{12} - \alpha_F + \tfrac{1}{2}(1 - x_{S1}) - \Lambda,$$
$$p^{BA}_{S2} = c_S + \tfrac{13}{12} - \alpha_F - \tfrac{3}{2}(1 - x_{S1}) - \Lambda, \qquad p^{AB}_{S2} = c_S + \tfrac{13}{12} - \alpha_F - \tfrac{3}{2}x_{S1} + \Lambda,$$
$$p^A_{F2} = c_F + 1 - \alpha_S + \Omega, \qquad p^B_{F2} = c_F + 1 - \alpha_S - \Omega.$$

Where $\Lambda \equiv \frac{3(2x_{S1}-1)(3-2\alpha_S(2\alpha_S+\alpha_F))}{4(9-2(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F))}$ and $\Omega \equiv \frac{(\alpha_S-\alpha_F)(2x_{S1}-1)}{4(9-2(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F))}$.

ODS Under this assumption, none among A's first period subscribers switches platform, therefore firms expect to meet x^B_2 agents in platform A and $1 - x^B_2$ in platform B. Plugging into equation (6) and putting together with (5), the market splitting cutoffs turn out to be the following:

$$x_2^B = \frac{1}{2} + \frac{(q_{S2}^{BB} - q_{S2}^{AB}) + \alpha_S(q_{F2}^B - q_{F2}^A)}{2(1-\alpha_S\alpha_F)},$$

$$x_2^A = \frac{1}{2} - \frac{\alpha_S}{2} + \frac{q_{S2}^{BA} - q_{S2}^{AA}}{2} + \frac{\alpha_S\alpha_F(q_{S2}^{BB} - q_{S2}^{AB}) + \alpha_S(q_{F2}^B - q_{F2}^A)}{2(1-\alpha_S\alpha_F)},$$

$$x_{F2} = \frac{1}{2} + \frac{\alpha_F(q_{S2}^{BB} - q_{S2}^{AB}) + (q_{F2}^B - q_{F2}^A)}{2(1-\alpha_S\alpha_F)}.$$

According to these expectations, platform A would keep all x_{S1} loyal subscribers, who are supposed to pay q_{S2}^{AA}. On the other hand, $x_2^B - x_{S1}$ are expected to switch from the rival and are going to pay price q_{S2}^{AB}. Accordingly, platform A solves the following problem:

$$\max_{q_{S2}^{AA}, q_{S2}^{AB}, q_{F2}^A} (q_{S2}^{AA} - c_S)x_{S1} + (q_{S2}^{AB} - c_S)(x_2^B - x_{S1}) + (q_{F2}^A - c_F)x_{F2},$$

under the constraint that $x_2^A \geq x_{S1}$. In turn, platform B only expects to keep $1 - x_2^B$ subscribers without attracting any new of them, thus solving the following:

$$\max_{q_{S2}^{BB}, q_{F2}^B} (q_{S2}^{BB} - c_S)(1 - x_2^B) + (q_{F2}^B - c_F)(1 - x_{F2}).$$

For what concerns the prices charged to inherited subscribers of platform B as well as to firms, the solution of the systems of first order conditions yields the following equilibrium values:

$$\begin{array}{ll} q_{S2}^{AB} = c_S + 1 - (1+\Psi)x_{S1} - \alpha_F, & q_{S2}^{BB} = c_S + 1 - (1-\Psi)x_{S1} - \alpha_F, \\ q_{F2}^A = c_F + 1 - \alpha_S + (2\alpha_S - \Gamma)x_{S1}, & q_{F2}^B = c_F + 1 - \alpha_S + \Gamma x_{S1}, \end{array} \tag{7}$$

where $\Psi \equiv \frac{3(1-\alpha_F\alpha_S)}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$ and $\Gamma \equiv \frac{2((4\alpha_S-\alpha_F)+\alpha_S^2(\alpha_S+2\alpha_F))}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$.

