Competitive Behaviour-Based Price Discrimination among Asymmetric Firms

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Abstract

This article studies the effects of Behaviour-Based Price Discrimination (BBPD) in a horizontally and vertically differentiated duopoly. In a two-period model, firms are allowed to condition their pricing policies on the past purchase behaviour of consumers. The paper shows two different types of equilibria depending on the strength of vertical differentiation. If the difference in quality is small enough, both firms steal each other's consumers and suffer a situation in which prices and profits are lower and the consumer surplus increases. When quality differentials are instead substantial, asymmetric behaviours arise: the high-quality firm sells its product to few consumers at a high price in the first period and then becomes aggressive in the second one. As a consequence, customers only move from the low-quality to the high-quality firm. If consumers are myopic, the ODS scenario is detrimental for them and beneficial for firms in relation to uniform pricing. If instead consumers are forward-looking, they and the low-quality firm are better off and the high-quality firm is worse off when BBPD is viable.

JEL codes: L1, D4

Keywords: Competitive BBPD, Vertical Differentiation, Switching

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1 Introduction

Customer's recognition represents an increasingly important matter in economics. Indeed, the development of big data and the availability to firms of consumers' sensible information have raised issues concerning consumers' privacy. Moreover, the improvements in obtaining and processing such information enable firms to infer preferences of consumers and to discriminate based on their past purchase behaviour (BBPD). Since this pricing strategy is being used frequently, it has captured the attention of many scholars,[1] whose main concern has been the understanding of its effect on firms' profits, consumer surplus and level of prices.

The present paper participates in this debate investigating the effects of BBPD when competing firms are assumed to be located at the endpoints of a Hotelling line and to offer goods of different qualities. In particular, in a two-period model, forward-looking firms can observe the purchase behaviour of consumers and thus are allowed to discriminate between old and new buyers. Depending on the relative strength of brands' vertical differentiation (difference in the quality of the good offered), the model exhibits two different equilibria. For weak vertical differentiation, unsurprisingly, the paper accords with the previous literature of BBPD with symmetric competitors: firms set prices in such a way that both steal each other's consumers in the second period (Two-Direction Switching, TDS). The model is able to replicate the results of Fudenberg and Tirole (2000) if the difference in quality is assumed to be zero and gives the essential welfare result of symmetric BBPD: consumers are better off and firms both worse off when price discrimination is feasible.

As soon as a certain level of vertical differentiation is reached, the paper adds important findings to the current literature. Specifically, firms converge to asymmetric pricing behaviours in the first period: the strong seller adopts a margin-focusing strategy and the low-quality rival conquers most of the market. As a consequence, buyers move in the second period only from the low-quality to the high-quality firm (One-Direction Switching, ODS). On top of that, when differentiation is very pronounced, ODS causes the exit of the small firm, that would have been active under uniform pricing.

This inter-temporally unbalanced equilibrium follows from the fact that vertical differentiation creates an important asymmetry in the incentives that each firm has in the first period. In particular, the stronger the vertical differentiation,

[1] Starting from Chen (1997), Villas-Boas (1999) and Fudenberg and Tirole (2000). Esteves (2009) provides an extensive and up-to-date survey on the existing literature in this field.

the more the high-quality firm best response is to use extreme pricing strategies, i.e., either to be very aggressive focusing only on current market share (and becoming monopolist in the first period) or to be benevolent focusing on margins and letting the rival conquer most of the Hotelling line (but becoming monopolist in the second period). Clearly, the first strategy is preferred when the rival sets a low price, since the fight becomes so hard to induce to lay down arms today, aware of the fact that this brings to cheap ODS tomorrow (as switchers are relatively close). On the other side, the low-quality firm anticipates that attracting consumers tomorrow will be more difficult as differentiation becomes stronger and it prefers to focus on conquering the largest possible market share in the first period. This pushes it to be aggressive and to pursue a market-share focusing strategy.

With these mechanisms in mind, the implications on profits and consumer surplus of the asymmetric equilibrium are straightforward. When consumers are myopic, BBPD becomes a very powerful tool for the high-quality seller, which is given the possibility to decide the destiny of the rival. At equilibrium, the high-quality firm decides to focus on margins and the low-quality firm enjoys a large market share. In this scenario, firms reach endogenously a sort of market sharing agreement, which allocates the surplus over time: the high-quality seller trades today's for tomorrow's market share and the low-quality firm does exactly the opposite. This turns out to be ex-post preferred by firms to the uniform pricing as it reduces price competition in the first period. Concerning the low-quality firm, the positive effect of reduced first-period competition compensates the disadvantage provoked by its exit (or cornering) in the second period. Consumers will be worse off as they suffer the reduced competition in terms of prices in the first period and in term of number of competitors in the second period.

When consumers are instead forward-looking, they anticipate tomorrow switching and the first period "elasticity of the demand" changes compared to the uniform pricing.[2] This is a standard result of BBPD literature: Fudenberg and Tirole (2000) discuss how the sensitivity to price of the market splitting location is weaker when consumers are forward-looking in relation to uniform pricing. This boosts first-period prices under BBPD. Oppositely, in the present model, when switching is uni-directional the elasticity to price is increased by BBPD.

[2]Technically speaking, the demand in the model is inelastic. Nevertheless, using the expression elasticity helps capture how market shares respond to marginal changes of prices in different ways moving from the uniform pricing to the case of BBPD with forward-looking consumers.

This induces the low-quality firm to be more aggressive than in the myopic case. As a consequence, the same force that lets the high-quality firm exert a strong power when consumers are myopic, becomes a curse when consumers are forward-looking. Namely, if in the first case differentiation helps the high-quality firm to make high margins and *lets the low-quality firm enjoy a weakened competition*, in the second one it imposes to the low-quality firm to be aggressive and *the high-quality firm suffers the increased competition* in the first period. The result overall is that the low-quality firm and the consumers are better off under the discriminatory pricing at the expenses of the high-quality firm.

Related Literature. The paper belongs naturally to the literature studying price discrimination in oligopolies, which generally agrees on a negative effect on firms' profits compared to uniform pricing. This is because the typical positive effect in the monopoly case (the so-called *Surplus Extraction* effect) is accompanied and often overturned by an intensification of competition in oligopolistic markets (*Business Stealing* effect). As a matter of fact, the information about brands preferences of consumers can be used in two different ways when markets are duopolistic. On the one hand, each firm wants to charge consumers belonging to its "strong" market (i.e., exhibiting relatively strong brand preference) with a high price, thus exploiting information in order to extract their surplus. On the other hand, a given seller also wants to set a low price in its "weak" market to steal rival's business. In the jargon used by Corts (1998), the market exhibits best-response asymmetry, as the"strong" market for a firm is "weak" for the competitor. In these cases, the firms' dominant strategy is to charge low prices in the rival's "strong" market and this, in turn, prevents the latter to fully extract surplus. In a very influential article, Thisse and Vives (1988) show that if firms know the precise location of each consumer and can accordingly engage in perfect price discrimination, then all prices might fall in relation to uniform pricing as the more distant firm is very aggressive in each location. For given prices offered by the rival, both firms find it profitable to discriminate, but this leads to a reduction in prices in the style of a prisoner's dilemma situation.

The paper is more specifically linked to the literature on BBPD, in which firms learn consumers' preferences by observing their purchase behaviour in the past rather than have full information about their locations. In Chen (1997),Villas-Boas (1999), Fudenberg and Tirole (2000) and Esteves (2010), the observation of consumers' identities allows sellers to distinguish between "strong" market (previous buyers) and "weak" market (rival's inherited consumers), as purchase reveals how much a consumer is inclined to buy one or the other product. The loss of firms and consequent gain of consumers are still there: as the latter can be identified and price discrimination is permitted, both sellers have incentives to steal each other's consumers and prices fall down. More recent articles have demonstrated how results may slightly or substantially differ under different settings. In a very recent paper, Colombo (2015) studies the incentives to price discriminate shown by a firm facing a discriminating competitor. He demonstrates that if consumers are myopic enough, the optimal choice is to commit to uniform prices even if the access to information about purchases of consumers is completely costless. Furthermore, Esteves and Reggiani (2014) show how increasing the demand elasticity reduces the negative impact of BBPD on firms' profits, while Chen and Pearcy (2010) demonstrate that when a weak correlation over time between preferences of consumers is assumed, then BBPD will actually be beneficial for firms and detrimental for consumers.

The intuition behind the present paper is that the welfare effect of BBPD depends crucially on the symmetry of the market: if firms are identical ex-ante and compete fiercely for switchers, they end up poaching the same number of consumers with the consequence of a lower level of prices and profits. In the analysis of their two-period model, for example, Fudenberg and Tirole (2000) need specifically to eliminate asymmetric subgames in order to provide their SPNE.[3] Namely, they do not take into account how an inherited market unbalanced in favour of one of the two firms may imply switching only from the dominant to the dominated firm.[4]

Other articles dealing with price discrimination in asymmetric duopolies have results directly comparable with the ones of this paper. As pointed out by Chen (2008), the effects of dynamic price discrimination change substantially from symmetric to asymmetric markets. In a considerably different approach from the present paper with regard to time horizon and consumers' preferences, he finds that price discrimination can be a tool for a low-cost firm to eliminate the less efficient competitor and if exit happens consumers are worse off compared to uniform pricing. Shaffer and Zhang (2002) propose a model where vertically

[3]From the article at page 639: "We will show that, provided that $|\theta^*|$ is not too large, the second-period equilibrium has this form: Both firms poach some of their rival's first-period customers, so that some consumers do switch providers". In their model $|\theta^*|$ represents the location of the time 1 indifferent agent in a Hotelling with firms symmetrically located around zero.

[4]See Gehrig et al. (2007) for an analysis of Fudenberg and Tirole (2000) second period with the past taken as given.

and horizontally differentiated firms are allowed to (costly) target consumers with one-to-one promotions (perfect price discrimination). They find that even though promotional offers intensify price competition they can result in a benefit in terms of market share and profits for the high-quality firm. In Liu and Serfes (2005), firms can costly acquire information about consumers-specific characteristics. They show that when information is not too costly, only the high-quality firm will buy it and engage in price discrimination, with the low-quality firm opting for a uniform price strategy at equilibrium. Differently from the last two articles, in the present model information cannot be acquired and price discrimination is only based to past purchase behaviour and, for strong vertical differentiation, price discrimination benefits the low-quality firm, as price competition is relaxed in the early stage. Gehrig et al. (2011, 2012) propose models in which the asymmetry of the firms is given by some inherited market dominance and firms are allowed to discriminate prices according to the (exogenous) purchase history of consumers.[5] Roughly speaking, their analysis is similar and allows for switching behaviours switching similar to the subgames of the model presented hereafter, which endogenises the purchase history of consumers.

The rest of the paper is organised as follows. The next section presents the main ingredients of the model. After, sections 3 and 4 are devoted to the analysis of the two benchmarks of uniform and discriminatory pricing. The two regimes are then compared in order to provide a welfare analysis on the effects of BBPD in Section 5. Finally, Section 6 contains some concluding remarks.

2 Description of the model

Two competing firms $i = H, L$ aim to sell a good to a population of customers assumed to be uniformly distributed along a unit segment. Firms locations are kept fixed at the end-points of this segment: firm H is located at $l^H = 0$ and L at $l^L = 1$. Sellers are vertically differentiated, as the qualities of the products they sell are different. For the sake of simplicity, firm H is assumed to sell the high-quality good. Formally, it is assumed that $q^H \geq q^L$, where q^k denotes the quality of the product offered by firm $k \in \{H, L\}$.

Consumers face a transportation cost normalised to 1 per unit of distance covered to reach the location of each firm and valuate linearly the quality of the

[5] In particular, Gehrig et al. (2011) provides the limit case of an entry model.

good they buy. According to these assumptions, the per-period utility of an agent located at x who buys good i will be given by:

$$U(x,i) = q^i - p^i - |x - l^i|. \tag{1}$$

Firms set prices in order to maximise profits, facing a unitary cost normalised to 0 in each time period and discounting the future at a factor $\delta < 1$. Each time period is composed of two stages. In stage (1.1) firms simultaneously set prices p_1^H and p_1^L and in stage (1.2) consumers decide upon purchase. In stage (2.1), firms simultaneously set prices knowing who bought which good in period 1: p_2^{iH} is defined as the price set by firm i for a consumer who bought good H in period 1, while p_2^{iL} is charged to L's inherited clients. In the last stage (2.2) consumers observe the new prices and buy again.

The following sections provide a complete analysis of the model. In particular, the next section introduces a benchmark case in which customer's recognition is not allowed, useful to isolate the effects of BBPD. The subsequent section describes the possible equilibria when firms are allowed to engage in BBPD.

3 Uniform Pricing

Assume there exists a ban on price discrimination or that customers' purchases cannot be observed. In this scenario, the utility of an agent buying respectively good H and good L will be:

$$U(x,H) = q^H - p^H - x, \quad U(x,L) = q^L - p^L - (1-x).$$

Accordingly, the indifferent consumer is located at:

$$\bar{x} = \frac{1}{2} + \frac{\Delta + p^L - p^H}{2}, \tag{2}$$

where $\Delta \equiv q^H - q^L$. We assume hereafter that q^H and q^L are high enough so that consumers of all locations prefer to buy one of the two products (full market coverage) and that the prices chosen by the two firms are not too different in order to get rid of situations in which one firm corners the market. Accordingly, the cutoff $\bar{x}$ determines a demand of $\bar{x}$ for firm H and $1-\bar{x}$ for firm L. Moreover, the attention is restricted only to cases in which the difference in quality is not too large to eject the low-quality firm out of the market. As can be clearly seen below, the necessary and sufficient condition for this to be the case is $\Delta < 3$, which allows firm L to charge an above-marginal-cost price at equilibrium. This assumption is maintained hereafter.

Anticipating the reaction of consumers, firms set prices in order to maximise the following static profits:

$$\pi^H = p^H \left(\frac{1}{2} + \frac{\Delta + p^L - p^H}{2} \right), \quad \pi^L = p^L \left(\frac{1}{2} - \frac{\Delta + p^L - p^H}{2} \right).$$

It is worth noticing that, in comparison with the standard Hotelling with equal qualities, firm H can charge higher prices as $\Delta > 0$ and the opposite happens to the low quality firm. Indeed, the equilibrium prices are the following:

$$p_u^H = 1 + \frac{\Delta}{3}, \quad p_u^L = 1 - \frac{\Delta}{3}.$$

They take into account both horizontal (through the transportation cost, 1) and vertical (through the term $\Delta/3$) differentiation. Specifically, 1 represents the market power that both firms enjoy on consumers, whereas $\Delta/3$ is the result of the competitive advantage that firm H enjoys because sells a higher quality product. The prices above result in the following static equilibrium profits:

$$\pi_u^H = \frac{(3+\Delta)^2}{18}, \quad \pi_u^L = \frac{(3-\Delta)^2}{18}.$$

Under uniform price in both periods, subgame perfect Nash equilibrium gives a replication of the static equilibrium, with the following overall profits:

$$\pi_u^H = \frac{(1+\delta)}{18}(3+\Delta)^2, \quad \pi_u^L = \frac{(1+\delta)}{18}(3-\Delta)^2. \tag{3}$$

4 Observation of Consumer Purchases and BBPD

In this section, first-period prices as well as the behaviour of first-period consumers are assumed to be observable to both firms when they choose second-period discriminatory prices. Subgame perfection is used as equilibrium concept.

4.1 Second-Period Subgames

In stage (2.2) consumers observe prices for loyalists and for switchers offered by both firms. In the inherited turf of firm H, a consumer prefers to buy again good

H rather than switch seller when $q^H - p_2^{HH} - x > q^L - p_2^{LH} - (1-x)$, which gives the following indifferent location:

$$\hat{x}_2^H = \frac{1}{2} + \frac{\Delta + p_2^{LH} - p_2^{HH}}{2}, \tag{4}$$

so that $\hat{x}^H$ agents buy again good H. Defining $\hat{x}_1$ as the inherited market share of firm H,[6] $\hat{x}_1 - \hat{x}_2^H$ agents will instead switch towards firm L. Concerning the turf of firm L, consumers compare $q^H - p_2^{HL} - x$ with $q^L - p_2^{LL} - (1-x)$. It means that all agents located on the right of

$$\hat{x}_2^L = \frac{1}{2} + \frac{\Delta + p_2^{LL} - p_2^{HL}}{2} \tag{5}$$

will buy again good L, whereas agents located in the interval $\left[\hat{x}_1, \hat{x}_2^L\right]$ will switch to firm H.

