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BROTHERS IN ALMS? COORDINATION BETWEEN NONPROFITS ON MARKETS FOR DONATIONS

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Abstract. Mission-driven nonprofit organizations compete for donations through fundraising activities. Such competition can lead to inefficient outcomes, if nonprofits impose externalities on each others' output. This paper studies the sustainability of fundraising coordination agreements, using a game-theoretic model of coalition formation. Three key characteristics determine the stability of cooperation: (i) the alliance formation rule, (ii) the extent to which fundraising efforts are strategic complements/substitutes, and (iii) whether deviation from the agreements is by an individual or by a group of nonprofits. We also characterize necessary and sufficient conditions for the stability of Pareto-optimal full coordination in fundraising.

Keywords: nonprofits, charitable giving, coordination, endogenous coalition formation, non-distribution constraint.
JEL codes: L31, D74, L44, C72.

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> ”On 21 December 1984, unable to resist the allure of Ethiopian famine pictures, World Vision ran an Australia-wide Christmas Special television show calling on the public in that country to give it funds. In so doing it broke an explicit understanding with the Australian Council of Churches that it would not run such television spectaculars in competition with the ACC’s traditional Christmas Bowl appeal. Such ruthless treatment of ’rivals’ pays, however: the American charity is, today, the largest voluntary agency in Australia ...” (Hancock 1989: 17)

1. Introduction

Private provision of public goods in modern economies is organized to a large extent by nonprofit organizations. These entities constitute a sector which employs, on average, 5.6 per cent, and in some countries - Netherlands, Belgium, Canada, U.K., Israel and Ireland - over 10 per cent of the economically active population (Salamon 2010).

An important part of the revenues of nonprofits comes from charitable donations. Given that nonprofits have to compete for donations through fundraising activities, these organizations can be considered as rational players on the philanthropic ’market’ (Andreoni 2006). Following this avenue, recent studies (Bilodeau and Slivinski 1997, Andreoni and Payne 2003, Castaneda et al. 2008, Aldashev and Verdier 2009) have modelled the nonprofit sector outcomes as equilibria of decentralized interaction between many nonprofit organizations, each of which maximizes a certain (usually impurely altruistic or "warm glow") objective function.

It is well known that one fundamental downside of this decentralized organization of the nonprofit sector is that competing for donations - when the aggregate amount of donations is relatively inelastic to fundraising - is socially wasteful (see, for instance, Rose-Ackerman 1982 and Aldashev and Verdier 2010). Occasionally, nonprofit organizations are able to overcome this problem, by designing voluntary cooperative agreements. A well-known example is the American United Way (Brilliant 1990). Other (rare) examples of successful coordination occur during humanitarian emergencies and have the form of umbrella organizations that conduct joint fundraising appeals: Disaster Relief Agency created by Dutch NGOs in 1993, Disasters Emergency Committee (DEC) in Britain (Smillie 1995: 116), and Belgian National Center for Development Cooperation (Similon 2009).

However, in general, constructing sustainable cooperation agreements that involve a substantial number of competing nonprofits is difficult. De Waal (1997) writes in his description of the development nonprofit sector: "[An organization that is] most determined to get the highest media profile obtains the most funds ... In doing so it prioritizes the requirements of fundraising: it follows the TV cameras, ... engages in picturesque and emotive programmes (food and medicine, best of all for children), it abandons scruples about when to go in and when to leave, and it forsakes cooperation with its peers for advertising its brand name." The opening quotation of this paper also presents an example of failed coordination agreement between emergency relief nonprofits in Australia.

This problem naturally calls for a policy intervention. However, given that nonprofit organizations are non-governmental entities, the "top-down" government intervention is unlikely to be effective, because it would be perceived as undermining the very essence of these organizations. Edwards and Hulme (1996) argue, for instance, that the stronger are the links of nonprofits with the government agencies, the less effective the nonprofits are in pursuing independently their missions. There might be some scope for indirect public policies, using the basic set of tools that governments usually employ towards the nonprofit sector (direct grants, matching grants that are given to nonprofits in proportion of private donations that they collect, tax deductibility of charitable donations, subsidizing or taxing the fixed costs of setting up a nonprofit, etc.). Finally, most countries have national associations of nonprofit organizations, and these platforms themselves can introduce some internal rules and codes of conduct for fundraising campaigns.

From an economist's point of view, this issue raises several questions: Why voluntary coordination between nonprofits is so difficult to attain? What can be done to facilitate such cooperation and to make it more sustainable? In this paper, we provide the first analysis of these questions. To do so, we build a model of endogenous nonprofit alliance formation, by exploiting game-theoretic tools used in the recent literature on endogenous coalition formation (see Bloch 2003, 2009, Yi 2003, Ray 2007, and Marini 2009 for surveys). In our two-stage model, at Stage 2 of the game, nonprofits engage in individual fundraising activities (with the opportunity cost being working on the project that contributes to their missions). Fundraising activity of one nonprofit can affect the donations collected by another nonprofit (either positively or negatively). Thus, nonprofits impose externalities on the each other's output. At Stage 1, nonprofits can form alliances, i.e. credibly commit to levels of fundraising that internalize the externalities among the alliance members.

The alliance formation takes place via the following process: each nonprofit announces an alliance to which it would like to belong; then, an alliance is formed on the basis of the profile of these announcements and according to a certain alliance-formation rule. We then study how outcomes differ under two alliance-formation rules. The first is the *unanimity* rule, which requires that all players of the alliance unanimously agree to form *that specific* alliance. Under this rule, one member's breaking away from an alliance automatically implies that all the remaining alliance members also become independent entities. The second is a milder *aggregative* rule, which only requires, for an alliance to form, that all its members have announced the same alliance (not necessarily the one that forms). Under this rule, upon a break-away by a member of an alliance, all remaining members continue to stick together. Given these rules, we investigate the conditions for the stability of the Pareto-efficient full-coordination agreement (the grand coalition of nonprofits) and other alliance structures, where stability is according to standard individual or coalitional equilibrium concepts (i.e. that no single nonprofit or a group of nonprofits have better alternatives in a different alliance structure).

We find that three key characteristics determine the stability of the voluntary coordination: (i) the alliance formation rule; (ii) whether the deviation from a given agreement can be done only individually or by a group of nonprofits; and (iii) the extent to which the fundraising efforts of nonprofits are strategic complements or substitutes (i.e. to which extent, for a given nonprofit, an increase in the fundraising effort of other nonprofits increases or decreases her incentives to conduct more fundraising). As we explain below, the extent of this strategic complementarity or substitutability is, in turn, determined by the technology of fundraising and whether donors perceive different nonprofits' projects as similar or as differentiated. These three features jointly affect the extent to which a deviation from the cooperative agreement influences the intensity of competition for donations, which, in turn, determines the relative benefits of deviation.

If the projects proposed by the nonprofits are considered by donors as highly differentiated, the main issue tackled by the nonprofits is relatively new and unknown to donors, and the fundraising technology allows for a relatively poor targeting of donors, the disciplining factors that help to sustain coordination are very weak, the stability of fundraising coordination agreements is extremely difficult to achieve. Contrarily, if the issue is well-known to donors, the technology of fundraising allows targeting donors rather well, and if donors consider the beneficiaries of the nonprofits' projects as being the same group (as, for example, is often the case during humanitarian emergencies), the strategic complementarity of fundraising efforts is very strong and this acts as a strong disciplining device; in that case, the Pareto-efficient full coordination of fundraising activities becomes a stable outcome (and, under some conditions, the only stable outcome). Finally, when the disciplining factors above are somewhat weaker (but not too weak), intermediate alliance structures (as, for example, one large alliance and one or several nonprofits refusing to join in) becomes a stable outcome.

To the best of our knowledge, there are no papers that theoretically analyze the cooperative agreements among nonprofits.[1] The existing literature (see, in particular, Rose-Ackerman 1982, Bilodeau and Slivinski 1997, Castaneda *et al.* 2008, Aldashev and Verdier 2009, Aldashev and Verdier 2010) mostly concentrates on fundraising competition between non-profits. Our paper contributes to this literature by studying the conditions under which the nonprofits successfully coordinate their fundraising activities, so as to reduce the negative consequences of the harmful competition for donations.

The rest of the paper is organized as follows. Section 2 presents the setup for both noncooperative and cooperative behavior of nonprofits; it also introduces the process of alliance formation and the necessary game-theoretic concepts of stability of a structure of alliances. Section 3 presents the main results of the paper, as well as the main intuitions in a simple four-player case. It also discusses the implications of our findings for real-life donation markets and briefly looks at how certain public policy tools might affect the nonprofit alliances. Section 4 discusses possible directions for future work and concludes. The technical proofs are relegated to the Appendix .

2. Setup of the model

2.1. **Donors.** Consider an economy with a continuum of donors whose total size is L. Donors consume a numeraire good and obtain a warm-glow benefit from giving to charitable causes served by a finite set $N = \{1, ..., n\}$ of nonprofit organizations, each having one charitable "project". The preferences of each donor are described by a simple linear-quadratic function, similar to the one used in industrial-organization models of oligopolistic competition (see, for example, Dixit 1979, Shubik and Levitan 1980, and Singh and Vives 1984):

$$U(C, \mathbf{d}) = C + \sum_{i=1}^{n} \omega_i d_i - \frac{\gamma}{2} \sum_{i=1}^{n} d_i^2 - \frac{1}{2} \left(\sum_{i=1}^{n} d_i \right)^2 .$$

Here, C is the consumption of a numeraire good, $\mathbf{d} = (d_i)_{i=1,..,n}$ is a vector of donations to nonprofit projects $i = 1, ..., n$, ω_i is the positive (or null) weight attached to giving to project i by each donor and γ a positive parameter capturing the degree of substitution between

[1]Gugerty (2008) and the papers in Gugerty and Prakash (2010) study, on the basis of case studies, the performance of several forms of nonprofit self-regulation, mainly aimed at increasing accountability towards donors.

donations to different projects (i.e. the extent to which donors perceive giving to projects of different nonprofits as differentiated goods).