Different reasoning holds for inherited subscribers of A, who are assumed to be loyal in the second period, making thus the platform A profit linearly increasing in q_{S2}^{AA}. This means that platform A wants to set the highest possible price to old subscribers compatible with the constraint. Moreover, at equilibrium the rival cannot find any profitable price undercut. In a two-sided market prices can be and, actually, often are below marginal cost. Therefore, in the current analysis firm B should be allowed to undercut the rival choosing a price in the latter's turf

q_{S1}^{BA} even lower than the marginal cost c_S. Nevertheless, in line with Armstrong and Wright (2007), the set of possible prices is here restricted to the positive reals, and thus the lowest price that can be charged is zero. Accordingly, the optimal q_{S2}^{AA} will be the higher possible given the constraint (namely, $x_2^A = x_{S1}$) and avoiding any possible price undercut from the rival ($q_{S2}^{BA} = 0$) which, rearranging terms, gives the following optimal price for A's loyal subscribers:

$$q_{S2}^{AA} = 1 + 2x_{S1}\left(\frac{\alpha_S(\alpha_F - \alpha_S)}{9 - (2\alpha_S + \alpha_F)(\alpha_S + 2\alpha_F)} - 1\right). \tag{8}$$

The prices described in equations (7) and (8) yield $x_2^A = x_{S1}$ and $x_B = \frac{1}{2} + \frac{3x_{S1}}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$.

5.2 Proof of Lemma 2

For the prices in point 1 of Proposition 1 to be an equilibrium, platforms must expect TDS to occur. To be consistent with these expectations, we need that $x_2^A < x_{S1} < x_2^B$. Given the equilibrium prices in point 1 of Proposition 1, the two cutoffs are:

$$x_2^A = \frac{1}{12} + \frac{x_{S1}}{2} + \frac{9(1 - 2x_{S1})}{12(9 - 2(2\alpha_F + \alpha_S)(\alpha_F + 2\alpha_S))},$$

$$x_2^B = \frac{5}{12} + \frac{x_{S1}}{2} + \frac{9(1 - 2x_{S1})}{12(9 - 2(2\alpha_F + \alpha_S)(\alpha_F + 2\alpha_S))}.$$

It is easy to check that $x_2^{A*} < x_{S1} < x_2^{B*}$ if and only if $\hat{x} < x_{S1} < 1 - \hat{x}$, where:

$$\hat{x} \equiv \frac{1}{6} + \frac{1}{12(9 - 2(2\alpha_F + \alpha_S)(\alpha_F + 2\alpha_S))}.$$

If the conditions above are not satisfied, then platforms expect ODS to occur, only towards A if $x_{S1} \leq \hat{x}$ and towards B if $x_{S1} \geq 1 - \hat{x}$. To prove point (iii), notice how:

$$\frac{\partial \hat{x}}{\partial \alpha_S} = -\frac{1}{(12(9 - 2(2\alpha_F + \alpha_S)(\alpha_F + 2\alpha_S)))^2}\left(-2(5\alpha_F + 4\alpha_S)\right) > 0,$$

$$\frac{\partial \hat{x}}{\partial \alpha_F} = -\frac{1}{(12(9 - 2(2\alpha_F + \alpha_S)(\alpha_F + 2\alpha_S)))^2}\left(-2(4\alpha_F + 5\alpha_S)\right) > 0.$$