Firms anticipate this reaction of consumers in term of purchase and set prices in stage (2.1). The analysis at this stage depends on the market shares $(\hat{x}_1, 1-\hat{x}_1)$ inherited from the first period, which determine the actual chances to have switching from one firm to the other one and the other way around. Differently from Fudenberg and Tirole (2000), who assume the inherited markets to be symmetric enough, here all possible subgames are analyzed in the backward-induction analysis of the model. In particular, we have subgames with two-direction switching (TDS) and subgames with switching only towards one of the two firms (one-direction switching or ODS).

When firms expect switching to occur in both directions, the thresholds described by equations (4) and (5) are located in such a way that prices can be found in both turfs such that $\hat{x}_2^H < \hat{x}_1 < \hat{x}_2^L$. When instead firms expect switching to occur only towards the high-quality firm (H), the thresholds above are located in such a way that $\hat{x}_1 \leq \hat{x}_2^H$ and $\hat{x}_1 < \hat{x}_2^L$. These two examples are depicted in the figure below.

Considering all the possible subgames, the following proposition contains all possible scenarios when the inherited base of customers $\hat{x}_1$ is considered exogenous.

Proposition 1. *When firms are allowed to price discriminate between old and rival's previous consumers, the second-period equilibrium prices are:*

[6] Notice that under the assumption of fully covered market, $1 - \hat{x}_1$ is the first period market share of firm L.

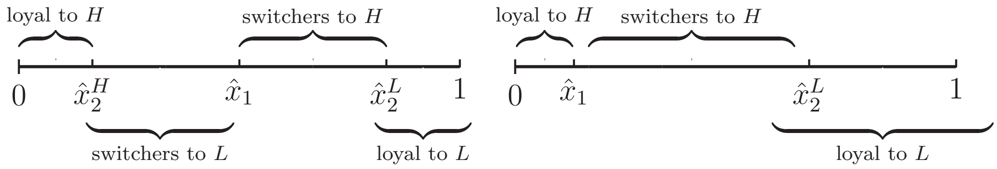


Figure 1: Different Switching Scenarios

$$\text{(i)} \quad \left.\begin{array}{ll} p_2^{HH} = 1 + \Delta - 2\hat{x}_1, & p_2^{LH} = 0, \\ p_2^{HL} = 1 + \frac{\Delta}{3} - \frac{4}{3}\hat{x}_1, & p_2^{LL} = 1 - \frac{\Delta+2}{3} + \frac{2}{3}(1 - \hat{x}_1) \end{array}\right\} \textit{ when } \hat{x}_1 \leq \frac{1+\Delta}{4}$$

$$\text{(ii)} \quad \left.\begin{array}{ll} p_2^{HH} = 1 + \frac{\Delta}{3} + \frac{2}{3}\hat{x}_1, & p_2^{LH} = 1 - \frac{\Delta}{3} - \frac{4}{3}(1 - \hat{x}_1), \\ p_2^{HL} = 1 + \frac{\Delta}{3} - \frac{4}{3}\hat{x}_1, & p_2^{LL} = 1 - \frac{\Delta+2}{3} + \frac{2}{3}(1 - \hat{x}_1) \end{array}\right\} \textit{ when } \hat{x}_1 \in \left(\frac{1+\Delta}{4}, \frac{3+\Delta}{4}\right)$$

$$\text{(iii)} \quad \left.\begin{array}{ll} p_2^{HH} = 1 + \frac{\Delta}{3} + \frac{2\hat{x}_1}{3}, & p_2^{LH} = 1 - \frac{\Delta}{3} - \frac{4}{3}(1 - \hat{x}_1), \\ p_2^{HL} = 0, & p_2^{LL} = 1 - \Delta - 2(1 - \hat{x}_1) \end{array}\right\} \textit{ when } \hat{x}_1 \geq \frac{3+\Delta}{4}$$

Proof. See mathematical appendix. ■

In order to better grasp the intuition behind Proposition 1 let us consider the equilibrium prices in point (ii). Unsurprisingly, a stronger vertical differentiation is associated with a competitive advantage in favour of the high-quality firm, whose equilibrium prices for old and new consumers are both increasing in Δ. Exactly the opposite relation exists between the prices of the low-quality firm and the vertical differentiation parameter.

On the other hand, the own inherited market share[7] affects positively the price a given firm charges to the old loyal consumers and negatively the one offered to the switchers. Intuitively, the relation between prices and market share follows directly from the effective power that the size of the first-period market creates in each turf for the "attacking" (else turf) and the "defending" firm (own turf). Clearly, the attack in the rival turf turns out to be more costly as the size of the market already conquered in the first period becomes higher. In other words, the price offered to the switchers should be lower when a lot of consumers were attracted in the first period, since the non-conquered portion is very far away in the Hotelling line. For extreme levels of the market share,[8] attracting new consumers is not profitable as it would require a below-marginal-cost price. These

[7]The market shares will be $\hat{x}_1$ for the high-quality firm and the remaining $1 - \hat{x}_1$ for the low-quality rival given the assumption of full market coverage.

[8]According to the proposition, this level will be $\frac{3+\Delta}{4}$ for firm H and $1 - \frac{1+\Delta}{4}$ for firm L.

cases are presented in points (i) and (iii), where respectively firm H and L prefer the dominating strategy of setting prices equal to the marginal cost (i.e., 0) in the rival turf.

Therefore, from the point of view of the defending firm, the higher the market share inherited from the past the weaker the price competition in its own turf, as the rival becomes less aggressive. For this reason, the equilibrium price for loyalists[9] is increasing in the inherited market share. In the extreme cases in which the attacking rival sets the price equal to the marginal cost (points (i) and (iii) in the proposition), then the optimal response of the defending firm is to offer to past consumers a price just sufficient not to lose any of them.

These equilibrium prices will determine peculiar switching behaviours of consumers. If the first-period market is balanced enough, then both firms succeed in finding profitable prices to offer to rival's consumers and both are able to attract (and respectively suffer the loss of) some new (old) consumers. If instead the market is strongly dominated by a firm in the first period, the dominating firm does not attract any rival consumers, even though it charges a price equal to the marginal cost. For this reason, switching will be one-direction towards the dominated firm. These results are formally presented in the following corollary:

Corollary 1. *Given the equilibrium prices in Proposition 1: (i) when* $\hat{x}_1 \leq \frac{\Delta+1}{4}$*, consumers only switch to firm H (*ODS*); (ii) when* $\hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$*, consumers switch from H to L and vice-versa (*TDS*); and (iii) when* $\hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}$*, consumers only switch to firm L (*ODS*).*

Proof. Plugging the equilibrium prices in proposition 1, it is easy to find the following cutoffs: (i) when $\hat{x}_1 \leq \frac{\Delta+1}{4}$, $\hat{x}_2^H = \hat{x}_1$ and $\hat{x}_2^L = \frac{\Delta+2\hat{x}_1+3}{6}$; (ii) when $\hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, $\hat{x}_2^H = \frac{\Delta+2\hat{x}_1+1}{6}$ and $\hat{x}_2^L = \frac{\Delta+2\hat{x}_1+3}{6}$; (iii) when $\hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}$, $\hat{x}_2^H = \frac{\Delta+2\hat{x}_1+1}{6}$ and $\hat{x}_2^L = 1 - \hat{x}_1$. ■

4.2 First Period

In stage (1.2) consumers observe prices and buy the good giving them the highest utility. In what follows, consumers are assumed to be myopic i.e., they only care about the utility they get at stage (1.2), without anticipating the second-period (possible) switching.[10] In what follows, only the case of myopic consumers is

[9] See prices p_2^{HH} and p_2^{LL} in point (ii) of proposition 1.

[10] As discussed in the book of Belleflamme and Peitz (2010), the effect of consumers' farsightedness is essentially that BBPD weakens price competition in the first period compared to

presented whereas the main features of the forward-looking consumers case are discussed in a separate paragraph and formally in the appendix.

Under myopia, the first-period indifferent consumer will be located at:

$$\hat{x}_1 = \frac{1}{2} + \frac{\Delta + p_1^L - p_1^H}{2}, \tag{6}$$

so that all agents to the left of the cutoff above buy the high-quality good and all agents to the right buy the low-quality good. Following a backward induction reasoning, in the initial stage (1.1) forward-looking firms correctly anticipate both purchase decisions in stage (1.2) and all possible subgames. Anticipating first-period purchase behaviour of consumers expressed by the cutoff in (6) and discounting future profits, firm H and L respectively maximize the following inter-temporal profits:

$$\pi_1^H + \delta\pi_2^H = p_1^H\hat{x}_1 + \delta\left[p_2^{HH}\min\left\{\hat{x}_2^H, \hat{x}_1\right\} + p_2^{HL}\max\left\{\hat{x}_2^L - \hat{x}_1, 0\right\}\right],$$

$$\pi_1^L + \delta\pi_2^L = p_1^L(1-\hat{x}_1) + \delta\left[p_2^{LL}\min\left\{1-\hat{x}_2^L, 1-\hat{x}_1\right\} + p_2^{LH}\max\left\{\hat{x}_1 - \hat{x}_2^H, 0\right\}\right].$$

Clearly, the future profits depend on the expectations firms have about tomorrow's movements of consumers. In particular, plugging the prices in proposition 1 and the resulting cutoffs expressed in the proof of Corollary 1, second-period profits depend on the inherited market share as follows:

$$\pi_2^H(\hat{x}_1) = \begin{cases} \pi_{2H}^H = \frac{\Delta^2+(9-2\hat{x}_1(10\hat{x}_1+3))+2\Delta(5\hat{x}_1+3)}{18} & \text{if firms expect } \hat{x}_1 \leq \frac{\Delta+1}{4}, \\ \pi_{2TDS}^H = \frac{\Delta^2+5(2\hat{x}_1^2-2\hat{x}_1+1)-2\Delta(\hat{x}_1-2)}{9} & \text{if } \hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right), \\ \pi_{2L}^H = \frac{(\Delta+2\hat{x}_1+1)^2}{18} & \text{if } \hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}. \end{cases}$$

for firm H. Similarly, firm L anticipates profits:

$$\pi_2^L(\hat{x}_1) = \begin{cases} \pi_{2H}^L = \frac{(\Delta+(2\hat{x}_1-3))^2}{18} & \text{if firms expect } \hat{x}_1 \leq \frac{\Delta+1}{4}, \\ \pi_{2TDS}^L = \frac{\Delta^2+5(2\hat{x}_1^2-2\hat{x}_1+1)-2\Delta(\hat{x}_1+1)}{9} & \text{if } \hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right), \\ \pi_{2L}^L = \frac{\Delta^2+\left(46\hat{x}_1-20\hat{x}_1^2-17\right)+2\Delta(5\hat{x}_1-8)}{18} & \text{if } \hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}. \end{cases}$$

uniform pricing because of a lower first-period elasticity to price. In the context of the present paper, which analyses all possible scenarios and not only the symmetric one, one can observe in the appendix how the farsightedness of consumers would bring to richer results because the responsiveness to price of the infra-marginal consumer is very strong in the asymmetric cases.

where the notation H, L, and TDS in the subscripts are used to indicate that switching is respectively towards firm H only, towards firm L only or towards both directions. The following paragraph is devoted to the description of best responses, that exhibit peculiar features due to the fact that firms can choose very different pricing strategies according to the inter-temporal objective they want to pursue. Subsequently, the equilibria of the model are presented, giving also some insights on the main characteristics of prices and switching behaviour. Finally, the last paragraph of this section discusses the case of forward-looking consumers, highlighting the main difference with the case of myopia.

Best Responses and Vertical Differentiation. In order to build up the best responses of firms, the following approach is used. For given price of the rival p_1^j, firm i has three different alternatives. Namely, it can optimally choose a price $p_{1H}^i(p_1^j)$, $p_{1TDS}^i(p_1^j)$ or $p_{1L}^i(p_1^j)$ leading respectively to the market splitting cutoff $\hat{x}_1$ in the interval $[0, \frac{1+\Delta}{4}]$ or $(\frac{1+\Delta}{4}, \frac{3+\Delta}{4})$ or $[\frac{3+\Delta}{4}, 1]$ and giving firm i the correspondent second-period profits. The correspondent profits in each of the three cases are then compared: the best response will be the one leading to the highest profit.

Albeit the complete construction of the best responses is left to the appendix, it is worth discussing their main features. The best-reply price will inter-temporally trade-off between today's profits (market share and per-consumer margin) and tomorrow's cost of poaching consumers. In particular, choosing an aggressive pricing strategy focusing on today's market share will entail a relative low per-consumer margin and makes the attraction of new consumers very costly. When a firm is particularly aggressive today, it conquers a large market and tomorrow only the rival succeed in attracting new consumers. The best response price in this case will be respectively

$$p_{1L}^H = \frac{(9-2\delta)}{18-2\delta} p_1^L + \frac{9-4\delta}{18-2\delta} + \frac{(9-4\delta)\Delta}{18-2\delta}$$

and

$$p_{1H}^L = \frac{(9-2\delta)}{18-2\delta} p_1^H + \frac{9-4\delta}{18-2\delta} - \frac{(9-4\delta)\Delta}{18-2\delta}.$$

In principle, when a firm is very aggressive this strategy can also lead to the corner case in which it becomes monopolist, i.e., $p_{1M}^H = p_1^L + \Delta - 1$ or $p_{1M}^L = p_1^H - \Delta - 1$.[11]

[11] As it will be shown afterwards, these prices are never part of an equilibrium but can be best responses for high levels of the rival's price.

On the other hand, choosing a benevolent pricing strategy that focuses on high margins on few consumers in the first period will make the attack of the rival turf less costly in the second period. Being benevolent today let a firm conquer of a small market today and benefit ODS tomorrow. The best response price in this case will be respectively

$$p_{1H}^H = \frac{(10\delta + 9)}{18 + 10\delta} p_1^L + \frac{9 + 13\delta}{18 + 10\delta} + \frac{\Delta}{2}$$

and

$$p_{1L}^L = \frac{(10\delta + 9)}{18 + 10\delta} p_1^H + \frac{9 + 13\delta}{18 + 10\delta} - \frac{\Delta}{2}.$$

Choosing a more inter-temporally balanced strategy will instead lead to tomorrow's TDS. In this case, the best response prices will be:

$$p_{1TDS}^H = \frac{(9 - 10\delta)}{18 - 10\delta} p_1^L + \frac{9}{18 - 10\delta} + \frac{(9 - 8\delta)\Delta}{18 - 10\delta}$$

and

$$p_{1TDS}^L = \frac{(9 - 10\delta)}{18 - 10\delta} p_1^H + \frac{9}{18 - 10\delta} - \frac{(9 - 8\delta)\Delta}{18 - 10\delta}.$$

The global best response is thus found by choosing the alternative leading to the highest profit among the three strategies described above. It turns out that the optimal pricing behaviours change dramatically according to the strength of the asymmetry present in the market.

In particular, when vertical differentiation is weaker than horizontal differentiation ($\Delta < 1$), the best responses have the following forms:

$$p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \le \hat{p}, \\ p_{1TDS}^H & \text{if } p_1^L \in (\hat{p}, \hat{p}_{LC}), \\ p_{1L}^H & \text{if } p_1^L \ge \hat{p}_{LC}, \end{cases} \quad p_1^L(p_1^H) = \begin{cases} p_{1L}^L & \text{if } p_1^H \le \tilde{p}, \\ p_{1TDS}^L & \text{if } p_1^H \in [\tilde{p}, \tilde{p}_{HC}], \\ p_{1H}^L & \text{if } p_1^H \ge \tilde{p}_{HC}, \end{cases}$$

where

$$\hat{p} \equiv \frac{\sqrt{(9\Delta+9)^2-(5\delta\Delta+5\delta)^2}}{30} + \frac{65\delta-15\delta\Delta}{90} - \frac{3\Delta+3}{10}, \qquad \tilde{p}_{LC} \equiv \frac{18-3\delta\Delta-5\delta}{9},$$
$$\tilde{p} \equiv \frac{\sqrt{(9-9\Delta)^2-(5\delta-5\delta\Delta)^2}}{30} + \frac{65\delta+15\delta\Delta}{90} + \frac{3\Delta-3}{10} \text{ and } \quad \tilde{p}_{HC} \equiv \frac{18+3\delta\Delta-5\delta}{9}$$

are the cutoff values of rival's prices that induce each firm to switch from one strategy (aggressive, balanced, benevolent) to another one.