The budget constraint of each donor with income I is:

$$C+\sum_{i=1}^{n} d_i = I.$$

Using the budget constraint, the individual optimum choice for each donor satisfies the first-order condition

$$\omega_i - \sum_{i=1}^{n} d_i - \gamma d_i - 1 = 0 \qquad \text{for all } i = 1, ..., n.$$

The marginal benefit of giving an additional dollar to nonprofit i is $\omega_i - \sum_{i=1}^{n} d_i - \gamma d_i$. The first term, as we discuss below, can be increased by fundraising effort of nonprofits. The second term captures the diminishing marginal utility of total donation. Finally, the third term depends on the degree of substitution between giving to different nonprofits: if γ is relatively high, and the available projects highly differentiated, the donor would need a substantially high component ω_i to induce him to give a large donation to nonprofit i. The marginal cost is 1, simply reflecting the forgone utility of consumption of numeraire good. Summing the first-order conditions over i, we obtain

$$\sum_{i=1}^{n} \omega_i - n = (\gamma + n) \sum_{i=1}^{n} d_i,$$

and thus,

$$\gamma d_i = \omega_i - 1 - \frac{\left(\sum_{i=1}^{n} \omega_i - n\right)}{\gamma + n}.$$

Then, the donation of each individual donor to nonprofit i can be written as

$$d_i = \frac{\omega_i (\gamma + n - 1) - \gamma - \sum_{j \neq i} \omega_j}{\gamma (\gamma + n)}. \tag{2.1}$$

2.2. **Nonprofit organizations.** Each nonprofit organization is founded by a "social entrepreneur". In terms of her motivation, a social entrepreneur is impurely altruistic (*à la* Andreoni 1989): she receives a warm-glow utility which increases (linearly, for simplicity) in the output of her nonprofit. In other words, she likes to see the objectives of the charitable sector advanced, but only if this advancement goes through *her* organization. This implies that the objective function of a nonprofit organization is maximizing the output of its project.

The production technology of the project of nonprofit i has two inputs: funds F_i and time τ_i:

$$Q_i(F_i, \tau_i) = F_i \cdot \tau_i. \tag{2.2}$$

Each social entrepreneur has an endowment of 1 unit of time. She can use this time either to work on the project or to collect funds:

$$1 = \tau_i + y_i, \tag{2.3}$$

where y_i denotes the amount of time devoted to fundraising. Therefore, time is fungible and the social entrepreneur faces a well-defined trade-off: more time spent on fundraising increases the funds that can be devoted to the project; however, this comes at the cost of reducing the *time* devoted to the project.

Time devoted to fundraising brings in funds: we assume that the weights ω_i in donors' utility function depend on the fundraising effort $y_i \geq 0$ of each nonprofit i:

$$\omega_i = \omega + y_i + \Delta\left(\sum_{j \neq i} y_j\right),$$

with $\omega \geq 1$ and $\Delta \lesseqqgtr 0$ expressing spillovers coming from rival nonprofits' fundraising.

In other words, we assume that fundraising activities by a specific nonprofit i increase donors' perceived importance of that nonprofit's project, and thus the marginal benefit from giving to this project.[2] Moreover, we assume that there may be some "awareness" spillovers of fundraising by other nonprofits $j \neq i$ on the willingness of each donor to give more funds to project i. For instance, being contacted by Greenpeace, a nonprofit working towards environmental issues, might raise the awareness of a donor about those issues. However, she might dislike the methods used by Greenpeace (and thus abstain from giving to this organization). Nevertheless, she now cares more about environment, and if contacted by another nonprofit in the same sector (e.g. World Wildlife Fund), she is much more willing to give to WWF, as compared to a scenario in which she was not contacted by Greenpeace in the first place. Thus, the fundraising activities of Greenpeace have created positive spillovers for WWF. Analytically, this is captured by the term $\Delta\left(\sum_{j \neq i} y_j\right)$ with $\Delta > 0$. Spillovers can also be negative (e.g. because of the irritation felt by donors, caused by massive solicitations by numerous charities; see Van Diepen et al. 2009a) and, in this case, $\Delta < 0$.

Given that nonprofits cannot, by law, distribute profits (Hansmann 1980, Weisbrod 1988), the social entrepreneur puts all the funds that she collects (net of the financial costs) into the project. Denoting, by $D_i = d_i L$, the donations collected by nonprofit i, the non-distribution constraint can be formally expressed by

$$D_i = f + cD_i + F_i,$$

where $0 \leq c < 1$ is the financial cost of collecting a unitary donation, $f > 0$ is the fixed cost of the nonprofit project, and F_i is the amount invested into the project. The non-distribution constraint pins down the amount of funds that a nonprofit invests into its project:

$$F_i = (1-c)D_i - f. \tag{2.4}$$

Using (2.1), (2.2) and (2.4), the objective function of nonprofit i can thus be expressed as a function of its fundraising effort and of the effort levels of other nonprofits as[3]:

$$Q_i\left(y_i, y_{-i}\right) = (1-c)\left(\alpha + \delta y_i - \beta \sum_{j \neq i} y_j\right)(1-y_i), \tag{2.5}$$

where

$$\alpha = \frac{L\left(\omega - 1\right)}{\gamma + n} - \frac{f}{1-c} \geq 0,\ \delta = \frac{L\left(\gamma + n - 1 + \gamma\Delta\right)}{\gamma\left(\gamma + n\right)} > 0,\ \beta = \frac{L(1-\gamma\Delta)}{\gamma\left(\gamma + n\right)} \lesseqgtr 0.$$

[2]This is similar to the assumption of persuasive advertising in models of industrial organization literature (see Section 2.2 of Bagwell 2007 for a detailed review).

[3]As usual we introduce the notation (y_i, y_{-i}) for the full vector of fundraising efforts, separating the effort y_i of a given nonprofit i from the vector of efforts $y_{-i} = (y_1, ..., y_{i-1}, y_{i+1}, ...y_n)$ of the other nonprofits different from i.

To guarantee the existence of interior fundraising equilibria in any possible alliance structure, we impose the following restrictions on the model parameters:[4]

$$\beta \in \left(\underline{\beta}, 0\right) \cup \left(0, \overline{\beta}\right), \text{ with } \underline{\beta} = -2\delta/(n-1),\ \overline{\beta} = (\delta - \alpha)/(n-1) \text{ and } \delta > \alpha \geq 0. \tag{2.6}$$

The key parameter of the model is β, that captures the direction and the intensity of the fundraising activity of other nonprofits on donations received (and then output) of nonprofit i. Mathematically, β is positive if $\gamma\Delta < 1$ (and is negative otherwise). Note that γ and Δ capture conceptually different characteristics: γ is a preference parameter that reflects the degree of substitution between projects in the utility function of donors, while Δ is a technological parameter that describes how fundraising activities of nonprofits affect donors' preferences for giving.

In terms of empirically observable measures, Δ and γ can be linked to three broad dimensions. The first is the nature of the cause towards which nonprofits operate. If the issue is relatively new and unknown to donors (e.g. health-related issues linked to the consumption of genetically modified food during the early anti-GMO campaigns), it is likely that the awareness-raising spillovers (Δ) are large. Otherwise (if the issue is such that most of the potential donors already have an extensive background information about the issue), the awareness-raising spillovers are likely to be minor.

The second is the dominant technology of soliciting donations. Suppose that the fundraising technology allows for precise targeting of potential donors on the basis of certain characteristics or behavior (as in case of online solicitations, when targeting is made through professional marketing firms on the basis of consumption patterns of potential donors). In this case, the spillovers are relatively small. Contrarily, if the technology of fundraising is essentially untargeted (as, for instance, in case of direct mailing), the spillovers (Δ) are likely to be high.

The third is the degree of perceived differentiation of nonprofits' projects by donors. For instance, if the beneficiaries of the projects of most nonprofits are the same and nonprofits work on the same dimension of development (as was often the case during humanitarian emergencies, such as the 2004 tsunami or the 2011 earthquake in Haiti), the donors consider most nonprofits' projects as close substitutes (γ is low). Contrarily, if each nonprofit put its efforts targeting a different group of beneficiaries, donors consider different nonprofits' projects as poor substitutes, and γ is relatively high.

Summing up, β is likely to be positive and large in absolute value (i.e. more fundraising by others strongly reduces the output yielded by nonprofit i) in donation markets populated by nonprofits whose projects concentrate on a relatively similar groups of beneficiaries (and work on similar dimensions), fundraising technologies allowing for good targeting, and projects centered around issues on which donors already have good information. On the other hand, β is likely to be either low or negative in markets with nonprofits aiming at highly diverse groups of beneficiaries (or conducting qualitatively different projects), where the fundraising technology is untargeted, and the issues in nonprofit mission require an active information-providing role for nonprofits.