5.3 Proof of Proposition 4

Formally, platform A and platform B set respectively prices p_{S1}^{A}, p_{F1}^{A} and p_{S1}^{B}, p_{F1}^{B}in order to maximize inter-temporal profits, i.e., solve:

$$\max_{p_{S1}^{A}, p_{F1}^{A}} p_{S1}^{A} x_{S1} + p_{F1}^{A} x_{F1} + \delta \pi_2^{A}(x_{S1}(p_{S1}^{A}, p_{F1}^{A}, p_{S1}^{B}, p_{F1}^{B})),$$

$$\max_{p_{S1}^{B}, p_{F1}^{B}} p_{S1}^{B} x_{S1} + p_{F1}^{B} x_{F1} + \delta \pi_2^{A}(x_{S1}(p_{S1}^{A}, p_{F1}^{A}, p_{S1}^{B}, p_{F1}^{B})),$$

where $\pi_2^A = \frac{71+72x_{S1}^2-66x_{S1}-36(\alpha_S+\alpha_F)}{72} + \frac{9(2x_{S1}-1)(4\alpha_F+2\alpha_S(\alpha_F+2\alpha_S-2)+4x_{S1}-5)}{72(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))}$ and $\pi_2^B = \frac{77+72x_{S1}^2-78x_{S1}-36(\alpha_F+\alpha_S)}{72} - \frac{9(2x_{S1}-1)(4\alpha_F+2\alpha_S(\alpha_F+2\alpha_S-2)-4x_{S1}-1)}{72(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))}$ are the profits they are going to receive tomorrow under the assumption of two-direction switching.

From the first-order conditions of this problem, the first period equilibrium prices will be the following:

1. subscription prices equal to: $p_{S1}^{A} = p_{S1}^{B} = c_S + 1 - \alpha_F + \frac{(3-2\alpha_S-\alpha_F)(\alpha_S-\alpha_F)}{3(9-2(2\alpha_S+\alpha_F)(2\alpha_F+\alpha_S)}$,
2. firms' prices equal to $p_{F1}^{A} = p_{F1}^{B} = t - \alpha_S$,

and since platforms charge the same prices, the market symmetrically splits in both sides, i.e., $x_{S1} = n_{F1} = 1/2$. Consequently, both platforms offer introductory prices to new users and TDS occurs, with corresponding equilibrium prices given by the ones in 3 and. Therefore, the numbers of subscribers switching from A to B and B to A are the same. In particular, users laying on the interval $\left(\frac{1}{2}, \frac{2}{3}\right)$ will switch from platform B to platform A and agents in $\left(\frac{1}{3}, \frac{1}{2}\right)$ towards the opposite direction. For what concerns firms, nothing changes from the first to the second period, as the prices charged to them as well as the total number of subscribers to each platform remain constant over time. Finally, the inter-temporal equilibrium profits are given by:

$$\Pi^A = \Pi^B = \hat{\Pi} = \frac{\delta\left(126-\alpha_F(\alpha_F(53-36\alpha_F)+90)-(62-126\alpha_F)\alpha_S^2-(\alpha_F(137-126\alpha_F)+72)\alpha_S+36\alpha_S^3\right)}{18(9-2(\alpha_F+2\alpha_S)(2\alpha_F+\alpha_S))} + \frac{9(2-\alpha_F-\alpha_S)+(9-2(\alpha_F+2\alpha_S)(2\alpha_F+\alpha_S))}{18(9-2(\alpha_F+2\alpha_S)(2\alpha_F+\alpha_S))}. \quad (9)$$

5.4 Proof of Proposition 5

A simple comparison of prices under BBPD (Propositions 1 and 4) with the ones with ban on BBPD (equation (4)) gives:

(i) $p^j_{F1} = p^j_{F2} = c_F + 1 - \alpha_S = \bar{p}^j_F$ with $j = \{A, B\}$,

(ii) $\underbrace{c_S + \frac{1}{3} - \alpha_F}_{p^{jj}_{S2}} < \underbrace{c_S + \frac{2}{3} - \alpha_F}_{p^{jj'}_{S2}} < c_S + 1 - \alpha_F = \bar{p}^j_S$ where $j \neq j'$,

(iii) $p^j_{S1} = c_S + 1 - \alpha_F + \frac{\delta(\alpha_S - \alpha_F)(3 - 2\alpha_S - \alpha_F)}{3(9 - 2(2\alpha_F + \alpha_S)(\alpha_F + 2\alpha_S))} \begin{cases} > c_S + 1 - \alpha_F = \bar{p}^j_S & \text{if } \alpha_S > \alpha_F, \\ < c_S + 1 - \alpha_F = \bar{p}^j_S & \text{otherwise.} \end{cases}$