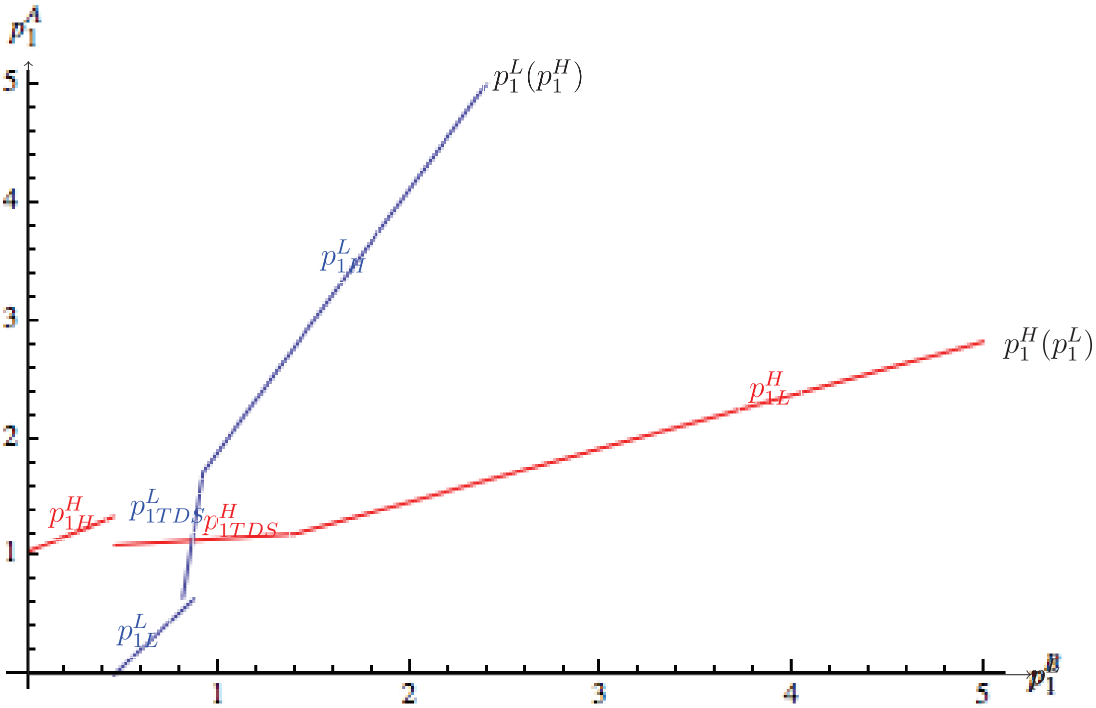


Figure 2: Best Responses: $\Delta < 1$

Intuitively, being aggressive (respectively benevolent) today is preferred when the rival is benevolent (aggressive). Indeed, if the rival sets a high price, being aggressive today does not entail a substantial cost in terms of lower margins. Therefore, since a firm is given the possibility to make high margins on a large market today, it does not care at all about tomorrow switching. Oppositely, when the rival is very aggressive, a seller will let it conquer a large part of the market enjoying uni-directional switching tomorrow. In other words, the seller lays down arms today when the fight becomes too hard, aware of the fact that this brings to a cheap conquest of rival territory tomorrow.

Finally, firm i will prefer p^i_{1TDS} when the rival chooses an intermediate price. In this segment, the best response is less steep than in the two extreme cases due to the fact that involves an inter-temporal balance of incentives. Namely, once a firm prefers to play a ODS equilibrium (either to itself or to the rival), then second period turns out to be less important because the change determined by a price cut (or increase) today does not involve sizeable changes in the movements of consumers tomorrow. Thus, the best response will be more sensitive too any price change of the rival compared to the TDS case, in which future movements of consumers are more crucial.[12]

As vertical differentiation is stronger than horizontal differentiation, ODS to firm L is no more a threat for the high-quality firm, as it is reachable only if the latter becomes a monopolist in the first period.[13] In particular, as the price charged by the low-quality firm reaches a cutoff (i.e., $p^L_1 > \hat{p}_M \equiv \frac{27+2\delta\Delta-10\delta-9\Delta}{9}$), p^H_{1TDS} is not optimal any more and firm L cannot enter the market in the first period. Moreover, the cutoff price $\hat{p}_M$ is decreasing in the level of vertical differentiation, meaning that the stronger the asymmetry the narrower the segment in which the high-quality firm finds it optimal the inter-temporally balanced strategy leading to TDS. This will determine the following firm H best response for increasing levels of asymmetry:

[12]Notice that the higher the discount factor, the less sensitive the best response today. When the discount factor is very high (i.e., $\delta > 9/10$), this tendency brings to situations in which prices are strategic substitute.This is because, when the future is very important, firms see a price cut of the rival as an opportunity of conquering a large market tomorrow rather than a threat of losing market share today. Accordingly, a price cut of the rival induces a firm to slightly increase the price today making high per-consumer margins, mainly focusing on the attraction of consumers tomorrow.

[13]Indeed, according to Corollary 1 ODS to L can be the case only if $\hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}$.

$$\text{if } \Delta \in [1, 3-\tfrac{12\delta}{9-2\delta}], \quad p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \le \hat{p}, \\ p_{1TDS}^H & \text{if } p_1^L \in (\hat{p}, \hat{p}_M), \\ p_{1M}^H & \text{if } p_1^L \ge \hat{p}_M, \end{cases}$$

$$\text{if } \Delta \in [3-\tfrac{12\delta}{9-2\delta}, \hat{\Delta}], \quad p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \le \hat{p}, \\ p_{1TDS}^H & \text{if } p_1^L \in (\hat{p}, \hat{p}_M), \\ p_{1H}^H & \text{if } p_1^L \in [\hat{p}_M, \hat{p}_H], \\ p_{1M}^H & \text{if } p_1^L > \hat{p}_H, \end{cases}$$

$$\text{if } \Delta \ge \hat{\Delta}, \qquad p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \le \hat{p}_H, \\ p_{1M}^H & \text{if } p_1^L > \hat{p}_H. \end{cases}$$

where $\hat{\Delta} \equiv 3 - \frac{12\left(\sqrt{81-25\delta^2}-(9-2\delta)\right)}{36-29\delta}$.[14]

On the other hand, firm L entry is prevented in the first-period if rival's price is lower than a certain level (i.e., $p_1^H < \tilde{p}_M \equiv \frac{10\delta-9+(9-2\delta)\Delta}{9}$). Again, since $\tilde{p}$ increases in Δ, increasing vertical differentiation entails that the segment in which TDS is possible is narrower. Formally the best response of firm L when $\Delta > 1$ will be:

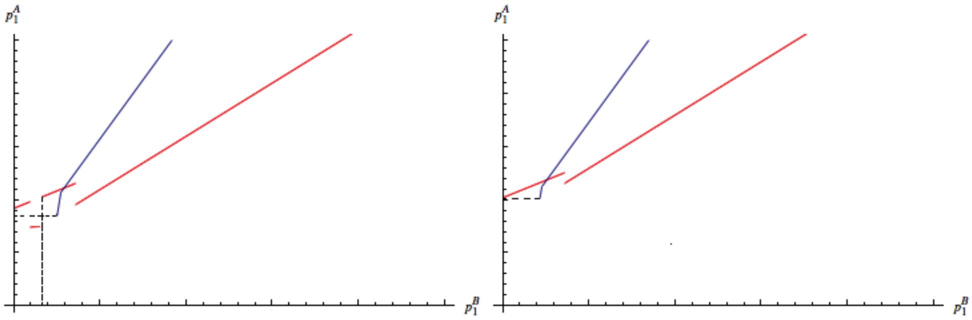
$$p_1^L(p_1^H) = \begin{cases} p_{1TDS}^L & \text{if } p_1^H \in [\tilde{p}_M, \tilde{p}_{HC}), \\ p_{1H}^L & \text{if } p_1^H \ge \tilde{p}_{HC}. \end{cases}$$

Figure 3 gives a graphical representation of the best responses when firms are more vertically than horizontally differentiated. Two different cases are depicted.[15] The left figure describes situations in which vertical differentiation is not so strong to eliminate completely the TDS price from the best response of the high-quality firm. In particular, TDS is optimally chosen in second tiny segment on the left the dashed vertical line. For a rival price on the right of the dashed line, the high-quality firm is given the possibility to choose between charging a high price today enjoying ODS tomorrow and conquering the entire market today. Clearly, the first solution is preferred for relatively low prices charged by the

[14] It is easy to verify that $\sqrt{81-25\delta^2}-(9-2\delta) > 0$ for all $\delta \in [0,1]$, meaning that $\hat{\Delta} < 3$.

[15] The case in which $\Delta \in [1, 3-\frac{12\delta}{9-2\delta}]$ is not represented in the figures as it is similar to the one with $\Delta < 1$. The only difference is that ODS to L can be the case only if the low-quality firm does not enter the market in the first period.

rival (and depicted in the third segment of the best response) as being aggressive would entail a too high loss in terms of per-consumer margins. Oppositely, when the rival is benevolent and sets a high price, the conquest of the entire Hotelling line in the first period turns out to be preferred (fourth segment in the best response). The right figure follows exactly the same reasoning, with the only difference that TDS is never chosen by the high quality firm. Graphically, the second segment in the left figure disappears completely and thus for low levels of firm L prices, only p_{1H}^{H} is chosen.



$\Delta \in [3 - \frac{12\delta}{9-2\delta}, \hat{\Delta}]$ $\qquad\qquad$ $\Delta > \hat{\Delta}$

Figure 3: Best Responses for Increasing Δ

To conclude, it is worth noticing a sudden discontinuity in the best response when switching to the ODS scenario.[16] This "jump" is due to a sharp change of strategy when we consider a rival's undercut. Assume a price of the rival just above the maximal level inducing a firm to use a benevolent pricing strategy enjoying ODS tomorrow. If the rival slightly lowers the price, the high-margins focused strategy becomes suddenly preferred to the alternative in which the market shares are inter-temporally balanced (or to the market-share focused solution). The discontinuity in the best response indicates that the focus on margins is particularly intense, as the responding firm will sudden increase the price.[17]

[16] In figure 2, this jump is present in the passage from the first (leading to ODS) to the second segment (leading to TDS) of the best responses of both firms. In figure 3, this jump is not present in the best response of the low-quality firm as ODS disappears. For the high-quality firm, we have three jumps, the first two when we pass from p_{1H}^{H} to p_{1TDS}^{H} and vice-versa and the second passing from p_{1H}^{H} to p_{1L}^{H}.

[17] As it will be clarified when discussing the forward-looking case, this jumps is made possible and preferred by the fact that myopic consumers are not affected by tomorrows switching, and

Existence and Uniqueness of Equilibria. In the analysis of equilibria, one should take into account two main aspects. On the one hand, it is important to see under which conditions asymmetric equilibria in which switching is one direction can be reached at equilibrium. This is an important novelty of the paper compared to the symmetric approach leading always to TDS proposed by Fudenberg and Tirole (2000), which result can be found in the present paper just setting vertical differentiation parameter $\Delta = 0$. On the other hand, due to the discontinuities in the best-response functions, equilibria might fail to exist or can be multiple. The following proposition summarise all possible scenarios

Proposition 2. *(Equilibria)*

1. *(TDS) If* $\Delta < 3 - \frac{8\delta}{9-4\delta} \equiv \bar{\Delta}$*, there exists an equilibrium in which the prices are:*
$$(p_{1TDS}^{H*}, p_{1TDS}^{L*}) = \left(1 + \tfrac{\Delta}{3} - \tfrac{4\delta\Delta}{81-60\delta}, 1 - \tfrac{\Delta}{3} + \tfrac{4\delta\Delta}{81-60\delta}\right)$$
resulting in TDS in the second period.

2. *(ODS) If* $\Delta > 3 - \min\left\{\frac{8\delta}{9-6\delta}, \frac{28\delta(2\delta+9)}{\delta(14\delta+27)+162}\right\} \equiv \underline{\Delta}$*, there exists an equilibrium in which prices are:*
$$(p_{1H}^{H*}, p_{1H}^{L*}) = \left(1 + \tfrac{\Delta}{3} + \tfrac{2\delta(22+11(1-\delta)+\Delta(1+5\delta)}{24\delta+81}, 1 - \tfrac{\Delta}{3} + \tfrac{\delta(15(1-\delta)+11\Delta+(10\Delta-7)\delta)}{24\delta+81}\right)$$
resulting in ODS to firm H in the second period.

3. *(Existence and Multiplicity) If the discount factor is not too high (i.e.,* $\delta <$ 0.93875*), then the two equilibria coexist in the interval* $[\underline{\Delta}, \bar{\Delta}]$*. Otherwise, no equilibrium exists in the interval* $[\bar{\Delta}, \underline{\Delta}]$*.*

Proof. See Appendix for a complete proof. ■

The proposition tells that we can observe two different sorts of equilibria. In the first one, both firm choose an inter-temporally balanced pricing strategy that leads to TDS (point 1.). When quality differentiation is rather substantial, asymmetric behaviours arise. Specifically, the high-quality firm finds it profitable to implement a benevolent pricing strategy, which allows for the obtainment of high first-period margins associated with second-period ODS. The following corollary of Proposition 2 formally explains how market is shared differently in the two equilibria and how this affects the second-period switching of consumers.

thus their responsiveness to price in the first period do not change passing from TDS to ODS scenarios.

Corollary 2. *(Market Shares & Switching)*

1. *When* $(p_{1TDS}^{H*}, p_{1TDS}^{L*})$ *are the equilibrium prices, then* $\hat{x}_1 = \frac{1}{2} + \frac{(9-4\delta)\Delta}{54-40\delta}$, $\hat{x}_2^H = \frac{1}{3} + \frac{2(3-2\delta)\Delta}{27-20\delta}$ *and* $\hat{x}_2^L = \frac{2}{3} + \frac{2(3-2\delta)\Delta}{27-20\delta}$. *In the second period* $\hat{x}_1 - \hat{x}_2^H = \frac{1}{6} - \frac{(3-4\delta)\Delta}{54-40\delta}$ *consumers switch from H to L and* $\hat{x}_2^L - \hat{x}_1 = \frac{1}{6} + \frac{(3-4\delta)\Delta}{54-40\delta}$ *move to the opposite direction.*

2. *When* $(p_{1H}^{H*}, p_{1H}^{L*})$ *are the equilibrium prices, then* $\hat{x}_1 = \hat{x}_2^H = \frac{1}{2} - \frac{(7\delta+9)\Delta-17\delta}{16\delta+54}$, *and* $\hat{x}_2^L = \frac{1}{2} + \frac{5\delta\Delta-3\delta+12\Delta+9}{16\delta+54}$. *In the second period,* $\min\left\{\hat{x}_2^L - \hat{x}_1, 1 - \hat{x}_1\right\}$ *consumers switch from L to A.*

Proof. The results are found by plugging the firs-period equilibrium prices into the cutoffs expressed in equations (4), (5) and (6). ■

The corollary above highlights two different scenarios. In the first one firms share first-period market in a relatively balanced way and both succeed in stealing rival consumers in the second period. In the second one, reached when vertical differentiation is important enough, we observe a sort of market sharing agreement, according to which firms allocate market shares and surplus over time in an asymmetric way. In particular, firm H pursues a benevolent pricing strategy, consisting in being inoffensive today in order to induce a favourable response of the low-quality rival. This strategy lets firm H make high unitary margins on a small number of consumers, allowing for the opportunity of a large market to conquer cheaply in the second period.

Unsurprisingly, this benevolent pricing strategy will lead to a mitigated price competition for increasing Δ compared both to the TDS and to the uniform pricing case. This result is formally stated in the following corollary:

Corollary 3. *For what concerns equilibrium prices, the following holds:*

(a) *high-quality firm:* $\frac{\partial p_{1H}^{H*}}{\partial \Delta} > \frac{\partial p_{1TDS}^{H*}}{\partial \Delta} > 0$ *and* $p_{1H}^{H*} > p_u^H$;

(b) *low-quality firm:* $\frac{\partial p_{1TDS}^{L*}}{\partial \Delta} < \frac{\partial p_{1H}^{L*}}{\partial \Delta} < 0$ *and* $p_{1H}^{L*} > p_u^L$.

Corollary 3 highlights how the equilibrium price set by the high-quality firm (respectively by the low-quality firm) is affected positively (negatively) by the difference in quality both in the TDS and in the ODS-to-H scenario. This simply follows from the fact that an increase in the vertical differentiation gives the high quality a stronger power. The difference between the two scenarios is that the positive effect on strong firm equilibrium price is amplified and the negative effect on weak firm's price is mitigated passing form TDS to ODS. This together with the fact that eh ODS-to-H equilibrium is associated to higher levels of vertical differentiation suggests that price competition is less severe when the asymmetric equilibrium arises.

Moreover, the ODS equilibrium entails a attenuation of competition in relation with the uniform pricing in two different ways. On the one hand, Corollary 3 shows that first-period prices are higher than the uniform price. This is the case because of the benevolent pricing strategy implemented by the high-quality firm. On the other hand, the equilibrium ODS to H in general entails the exit of the low quality firm which cannot find any profitable way to compete in the second period. This result is formally explained in the following corollary.