Recent empirical work measures the extent of spillovers and complementarities in nonprofit fundraising. Van Diepen et al. (2009b) analyze data on direct mailing solicitations for charitable giving in the Netherlands and find that solicitations by different charities are

[4]Since for $\beta = 0$ there is no strategic interaction between nonprofits, we exclude this value from our analysis.

short-run complements (i.e. awareness spillovers seem to be present in the short run) and long-run substitutes (i.e. spillovers die out in the long run). This is consistent with our interpretation above: in the short run, donors do not have enough information about the issue raised by a nonprofit asking for a donation, thus the awareness-raising effect of such solicitation probably benefits other nonprofits; in the long run, donors accumulate enough information about the issue, and thus the total size of the donation market becomes constant. Two subsequent papers (Lange and Stocking 2012 and Reinstein 2012) that analyze the same question using experimental data find additional interesting results. In a field experiment, Lange and Stocking (2012) find that providing some donors of one charity with the opportunity to be included in the stream of solicitations of another charity does not decrease their donation to the original charity (if anything, such donations actually increase). In a lab experiment, Reinstein (2012) finds, instead, that providing more promotional information about some charity or reducing the price of giving to that charity leads to an increase of donations to this charity and to a similar-size decrease of donations to the charities with similar goals. The framework that we develop in this paper allows to encompass both the cases of complements and substitutes.

2.3. **Non-cooperative fundraising equilibrium between individual nonprofits.** Before analyzing the fundraising coordination agreements between nonprofits, we first consider the situation in which every nonprofit acts individually and non-cooperatively, with the objective of maximizing its output (by choosing the amount of time it devotes to fundraising, y_i), taking as given other nonprofits' fundraising efforts, y_{-i}. In other words, for every $i = 1, ..., n$, the problem is

$$\max_{y_i} Q_i\left(y_i, y_{-i}\right) = \max_{y_i} \ (1-c)\left(\alpha + \delta y_i - \beta \sum_{j \neq i} y_j\right)(1-y_i). \tag{2.7}$$

First-order conditions for an interior equilibrium of the game played among nonprofits, denoted $y = (y_1, y_2, .., y_n)$, imply that for every $i = 1, ..., n$:

$$\frac{\partial Q_i\left(y_i, y_{-i}\right)}{\partial y_i} = \underbrace{\delta(1-y_i)(1-c)}_{\text{marginal fundraising benefit}} - \underbrace{(1-c)(\alpha + \delta y_i - \beta \sum_{j \neq i} y_j)}_{\text{marginal fundraising cost}} = 0\,. \tag{2.8}$$

Intuitively, at the equilibrium each nonprofit equates the marginal benefit of additional fundraising (in terms of project output) to its marginal (opportunity) cost. Using symmetry, it is easy to obtain the following Nash equilibrium fundraising levels:

$$y_i^N = \frac{\delta - \alpha}{2\delta - \beta\left(n-1\right)}, \tag{2.9}$$

and Nash equilibrium payoffs of nonprofits as

$$Q_i\left(y^N\right) = \frac{\left(1-c\right)\delta\left(\delta + \alpha - \beta\left(n-1\right)\right)^2}{\left(2\delta - \beta\left(n-1\right)\right)^2}. \tag{2.10}$$

The parameters constraints introduced in (2.6) guarantee the existence of an interior equilibrium $\overline{y}_i \in (0, 1)$; they also make sure that all individual nonprofits' best-reply functions satisfy the contraction property:

$$|\beta| < \frac{2\delta}{(n-1)}. \tag{2.11}$$

It is easy to check that $Q_i(y_i, y_{-i})$ is strictly concave in y_i, given that $c < 1$ and $\delta > 0$. How the output of nonprofit i is affected by fundraising activities of some other nonprofit j crucially depend on the sign of β:

$$\frac{\partial Q_i(y_i, y_{-i})}{\partial y_j} = -\beta(1-c)(1-y_i) \lesseqgtr 0 \text{ for } \beta \gtreqless 0, \text{ for all } j \neq i.$$

Moreover,

$$\frac{\partial^2 Q_i(y_i, y_{-i})}{\partial y_i \partial y_j} = \beta(1-c) \gtreqless 0 \text{ for } \beta \gtreqless 0. \tag{2.12}$$

Thus, when $\beta > 0$ (more fundraising by others reduces the donations to i), nonprofits impose negative externalities on each other's output, and fundraising efforts are *strategic complements* (in other words, any increase in rivals' fundraising raises the incentives for a nonprofit to devote more time to fundraising). Contrarily, when $\beta < 0$ (more fundraising by others increases donations given to i), nonprofits impose positive externalities on each other's output, whereas fundraising efforts are *strategic substitutes*: higher effort by rival nonprofits increase the donations given to i, thus rising its opportunity cost of diverting an additional unit of time from the project to the fundraising activity. This feature makes our model of nonprofit competition qualitatively different from the usual differentiated product oligopolistic-competition models of industrial organization, where negative externalities are associated with strategic substitutes (firms compete by setting quantities) and positive externalities are associated with strategic complements (price competition).

2.4. **Fundraising equilibrium between nonprofit alliances.** Next, let us analyze the situation in which nonprofits have organized their common actions in alliances $A \subset N$. The *grand coalition* N is the largest possible alliance and corresponds to the full coordination among nonprofits in the sector. Alternatively, nonprofits can coordinate their actions to some intermediate levels. These are expressed by alliance structures (i.e. partitions) representing a collection of nonprofits in alliances having null intersection and summing up to N, denoted with $S = (A_1, ..., A_k, ..., A_m)$ with $m \leq n$.

To obtain a well-defined interaction for nonprofits forming alliances A_k in all feasible coalition structures $S \in \mathcal{S}$ (where $\mathcal{S}$ denotes the set of all feasible alliance structures S) we assume that members inside A_k fully coordinate their individual actions, and the alliance payoff is simply expressed as $Q_{A_k} = \sum_{i \in A_k} Q_i(y_i, y_{-i})$, i.e. the sum of every nonprofit's project output. Note that in our setting the main benefit for nonprofits to create alliances is to coordinate their fundraising effort levels, thus internalizing the externalities (negative or positive) that the nonprofits impose on each other's output. In this case, the creation of an alliance always generates a positive spillover (or *coalitional externality*) on remaining nonprofits. Consequently, the outcome obtained by the grand coalition is Pareto-efficient (from the point of view of nonprofits' objective functions and project beneficiaries, if the payoffs of the latter monotonically increase with total nonprofits' output).

Moreover, we assume for simplicity that within every alliance there is an equal-sharing allocation rule, and no side-payments are allowed between different alliances.[5] In addition,

[5]If side payments between nonprofits are allowed, full cordination would always trivially be the unique coalitional equilibrium. Note also that with symmetric players (as in our model) the equal-sharing allocation rule can be obtained endogenously by assigning to every participant, as in Hart and Kurz (1983), the Owen value (Owen 1977) for transferrable-utility games (i.e. the Shapley value with *a priori* coalition structures).

every nonprofit alliance behaves *à la* Nash against rival alliances of nonprofits and therefore acts to maximize the sum of the joint output of all its members, taking as given the actions of nonprofits that do not belong to this alliance. Formally, for every $i \in A_k$ and $A_k \in S$, the objective function is

$$\max_{(y_i)_{i \in A_k}} \sum_{i \in A_k} (1-c)\,(\alpha + \delta y_i - \beta \sum_{j \neq i} y_j)(1-y_i).$$

The first-order condition of this problem implies, for every member of the alliance, $i \in A_k$ in a generic partition of m alliances $S = (A_1, ..., A_k, ..., A_m)$,

$$\underbrace{\delta(1-y_i)(1-c) - \textstyle\sum_{j \in A_k \backslash \{i\}} \beta(1-y_j)(1-c)}_{\text{social marginal fundraising benefit}} - \underbrace{(1-c)(\alpha + \delta y_i - \beta \sum_{j \neq i} y_j)}_{\text{marginal fundraising cost}} = 0. \tag{2.13}$$

This expression indicates that every nonprofit participating in alliance A_k sets its fundraising level to equate the marginal cost of fundraising to the marginal (coalitional) benefit. Using the symmetry of nonprofits inside each alliance A_k, the generic best-reply of a nonprofit $i \in A_k$ in $S = (A_1, A_2, ..., A_m)$, given the symmetric output y_h of every nonprofit in A_h, writes as:

$$y_i(\sum_{h \neq k} a_h y_h) = \frac{1}{2} \frac{\left(\delta - \alpha + \beta \sum_{h \neq k} a_h y_h - \beta\,(a_k - 1)\right)}{\delta - \beta(a_k - 1)},$$

where a_k denotes the size of the alliance A_k in S. It can be easily checked that conditions (2.11) and (2.6) ensure the existence of a unique interior fundraising equilibrium for any nonprofit alliance structure.[6] We denote such equilibrium vector as $\overline{y}(S)$.