For the result in point *(iv)*, let us just compare BBPD profits in (9) and benchmark profits in (4). The difference between the two:

$$\hat{\Pi} - \bar{\Pi} = \frac{\alpha_F + \alpha_S}{2} - \left(\frac{\delta\big(\alpha_F(\alpha_F(36\alpha_F - 19) - 72) + 2(63\alpha_F - 5)\alpha_S^2 + (\alpha_F(126\alpha_F - 43) - 90)\alpha_S + 36\alpha_S^3 + 36\big)}{36(\alpha_F + 2\alpha_S)(9 - 2\alpha_F - \alpha_S)} \right)$$

is always negative under the assumption of single-homing ($1 > \max\{\alpha_S, \alpha_F\}$) and concave profits ($1 > 2(\alpha_S + \alpha_F)^2$).

References

Ambrus, A. and Argenziano, R. (2009). Asymmetric networks in two-sided markets. *American Economic Journal: Microeconomics*, 1(1):17–52.

Angelucci, C., Cage, J., and Nijs, R. D. (2013). Price discrimination in a two-sided market: Theory and evidence from the newspaper industry. *Available at SSRN 2335743*.

Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics*, 37(3):668–691.

Armstrong, M. and Wright, J. (2007). Two-sided markets, competitive bottlenecks and exclusive contracts. *Economic Theory*, 32(2):353–380.

Caillaud, B. and De Nijs, R. (2014). Strategic loyalty reward in dynamic price discrimination. *Marketing Science*.

Caillaud, B. and Jullien, B. (2003). Chicken & egg: Competition among intermediation service providers. *RAND Journal of Economics*, 34(2):309–28.

Chen, Y. (2008). Dynamic price discrimination with asymmetric firms. *Journal of Industrial Economics*, 56(4):729–751.

Esteves, R.-B. (2010). Pricing with customer recognition. *International Journal of Industrial Organization*, 28(6):669–681.

Ferrando, J., Gabszewicz, J. J., Laussel, D., and Sonnac, N. (2008). Intermarket network externalities and competition: An application to the media industry. *International Journal of Economic Theory*, 4(3):357–379.

Fudenberg, D. and Tirole, J. (2000). Customer poaching and brand switching. *RAND Journal of Economics*, 31(4):634–657.

Gabszewicz, J. J. and Wauthy, X. Y. (2014). Vertical product differentiation and two-sided markets. *Economics Letters*, 123(1):58 – 61.

Gehrig, T., Shy, O., and Stenbacka, R. (2007). *Market Dominance and Behaviour-based Pricing Under Horizontal and Vertical Differentiation.* Centre for Economic Policy Research.

Gil, R. and Riera-Crichton, D. (2011). Price discrimination and competition in two-sided markets: Evidence from the spanish local tv industry. IESE Research Papers D/894, IESE Business School.

Liu, Q. and Serfes, K. (2005). Imperfect price discrimination in a vertical differentiation model. *International Journal of Industrial Organization*, 23(5-6):341–354.

Liu, Q. and Serfes, K. (2013). Price discrimination in two-sided markets. *Journal of Economics & Management Strategy*, 22(4):768–786.

Rochet, J.-C. and Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the European Economic Association*, 1(4):990–1029.

Shaffer, G. and Zhang, Z. J. (2002). Competitive One-to-One Promotions. *Management Science*, 48(9):1143–1160.

Taylor, C. R. (2003). Supplier surfing: Competition and consumer behavior in subscription markets. *RAND Journal of Economics*, 34(2):223–46.

Villas-Boas, J. M. (1999). Dynamic competition with customer recognition. *RAND Journal of Economics*, 30(4):604–631.