Corollary 4. *(Exit of the low quality firm) If* $\Delta > \max\{\frac{11\delta+18}{5\delta+12}, \underline{\Delta}\}$*, then ODS to H determines the exit of the low quality firm from the market.*

Proof. Take the $\hat{x}_2^L = \frac{1}{2} + \frac{5\delta\Delta-3\delta+12\Delta+9}{16\delta+54}$ resulting from $(p_{1H}^{H*}, p_{1H}^{L*})$. It holds that: $\frac{1}{2} + \frac{5\delta\Delta-3\delta+12\Delta+9}{16\delta+54} > 1 \Leftrightarrow \Delta > \frac{11\delta+18}{5\delta+12}$. Since ODS-to-H equilibrium exists only if $\Delta > \underline{\Delta}$, the result of the corollary is proved. ■

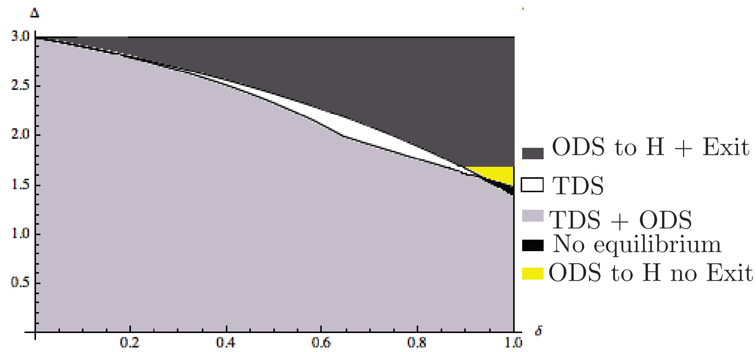


Figure 4: Equilibria with myopic consumers.

The result in the corollary above is due by the fact that the high-quality firm conquers a very small market in the first period, with the consequence that competing in the rival's territory in the second period becomes very easy. This gives the high-quality firm a way to profitably attack all the rival turf and conquer the entire Hotelling segment. For the sake of completeness, when the discount factor is very high ($\delta > 6/7$), there exist situations in which the low-quality firm survives, albeit it is relegated to a very tiny corner of the market. As can be seen in Figure 4, these instances can arise only for very specific combinations of vertical differentiation and discount factor.

Forward-Looking Consumers: Best Responses and Equilibria. This paragraph is devoted to a brief description of best responses and equilibria under consumers' farsightedness, providing a discussion about the main differences between myopic and forward-looking consumers. For all technical detail, the reader is invited to read the appendix.

In the present setting, a forward-looking consumer should be able to correctly anticipate tomorrow's switching scenarios (TDS or ODS). In order to find the indifferent first-period consumer, the following approach is used. When consumers observe prices p_1^H, p_1^L offered by firms, they become aware about which game firms are playing in terms of tomorrow's switching (TDS or ODS). Accordingly, the indifferent period-one consumer will locate differently according to her expectations. Let us assume that TDS is expected. The rational consumer indifferent in period one foresees that if she buys good H in time 1, she will switch to L in the next period and vice-versa. As proved in the appendix, this consumer will be located at:

$$\hat{x}_{1TDS}^{FL} = \frac{1}{2} + \frac{(3-\delta)\Delta + 3(p_1^L - p_1^H)}{2\delta + 6}. \tag{7}$$

When ODS to H is expected, the rational consumer foresees that if she buys good H in time 1, she will buy it again in the second period, whereas if she purchases product L, she will switch to H in the subsequent period. As proved in the appendix, this consumer will be located at:

$$\hat{x}_{1H}^{FL} = \frac{1}{2} + \frac{(3-2\delta)\Delta + 3(p_1^L - p_1^H) + \delta}{6 - 2\delta}. \tag{8}$$

The main difference between expecting TDS and ODS-to-H is that the indifferent consumer is more sensitive to price in the second case. On top of that, as expressed in the following lemma, the "elasticity of first period demand" in relation to uniform pricing and myopic consumers is respectively reduced when TDS occurs and increased when ODS occurs.

Lemma 1. *If consumers foresee the future switching scenarios, the location of the indifferent consumer is more sensible to price changes when tomorrow's switching* *is expected to be uni-directional, i.e.,*

$$|\frac{\partial \hat{x}^{FL}_{1H}}{\partial p^i_1}| = \frac{3}{6-2\delta} > |\frac{\partial \hat{x}_1}{\partial p^i_1}| = \frac{1}{2} = |\frac{\partial \bar{x}}{\partial p^i_1}| > \frac{3}{6+2\delta} = |\frac{\partial \hat{x}^{FL}_{1TDS}}{\partial p^i_1}|$$

with $i \in \{H, L\}$.

The result presented in Lemma 1 is not surprising. Assume two different equilibria in which the market splits at given locations $\hat{x}^{FL}_{1H}$, $\hat{x}^{FL}_{1TDS}$. *Ceteris paribus*, consider an increase in the price of the low-quality firm. This will have the effect to change the optimal behaviour of agents located just on the right of the cutoffs, who will decide to buy good H rather than good L. This will move the cutoff towards right. Clearly, the responsiveness of consumers to the increase in the price of firm L is stronger as long as consumers are closely firm H's location. Thus, since $\hat{x}^{FL}_{1H} < \hat{x}^{FL}_{1TDS}$, a higher number of agents will change buying firm H's product in the ODS-to-H compared to the TDS case.

Compared to the non-discriminatory regime, the "elasticity" changes due to the fact that forward-looking consumers take into account not only the direct impact of a price variation,[18] but also the indirect effect of a variation over the second period's prices. Colombo (2015) provides a very accurate and precise explanation of this effect in the TDS case with symmetric firms and points out how the demand "elasticity" is lower under BBPD.[19] Oppositely, when ODS is assumed to be the case, consumers anticipate that tomorrow's discounted prices will be less attractive as firm H will not need to lower the price too much to attract switchers. As a result, the first period benefit from switching after a price decrease is higher than the uniform case.

This result is very important to understand the discontinuity in the best responses that we can observe in figure 5. Differently from the myopic case, discontinuities emerge when the price of the rival is high rather than when it is low. In particular, when the price of the rival reaches a threshold, we have the passage from TDS to an aggressive best-response leading to uni-directional switching towards the rival. When this change happens, a forward-looking consumer anticipates it. And, since the responsiveness to price of the indifferent consumer

[18] Notice that they would consider only this direct effect both in the uniform pricing regime and in the myopic-consumers case.

[19] Studying an increase in the price of firm L, he concludes the following: "It follows that the first-period benefit from shifting from firm L to firm H is lower when future is taken into account. Hence, the higher δ is, the lower is the benefit from shifting after a first-period price decrease."

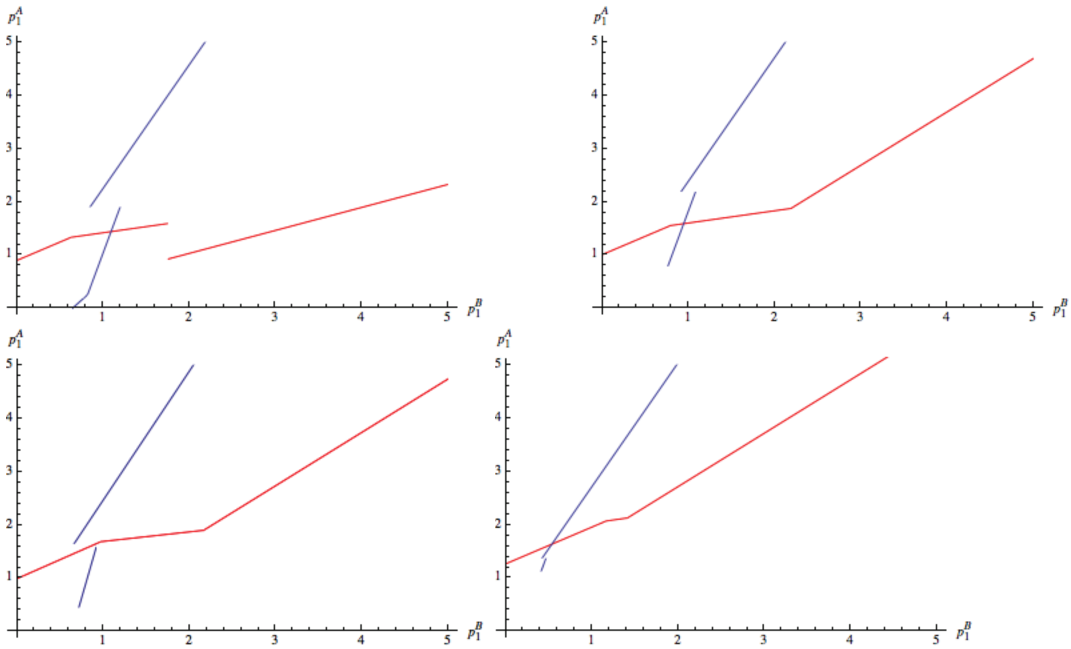


Figure 5: Best Responses for Increasing Δ: Forward-Looking Consumers.

becomes suddenly stronger, a firm should sudden lower the price in order to attract a lot of consumer. For this reason we observe jumps downwards when when rival's price reaches a certain level. This also entails that the best response of the high-quality firm becomes continuous when the vertical differentiation is stronger than horizontal differentiation and ODS to L is not a threat (last three plots of Figure 5).[20]

Concerning the shape of the best response prices and the relation with the strength of vertical differentiation, the essential ingredients are still there. In particular, Figure 5 shows how firms trade-off between first-period market share and second-period switching by being essentially aggressive in pricing when the rival is benevolent and benevolent when the rival is aggressive. An inter-temporally balanced TDS is instead chosen when the rival price is intermediate.

Similarly to the case of myopic consumers, the best response is less steep in the

[20]As a matter of fact, the continuity of firm H optimal price together with the fact that both best responses are always upward sloping is what technically determines the non-emergence of multiple equilibria, as it can never happen that the two functions cross each other more than once.

latter intermediate case than in the former extreme ones. The only difference is that, no matter the discount factor, prices are always strategic complement. This difference is due to the fact that, since consumers correctly anticipate tomorrow's switching, any price cut today would bring to less substantial movements of consumers in the second-period compared to the myopic-consumer case. For this reason, even when the future is very important, a tentative undercut of the rival does never push a firm to increase the price today focusing on the attraction of consumers in the second-period, because the change in the size of this set of consumers is not as important as in the myopic consumers case. The equilibria are summarised in the following proposition.

Proposition 3. *(Equilibria with Forward-Looking Consumers)*

1. *If* $\Delta < 3 - \frac{20\delta}{9+3\delta} \equiv \bar{\Delta}_{FL}$*, there exists a unique equilibrium in which prices are:*

$$(p^{H*}_{1TDS}, p^{L*}_{1TDS}) = \left(1 + \frac{\Delta}{3} + \frac{\delta}{3} - \frac{(13-9\delta)\delta\Delta}{81-33\delta}, 1 - \frac{\Delta}{3} + \frac{\delta}{3} + \frac{(13-9\delta)\delta\Delta}{81-33\delta}\right),$$

resulting in TDS in the second period.

2. *If* $\Delta > 3 - \min\left\{\frac{20\delta}{9+3\delta}, \frac{20\delta\left(9\left(3\sqrt{(9-2\delta)(9-4\delta)}+227\right)-\delta\left(4\delta+\sqrt{(9-2\delta)(9-4\delta)}+1029\right)\right)}{3(1296-\delta(279-\delta(\delta+138)))}\right\} \equiv \underline{\Delta}_{FL}$*, there exists a unique equilibrium in which prices are:*

$$(p^{H*}_{1H}, p^{L*}_{1H}) = \left(1 + \frac{\Delta}{3} + \frac{4\delta(12-9\delta-(5-\delta)\Delta)}{3(27-\delta)}, 1 - \frac{\Delta}{3} + \frac{\delta((\delta+29)\Delta-21(\delta+1))}{3(27-\delta)}\right),$$

resulting in ODS to firm H in the second period.

3. *For* $\delta > 0.6871$*, no equilibrium exists in the interval* $[\bar{\Delta}_{FL}, \underline{\Delta}_{FL}]$.

Similarly to the myopic case, when vertical differentiation is substantial, the inter-temporally balanced strategy leading to TDS gradually disappears from the best responses of the two firms. Nevertheless, some important differences emerge. First, the region in which equilibria are multiple disappears. In particular, if the discount factor is sufficiently low ODS to H emerges as a unique equilibrium for strong vertical differentiation and TDS for weak vertical differentiation. On the other hand, as in the case of myopia, when the discount factor is high and the vertical differentiation not sufficiently strong, the equilibrium fails to exist.

Secondly, the elasticity of the demand today changes according to the fact consumers anticipate tomorrow switching. This will lead to an asymmetric equilibrium equivalent to the on of the myopic case, with firm L relatively aggressive, firm H relatively benevolent and ODS-to-H in the second period. But since the elasticity of consumers is stronger, firm L is much more aggressive, making the benevolent pricing strategy implemented by the high-quality firm less effective in terms of reduction of price competition. In other words, vertical differentiation in association with BBPD pushes the low-quality firm to be more aggressive today and to let the rival enjoy ODS tomorrow. All these results are formally are expressed in the following corollary.

Corollary 5. *For what concerns equilibrium prices, the following holds:*

(a) high-quality firm: $\frac{\partial p_{1TDS}^{H*}}{\partial \Delta} > \frac{\partial p_{1H}^{H*}}{\partial \Delta} > 0$ *and* $p_{1H}^{H*} < p_u^H$;

(b) low-quality firm: $\frac{\partial p_{1TDS}^{L*}}{\partial \Delta} < \frac{\partial p_{1H}^{L*}}{\partial \Delta} < 0$ *and* $p_{1H}^{L*} > p_u^L$.

Similarly to the case of myopic consumers, Corollary 5 shows that the equilibrium price set by the high-quality firm (respectively by the low-quality firm) is affected positively (negatively) by the difference in quality both in the TDS and in the ODS-to-H scenario. As in the case of myopic consumers, the negative effect on weak firm's price is mitigated passing form TDS to ODS. Things change instead dramatically for the high-quality firm, for which the positive effect of Δ on the equilibrium price is attenuated going from the TDS to the the ODS-to-H equilibrium. This is due to the fact that the implementation of the strategy of benevolent pricing makes the consumer more elastic to price changes compared to the inter-temporally balanced pricing entailing TDS. Clearly, since the asymmetric inter-temporal strategy is anticipated by consumers, the high-quality firm enjoys less power than in the case of myopic consumers. Another proof of this reduction of power is that the first-period price of the high-quality firm is lower than the uniform price.

Concerning the exit of the low-quality firm, results are very similar to the myopic case, as shown in the following corollary.

Corollary 6. *(Market Shares, Switching & Exit)*

1. *When* $(p_{1TDS}^{H*}, p_{1TDS}^{L*})$ *are the equilibrium prices, then* $\hat{x}_1 = \frac{1}{2} + \frac{(9-7\delta)\Delta}{27-11\delta}$, $\hat{x}_2^H = \frac{1}{3} + \frac{3(2-\delta)\Delta}{27-11\delta}$ *and* $\hat{x}_2^L = \frac{2}{3} + \frac{3(2-\delta)\Delta}{27-11\delta}$. *In the second period* $\hat{x}_1 - \hat{x}_2^H = \frac{1}{6} - \frac{(\delta+3)\Delta}{54-22\delta}$ *consumers switch from H to L and* $\hat{x}_2^L - \hat{x}_1 = \frac{1}{6} + \frac{(\delta+3)\Delta}{54-22\delta}$ *move to the opposite direction.*

2. *When* $(p_{1H}^{H*}, p_{1H}^{L*})$ *are the equilibrium prices, then* $\hat{x}_1 = \hat{x}_2^H = \frac{1}{2} + \frac{\delta(\Delta-14)+9\Delta}{2(27-\delta)}$*, and* $\hat{x}_2^L = \frac{1}{2} + \frac{12\Delta+9-5\delta}{2(27-\delta)}$*. In the second period,* $\min\left\{\hat{x}_2^L - \hat{x}_1, 1-\hat{x}_1\right\}$ *consumers switch from L to A.*

3. *If* $\Delta > \max\{\frac{2\delta+9}{6}, \underline{\Delta}_{FL}\}$*, then ODS to H determines the exit of the low quality firm from the market.*

Proof. The first two results are found by plugging the first-period equilibrium prices into the cutoffs expressed in equations (4), (5), (7) and (8). For the result in point 3., take the $\hat{x}_2^L = \frac{3(2(\Delta+3)-\delta)}{27-\delta}$ resulting from $(p_{1H}^{H*}, p_{1H}^{L*})$. It holds that: $\frac{3(2(\Delta+3)-\delta)}{27-\delta} > 1 \Leftrightarrow \Delta > \frac{2\delta+9}{6}$. Since ODS-to-H equilibrium exists only if $\Delta > \underline{\Delta}_{FL}$, the result of the corollary is proved. ■

5 Welfare Analysis

The current section presents the effects of BBPD on both firms' and consumers' welfare. In order to provide this analysis, profits and consumer surplus resulting under customer's recognition are compared with the benchmark case of no BBPD or no customer's recognition, which serves to isolate the effect of price discrimination.