Comparing conditions (2.8) and (2.13), we see that when nonprofits' fundraising activities impose positive (negative) externalities on each other's output, the level of fundraising chosen by an alliance member is higher (lower) than that of a nonprofit playing non-cooperatively. When the grand coalition $S = \{N\}$ forms (the alliance of size $|\{N\}| = a_N = n$), the equilibrium fundraising effort y_i^G for each i is

$$y_i^G = \overline{y}_i\,(\{N\}) = \frac{1}{2} \frac{\delta - \alpha - \beta\,(n-1)}{\delta - \beta\,(n-1)}, \tag{2.15}$$

with output

$$Q_i^G = Q_i(\overline{y}\,(\{N\})) = \frac{1}{4} \frac{(\alpha + \delta - \beta\,(n-1))^2\,(1-c)}{\delta - \beta\,(n-1)}. \tag{2.16}$$

If nonprofits impose positive externalities on each others' output (i.e. $\beta < 0$), equilibrium fundraising efforts under the grand coalition are higher than those in the noncooperative all-singleton Nash equilibrium (2.9) (i.e. $y_i^G > y_i^N$). Analogously, $y_i^G < y_i^N$ when $\beta > 0$. Note that the boundaries imposed on parameters in (2.6) imply that $y_i^G > 0$ for $\beta < 0$

[6] Since a sufficient condition for the contraction property is (see, for instance, Vives 2000: 47):

$$\frac{\partial^2 Q_i}{\partial\,(y_i)^2} + \sum_{j \neq i} \left| \frac{\partial^2 Q_i}{\partial y_i \partial y_j} \right| < 0,$$

using (2.13), the above condition requires that

$$|\beta| < \frac{2\delta}{a_1 + ... + a_{k-1} + a_{k+1} + ... + a_m + 2\,(a_k - 1)}, \tag{2.14}$$

and this is always respected if (2.11) holds.

and $y_i^N < 1$ for $\beta > 0$. Therefore, these boundaries are sufficient to guarantee an interior fundraising effort to every nonprofit in any possible alliance structure S.

2.5. **Nonprofit alliance formation stage.** As our ultimate objective is to understand the strategic behavior of nonprofit in terms of creating and breaking cooperative agreements, we need to introduce two further elements into our analysis: how alliances are formed and what do we intend by stability of a given nonprofit alliance structure.

We adopt a very simple approach to the alliance formation process: it is a simultaneous-move game in which every nonprofit i announces a (non-empty) alliance $A_k \subset N$ to which it would like to belong. Thus, for every profile of announcements, denoted $\sigma = (\sigma_1, \sigma_2, ..., \sigma_n)$, declared by the n nonprofits, an alliance structure $S = \{A_1, A_2, ..., A_m\}$ is induced.

Clearly, the rule according to which an alliance structure originates from a profile of announcements is a key issue for predicting which nonprofit alliances will emerge in equilibrium. One possibility is to assume that a particular alliance emerges if and only if all its (future) members announce exactly this particular alliance. We call this rule the *unanimity rule.* Formally, the partition

$$S^U(\sigma) = \{A_1(\sigma), A_2(\sigma), ..., A_m(\sigma)\},$$

is such that every nonprofit i belongs to A_k $(k = 1, 2, ..., m)$ if and only if $\sigma_i = \{A_k\}$ for all $i \in A_k$ and stays as singleton otherwise.

Another possibility is to assume that a nonprofit alliance emerges if and only if all its (future) members announce the same alliance. The difference with respect to the unanimity rule is that this announced alliance may, in general, differ from the alliance that will form. We call this rule the *aggregative rule.*[7] Formally:

$$S^A(\sigma) = \{A_1(\sigma), A_2(\sigma), ..., A_m(\sigma)\},$$

such that every nonprofit i belongs to A_k $(k = 1, 2, .., m)$ if and only if $\sigma_i = \sigma_j$ for all $i, j \in A_k$ and stays as singleton otherwise.

The two rules generate different partitions after a deviation from a given alliance structure by an individual organization or an alliance of nonprofits. Under the unanimity rule, a deviation induces the remaining organizations in the alliance to split up into singletons. Contrarily, under the aggregative rule the remaining nonprofits continue to stick together. All nonprofits understand this; therefore, the strategic incentives to announce a given alliance might differ under the two rules.[8]

Of course, in reality, the alliance formation process among nonprofits is much more complex: it involves meetings, proposals and counter-proposals, back-and-forth negotiations, and so on. We can think of the rules described above as a simplified representation of this complicated process, which captures some of its basic features. Which alliance formation rule represents better the reality depends on the institutional environment in which nonprofits operate. Most coordination agreements - e.g. umbrella associations - operate in the fashion

[7]The unanimity rule was first introduced by von Neumann and Morgenstern (1944). It is also sometimes called the *gamma-rule*, whereas the aggregative rule is also known as the *delta-rule* of coalition formation (Hart and Kurz 1983).

[8]The easiest way to illustrate the difference between the two rules is as follows: suppose there are three nonprofit organizations: i, j, k. Let nonprofits i and j announce the grand coalition: $\sigma_i = \sigma_j = \{i, j, k\}$, whereas nonprofit k announces $\sigma_k = \{k\}$. Under the unanimity rule, this profile of announcements will result in all nonprofits playing as singletons: $\mathcal{S}^U(\sigma) = (\{i\}, \{j\}, \{k\})$. Instead, under the aggregative rule, the same profile of announcements results in the structure $\mathcal{S}^A(\sigma) = (\{i, j\}, \{k\})$.

which is best described by the aggregative rule: if one nonprofit decides to exit the agreement, usually the remaining nonprofits continue to operate under the umbrella. In some cases, however, the fact that some organization exits the alliance might lead to the break-up of the remaining alliance: this occurs if the coordination is such that nonprofits put their productive assets into a common activity in a highly complementary fashion. In our model, this would correspond to a more restrictive unanimity rule.

Next, we can define the notion of stability of a given alliance structure. An alliance structure is *Nash-stable* when it is induced by an announcement profile that is a Nash equilibrium of a given game of alliance formation (i.e. it is robust to individual deviations). An alliance structure is *coalitionally stable* when the profile of announcements is robust to deviations by any alliance of nonprofits (i.e. is strong Nash). Let

$$Q_i(\sigma) \equiv Q_i(\overline{y}\left(S\left(\sigma\right)\right)) \text{ for every } i \in A_k \text{ and } A_k \in S,$$

denote the payoff of a nonprofit belonging to alliance A_k at the fundraising equilibrium $\overline{y}$ when the alliance structure $S\left(\sigma\right) = \{A_1, A_2, ..., A_m\}$ has been induced by the announcement profile $\sigma = (\sigma_1, \sigma_2, ..., \sigma_n)$. As mentioned earlier, we assume that nonprofits in an alliance share equally the alliance output and, therefore, $Q_i(\sigma) = Q_{A_k}(\sigma)/a_k$. Using this shortcut in notation, we can define formally the two distinct concepts of stability of alliance structures.

Definition 1. *(Nash stability) The alliance structure $S = \{A_1, A_2, ..., A_m\}$ is Nash-stable if $S = S(\overline{\sigma})$ for some $\overline{\sigma}$ such that for every organization $i \in N$ and every feasible alternative announcement σ_i'*

$$Q_i(\overline{\sigma}) \geq Q_i(\sigma_i', \overline{\sigma}_{N\backslash\{i\}}).$$

Definition 2. *(Coalitional stability) The alliance structure $S = \{A_1, A_2, ..., A_m\}$ is coalitionally stable if $S = S(\widetilde{\sigma})$ for some $\widetilde{\sigma}$, such that there exists no alliance $A \subset N$ with an alternative joint announcement σ'_A such that*

$$Q_i(\sigma'_A, \widetilde{\sigma}_{N\backslash A}) \geq Q_i(\widetilde{\sigma}) \textit{ for all } i \in A$$

and

$$Q_h(\sigma'_A, \widetilde{\sigma}_{N\backslash A}) > Q_h(\widetilde{\sigma}) \textit{ for at least one } h \in A.$$

It is clear that $\overline{\sigma} = (\overline{\sigma}_1, \overline{\sigma}_2, ..., \overline{\sigma}_n)$ corresponds to a Nash equilibrium of the announcement game and $\widetilde{\sigma} = (\widetilde{\sigma}_1, \widetilde{\sigma}_2, ..., \widetilde{\sigma}_n)$ to a coalitional (or strong Nash) equilibrium of the same game. Note also that the coalitional stability is a highly demanding stability concept. By requiring the stability with respect to every alternative profile of announcements (including the one formulated by the grand coalition), it imposes the Pareto optimality on the resulting allocation. In fact, any Pareto-inefficient outcome would be objected by the grand coalition of nonprofits.

3. Main results

Our aim in this section is to establish which specific conditions make an alliance structure between nonprofits individually and coalitionally stable. As we will see, two features crucially shape the stability of alliance structures: (i) the nature of strategic interdependence in fundraising activities, that can be either strategic *complements* or *substitutes*; and (ii) the mechanism of formation and dissolution of nonprofit alliances. We divide the analysis that follows into two sections: the first is devoted to the analysis of individual (or Nash) stability of alliance structures, whereas the second - to the analysis of their coalitional stability. Within each section, we first consider the case when fundraising activities are strategic complements ($\beta > 0$) and then turn to the case of strategic substitutes ($\beta < 0$). Analogously, in each section we consider first the mechanism of coalition formation that makes the stability of alliances easier (the unanimity rule) and then progressively switch to the one that makes coordination harder (i.e. the aggregative rule). We establish that when the level of strategic complementarity decreases (or the level of strategic substitutability increases) and the alliance formation rule becomes looser, we move from settings in which there is an easy coordination of every alliance structure to ones in which guaranteeing the stability of even the simplest agreements is difficult. We illustrate these formal observations throughout with a simple four-nonprofit example, and discuss the implications of these results for the feasibility of fundraising coordination between nonprofits operating in real-life donation markets.

3.1. **The Nash stability of alliance structures.** In the previous sections we saw that under $\beta > 0$ (i.e. when a more intense fundraising activity by other nonprofits reduces the donations received by a given nonprofit) the fundraising efforts are *strategic complements*, meaning that an increase in rivals' fundraising enhances the incentive of a nonprofit to devote more time to fundraising. We show below that in this setting strategic complementarity represents an important natural driver for the stability of coordination between nonprofits: if a group of nonprofits increase its fundraising, its competitors will increase their fundraising in response. Under negative fundraising externalities, the alliances exert less fundraising effort as compared to individual nonprofits. Hence, when an alliance breaks apart, it leaves all nonprofits unambiguously worse off. This negative effect helps to discipline fundraising coordination. When, conversely, $\beta < 0$, fundraising activities exert a positive impact on rivals and are strategic substitutes: every nonprofit has an incentive to reduce its fundraising effort in response to an increase in fundraising by rival nonprofits. In this case, nonprofits prefer to enjoy the existing positive fundraising spillovers emanating from the actions of its rivals, which leads in equilibrium to a greater tendency to remain independent (this tendency is particularly strong in large alliances).

3.1.1. *Nash stability under the unanimity rule.* We start by establishing a result on the Nash stability of any structure under the unanimity rule, provided that fundraising efforts are strategic complements.

Proposition 1. *When fundraising efforts are strategic complements ($\beta > 0$), all alliance structures $S \in \mathcal{S}$ are Nash stable under the unanimity rule of coalition formation.*

Proof. See the Appendix. □

Intuitively, the unanimity rule imposes that for any given alliance, a deviation by an individual member breaks up the alliance completely (into singletons). The alliance was internalizing the externalities that its members were imposing on each other, by keeping the fundraising activities contained. Upon the break-up, given that this containment is no longer in place, all ex-members increase their fundraising. However, because of the strategic complementarity, this also increases the incentive for all other alliances (or singleton nonprofits) to increase their fundraising, and - because of negative externalities - this unambiguously harms all the ex-members of the first alliance, including the nonprofit that chose to deviate.

Therefore, deviating is not individually rational; thus, no nonprofit would deviate from a given alliance structure.

The above proposition underlines how the strategic complementarity leads to the multiplicity of Nash-stable alliance structures. Since any deviation from an alliance agreement leads to a full break-up of the alliance and triggers a fundraising war, the unanimity rule turns out to be extremely disciplining, for any alliance structure to start with.

A similar logic easily allows us to see that, since the grand coalition is Pareto-efficient, it is by definition robust to deviation by singleton nonprofit and, therefore, Nash stable under the unanimity rule, regardless of whether fundraising efforts are strategic complements or substitutes.

Proposition 2. *Under the unanimity rule of coalition formation, the grand coalition* $\{N\}$ *is Nash stable for any* $\beta \in \left(\underline{\beta}, 0\right) \cup \left(0, \overline{\beta}\right)$.

This proposition has an important empirical implication. As noted in the introduction, we rarely observe stable full coordination agreements between nonprofits. Proposition 2 thus implies that strategic complementarity or substitutability alone cannot explain why such agreements are so rarely stable in real life, and, therefore, the explanation must lie elsewhere (e.g. in the alliance formation rule or the possibility of coalitional deviations, as we argue below).

What can we say about other alliance structures if the fundraising activities are strategic substitutes? The next proposition shows that when the strategic substitutability in fundraising efforts is sufficiently strong, the nonprofits find it rarely attractive to form alliances of relatively small size.

Proposition 3. *Under the unanimity rule of coalition formation, the alliance structures of the form* $S = (\{A\}, \{j\}_{j \in N \backslash A})$ *fail to be Nash stable if* $\underline{\beta} < \beta < \widehat{\beta}(n, a)$, *with* $\widehat{\beta}(n, a) < 0$.

Proof. See the Appendix. □

In other words, all alliance structures in which a coalition A competes with all other nonprofits playing as singletons fail to be individually rational if $\beta < \widehat{\beta}(n, a) < 0$, where a denotes the size of A and n the number of existing nonprofits.

Intuitively, when the strategic substitutability of fundraising efforts is sufficiently strong, individual deviations from an alliance (that plays against singletons) become convenient for members of the alliance. Remember that the members of the alliance exert higher fundraising effort than singletons outside it, and that fundraising externalities are positive. Thus, breaking away has two effects on the payoffs of the deviator (that go in the opposite direction): (i) the deviator reduces its the fundraising effort and thus can devote more time to its project; and (ii) such a deviation negatively affects the funds it collects because each nonprofit spends less fundraising effort in the resulting all-singleton equilibrium. The first effect operates in favor of breaking away, whereas the second against it. The two effects cancel each other exactly when $\beta = \widehat{\beta}$.

The above analysis highlights analogies and differences between the mechanism underlying the agreements among nonprofits in markets for donations and those among traditional firms in oligopolies (e.g., Salant *et al.* 1983 and Davidson and Deneckere 1985). In both cases, alliances are easier to form when decision variables are strategic complements (i.e. fundraising for nonprofits and prices for oligopolists). However, in the case of nonprofits the larger alliances are likely to form when the interaction between organizations (via increasing fundraising) exerts a negative impact on rivals. For firms, instead, this occurs when increases in prices impose a positive effect on rivals. Conversely, alliances among nonprofits are harder to form when fundraising efforts are strategic substitutes and generate positive externalities on outputs of nonprofits, whereas for oligopolists alliances are more difficult to form when firms compete in quantities, and such competition generates negative externalities for rivals. Thus, the main similarity is that for nonprofits as well as for firms, collusion is more likely to result when individual choices (i.e. prices for firms and fundraising efforts for nonprofits) reinforce each other (there are strategic complementarities). The main difference is that for firms such situations occur when more competition (in prices) is socially desirable, while for nonprofits this happens when more competition (in fundraising) is socially wasteful.

3.1.2. *A four-nonprofit example.* Let's illustrate the above mechanisms with the help of a simple four-player example. Consider an economy with $n = 4$ nonprofits and $L = 1$ donors, and let, for simplicity, the parameters of the model be normalized as follows: $\alpha = 0$ and $\delta = 1$ (see the Appendix for details). In this case, it is easy to calculate that the strategic complementarity or substitutability is fully determined by the substitutability of nonprofits' projects in the eyes of donors:

$$\beta = \frac{1-\gamma}{\gamma} \gtreqless 0 \Longleftrightarrow \gamma \lesseqgtr 1.$$

The boundaries for β are easily obtained as $\underline{\beta} = -2/(n-1) = -0.66$ and $\overline{\beta} = 1/(n-1) = 0.33$. The set of all possible alliance structures is the following (the sizes of alliances are denoted in curly brackets):

$$\mathcal{S} = \left[(\{4\}), (\{3\}, \{1\}), (\{2\}, \{2\}), (\{2\}, \{1\}, \{1\}), (\{1\}, \{1\}, \{1\}, \{1\})\right].$$

Figure 1 shows the alliances structures that can be induced by a single nonprofit when it deviates from a given alliance structure.

[Figure 1 about here]

Thus, using the equilibrium payoffs (shown in the Appendix), we can check that, for every $\beta \in (\underline{\beta}, 0) \cup (0, \overline{\beta})$, any nonprofit prefers to stay either in the grand coalition or in the structure $(\{3\}, \{1\})$, rather than to break away:

$$\begin{aligned} Q_i(\widetilde{y}(\{4\})) - Q_i(\overline{y}(\{1\}, \{1\}, \{1\}, \{1\})) &> 0, \\ \underbrace{Q_i(\overline{y}(\{3\}, \{1\}))}_{i \in \{3\}} - Q_i(\overline{y}(\{1\}, \{1\}, \{1\}, \{1\})) &> 0. \end{aligned}$$

Thus, once formed, such a structure is Nash stable for any value of β. This is not true for other alliance structures. In particular, if strategic substitutability is strong enough, a nonprofit prefers breaking up from the structure $(\{2\}, \{1\}, \{1\})$ or $(\{2\}, \{2\})$:

$$\underbrace{Q_i(\overline{y}(\{2\}, \{1\}, \{1\}))}_{i \in \{2\}} - Q_i(\overline{y}(\{1\}, \{1\}, \{1\}, \{1\})) \gtreqless 0,$$

and the sign of the inequality depends on $\beta \gtreqless \widehat{\beta} = -0.30$; similarly,

$$Q_i(\overline{y}(\{2\}, \{2\})) - \underbrace{Q_j(\overline{y}(\{2\}, \{1\}, \{1\}))}_{j \in \{1\}} \gtreqless 0$$

for $\beta \gtreqless \widetilde{\beta} = -0.36$.

Note that in all expressions above, the first terms of the differences denote the per-capita payoffs of nonprofits in the various alliance structures whereas the second terms denote the corresponding payoffs obtained in the alliance structure induced by any deviating nonprofit under the unanimity rule.

3.1.3. *Nash stability under the aggregative rule.* As noted earlier, the unanimity rule is a fairly demanding restriction on the alliance formation process. The more realistic aggregative rule implies that an individual deviation does not automatically lead to the break-up of the alliance of the deviator. In the next proposition, we prove that the grand coalition is Nash stable under the aggregative rule of coalition formation only when strategic complementarity is sufficiently strong.