5.1 Myopic Consumers

Firms' Profits. First, let us consider the effects of BBPD on firms' profits. According to the results shown in Proposition (2), firms will enjoy different profits at equilibrium according to the difference in quality and the occurrence of TDS,ODS and exit of the low-quality seller. Leaving all technical details to the appendix, the comparison of profits attained under BBPD with the ones resulting in the benchmark case of Section 3, it is very easy to verify the following proposition.

Proposition 4. *If consumers are myopic, price discrimination according to past purchase behaviour will be:*

(i) detrimental for both firms if TDS occurs and $\Delta \in [0, \min\{\bar{\Delta}, \tilde{\Delta}\})$*,*

(ii) beneficial for the low-quality firm and detrimental for the high-quality firm if $\bar{\Delta} > \tilde{\Delta}$ *and* $\Delta \in [\tilde{\Delta}, \bar{\Delta}]$

(iii) beneficial for both firms if ODS occurs and $\delta < 0.98$.

$\tilde{\Delta} \equiv \frac{60\delta - 81\sqrt{(27-20\delta)^2(16\delta(20\delta-61)+765)}}{2(4\delta(20\delta-61)+189)}$ *represents the minimal level of asymmetry needed for the low-quality seller to be better off in the TDS scenario.*

Proof. See Appendix. ■

Point *(i)* tells that low levels of vertical differentiation yield the firms-damaging scenario shown in the traditional literature of BBPD. In particular, assuming Δ to be zero replicates the results of Fudenberg and Tirole (2000), demonstrating de facto that their assumption of symmetry is not needed, as firms reach the symmetric equilibrium endogenously.

Things change radically when firms are assumed to be sufficiently asymmetric. In particular, as vertical differentiation becomes stronger, the high-quality firm is given the choice to decide the destiny of the low-quality competitor. The equilibrium price configuration sees the high-quality firm implementing a fat cat strategy, consisting in being inoffensive today in order to induce a favourable response of the rival. This scenario will be always profit-enhancing compared to the uniform pricing case because reduces price competition in the first period and yields the exit (or, at least, the cornering) of the low quality opponent. Therefore, the high-quality firm enjoys high margins on a small market in the first period and conquers the entire second-period market without excessive effort in terms of prices.

This strategy gives the lower-quality rival the opportunity to obtain a large first-period market share, without the need to charge an extremely low price. This is clearly beneficial in the early competition, but it becomes harmful in the second period. Indeed, the high-quality firm steals a lot of consumers and the low-quality firm only loses market share. Moreover, this ODS result leads in mostly of the cases to the exit. The balance between these two opposite effects is always positive for the low quality seller, no matter if it survives or exits the market. At a first sight, the fact that firm L is happier out of the market can be counterintuitive. Nevertheless, when the difference in quality is very pronounced, a situation without customer's recognition is not so appealing for this firm, which would conquer a niche of the market in any case. Therefore, the ODS scenario gives to the low-quality firm the possibility to get a level of profits that would not be reached in the benchmark case, not even in two periods.

Consumer Surplus. This paragraph provides the analysis on the effects of BBPD on consumer surplus. In what follows, $U^{ij}(x)$ refers to the inter-temporal utility of a consumer located at x who buys good j in the first period and good i in the second one, with possibly $i \neq j$ in case of switching in the second period.[21] This utility will be equal to:

$$U^{ij}(x) = q^i + q^j - p_1^i - p_2^{ij} - |x - l^i| - |x - l^j| \text{ where } i, j \in \{H, L\}.$$

Indeed, the consumer above enjoys quality i product in the first period and quality j in the second one, pays the respective prices and bears the respective transportation cost to reach the supplier location. If $i = j$, the consumer above is loyal to firm j in both periods so that the total transportation cost becomes $2|x - l^j|$, l^j being the location of firm j. Oppositely, when $i \neq j$ the consumer switches from firm j to firm i in the second period and the transportation cost is faced in the whole segment. The total consumer surplus is thus given by the sum of utilities of all consumers:

$$CS = \int_0^{l^H} U^{HH}(x)dx + \int_{l^L}^{1} U^{LL}(x)dx + \int_{s^L}^{l^L} U^{HL}(x)dx + \int_{l^H}^{s^H} U^{LH}(x)dx,$$

where $l^H = \min\{\hat{x}_1, \hat{x}_2^H\}$, $l^L = \max\{\hat{x}_1, \hat{x}_2^L\}$, $s^L = \min\{\hat{x}_1, \hat{x}_2^L\}$ and $s^H = \max\{\hat{x}_1, \hat{x}_2^H\}$. The first two terms in the sum represents the surplus for loyalist whereas the second ones are taken by switchers. Comparing the consumer surplus under BBPD with the one obtained under uniform pricing, we find the net effect of BBPD on surplus, which is presented in the following proposition

Proposition 5. *If consumers are myopic, price discrimination according to past purchase behaviour:*

(i) will increase consumer surplus if TDS happens,

(ii) will decrease consumer surplus if ODS happens.

Proof. See Appendix. ■

The intuition behind this very sharp result is nothing but the other side of the coin of the discussion made for firms' profits. The more symmetric firms are, the higher the odds that BBPD brings to an intensification of competition benefiting consumers in terms of lower prices. When instead vertical differentiation becomes strong, consumer surplus is gradually eroded due to the mitigated first-period price competition and to the exit of the low-quality firm.

[21] When consumers are myopic, their discount factor in the first period is implicitly assumed to be 0. Thus the consumer's utility is computed as a non-weighted sum of per-period utilities.

5.2 Forward-Looking Consumers

Again, in order to look at the effects of BBPD on firms' profits, we compare BBPD profits with the ones resulting in the benchmark case of Section 3 The results are contained in the following proposition.

Proposition 6. *If consumers are forward-looking, price discrimination according to past purchase behaviour will be:*

(i) detrimental for both firms if TDS occurs and $\Delta \in [0, \min\{\bar{\Delta}_{FL}, \tilde{\Delta}_{FL}\})$*,*

(ii) beneficial for the low-quality firm and detrimental for the high-quality firm if ODS-to-H occurs or if TDS occurs and $\Delta \in [\tilde{\Delta}_{FL}, \bar{\Delta}_{FL}]$*.*

$\tilde{\Delta}_{FL} \equiv 3 - \frac{(27-11\delta)\left(2\sqrt{117-\delta(37-8\delta)}-3(7-\delta)\right)}{27-\delta(23\delta-22)}$ *represents the minimal level of asymmetry needed for the low-quality seller to be better off in the TDS scenario.*

Proof. See Appendix. ■

Proposition 6 highlights how the high-quality firm suffers the BBPD equilibrium when this leads to ODS-to-H. In particular, the same mechanism that gives a very powerful tool in the hands high-quality firm when consumers are myopic turns out to be a condemn when consumers are forward-looking. Here, the low-quality firm sets a very low price in the first period and the high-quality firm is "forced" to postpone the attack to tomorrow. Oppositely,bwhen consumers are myopic the high-quality firm sets a high price in the first period enjoying ODS tomorrow and the rival "enjoys" the reduced price competition in the first period. This strategy gives the lower-quality rival the opportunity to obtain a large first-period market share and this is always positive and more than compensate the lost of market share suffered in the second period.

On the other hand, the loss suffered by the high-quality firm is captured not only by the low-quality firm but also by the consumers.

Proposition 7. *If consumers are forward-looking consumers, price discrimination according to past purchase behaviour will always increase consumer surplus.*

Proof. See Appendix. ■

Indeed, in aggregate terms the enjoy a higher surplus than under the uniform pricing due to the fact that most of them pay a relative low price in the first period to the low-quality firm and then switch at a relative low price to the high-quality supplier.

6 Conclusion

Despite the issues in terms of privacy created by the access of firm to consumer specific information, BBPD literature has been in favour of consumers recognition due to the fact that consumers benefit from it in terms of lower prices and increased competition. In particular, the main message of Fudenberg and Tirole (2000) is the same sent by the traditional price discrimination literature in oligopolistic markets: once firms can discriminate prices, they suffer a more intense competition, leading to lower prices and a positive effect for consumer surplus.

This paper participates to the debate arguing that the result above does not necessarily hold anymore if firms sell good of different qualities. As expected, the model gives the same results as Fudenberg and Tirole (2000) when firms are not (enough) vertically differentiated. However, if firms are different enough regarding the quality of the good sold then asymmetric pricing behaviours arise. This will be the case when vertical differentiation is sufficiently strong.

Specifically, the high-quality firm optimally chooses to set a high price in the first period serving a small number of consumers attaining high per-consumer margins and then to compete fiercely in the second period attracting a lot of new and keeping all previous consumers. This outcome turns out to be ex-post preferred to uniform pricing as the renounce in terms of first-period demand exceeds the gain in margin at that time and the ODS in the second period. Moreover, the high-quality firm becomes a monopolist or , at least, conquers a market share very close to one in the thanks to BBPD.

The low-quality seller responds to this strategy in a yielding way, as it simply takes what is left by the high quality competitor. Nonetheless, this accommodating behaviour turns out to be positive, as in the early competition low-quality firm can enjoy a large part of demand charging a price relatively high compared to uniform pricing. The positive effect is not neutralised even when this asymmetric scenario leads to the exit of the low-quality firm from the market. In any case, the possible exit of a firm caused by price discrimination suggests the natural policy implication that an authority which main concern is to preserve competition should block BBPD as it reduces the number of firms active in the market.

All these considerations give peculiar effects on the consumer surplus, which is clearly enhanced by BBPD only under low enough vertical differentiation. Indeed, when the asymmetric result of ODS to the high-quality firm occurs, BBPD may results in a reduction of consumer surplus compared to the uniform price regime. As a consequence, the second policy implication of the paper is that if consumer surplus is the antitrust authority's criterion, then BBPD should be banned in markets exhibiting strong vertical differentiation.

Appendix

Proof of Proposition 1

Let us analysis second period pricing decisions. Given the cutoffs $\hat{x}_2^H$ and $\hat{x}_2^L$ in equation (4) and (5), firms solve the following problem in A's turf:

$$\max_{p_2^{HH}} p_2^{HH} \hat{x}_2^H = p_2^{HH} \left(\tfrac{1}{2} + \tfrac{\Delta + p_2^{LH} - p_2^{HH}}{2} \right),$$
$$\max_{p_2^{LH}} p_2^{LH} \left(\hat{x}_1 - \hat{x}_2^H \right) = p_2^{LH} \left(\hat{x}_1 - \tfrac{1}{2} - \tfrac{\Delta + p_2^{LH} - p_2^{HH}}{2} \right).$$

Solving the maximization problem, firm H's best response turns out to be:

$$p_2^{HH} = \frac{1 + \Delta + p_2^{LH}}{2},$$

and firm L best response is to set a price

$$p_2^{LH} = \hat{x}_1 + \frac{p_2^{HH} - 1 - \Delta}{2}.$$

which give the following equilibrium prices:

$$p_2^{HH} = \frac{\Delta + 2\hat{x}_1 + 1}{3} \text{ and } p_2^{LH} = \frac{4\hat{x}_1 - 1 - \Delta}{3}.$$

Doing the same in L's turf yields:

$$p_2^{HL} = \frac{\Delta + 3 - 4\hat{x}_1}{3} \text{ and } p_2^{LL} = \frac{3 - 2\hat{x}_1 - \Delta}{3}.$$

Notice that charging a price $p_2^{LH} < 0$ is a dominated strategy for firm L. Therefore,whenever the equilibrium price $p_2^{LH} < 0$ then the best option for this firm is to set $p_2^{LH} = 0$. This will happen when $\frac{4\hat{x}_1 - 1 - \Delta}{3} \leq 0 \Leftrightarrow \hat{x}_1 \leq \frac{\Delta+1}{4}$.

When $\hat{x}_1 \leq \frac{\Delta+1}{4}$ it follows that $p_2^{LH} = 0$ and thus the best response of firm H is to charge the maximal possible price compatible with not to lose the marginal consumer located at $\hat{x}_1$, i.e., a price p_2^{HH} such that $q^H - p_2^{HH} - \hat{x}_1 = q^L - 0 - (1 - \hat{x}_1)$, which gives $p_2^{HH} = \Delta + 1 - 2\hat{x}_1$. This will give equilibrium prices when $\hat{x}_1 \leq \frac{\Delta+1}{4}$:

$$\begin{array}{ccc} p_2^{HH} = \Delta + 1 - 2\hat{x}_1 & \text{and} & p_2^{LH} = 0; \\ p_2^{HL} = \frac{\Delta+(3-4\hat{x}_1)}{3} & \text{and} & p_2^{LL} = \frac{3-2\hat{x}_1-\Delta}{3}. \end{array}$$

In this case switching is one direction towards firm H. Similarly, firm H will set $p_1^{HL} = 0$ when $\frac{\Delta+3-4\hat{x}_1}{3} \leq 0 \Leftrightarrow \hat{x}_1 \geq \frac{\Delta+3}{4}$.

When $\hat{x}_1 \geq \frac{\Delta+3}{4}$ it follows that $p_2^{LH} = 0$ and thus the best response of firm L is to charge the maximal possible price compatible with not to lose the marginal consumer located at $\hat{x}_1$, i.e., a price p_1^{LL} such that $q^H - 0 - \hat{x}_1 = q^L - p_2^{LL} - (1 - \hat{x}_1)$, which gives $p_2^{LL} = 2\hat{x}_1 - 1 - \Delta$. This will give equilibrium prices when $\hat{x}_1 \leq \frac{\Delta+3}{4}$:

$$\begin{array}{ccc} p_2^{HH} = \frac{\Delta+2\hat{x}_1+1}{3} & \text{and} & p_2^{LH} = \frac{4\hat{x}_1-1-\Delta}{3}; \\ p_2^{HL} = 0 & \text{and} & p_2^{LL} = 2\hat{x}_1 - 1 - \Delta. \end{array}$$

Notice that if $\Delta > 1$, this scenario with ODS to firm L cannot be reached unless. Summarizing, ODS to L can be the case only if $\Delta < 1$ or $\hat{x}_1 = 1$.

Construction of the Best Replies.

Firm H best response.

(i) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta+p_1^L-p_1^H}{2} \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, *TDS* occurs and firm *H* enjoys the following second period profits: $\pi_{2TDS}^H = \frac{\Delta^2+5(2\hat{x}_1^2-2\hat{x}_1+1)-2\Delta(\hat{x}_1-2)}{9}$. Accordingly, firm H solves $\max_{p_1^H} \pi_{TDS}^H = \max_{p_1^H} p_1^H \hat{x}_1 + \delta \pi_{2TDS}^H$ under the constraints $\frac{\Delta+t}{4t} < \hat{x}_1 < \frac{\Delta+3t}{4t}$. The first order condition of this problem gives:

$$p_{1TDS}^H = \frac{(9-10\delta)}{18-10\delta} p_1^L + \frac{9}{18-10\delta} + \frac{(9-8\delta)\Delta}{18-10\delta},$$

with correspondent $\hat{x}_1 = \frac{9p_1^L-2\delta(\Delta+5)+9(\Delta+1)}{36-20\delta}$. If $\Delta \leq 1$, then $\hat{x}_1 \in (0,1)$ and constraints are met if $\frac{5\delta-3\delta\Delta}{9} \equiv \hat{p}_{HC} < p_1^L < \hat{p}_{LC} \equiv \frac{18-3\delta\Delta-5\delta}{9}$. The correspondent profit will be:

$$\pi_{TDS}^H = \frac{(9p_1^L-9(\Delta+1))^2-18p_1^L(2\delta(\Delta+5))-4\delta^2(3\Delta+5)^2+36\delta(\Delta(\Delta+2)+5)}{72(9-5\delta)}.$$

We have three more cases to consider.