Proposition 4. *Let $n \geq 3$ and fundraising efforts be strategic complements. The grand coalition of nonprofits $\{N\}$ is Nash stable under the aggregative rule of coalition formation* ***if and only if*** *strategic complementarity is strong enough, i.e. $\beta \geq \beta^*$, with*

$$\beta^* = \frac{\delta\left(8n - 10 - 2n^2\right) + 2\delta\sqrt{28 - 44n + 27n^2 - 8n^3 + n^4}}{n-1}.$$

Proof. See the Appendix. □

For intuition, consider a setting in which fundraising efforts are strategic complements. Under the aggregative rule, following a deviation by a nonprofit from the full coordination agreement, the remaining nonprofits remain in the alliance. Although in this slightly smaller alliance, each member now exerts a slightly higher fundraising effort, such an increase is much less aggressive, as compared to the situation under the unanimity rule. Thus, the disincentives to deviate are definitely weaker, and become insufficient to discipline the potential deviators except when the strategic complementarity is strong enough (i.e. when $\beta \geq \beta^*$).

An implication of this proposition is that if the norm of respecting the agreements is relatively well respected in the nonprofit community, this can paradoxically reduce the viability of the Pareto-efficient full fundraising coordination. Such settings are appropriately described by the aggregative rule, and although most members of a coordination agreement are willing to keep respecting it even if some members do not, this norm reduces the disincentives of the potential deviators and thus weakens the discipline that, contrarily, is present in a community where an agreement fully loses its power as soon as one member deviates.

3.1.4. *Four-nonprofit example (continued).* Figure 2 illustrates the alliance structures that can be induced by single nonprofits under the aggregative rule in the four-player case.

[Figure 2 about here]

The stability of several alliance structures remains the same as under the unanimity rule (for instance, $(\{2\}, \{2\})$ and $(\{2\}, \{1\}, \{1\})$): these structures, which were proven to be stable under the unanimity rule for sufficiently high levels of β (respectively, for $\beta > \widehat{\beta}$ and $\beta > \widetilde{\beta}$), remain stable also over the same thresholds also under the aggregative rule. However, some other alliance structures fail to be stable for lower levels of β under the aggregative rule, as compared to the same levels under the unanimity rule. More precisely:

$$Q_i(\widetilde{y}(\{4\})) - \underbrace{Q_j(\overline{y}(\{3\},\{1\}))}_{j\in\{1\}} \gtreqless 0 \Leftrightarrow \beta \gtreqless \beta^* = 0.194$$

$$\underbrace{Q_i(\overline{y}(\{3\},\{1\}))}_{i\in\{3\}} - \underbrace{Q_j(\overline{y}(\{2\},\{1\},\{1\}))}_{j\in\{1\}} \gtreqless 0 \Leftrightarrow \beta \gtreqless 0.$$

The first expression indicates that the grand coalition is stable if and only if $\beta > 0.194$. The second expression states that the stability of $(\{3\},\{1\})$ holds only if fundraising activities are strategic complements. As discussed above, for sufficiently high levels of β, individual deviations still lead to sufficiently large increases in fundraising by the remaining members and other alliances, and thus imply a strong enough negative impact on nonprofits' payoffs. For this reason, intermediate alliances structures are individually stable even under the less strict aggregative rule of alliance formation. However, as soon as fundraising activities becomes strategic substitutes, the individual deviation by a nonprofit does not lead to an aggressive increase in fundraising by other members of the alliance (and, as a response, by other alliance members). Thus, the disincentives to deviate become generally weaker, and only few alliance structures remain stable under the aggregative rule (see Figure 2).

Aggregative rule is definitely more realistic in the context of nonprofits: it is difficult to imagine nonprofit communities with a very weak norm of respecting coordination agreements, such that a full break-up follows an individual deviation of one of its members. However, we still observe that the multiplicity of stable equilibria remains for large sections of parameters. Thus, we need to dig further to explain the puzzle of absence of Pareto-efficient nonprofit coordination in real life. We do so next, by allowing for richer and more realistic possibilities of deviation from agreements.

3.2. **Coalitional stability of nonprofits alliances.** To be coalitionally stable, alliance structures different from the grand coalition have to be Pareto-optimal and Nash stable against the deviations of *any coalition* of nonprofits. Thus, the *coalitional* stability is a more demanding condition than Nash stability, and the set of stable alliance structures is likely to be smaller. A key question is whether (and when) the grand coalition ceases to be stable once we allow for deviations by coalitions of nonprofits.

In what follows, we prove three major results on the coalitional stability under the unanimity rule: (i) the grand coalition is always coalitionally stable regardless of the strategic complementarity or substitutability of fundraising efforts; (ii) if strategic complementarity is sufficiently strong, it is extremely difficult to encounter coalitionally stable alliance structures (of the form "an alliance versus all remaining nonprofits as singletons") other than the grand coalition; (iii) for lower levels of β, other alliance structures can be coalitionally stable, but have to be asymmetric, i.e. made of alliances of different size.

Under the aggregative rule, our analysis shows that coalitional stability is unlikely and notably the grand coalition ceases to be coalitionally stable when fundraising efforts are strategic substitutes. We then show that a possible way to achieve stability at intermediate levels of complementarity (or even when fundraising efforts are strongly substitutable) is to impose a constraint on the way nonprofits alliances form and dissolve. In particular, we prove that when a *majority breaking protocol* is imposed on a certain alliance structure, i.e. a rule permitting only to majorities of the members to defect from the agreement, the coalitional stability of full coordination can be guaranteed even under the aggregative rule.

3.2.1. *Coalitional stability of the grand coalition.* To prove our result on the coalitional stability of the grand coalition under the unanimity rule, we proceed in two steps. First, we show (in the Appendix) that at the fundraising equilibrium the members of a smaller coalition receive a higher payoff than the members of a bigger coalition. Intuitively, this comes from the fact that in the competition for fundraising, smaller coalitions tend to free-ride on bigger ones which internalize more the negative effect of fundraising effort of its members on each other. Using this result, we then easily show that, regardless of the type of strategic interaction (i.e. the sign of β), the grand coalition is always coalitionally stable under the unanimity rule.

Proposition 5. *Let alliance formation occur by unanimity rule. Then, the grand coalition is always coalitionally stable.*

Proof. A group of nonprofits (or an individual nonprofit) has an interest in deviating from the full coordination agreement only if by doing so it obtains a higher payoff. Under the unanimity rule, this deviation implies that the remnant of the grand coalition breaks down into singleton nonprofits. In the Appendix we show that the non-deviant nonprofits (which now find themselves as singletons) are better off than those in the deviating group. However, this would mean that the sum of payoffs of organizations in deviating and non-deviant groups must be higher than the sum of payoffs in the grand coalition, which is impossible, given that the grand coalition is Pareto-efficient. □

For intuition, suppose the norm of respecting the agreements is relatively weak (i.e. the unanimity rule describes the situation fairly well). Let a group of nonprofits decide to break away from the full coordination agreement, by forming another separate agreement. Given the weakness of the respect of agreements, the remaining non-deviating nonprofits all start to conduct their fundraising activities alone. Although each of them might be worse off than under the full-coordination agreement, any of them does better than any member of the new agreement. However, because of the Pareto efficiency of the grand coalition, the members of this new alliance are necessarily worse off in this situation as compared to the one under the grand coalition, and this should definitely discourage them from breaking away at the first place.

Our second result concerns the conditions for uniqueness of the grand coalition as a coalitionally stable alliance structure.

Proposition 6. *If the strategic complementarity in fundraising activities is sufficiently strong (i.e.* $\overline{\beta} > \beta \geq \beta^* > 0$*), the grand coalition is the* ***unique*** *alliance structure of the form* $S = (\{A\}, \{j\}_{j\in N\setminus A})$ *to be coalitionally stable under the unanimity rule of coalition formation.*

Proof. See the Appendix. □

The intuition behind this proposition is somewhat similar to the one above. Given the sufficiently strong complementarity in fundraising efforts, gains from full coordination are very large. Moreover, all members of a coalition of any size that plays against singletons would like to be part of the grand coalition, in order to reduce the tough competition (implied by high β) and to capture the gains from full coordination. Also, even the nonprofits that play as singletons obtain in this case a larger payoff under full coordination than that of remaining at the fringe. Given that a deviation by coalitions (including the grand coalition) is possible, this guarantees that any coalition that plays against singletons is not stable (in particular, against the coordinated deviation by all the nonprofits); thus, the grand coalition is the only coalitionally stable structure of the form "one coalition against singletons".

The empirical implications of this proposition are interesting. We definitely do not observe full coordination as the unique long-run outcome in donation markets. Given the theoretical uniqueness of the full coordination (among "one large against many small" structures) under the conditions described above, this implies that such conditions (jointly) must not hold in reality. In other words, either the real-life strategic complementarities are never too strong, or there must be at least some respect of the norm of coordination agreements in the nonprofit community.

What happens when the level of fundraising complementarity decreases? Under the unanimity rule, plausibly, alliance structures other than full coordination become coalitionally stable. However, we still can rule out some alliance structures. The next proposition shows that, in order to be stable, such structures have to be necessarily asymmetric, i.e. made of asymmetric alliances.

Proposition 7. *Regardless of the rule of alliance formation, no partition of nonprofits* $S^E = \{A_1, A_2, ..., A_m\}$ *, such that every alliance possesses the same size* $a_1 = a_2 = ... = a_m$*, can be coalitionally stable.*

Proof. See the Appendix. □

An alliance structure with alliances of equal size is not coalitionally stable because, given symmetry, all nonprofits under such a structure are worse off as compared to the outcomes under full coordination. Therefore, such a structure is not robust to a concerted grand-coalition deviation announcement. If coalitionally stable confederations of nonprofits different from the grand coalition exist, they have to be formed by asymmetric alliances.

This proposition implies that, for instance, two similar-sized fundraising confederations of nonprofits cannot be observed as a long-run equilibrium outcomes: all nonprofits in both confederations sooner or later realize that there is scope for a beneficial negotiation across confederations to form one large full-coordination agreement. Similarly, it implies that we should never observe every nonprofit conducting its fundraising on its own: there will always be attempts to build fundraising-coordination agreements by at least some nonprofits.

3.2.2. *Coalitional stability of intermediate-size coalitions.* So far we know that, when the strategic complementarity in fundraising activities is sufficiently weak, alliance structures other than the full-coordination agreement can be coalitionally stable under the unanimity rule. Specifically, proposition 6 has proven that for $\beta \geq \beta^* > 0$ all members of the grand coalition receive a higher payoff than when they play as singletons against the coalition of all remaining players. This also means that for $\beta < \beta^*$the alliance structure $(\{N-1\}, \{1\})$ is necessarily coalitionally stable under the unanimity rule. The reason is that now some nonprofits prefer to remain independent and do not have an interest in joining the grand coalition. In particular, the next proposition finds a threshold of β (denoted β^{**}) below which the fringe of singletons competing with a generic alliance A does not find it attractive to form the grand coalition.

Proposition 8. *If* β *is sufficiently small (i.e.* $\beta^* > \beta^{**} > \beta > \underline{\beta}$*), all alliance structures of the form* $S = \left(\{A\}, \{j\}_{j \in N \setminus A}\right)$ *with* $A \subset N$*, are coalitionally stable under the unanimity rule of coalition formation.*

Proof. See the Appendix. □

The intuition behind this proposition is as follows. Intermediate alliance structures can be coalitionally stable for two reasons. On the one hand, the nonprofits that have formed alliance A cannot succeed to convince the fringe of singletons to form the grand coalition; in other words, forming a larger agreement is beyond reach. On the other hand, since competition is still very harmful at this level of β, all coalitional and individual deviations from the existing alliances are detrimental to nonprofits' payoffs; therefore, there is no interest in breaking up the existing intermediate-size agreement.

If the norm of respecting the fundraising coordination agreements is relatively weak in the nonprofit community (i.e. the unanimity rule applies), and, moreover, the strategic complementarity in fundraising is not too strong, we can observe stable alliance structures with an intermediate-sized coalition (plus other nonprofits as singletons). The weakness of the norm makes sure that the members of the alliance do not break away from it. However, the allure of an even weaker fundraising competition is not sufficiently strong for the alliance to convince the other (fringe) nonprofits to join in.

3.2.3. *Four-nonprofit example (continued).* Using the payoffs of the four-nonprofit example, we can easily check that

$$Q_i(\widetilde{y}(\{4\})) - Q_i(\overline{y}(\{2\},\{2\})) = \frac{1}{4}\frac{(1-3\beta)(1-c)\beta^2}{(2\beta-1)^2} > 0,$$

whereas, as we have seen earlier,

$$Q_i(\widetilde{y}(\{4\})) - Q_i(\overline{y}(\{1,1,1,1\})) > 0.$$

This illustrates Proposition 7: no symmetric intermediate-size alliances can be stable against coalitional deviations.

We can also easily establish that

$$\begin{aligned} Q_i(\widetilde{y}(\{4\})) - \underbrace{Q_j(\overline{y}(\{3\},\{1\}))}_{j\in\{1\}} &\gtreqless 0 \Leftrightarrow \beta^* \gtreqless 0.194, \\ Q_i(\widetilde{y}(\{4\})) - \underbrace{Q_j(\overline{y}(\{2\},\{1\},\{1\}))}_{j\in\{1\}} &\geq 0 \Leftrightarrow \beta > \underline{\beta}. \end{aligned} \tag{3.1}$$

Thus, within the range $\beta \in (\beta^*, \overline{\beta})$ the grand coalition is the only coalitionally stable alliance structure under the unanimity rule. When, conversely, the level of fundraising complementarity becomes lower, other intermediate alliance structures (in our example, $(\{3\},\{1\})$) become stable. One can verify that

$$\begin{aligned} \underbrace{Q_i(\overline{y}(\{3\},\{1\}))}_{i\in\{3\}} - \underbrace{Q_i(\overline{y}(\{2\},\{1\},\{1\}))}_{i\in\{2\}} &> 0 \text{ for } \beta \in (\underline{\beta},0) \cup (0,\overline{\beta}), \\ \underbrace{Q_i(\overline{y}(\{3\},\{1\}))}_{i\in\{3\}} - Q_i(\overline{y}(,\{1\},\{1\},\{1\},\{1\})) &> 0 \text{ for } \beta \in (\underline{\beta},0) \cup (0,\overline{\beta}), \text{ and} \\ \underbrace{Q_i(\overline{y}(\{3\},\{1\}))}_{i\in\{3\}} - \underbrace{Q_j(\overline{y}(\{2\},\{1\},\{1\}))}_{j=\{1\}} &\gtrless 0 \text{ for } \beta \gtrless 0. \end{aligned} \tag{3.2}$$

The first two inequalities jointly imply that the structure $(\{3\}, \{1\})$ is coalitionally stable for any β in the feasible range under the unanimity rule. No other intermediate alliance structures are coalitionally stable. This is illustrated in Figure 3.

[Figure 3 about here]

The last inequality of (3.2) implies that $(\{3\}, \{1\})$ is also coalitionally stable for any $\beta \geq 0$ under the aggregative rule. Furthermore, equation (3.1) also shows us that under the aggregative rule, for $\beta < \beta^*$ the grand coalition stops to be coalitionally stable. This might be the explanation behind the case of World Vision that we discussed in the introduction: there was an explicit agreement to coordinate fundraising (between the World Vision and Australian Council of Churches); the World Vision broke away from this agreement; thus, a structure of "one large alliance versus a singleton" was formed (and remained stable). Note that this occurred despite being in a situation of humanitarian emergency, which - as we have discussed above - results in a relatively low value of γ; however, the fundraising technology used in 1980s was still essentially untargeted (e.g. direct mail), thus generating substantial spillovers. This results in a high value of Δ. Together with a low γ, this maps into an intermediate value of β (in particular, $\beta < \beta^*$).

In addition, when fundraising activities are strategic substitutes ($\beta < 0$), it is impossible for any type of alliance structure to remain stable under the aggregative rule (see Figure 4). Any structure is vulnerable to deviation towards some other structure. In particular, $(\{4\})$ is vulnerable to a deviation to $(\{3\}, \{1\})$ (from the first inequality in (3.1)), and $(\{1, 1, 1, 1\})$ is always dominated by $(\{4\})$. In turn,$(\{3\}, \{1\})$ is vulnerable to a deviation to $(\{2\}, \{1\}, \{1\})$ (from the third inequality in (3.2)), while $(\{2\}, \{1\}, \{1\})$ is vulnerable to a joint deviation to $(\{4\})$ (from the second inequality of (3.1)).

[Figure 4 about here]

The above analysis of our simple example highlights a further interesting point. When strategic complementarity in fundraising activities is sufficiently weak (for instance, if fundraising efforts are strategic substitutes), we might observe a lot of dynamics in formation and break-up of fundraising coordination agreements. The whole nonprofit sector then might exhibit behavior that looks like it is unable to build lasting alliances (but, at the same time, keep trying to organize itself into alliances).

3.2.4. *Coalitional stability under the aggregative rule: majority-breaking protocol.* The analysis and the example above suggests that the lack of stability in fundraising coordination agreements is essentially driven by allowing for (any kinds of) coalitional deviations, and that this is particularly powerful under the aggregative rule. One may wonder to which extent some stability (in particular, of the grand coalition) can be attained if we were to impose some restrictions on the way in which a given alliance could dissolve. Below we show that the stability of the grand coalition is restored when the decision to dissolve any alliance can be taken only by the *majority* of its members. In this case, every deviating alliance of nonprofits by definition consists of a number of members greater or equal than $n/2$.[9]

Definition 3. *A majority breaking protocol holds in any arbitrary alliance of nonprofits $A \subset N$ if and only if the decision to deviate from A can be taken only by a majority of its members.*

[9] Any game of coalition formation based on a unanimity or aggregative rule can be constrained to a majority protocol by simply restricting the coalitional payoff of every coalition $A \subset N$ to be equal to zero for $|A| < |N|/2$ as happens in majority games. See Ray 2007: 289, for a detailed discussion.