- When $\Delta > 1$, the constraint $\hat{x}_1 < \frac{\Delta+3}{4}$ is non-binding. In this case, whenever p_1^L is such that $\hat{x}_1(p_{TDS}^H, p_1^L) \geq 1$, i.e. $p_1^L \geq \hat{p}_M \equiv \frac{2\delta\Delta-10\delta-9\Delta+27}{9}$, TDS cannot occur tomorrow and firm H becomes a monopolist setting the price p_1^H such that $\hat{x} = 1$ or $p_{1M}^H = \Delta + p_1^L - 1$ and resulting profit of $\pi_M^H = \frac{1}{18}\delta(\Delta+3)^2 + \Delta + p_1^L - 1$, where the subscript M stays for monopoly of firm H.
- If $\Delta \leq 1$, the constraint $\hat{x}_1 < \frac{\Delta+3}{4}$ turns out to be binding when $p_1^L \geq \hat{p}_{LC}$, p_1^H is such that $\hat{x}_1 = \frac{\Delta+3}{4}$, or $p_{1LC}^H = \frac{\Delta+2p_1^L-1}{2}$, leading to ODS to L. The profit overall will be:

$$\pi_{LC}^H = \frac{1}{72}\left(18(\Delta+3)p_1^L + \delta(3\Delta+5)^2 + 9(\Delta-1)(\Delta+3)\right).$$

- Finally, if $p_1^L \leq \hat{p}_{HC}$, then $\hat{x}_1 \leq \frac{\Delta+1}{4}$. It means that p_1^H is such that the constraint is binding, i.e., $p_{1HC}^H = \frac{\Delta+2p_1^L+1}{2}$, , leading to ODS to L. The correspondent profit will be:

$$\pi_{HC}^H = \frac{1}{72}\left(\delta(9\Delta(\Delta+2)+25) + 9(\Delta+1)^2 + 18(\Delta+1)p_1^L\right).$$

(ii) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta+p_1^L-p_1^H}{2} \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm H, which receives profit $\pi_{2H}^H = \frac{\Delta^2+(9-2\hat{x}_1(10\hat{x}_1+3))+2\Delta(5\hat{x}_1+3)}{18}$. The maximisation problem will be the following $\max_{p_1^H} \pi_H^H = \max_{p_1^H} p_1^H\hat{x}_1 + \delta\pi_{2H}^H$ under the constraint $\hat{x}_1 \leq \frac{\Delta+1}{4}$. The first order condition of this problem gives:

$$p_{1H}^H = \frac{(10\delta+9)}{18+10\delta}p_1^L + \frac{9+13\delta}{18+10\delta} + \frac{\Delta}{2},$$

with correspondent $\hat{x}_1 = \frac{9p_1^L+5\delta\Delta-3\delta+9\Delta+9}{20\delta+36}$. Constraint is met and $\hat{x}_1 \in (0,1)$ if $0 < p_1^L < \hat{p}_H \equiv 8\delta/9$. The correspondent profit will be:

$$\pi_H^H = \frac{1}{8}\left(\frac{6\delta\left(5-p_1^L\right)+9\left(p_1^L+1\right)^2+21\delta^2}{5\delta+9} + 2\Delta\left(p_1^L+\delta+1\right) + (\delta+1)\Delta^2\right).$$

If $p_1^L < \hat{p}_H$, $\hat{x}_1 \geq \frac{\Delta+1}{4}$ and thus firm H sets a price such that $\hat{x}_1 = \frac{\Delta+1}{4}$, i.e., p_{1HC}^H. Moreover, since $\pi_{2H}^H(\hat{x}_1 = \frac{\Delta+1}{4}) = \pi_{2TDS}^H(\hat{x}_1 = \frac{\Delta+1}{4})$, this profit turns out to be π_{HC}.

(iii) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta + p_1^L - p_1^H}{2} \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm *L*. This case can exist only if $\Delta \leq t$ or, when $\Delta > t$, if $\hat{x}_1 = 1$. ODS to L would give firm *H* a second period profit of $\pi_{2L}^H = \frac{(\Delta + 2t\hat{x}_1 + 1)^2}{18t}$.

- If $\Delta \leq 1$, then firm H solves $\max_{p_1^H} \pi_L^H = \max_{p_1^H} p_1^H \hat{x}_1 + \delta \pi_{2L}^H$ under the constraint $\hat{x}_1 \geq \frac{\Delta+3}{4}$. The first order condition of this problem gives:
$$p_{1L}^H = \frac{(9-2\delta)}{18-2\delta} p_1^L + \frac{9-4\delta}{18-2\delta} + \frac{(9-4\delta)\Delta}{18-2\delta},$$
with correspondent $\hat{x}_1 = \frac{9p_1^L + (2\delta+9)(\Delta+1)}{4(9-\delta)}$. The constraint is met if $p_1^L \geq \frac{18-3\delta\Delta-5\delta}{9}$, for other prices ODL cannot occur. The resulting profit will be:
$$\pi_L^H = \frac{(8\delta+9)(\Delta+1)^2 + 9(p_1^L)^2 + 2(2\delta+9)(\Delta+1)p_1^L}{8(9-\delta)}.$$
If $p_1^L \leq \hat{p}_{LC}$, $\hat{x}_1 \leq \frac{\Delta+3}{4}$ and thus firm *H* sets a price such that $\hat{x}_1 = \frac{\Delta+1}{4}$, i.e., p_{1LC}^L. Moreover, since $\pi_{2L}^H(\hat{x}_1 = \frac{\Delta+3}{4}) = \pi_{2TDS}^H(\hat{x}_1 = \frac{\Delta+3}{4})$, this profit turns out to be π_{LC}^H. The case in which $\hat{x}_1 = 1$ has been already discussed in the TDS case.

Up to now, we obtained all possible best responses of firm *H* within each regime. In order to build up the global best response, we must compare profits across regimes in each segment. We have 3 possible cases:

1. If $\Delta < 1$, then we have the following segments:
   (a) $p_1^L \leq \hat{p}_{HC}$. $\implies$ best response p_{1H}^H.
   (b) $p_1^L \in (\hat{p}_{HC}, \hat{p}_H)$ If $p_1^L > \frac{\sqrt{(9\Delta+9)^2 - (5\delta\Delta+5\delta)^2}}{30} + \frac{65\delta - 15\delta\Delta}{90} - \frac{3\Delta-3}{10} \equiv \hat{p}$, then $\pi_{TDS}^H > \pi_H^H$. Otherwise, $\pi_H^H > \pi_{TDS}^H$.
   (c) $p_1^L \in [\hat{p}_H, \hat{p}_{LC})$. $\implies$ best response p_{1TDS}^H.
   (d) $p_1^L \geq \hat{p}_{LC}$. $\implies$ best response p_{1L}^H.
2. $\Delta \in [1, 3 - \frac{12\delta}{9-2\delta}]$, then $\hat{p}_H < \hat{p}_M$. In segments (a) and (b) nothing changes. Above $\hat{p}_H$ we have:
   (c.i) $p_1^L \in [\hat{p}_H, \hat{p}_M]$. $\implies$ best response p_{1TDS}^H.

(d.i) $p_1^L \geq \hat{p}_M$. $\implies$ best response p_{1M}^H.

3. If $\Delta > 3 - \frac{12\delta}{9-2\delta}$ then $\hat{p}_M > \hat{p}_H$In segments (a) nothing changes compared to point 1. Above $\hat{p}_{HC}$ we have:

(b.ii) $p_1^L \in (\hat{p}_{HC}, \hat{p}_M)$. If $\Delta < 3 - \frac{12\left(\sqrt{81-25\delta^2}-(9-2\delta)\right)}{36-29\delta} \equiv \hat{\Delta}$, then $\pi_{TDS}^H > \pi_H^H$ for $p_1^L > \hat{p} \implies$ best response p_{1TDS}^H. Otherwise, the best response is always p_{1H}^H.

(c.ii) $p_1^L \in [\hat{p}_M, \hat{p}_H]$. $\pi_H^H > \pi_M^H \implies$ best response p_{1H}^H.

(d.ii) $p_1^L \geq \hat{p}_H$. $\pi_M^H > \pi_{HC}^H \implies$ best response p_{1M}^H.

Putting together all the results above, the best response will depend on the size of Δ. Indeed, the best response of firm H will be the following:

$$p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}, \\ p_{1TDS}^H & \text{if } p_1^L \in (\hat{p}, \hat{p}_{LC}), \\ p_{1L}^H & \text{if } p_1^L \geq \hat{p}_{LC}, \end{cases} \quad p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}, \\ p_{1TDS}^H & \text{if } p_1^L \in [\hat{p}, \hat{p}_M], \\ p_{1M}^H & \text{if } p_1^L > \hat{p}_M, \end{cases}$$

when $\Delta < 1$, when $\Delta \in [1, 3 - \frac{12\delta}{9-2\delta}]$,

$$p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}, \\ p_{1TDS}^H & \text{if } p_1^L \in [\hat{p}, \hat{p}_M], \\ p_{1H}^H & \text{if } p_1^L \in [\hat{p}_M, \hat{p}_H], \\ p_{1M}^H & \text{if } p_1^L > \hat{p}_H, \end{cases} \quad p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}_H, \\ p_{1M}^H & \text{if } p_1^L > \hat{p}_H, \end{cases}$$

when $\Delta \in [3 - \frac{12\delta}{9-2\delta}, \hat{\Delta}]$, when $\Delta > \hat{\Delta}$.

Notice that $3 - \frac{12\delta}{9-2\delta} < \hat{\Delta}$ for any discount factor.

Firm L best response.

(i) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta + p_1^L - p_1^H}{2} \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, *TDS* occurs and firm L enjoys a second period profit of $\pi_{2TDS}^L = \frac{\Delta^2 + 5(2\hat{x}_1^2 - 2\hat{x}_1 + 1) - 2\Delta(\hat{x}_1+1)}{9}$. Accordingly, firm L solves $\max_{p_1^L} \pi_{TDS}^L = \max_{p_1^L} p_1^L(1-\hat{x}_1) + \delta\pi_{2TDS}^L$ under the constraints $\frac{\Delta+1}{4} < \hat{x}_1 < \frac{\Delta+3}{4}$. The first order condition of this problem gives:

$$p_{1TDS}^L = \frac{(9-10\delta)}{18-10\delta} p_1^H + \frac{9}{18-10\delta} - \frac{(9-8\delta)\Delta}{18-10\delta},$$

with resulting $\hat{x}_1 = \frac{9(\Delta+3)-9p_1^H-2\delta(\Delta+5)}{4(9-5\delta)}$. If $\Delta \leq 1$, then constraints are met if $\frac{3\delta\Delta+5\delta}{9} \equiv \tilde{p}_{LC} < p_1^H < \tilde{p}_{HC} \equiv \frac{3\delta\Delta-5\delta+18}{9}$. The correspondent profit will be:

$$\pi_{TDS}^L = \frac{(9p_1^H-9(\Delta-1))^2+18p_1^H(2\delta(\Delta-5)+4\delta^2(5-3\Delta)^2+36\delta((\Delta-2)\Delta-5)}{72(9-5\delta)}.$$

We have three more cases to consider.

- If $\Delta > 1$ the constraint $\hat{x}_1 < \frac{\Delta+3}{4}$ is non-binding. Whenever $p_1^H < \tilde{p}_M$, $\hat{x}_1 \geq 1$ firm L cannot enter the market.
- If $p_1^H \geq \tilde{p}_{HC}$, then first constraint is not satisfied and thus p_1^L will be such that $\hat{x}_1 = \frac{\Delta+1}{4}$ or equivalently $p_{1HC}^L = \frac{2p_1^H-1-\Delta}{2}$. In this case the profit will be:

$$\pi_{HC}^L = \frac{1}{72}\left(\delta(5-3\Delta)^2 + 9(\Delta-3)(\Delta+1) - 18(\Delta-3)p_1^H\right).$$

- When $\Delta \leq 1$ and $p_1^H \leq \tilde{p}_{LC}$, then the second constraint is not satisfied and thus p_1^L will be such that $\hat{x}_1 = \frac{\Delta+3}{4}$ or equivalently $p_{1LC}^L = \frac{2p_1^H+1-\Delta}{2}$. The correspondent profit will be:

$$\pi_{LC}^L = \frac{1}{72}\left(9(\delta+1)\Delta^2 + 25\delta - 18\Delta(\delta+p_1^H+1) + 18p_1^H + 9\right).$$

(ii) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta+p_1^L-p_1^H}{2} \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm H and firm L gets $\pi_{2H}^L = \frac{(\Delta+(2\hat{x}_1-3))^2}{18}$. The maximisation problem will be $\max_{p_1^L} \pi_H^L = \max_{p_1^L} p_1^L(1-\hat{x}_1) + \delta\pi_{2H}^L$ under the constraint $\hat{x}_1 \leq \frac{\Delta+1}{4}$. The first order condition of this problem gives:

$$p_{1H}^L = \frac{(9-2\delta)}{18-2\delta}p_1^H + \frac{9-4\delta}{18-2\delta} - \frac{(9-4\delta)\Delta}{18-2\delta},$$

with correspondent $\hat{x}_1 = \frac{2\delta\Delta-6\delta+9\Delta-9p_1^H+27}{36-4\delta}$. Constraint is met if $p_1^H \geq \hat{p}_{HC}$ and the correspondent profit will be:

$$\pi_H^L = \frac{(8\delta+9)(\Delta-1)^2 + 9(p_1^H)^2 - 2(2\delta+9)(\Delta-1)p_1^H}{8(9-\delta)}.$$

If $p_1^H \leq \hat{p}_{HC}$, then $\hat{x}_1 \geq \frac{\Delta+1}{4}$ and thus firm L sets a price such that $\hat{x}_1 = \frac{\Delta+1}{4}$, i.e., p_{1HC}^H. Moreover, since $\pi_{2H}^L(\hat{x}_1 = \frac{\Delta+1}{4}) = \pi_{2TDS}^L(\hat{x}_1 = \frac{\Delta+1}{4})$, the profit will be π_{HC}^L.

(iii) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta + p_1^L - p_1^H}{2} \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm L. This case can exist only if $\Delta < 1$ or, when $\Delta > 1$, if $\hat{x}_1 = 1$. ODS to L would give firm L a second period profit of $\pi_{2L}^L = \frac{\Delta^2 + \left(46\hat{x}_1 - 20\hat{x}_1^2 - 17\right) + 2\Delta(5\hat{x}_1 - 8)}{18}$.

If $\Delta < 1$, then firm L maximizes $\max_{p_1^L} \pi_L^L = \max_{p_1^L} p_1^L(1 - \hat{x}_1) + \delta \pi_{2L}^L$ under the constraint $\hat{x}_1 \geq \frac{\Delta+3}{4}$. The first order condition of this problem gives:

$$p_{1L}^L = \frac{(10\delta + 9)}{18 + 10\delta} p_1^H + \frac{9 + 13\delta}{18 + 10\delta} - \frac{\Delta}{2},$$

with correspondent $\hat{x}_1 = \frac{27 - 9p_1^H + 5\delta\Delta + 23\delta + 9\Delta}{20\delta + 36}$. Constraint is met if $p_1^H \leq \tilde{p}_L \equiv \frac{8\delta}{9}$ and the correspondent profit will be:

$$\pi_L^L = \frac{1}{8}\left(\frac{21\delta^2 + 6\delta(5 - p_1^H) + 9(p_1^H + 1)^2}{5\delta + 9} + (\delta + 1)\Delta^2 - 2\Delta(\delta + p_1^H + 1)\right).$$

If the constraint is not satisfied (i.e., $p_1^H \geq \tilde{p}_L$), then we are back to the case with price p_{1LC}^L and profit π_{LC}^L. If instead $\hat{x}_1 = 1$, firm L entry is prevented.

Up to now, we obtained all possible best responses of firm L within each regime. In order to build up the global best response, we must compare profits across regimes in each segment. We have two cases:

1. If $\Delta \leq 1$, then $\tilde{p}_L \geq \tilde{p}_{LC}$. We will have four segments:
   (a) $p_1^H < \tilde{p}_{LC}$ $\pi_L^L > \pi_L^L \implies p_{1L}^L$
   (b) $p_1^H \in (\tilde{p}_{LC}, \tilde{p}_L)$. If $p_1^H < \frac{\sqrt{-25\delta^2\Delta^2 + 50\delta^2\Delta - 25\delta^2 + 81\Delta^2 - 162\Delta + 81}}{30} + \frac{15\delta\Delta + 65\delta + 27\Delta - 27}{90} \equiv \tilde{p}$, then $\pi_L^L > \pi_{TDS}^L$. When $p_1^H > \hat{p} \implies \pi_{TDS}^L > \pi_{1L}^L$.
   (c) $p_1^H \in (\tilde{p}_L, \tilde{p}_{HC})$. $\pi_{TDS}^L > \pi_L^L \implies p_{1TDS}^L$.
   (d) $p_1^H \geq \tilde{p}_{HC}$. $\pi_H^L > \pi_{HC}^L \implies p_{1H}^L$.
2. $\Delta \geq 1$, then in the last segment nothing changes compared to the case with $\Delta < 1$. For $p_1^H \leq \tilde{p}_{HC}$
   (a.i) $p_1^H < \tilde{p}_M$. Firm L is out of the market.
   (b.i) $p_1^H \in (\tilde{p}_M, \tilde{p}_{HC})$. $\pi_{TDS}^L > \pi_L^L \implies p_{1TDS}^L$.

Putting together all the results above, the best response of firm L is

$$p_1^L(p_1^H) = \begin{cases} p_{1L}^L & \text{if } p_1^H \leq \tilde{p}, \\ p_{1TDS}^L & \text{if } p_1^H \in [\hat{p}, \tilde{p}_{HC}], \\ p_{1H}^L & \text{if } p_1^H > \tilde{p}_{HC}, \end{cases} \quad \Delta < 1, \qquad p_1^L(p_1^H) = \begin{cases} p_{1TDS}^L & \text{if } p_1^H \in (\tilde{p}_M, \tilde{p}_{HC}), \\ p_{1H}^L & \text{if } p_1^H > \tilde{p}_{HC}, \end{cases} \quad \Delta \geq 1.$$

Proof of Proposition 2

Existence and uniqueness of the equilibria. We have three cases.

- **TDS scenario**. The only couple of prices generating this scenario is $\boldsymbol{(p_{1TDS}^{H*}, p_{1TDS}^{L*}) = \left(1 + \frac{\Delta}{3} - \frac{4\delta\Delta}{81-60\delta}, 1 - \frac{\Delta}{3} + \frac{4\delta\Delta}{81-60\delta}\right)}$. **This is an equilibrium whenever $\boldsymbol{\Delta < 3 - \frac{8\delta}{9-4\delta} \equiv \bar{\Delta}}$.** If $\Delta \geq \bar{\Delta}$, then the market splitting cut-off will be located above 1, so that TDS cannot be the case in the second period.

- **ODS to H scenario**. The only couple of prices is:

  $$(p_{1H}^{H*}, p_{1H}^{L*}) = \left(1 + \frac{\Delta}{3} + \frac{2\delta(22+11(1-\delta)+\Delta(1+5\delta)}{24\delta+81}, 1 - \frac{\Delta}{3} + \frac{\delta(15(1-\delta)+11\Delta+(10\Delta-7)\delta)}{24\delta+81}\right).$$

  From this situation, firm L would never deviate provided that $\Delta > 3 - \frac{8\delta}{9-6\delta}$ since $p_{1H}^{H*} > \tilde{p}_{HC}$. On the other hand, firm H does not deviate (i.e., $p_{1H}^L \in [\hat{p}_M, \hat{p}_A]$) whenever $\Delta > 3 - \min\left\{\frac{8\delta}{9-6\delta}, \frac{28\delta(2\delta+9)}{\delta(14\delta+27)+162}\right\} \equiv \underline{\Delta}$. Summarizing, **if $\boldsymbol{\Delta \geq \underline{\Delta}}$, this is always an equilibrium**.

- **ODS to L scenario**. Two cases:

  1. When $\Delta < 1$, only one couple of prices can lead to this scenario, i.e., $(p_{1L}^H, p_{1L}^L) = \left(\frac{27(\Delta+3)-\delta(10\delta\Delta+22\delta+3\Delta-39)}{24\delta+81}, \frac{-2\delta(5\delta\Delta+11\delta+3\Delta-45)+27(3-\Delta)}{24\delta+81}\right)$. This cannot be an equilibrium because the best response of firm H to p_{1L}^L is p_{1TDS}^H.
  2. If $\Delta \geq 1$, the only possibility is to have a monopoly of firm H in the first period, choosing price $p_{1M}^H = p_1^L + \Delta - 1$. This strategy is effective (i.e., firm L cannot enter the market) only if $p_1^L + \Delta - 1 < \tilde{p}_M = \frac{10\delta+9\Delta-2\delta\Delta-9}{9} \Leftrightarrow p_1^L < \frac{10\delta-2\delta\Delta}{9}$. Firm H always deviates to ODS when $\Delta > \frac{9}{7}$ because $\frac{10\delta-2\delta\Delta}{9} < \frac{8\delta}{9} = \hat{p}_H$ and when $\Delta < \frac{9}{7}$ because $\frac{10\delta-2\delta\Delta}{9} < \frac{27+2\delta\Delta-10\delta-9\Delta}{9} = \hat{p}_M$.

Case of forward-looking consumers

When consumers are forward-looking, they take into account the possibility of tomorrow's switching. H forward looking consumer buys good i today and potentially switches to firm j enjoying a discount price. This will give him utility $U^i(x) = q^i - p_1^i - |x - l^i| + \delta(q^j - p_2^{ij} - |x - l^j|)$.

Firm H best response.

(i) If $\hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$. Compared to the case of myopia, the rational consumer who is indifferent in period 1 anticipates that if she buys product H in period 1, she will switch to product L in period 2, whereas if she chooses product L in period 1 she will switch to product H in period 2. Thus, the indifferent consumer is located in the $\hat{x}_1$ such that

$$q^H - p_1^H - \hat{x}_1 + \delta\left[q^L - p_2^{LH} - (1 - \hat{x}_1)\right] = q^L - p_1^L - (1 - \hat{x}_1) + \delta(q^H - p_2^{HL} - \hat{x}_1)$$

where p_2^{LH} and p_2^{HL} are the ones in point (ii) of proposition 1. Rewriting:

$$\hat{x}_1 = \frac{1}{2} + \frac{(3-\delta)\Delta + 3(p_1^L - p_1^H) - \delta\Delta}{2\delta + 6}. \tag{9}$$

Following the same notation used in the construction of best replies of the myopic case, we find the following:

Prices

$$p_{1TDS}^H = \frac{(9-7\delta)p_1^L + (\delta(3\delta-8)+9)\Delta + (\delta+3)^2}{18-4\delta}, \quad p_{1M}^H = \Delta + p_1^L - 1 - \frac{\delta(\Delta+1)}{3},$$
$$p_{1LC}^H = \frac{-3\delta\Delta - \delta + 3\Delta + 6p_1^L - 3}{6}, \qquad p_{1HC}^H = \frac{-3\delta\Delta + \delta + 3\Delta + 6p_1^L + 3}{6}.$$

Profits

$$\pi_{TDS}^H = \frac{9\left(3p_1^L - \delta\Delta + \delta + 3\Delta + 3\right)^2 + 12\delta(3\Delta(\delta(\Delta+4)+\Delta+2) + 5(\delta+3) - 3(\Delta+5)p_1^L) - 4\delta^2(3\Delta+5)^2}{72(9-2\delta)},$$

$$\pi_{LC}^H = \frac{18(\Delta+1)p_1^L + 4\delta(3\Delta+7) + 9(\Delta+1)^2}{72}, \quad \pi_{HC}^H = \frac{18(\Delta+3)p_1^L + 16\delta + 9(\Delta-1)(\Delta+3)}{72},$$
$$\pi_M^H = p_1^L + (\Delta - 1) + \frac{(\Delta-1)\Delta\delta + 2\delta}{9}.$$

Cutoffs

$$\hat{p}_{HC} = \frac{\delta(3\Delta+5)}{9}, \quad \hat{p}_{LC} \equiv \frac{3\delta\Delta + \delta + 18}{9}.$$

(ii) If $\hat{x}_1 \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm H. Here, the rational consumer who is indifferent in period 1 anticipates that if she buys product H in period 1, she will buy it again in period 2, whereas if she chooses product L in period 1 she will switch to product H in period 2. Thus, the indifferent consumer is located in the $\hat{x}_1$ such that

$$q^H - p_1^H - \hat{x}_1 + \delta\left[q^H - p_2^{HH} - \hat{x}_1\right] = q^L - p_1^L - (1-\hat{x}_1) + \delta(q^H - p_2^{HL} - \hat{x}_1)$$

where p_2^{HH} and p_2^{HL} are the ones in point (ii) of proposition 1. Rearranging:

$$\hat{x}_1 = \Delta + \frac{3(1 + p_1^L - \Delta - p_1^H)}{2(3-\delta)}.$$

Using the same notation of the myopia case, we will have:

$$p_{1H}^H = \tfrac{(7\delta+9)p_1^L - \delta(3\delta\Delta+\delta+4\Delta-10)+9(\Delta+1)}{18+4\delta}, \quad \hat{p}_H = \tfrac{\delta(3\Delta+5)}{9},$$
$$\pi_H^H = \tfrac{9\left(p_1^L\right)^2 - 2p_1^L(\delta(\Delta+3)-9(\Delta+1))+\delta^2(\Delta+3)^2+2\delta(\Delta+3)(\Delta+5)+9(\Delta+1)^2}{8(9+2\delta)}.$$

(iii) If $\hat{x}_1 \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm L. The rational consumer who is indifferent in period 1 anticipates that if she buys product L in period 1, she will buy it again in period 2, whereas if she chooses product H in period 1 she will switch to product L in period 2. Thus, the indifferent consumer is located in the $\hat{x}_1$ such that

$$q^H - p_1^H - \hat{x}_1 + \delta\left[q^L - p_2^{LH} - (1-\hat{x}_1)\right] = q^L - p_1^L - (1-\hat{x}_1) + \delta\left[q^L - p_2^{LL} - (1-\hat{x}_1)\right],$$

where p_2^{LH} and p_2^{LL} are the ones in point (ii) of Proposition 1. Rearranging:

$$\hat{x}_1 = \Delta + 1 - \frac{3\left(p_1^H - p_1^L + \Delta + 1\right)}{2\left(3-\delta\right)}.$$

Using the same notation of the myopia case, we will have:

$$p_{1L}^H = \tfrac{(9-5\delta)p_1^L - (1-\delta)(9-4\delta)(\Delta+1)}{18-8\delta}, \quad \pi_L^H = \tfrac{2(9-4\delta)(\Delta+1)p_1^L + 9\left(p_1^L\right)^2 + (9-4\delta)(\Delta+1)^2}{8(9-4\delta)},$$
$$\hat{p}_L = 2 - \tfrac{8\delta}{9}.$$

Doing the same analysis done for the myopic consumers' case, we can distinguish four possible cases.

1. If $\Delta < 1$, then we have the following segments:

   (i) $p_1^L \leq \hat{p}_H$. $\pi_H^H > \pi_{LC}^H, \pi_{HC}^H \implies$ best response p_{1H}^H.

   (ii) $p_1^L \in (\hat{p}_H, \hat{p}_L)$. $\pi_{TDS}^H > \pi_{HC}^H, \pi_{LC}^H \implies p_{1TDS}^H$.

   (iii) $p_1^L \in [\hat{p}_L, \hat{p}_{LC}]$. $\pi_{TDS}^H > \pi_L^H$ if

   $$p_1^L < \hat{p} \equiv \tfrac{1}{18}\left(4\delta(3\Delta+5) + 3\left(\sqrt{(2\delta-9)(4\delta-9)(\Delta+3)^2} - 9\Delta - 15\right)\right),$$

   the opposite is true otherwise.

   (iv) $p_1^L > \hat{p}_L$. $\pi_L^H > \pi_{HC}^H, \pi_{LC}^H \implies p_{1L}^H$.

2. If $3 > \Delta > 1$, then the best response remains unchanged in segment (i). For $p_1^L \geq \hat{p}_H$, we have the following segments:

   (ii) $p_1^L \in (\hat{p}_H, \hat{p}_M)$. $\pi_{TDS}^H > \pi_{HC}^H \implies p_{1TDS}^H$.

   (iii) $p_1^L > \hat{p}_M$. $\pi_M^H > \pi_{HC}^H \implies p_{1MH}^H$.

Putting together all the results above, the best response will depend on the size of Δ. Namely, the best response of firm H will be:

$$p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}_H, \\ p_{1TDS}^H & \text{if } p_1^L \in (\hat{p}_H, \hat{p}), \\ p_{1L}^H & \text{if } p_1^L \geq \hat{p}, \end{cases}$$

when $\Delta < 1$,

$$p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}_H, \\ p_{1TDS}^H & \text{if } p_1^L \in [\hat{p}_H, \hat{p}_M], \\ p_{1M}^H & \text{if } p_1^L > \hat{p}_M, \end{cases}$$

when $\Delta \in [1, 3]$.

Firm L best response.

(i) If $\hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$. Doing the same as done for the high-quality firm and using the same notation of the myopic case, we find:

Prices

$$p_{1TDS}^L = \frac{(9-7\delta)p_1^H - (9-\delta(8-3\delta))\Delta + (\delta+3)^2}{18-4\delta}, \quad p_{1M}^L = \Delta + p_1^L - 1 - \frac{\delta(\Delta+1)}{3},$$
$$p_{1LC}^L = \frac{-3\delta\Delta - \delta + 3\Delta + 6p_1^L - 3}{6}, \qquad p_{1HC}^L = \frac{-3\delta\Delta + \delta + 3\Delta + 6p_1^L + 3}{6}.$$

Profits

$$\pi^L_{TDS} = \frac{9\left(3p_1^H+(\delta-3)\Delta+\delta+3\right)^2+12\delta\left(3(\Delta-5)p_1^H+3\Delta(\delta(\Delta-4)+\Delta-2)+5(\delta+3)\right)-4\delta^2(5-3\Delta)^2}{72(9-2\delta)},$$

$$\pi^L_{HC} = \frac{18(3-\Delta)p_1^H+16\delta-9(3-\Delta)(\Delta+1)}{72}, \quad \pi^L_{LC} = \frac{18(1-\Delta)p_1^H+4\delta(7-3\Delta)+9(1-\Delta)^2}{72}.$$

Cutoffs

$$\tilde{p}_{LC} = \frac{\delta(5-3\Delta)}{9}, \quad \tilde{p}_{HC} = \frac{18-3\delta\Delta+\delta}{9}, \quad \tilde{p}_M = \Delta + \frac{\delta(7-5\Delta)}{9} - 1.$$

(ii) If $\hat{x}_1 \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm H. Doing the same as done for the high-quality firm and using the same notation of the myopic case, we find:

$$p^L_{1H} = \frac{(9-5\delta)p_1^H+(1-\delta)(9-4\delta)(1-\Delta)}{18-8\delta}, \quad \tilde{p}_H = 2 - \frac{8\delta}{9}, \quad \pi^L_H = \frac{2(9-4\delta)(1-\Delta)p_1^H+9\left(p_1^H\right)^2+(9-4\delta)(\Delta-1)^2}{8(9-4\delta)}.$$

(iii) If $\hat{x}_1 \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm L. Doing the same as done for the high-quality firm and using the same notation of the myopic case, we find:

$$p^L_{1L} = \frac{(7\delta+9)p_1^H+\delta(\delta(3\Delta-1)+4\Delta+10)-9\Delta+9}{4\delta+18}, \quad \tilde{p}_L = \frac{\delta(5-3\Delta)}{9},$$

$$\pi^L_L = \frac{2p_1^H(\delta(\Delta-3)-9\Delta+9)+9\left(p_1^H\right)^2+\delta^2(\Delta-3)^2+2\delta(\Delta-5)(\Delta-3)+9(\Delta-1)^2}{8(2\delta+9)}.$$

We have two cases:

1. If $\Delta \leq 1$, we will have four segments:
   (a) $p_1^H < \tilde{p}_{LC}$. $\pi^L_L > \pi^L_{HC}, \pi^L_{LC} \implies p^L_{1L}$
   (b) $p_1^H \in (\tilde{p}_{LC}, \tilde{p}_H)$. $\pi^L_{TDS} > \pi^L_{LC}, \pi^L_{HC} \implies p^L_{1TDS}$.
   (c) $p_1^H \in (\tilde{p}_H, \tilde{p}_{HC})$. $\pi^L_{TDS} > \pi^L_H$ if

   $$p_1^H < \tilde{p} \equiv \frac{1}{18}\left(3\left(\sqrt{(2\delta-9)(4\delta-9)(\Delta-3)^2}+9\Delta-15\right)+4\delta(5-3\Delta)\right),$$

   the opposite is true otherwise.
   (d) $p_1^H > \tilde{p}_{HC}$. $\pi^L_H > \pi^L_{HC}, \pi^L_{LC} \implies p^L_{1H}$
2. $\Delta \geq 1$, then

(a) $p_1^H < \tilde{p}_M$. Firm L is out of the market.

(b) $p_1^H \in (\tilde{p}_M, \tilde{p}_H)$. $\pi^L_{TDS} > \pi^L_{HC} \implies p^L_{1TDS}$.

(c) $p_1^H \in (\tilde{p}_H, \tilde{p}_{HC})$. $\pi^L_{TDS} > \pi^L_H$ if $p_1^H < \tilde{p}$.

(d) $p_1^H > \tilde{p}_{HC}$. $\pi^L_H > \pi^L_{HC} \implies p^L_{1H}$.

Putting together all the results above, the best response of firm L is

$$p_1^L(p_1^H) = \begin{cases} p^L_{1L} & \text{if } p_1^H \leq \tilde{p}_{LC}, \\ p^L_{1TDS} & \text{if } p_1^H \in (\tilde{p}_{LC}, \tilde{p}), \\ p^L_{1H} & \text{if } p_1^H \geq \tilde{p}, \end{cases}$$

when $\Delta \leq 1$,

$$p_1^L(p_1^H) = \begin{cases} p^L_{1TDS} & \text{if } p_1^H \in (\tilde{p}_M, \tilde{p}), \\ p^L_{1H} = & \text{if } p_1^H > \tilde{p}, \end{cases}$$

when $\Delta > 1$.

Existence and uniqueness of the equilibria. We have three cases.

- **TDS scenario**. The only couple of prices generating this scenario is $\boldsymbol{(p^{H*}_{1TDS}, p^{L*}_{1TDS}) = \left(1 + \frac{\Delta}{3} + \frac{\delta}{3} - \frac{(13-9\delta)\delta\Delta}{81-33\delta}, 1 - \frac{\Delta}{3} + \frac{\delta}{3} + \frac{(13-9\delta)\delta\Delta}{81-33\delta}\right)}$. When $\Delta < 1$, both firms are on their best responses. If $\Delta \geq 1$, then firm H always deviates if $\Delta > 3 - \frac{20\delta}{9+3\delta} \equiv \bar{\Delta}_{FL}$, since $p^{L*}_{1TDS} < \hat{p}_H$. For what concern firm L, $\Delta > \bar{\Delta}_{FL}$ implies $p^{H*}_{1TDS} \leq \tilde{p}_M$. Summarizing, **This is an equilibrium iff $\boldsymbol{\Delta < \bar{\Delta}_{FL}}$**.

- **ODS to H scenario**. Given the best responses, the candidate equilibrium prices will be $\boldsymbol{(p^{H*}_{1H}, p^{L*}_{1H}) = \left(1 + \frac{\Delta}{3} + \frac{4\delta(12-9\delta-(5-\delta)\Delta)}{3(27-\delta)}, 1 - \frac{\Delta}{3} + \frac{\delta((\delta+29)\Delta-21(\delta+1))}{3(27-\delta)}\right)}$. When $\Delta < 1$, this is not an equilibrium. When $\Delta > 1$, the monotonicity and continuity of firm H best response implies that whenever TDS can be an equilibrium, ODS to H cannot. Therefore, a necessary condition for ODS to H to be an equilibrium is that $\Delta > \bar{\Delta}_{FL}$. From this situation, firm L would never deviate when $\delta \leq \frac{9\left(2\sqrt{103}-19\right)}{17} \approx 0.6871$. Otherwise, it is needed the stricter condition that

$$\Delta > 3 - \frac{20\delta\left(9\left(3\sqrt{(9-2\delta)(9-4\delta)}+227\right)-\delta\left(4\delta+\sqrt{(9-2\delta)(9-4\delta)}+1029\right)\right)}{3(1296-\delta(279-\delta(\delta+138)))},$$

otherwise $p^{H*}_{1H} > \tilde{p}$. On the other hand, firm H does not deviate (i.e., $p^{L*}_{1H} < \hat{p}_H$) whenever $\Delta > \bar{\Delta}_{FL}$. Therefore, **if**

$$\Delta \geq 3 - \min\left\{\frac{20\delta}{9+3\delta}, \frac{20\delta\left(9\left(3\sqrt{(9-2\delta)(9-4\delta)}+227\right)-\delta\left(4\delta+\sqrt{(9-2\delta)(9-4\delta)}+1029\right)\right)}{3(1296-\delta(279-\delta(\delta+138)))}\right\} \equiv \underline{\Delta}_{FL},$$

this is always an equilibrium.

- **ODS to L scenario**. Two cases:

  1. When $\Delta < 1$, only one couple of prices can lead to this scenario, i.e., $(p_{1L}^H, p_{1L}^L) = \left(\frac{27(\Delta+3-\delta^2(\Delta+21)-6\delta(5\Delta+4))}{3(27-\delta)}, \frac{(9-4\delta)(\delta(\Delta+9)-3\Delta+9)}{3(27-\delta)}\right)$. These prices cannot be an equilibrium because $p_{1L}^{H*} < \hat{p}$.
  2. If $\Delta \geq 1$, the only possibility is to have a monopoly of firm H in the first period, choosing price $p_{1MH}^H = p_1^L + \Delta - 1$. This strategy is effective (i.e., firm L cannot enter the market) only if $p_1^L + \Delta - 1 < \frac{-2\delta\Delta+10\delta+9\Delta-9}{9} \Leftrightarrow p_1^L < \frac{10\delta-2\delta\Delta}{9}$. Firm H always deviates when $\Delta > \frac{9}{7}$ because $\frac{10\delta-2\delta\Delta}{9} < \frac{8\delta}{9}$ (the minimal price charged by the rival to find profitable the monopoly case) and when $\Delta < \frac{9}{7}$ because $\frac{10\delta-2\delta\Delta}{9} < \frac{2\delta\Delta-10\delta-9\Delta+27}{9}$.

Proof of Proposition 4

Under BBPD, three different cases may arise:

(a) Exit of the low-quality firm ($\Delta > \frac{11\delta+18}{5\delta+12}$). In this case, firm H and firm L respectively get:

$$\pi_E^H = \frac{\delta^3(\Delta(133\Delta-358)+789)+66\delta^2(\Delta(8\Delta+3)+27)+162\delta(\Delta+3)(4\Delta+5)+243(\Delta+3)^2}{6(8\delta+27)^2},$$

$$\pi_E^L = \frac{\left(81++39\delta-22\delta^2+(2\delta-3)(5\delta+9)\Delta\right)(\delta(25-7\Delta)+9(3-\Delta))}{6(8\delta+27)^2}.$$

It is easy to verify that $\pi_E^H - \pi_u^H$ and $\pi_E^L - \pi_u^L$ are both positive if $\Delta > \frac{11\delta+18}{5\delta+12}$.

(b) ODS with the low-quality firm active. Since $\underline{\Delta} < \frac{11\delta+18}{5\delta+12}$ only if $\delta > 6/7$, this case can exist only if $\delta > 6/7$. Here firm H gets $\pi_H^H = \frac{\delta^3(\Delta(103\Delta-82)+327)+\delta^2\left(501\Delta^2+846\Delta+729\right)+108\delta(\Delta+}{6(8\delta+27)^2}$

which is higher than π_u^H if $\delta <\approx 0.978601$. When the discount factor is very close to 1, it will be higher only if

$$\Delta > \frac{9}{7} - \frac{30(251\delta+540)}{7\delta(245\delta+1007)+7749} + 30\sqrt{\frac{(5\delta+9)(8\delta+27)^2}{(\delta(245\delta+1007)+1107)^2}},$$

lower otherwise. Since this case is very specific, we assume a discount factor reasonably lower than 0.978601.

On the other hand, firm L is always better off under the discriminatory regime, as the profit it gets, i.e.,

$$\pi_H^L = \frac{\delta^3(\Delta+17)(5\Delta-11)+3\delta^2(\Delta(83\Delta-354)+523)+54\delta(\Delta(11\Delta-52)+75)+243(\Delta-3)^2}{6(8\delta+27)^2}$$

is always higher that π_u^L.

(c) TDS. In this case, both firms are strictly worse off under the discriminatory regime. Indeed, the profits they get:

$$\pi_{TDS}^H = \frac{80\delta^3(3\Delta+5)^2-72\delta^2(\Delta(23\Delta+60)+25)+81\delta(\Delta(5\Delta-22)-75)+729(\Delta+3)^2}{18(27-20\delta)^2},$$
$$\pi_{TDS}^L = \frac{80\delta^3(5-3\Delta)^2-72\delta^2(\Delta(23\Delta-60)+25)+81\delta(\Delta(5\Delta+22)-75)+729(\Delta-3)^2}{18(27-20\delta)^2},$$

are both lower than the respective profits resulting under the uniform pricing if $\Delta < \frac{60\delta-81\sqrt{(27-20\delta)^2(16\delta(20\delta-61)+765)}}{2(4\delta(20\delta-61)+189)} \equiv \tilde{\Delta}$. Otherwise, the low-quality firm is better off and the high-quality worse off under the discriminatory regime.

Proof of Proposition 5

The benchmark case. If BBPD is not viable, prices are equal in both periods and there is not switching. By simple computation, the total surplus will be:

$$CS_u = \int_0^{\bar{x}} U^{HH}(x)dx + \int_{\bar{x}}^1 U^{LL}(x)dx = q^H + q^L - \frac{45-\Delta^2}{18},$$

where $U_u^{ii}(x) = 2\left(q^i - p_u^i - |x - l^i|\right)$ represents the utility of buying in the two period good i paying the non-discriminatory price.

Discriminatory Price. Under BBPD, three different cases may arise:

(a) Exit of the low-quality firm .

$$CS_E = \int_0^{\hat{x}_1} U_H^{HH}(x)dx + \int_{\hat{x}_1}^{1} U_H^{HL}(x)dx = \frac{\left(24q^H + 30q^L - 2\Delta^2\right)}{27} - 2,$$

where U_H^{ij} is simply the utility of buying good j in the first period and good i in the second when prices are the one leading to a scenario in which only the high-quality firm poaches rival's consumers. Compared with CS_u, this is always lower.

(b) ODS with the low-quality firm active. In this case, the total surplus will be:

$$\begin{aligned} CS_H &= \int_0^{\hat{x}_1} U_H^{HH}(x)dx + \int_{\hat{x}_1}^{\hat{x}_2^L} U_H^{HL}(x)dx + \int_{\hat{x}_2^L}^{1} U_H^{LL}(x)dx \\ &= \tfrac{1}{486}\left(582q^L + 147q^H - (16 + 81q^H)\Delta^2 - 954\right). \end{aligned}$$

This is always lower than the benchmark case of uniform pricing.

(c) TDS. In this case, the total surplus will be:

$$\begin{aligned} CS_{TDS} &= \int_0^{\hat{x}_2^H} U_{TDS}^{HH}(x)dx + \int_{\hat{x}_2^H}^{\hat{x}_1} U_{TDS}^{LH}(x)dx + \int_{\hat{x}_1}^{\hat{x}_2^L} U_{TDS}^{HL}(x)dx + \int_{\hat{x}_2^L}^{1} U_{TDS}^{LL}(x)dx \\ &= \tfrac{1}{225}\left(\tfrac{243\Delta^2}{27-20\delta} - \tfrac{1053\Delta^2}{(27-20\delta)^2} - \Delta^2 + 225(q^H + q^L) - 475\right), \end{aligned}$$

where U_{2TDS}^{ij} is simply the utilities of buying good j in the first period and good i in the second when prices are the one leading to a two-direction switching. Compared with CS_u, this is always higher.

Proof of Proposition 6

Under BBPD, three different cases may arise:

(a) Exit of the low-quality firm ($\Delta >$). In this case, firm H and firm L respectively get:

$$\pi_E^H = \frac{\delta^3(13-\Delta)(87-7\Delta)-3\delta^2(603+\Delta(240-23\Delta))+81\delta(\Delta+3)(\Delta+9)+243(\Delta+3)^2}{6(27-\delta)^2}$$
$$\pi_E^L = \frac{(\delta(13-\Delta)+9(3-\Delta))(\delta(\delta+30)\Delta-3\delta(7\delta+8)+27(3-\Delta))}{6(27-\delta)^2}.$$

It is easy to verify that $\pi_E^H - \pi_u^H < 0$ and $\pi_E^L - \pi_u^L > 0$.

(b) ODS with the low-quality firm active. Here firm H gets

$$\pi_H^H = \frac{\delta^3(\Delta(7\Delta-166)+1023)+3\delta^2(\Delta(11\Delta-126)-801)+189\delta(\Delta+3)^2+243(\Delta+3)^2}{6(\delta-27)^2},$$

which is always lowerer that π_u^H.

On the other hand, firm L is always better off under the discriminatory regime, as the profit it gets, i.e.,

$$\pi_H^L = \frac{2\delta\Big(-(\delta(\delta+16)+27)\Delta^2+3\delta(9\delta+59)\Delta-9\delta(19\delta+24)+486(\Delta-1)\Big)}{9(\delta-27)^2}$$

is always higher that π_u^L.

(c) TDS. In this case, both firms are strictly worse off under the discriminatory regime. Indeed, the profits they get:

$$\pi_{TDS}^H = \frac{4\delta^3(3\Delta(12\Delta+55)+242)-9\delta^2(\Delta(55\Delta+246)+407)+162\delta((\Delta-2)\Delta+3)+729(\Delta+3)^2}{18(27-11\delta)^2},$$
$$\pi_{TDS}^L = \frac{4\delta^3(3\Delta(12\Delta-55)+242)-9\delta^2(\Delta(55\Delta-246)+407)+162\delta(\Delta(\Delta+2)+3)+729(\Delta-3)^2}{18(27-11\delta)^2},$$

are both lower than the respective profits resulting under the uniform pricing if $\Delta < 3 - \frac{(27-11\delta)\left(2\sqrt{117-\delta(37-8\delta)}-3(7-\delta)\right)}{27-\delta(23\delta-22)} \equiv \tilde{\Delta}_{FL}$. Otherwise, the low-quality firm is better off and the high-quality worse off under the discriminatory regime.

Proof of Proposition 5.2

The benchmark case. If BBPD is not viable, prices are equal in both periods and there is not switching. By simple computation, the total surplus will be:

$$CS_u^{FL} = \int_0^{\bar{x}} U^{HH}(x)dx + \int_{\bar{x}}^1 U^{LL}(x)dx = (1+\delta)\left(q^H + q^L - \frac{45-\Delta^2}{36}\right),$$

where $U_u^{ii}(x) = (1+\delta)\left(q^i - p_u^i - |x - l^i|\right)$ represents the utility of buying in the two period good i paying the non-discriminatory price.

Discriminatory Price. Under BBPD, three different cases may arise:

(a) Exit of the low-quality firm .

$$CS_E^{FL} = \int_0^{\hat{x}_{1H}^{FL}} U_H^{HH}(x)dx + \int_{\hat{x}_{1H}^{FL}}^{1} U_H^{HL}(x)dx =$$
$$\frac{81\left(18\left(q^H+q^L\right)+\Delta^2-45\right)-\delta^2\left(98q^H+114q^L+23\Delta^2-1703\right)}{4(27-\delta)^2}$$
$$+\frac{18\delta\left(70q^H+7\Delta^2+80\text{qB}-174\right)-2\delta^3\left(287-2q^H-48q^L+2\Delta^2\right)}{4(27-\delta)^2}$$

where U_H^{ij} is simply the utility of buying good j in the first period and good i in the second when prices are the one leading to a scenario in which only the high-quality firm poaches rival's consumers. Compared with CS_u, this is always higher.

(b) ODS with the low-quality firm active. In this case, the total surplus will be:

$$CS_H^{FL} = \int_0^{\hat{x}_{1H}^{FL}} U_H^{HH}(x)dx + \int_{\hat{x}_{1H}^{FL}}^{\hat{x}_2^L} U_H^{HL}(x)dx + \int_{\hat{x}_2^L}^{1} U_H^{LL}(x)dx$$
$$= \frac{\delta^2\left(1847-194q^H-18q^L-23\Delta^2\right)+18\delta\left(46q^H+104q^L+15\Delta^2-156\right)-2\delta^3\left(279-2q^H+2\Delta^2-48\Delta\right)+81\left(\Delta^2-18\Delta-45\right)}{4(27-\delta)^2}.$$

This is always higher than the benchmark case of uniform pricing.

(c) TDS. In this case, the total surplus will be:

$$CS_{TDS} = \frac{729((\Delta-18)\Delta-45)+\delta^3\left(243\Delta^2+2046q^H+2310q^L-5203\right)-9\delta^2\left(89\Delta^2+954q^H+938q^L-2233\right)}{36(27-11\delta)^2},$$
$$+\frac{81\delta\left(13\Delta^2+42q^H+18q^L-57\right)}{36(27-11\delta)^2}$$

where U_{2TDS}^{ij} is simply the utilities of buying good j in the first period and good i in the second when prices are the one leading to a two-direction switching. Compared with CS_u, this is always higher.

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