We can easily apply Lemma 2 (in the Appendix) to every alliance structure $S^A = (\{A\}, \{N \backslash A\})$, in which $a \geq n/2$. In this case, for alliance structure S^A the following results hold: (i) $\overline{y}_i \leq \overline{y}_j$ under negative externalities in fundraising, and (ii) $\overline{y}_i \geq \overline{y}_j$ under positive externalities in fundraising, for every nonprofit $i \in A$ and $j \in N \backslash A$. As a result, $Q_i\left(\overline{y}\left(S^A\right)\right) \leq Q_j\left(\overline{y}\left(S^A\right)\right)$, for every nonprofit $i \in A$ and $j \in N \backslash A$. This implies, in turn, the following:

Proposition 9. *Let $\beta \in \left(\underline{\beta}, 0\right) \cup \left(0, \overline{\beta}\right)$ and a majority breaking protocol hold in N. Then, under the aggregative rule of alliance formation, full coordination is coalitionally stable.*

Proof. By Definition 3, only a majority of nonprofit can break the grand coalition. Above, we have shown that $Q_i\left(\overline{y}\left(S^A\right)\right) \leq Q_j\left(\overline{y}\left(S^A\right)\right)$. Then, applying Definition 5 and Lemma 2 implies that full coordination is coalitionally stable. □

The intuition for the above results is as follows. Since a majority breaking protocol is imposed by assumption to the grand coalition of nonprofits, the only feasible deviations that can concretely take place are those made by groups of nonprofits with a size greater or equal than half of all the nonprofits operating in the market. In this case, under the aggregative rule of alliance formation the deviating group (A) will compete against a smaller alliance $N \backslash A$ and, therefore, any of its member $i \in A$ will exert a fundraising effort greater than any $j \in N \backslash A$ under positive externalities (or a smaller one, under negative externalities). The fact that small groups are relatively more advantaged than big groups in the fundraising market depends on the symmetry of players and on the presence of externalities. As a result, we have (as shown in the Appendix) that at the fundraising equilibrium every nonprofit in $N \backslash A$ is better off than any nonprofit belonging to A. Therefore, by the efficiency of the grand coalition, it is impossible that a deviating majority of nonprofits improves upon its allocation of output obtained in the grand coalition.

3.3. **Implications and public policies.** What do all the above results jointly imply for the fundraising coordination by nonprofits operating in real-life donation markets?

First, when the projects proposed by the nonprofits are considered by donors as highly differentiated (γ is high), when the main issue tackled by the nonprofits is relatively new and unknown to donors, and when the fundraising technology allows for a relatively poor targeting of donors (i.e. there are substantial awareness spillovers and Δ is relatively high), nonprofits fundraising activities generate large positive externalities on each other, and the incentives to free-ride on others' fundraising are strong. In such settings, the disciplining factors (that help to sustain coordination) are very weak, and we should observe very little stability of fundraising coordination agreements.

Second, and contrarily, when the issue is well-known to donors, the technology of fundraising allows targeting donors rather well (low Δ), and when donors consider the beneficiaries of the nonprofits' projects as being the same group (low γ), the strategic complementarity of fundraising efforts is very strong and fundraising generates strong negative externalities on nonprofits' payoffs. Generally, these factors acts as strong disciplining device, and the grand coalition becomes a stable outcome (and sometimes the only stable outcome). This might explain, for instance, why all the examples of successful coordination that we talk about in the introduction have occurred during humanitarian emergencies.

Finally, when the factors that we mentioned above become somewhat weaker, intermediate alliance structures (as, for example, one large alliance and one or several nonprofits refusing to join in) can become a stable outcome.

In general, our results indicate that improvements in targeting of fundraising (for example, fundraising on the Internet, that exploits the past search behavior of a potential donor for deciding whether to solicit her donation or not) should lead to more stability of coordination agreements. Also, across sectors in which nonprofits operate, we should observe less stability of fundraising coordination agreements where nonprofits have had sufficient time to establish to distinguish their intended beneficiaries as very specific groups, as compared to sectors where such differentiation has not yet occurred.

Concerning public policies, our analysis conducted so far raises a natural question: can public policies affect the incentives of nonprofits to form stable fundraising coordination agreements? We can now briefly discuss the effects of various public policies, and start by looking at two widely used policies towards nonprofits: direct grants and matching grants (i.e. grants that supplement nonprofit funding in proportion to the funds raised from private donations).

Denote with s the amount of direct grant that a i-th nonprofit receives, and with m the match ratio of the matching grant. Then, its non-distribution constraint (2.4) now writes as

$$F_i = (1-c)(1+m)d_i L - f + s, \tag{3.3}$$

and its output becomes

$$Q_i(y_i, y_{-i}; m, s) = (1-y_i)\,(1-c)\,(1+m)(\widetilde{\alpha} + \delta y_i - \beta \sum_{j\neq i} y_j), \tag{3.4}$$

where

$$\widetilde{\alpha} = \frac{L\,(\omega-1)}{\gamma+n} - \frac{f-s}{(1-c)(1+m)}.$$

It is easy to see that now

$$\frac{\partial^2 Q_i}{\partial y_i \partial y_j} = \beta(1+m)(1-c) \gtreqless 0 \text{ for } \beta \gtreqless 0. \tag{3.5}$$

From (3.5), one clearly observes that direct grants or matching grants do not affect whether fundraising efforts are strategic complements or substitutes. Indeed, direct grants do not appear in equation (3.5) at all, while matching grants have an effect on the absolute level but not on the sign of the cross derivative $\partial^2 Q_i/\partial y_i \partial y_j$. Moreover, since matching grants enter in a similar manner in every expressions of a nonprofit's payoff, they do not influence the incentives to maintain or to break away from an agreement, and thus, ultimately, they do not cause any direct change on the stability of alliance structures. Thus, in our framework it turns out that, while grants affect the equilibrium levels of fundraising effort of nonprofits[10], they do not substantially influence the pattern of alliance formation among these organizations.

Other policy interventions, however, might be more effective. For instance, some public policies can directly affect the strategic complementarity of fundraising efforts of nonprofits, via β. For instance, if the awareness campaigns about the issues towards which most nonprofits operate is done by the public sector entities (e.g. the ministry of health), then such a policy reduces the awareness spillovers of fundraising activities of nonprofits (Δ decreases). Alternatively, if government subsidizes the cost of fundraising campaigns via a technology that allows for precise targeting of donors, e.g. consumer-analytics based solicitations via Internet (as compared to non-targeting technologies, such as direct mailing), this also would

[10]Both direct and matching grants have a level effect on equilibrium fundraising efforts that operates through $\widetilde{\alpha}$.

reduce the spillovers (and thus lower the value of Δ). Both such policies would increase the strategic complementarity of fundraising and, according to our model, stabilize the large coordination agreements between nonprofits.

4. Conclusion

As some nonprofit practitioners write, "Umbrella campaigns, such as the United Way, have traditionally been one means of reducing wasteful competition among nonprofits. But [...] pressures to allow greater donor choice have had the effect of reviving such competition. Although it may be desirable in principle to reduce incentives for organizations to engage in socially wasteful competition, it is more challenging in practice to develop either new social institutions, or policies that limit such spending" (Cordes and Rooney 2003). Our model provides indications on what helps and what undermines the stability of nonprofit cooperation, by taking into account the strategic effects and incentives.

The main message of our analysis is that the stability of voluntary coordination agreements between nonprofits depends on (i) the nature of strategic interactions in nonprofit fundraising (the extent of strategic substitutability or complementarity), (ii) on what happens to the remaining alliance members upon a break-away by one member (or a group of members), and (iii) on whether such deviations are made only by individual nonprofits or by groups of organizations. The first feature, i.e. the nature of strategic interactions in fundraising, varies between donation markets and crucially depends on the technology of fundraising (the extent of awareness spillovers and the degree of perceived substitutability of nonprofits' projects by donors). The second feature depends on the expectational context of the nonprofit sector (the norm - or even formal procedures - concerning the acceptability of ending the coordination agreement upon an exit by only one or a few members). We connect these various features into a common framework that can help to address public actions that might improve the stability of nonprofit coordination when such coordination is desirable from the social point of view.

One important caveat is worth mentioning. The discussion of efficiency throughout the model assumes that we look at the efficiency taking only the payoffs of nonprofits into account. The beneficiaries of the nonprofit projects are not portrayed explicitly. It is quite possible that in a more complete model (i.e. the one that includes the beneficiaries as active players), the analysis of efficiency substantially differs from the one developed here. For instance, in a setting with the Samaritan's dilemma (Buchanan 1975), the efficient output level by the nonprofits would be lower than the one derived here.

While our model concentrates on the nonprofit interaction along the fundraising dimension, there are several other dimensions along which nonprofit compete and (possibly) coordinate. These include, for instance, physical location of operations between and within developing (beneficiary) countries (Koch 2007), and emphasis on urgent versus long-run projects (Brown and Minty 2006)). Also, some interesting case studies (Gugerty 2008, Gugerty and Prakash 2010) underline that one important role that voluntary agreements between nonprofits play is that of enhancing the accountability towards donors. This indicates that there exist some key informational asymmetries between nonprofits and donors, and the desire to solve them might result in an incentive (and sustain) to form such agreements. Both these avenues call for future work that would integrate these dimensions of the donation markets into a framework similar to the one built in this paper.

5. Appendix

5.1. **Four-player example.** Let us apply the model to a simple economy with $n = 4$ nonprofits and $L = 1$ donors. Normalize the parameters of the model as follows:

$$f = \frac{(1-c)(\omega-1)}{\gamma+4}, \text{ which implies } \alpha = 0, \text{ and } \delta = \frac{(\gamma+3+\gamma\Delta)}{\gamma(\gamma+4)} = 1.$$

Using (2.1), the equilibrium amount of donations received by every nonprofit $i \in N$ becomes

$$d_i = \frac{\omega_i(\gamma+3) - \gamma - \sum_{j\neq i}\omega_j}{\gamma(\gamma+4)}.$$

Given the above normalization,

$$\Delta = \frac{3(\gamma-1)+\gamma^2}{\gamma},$$

and, therefore,

$$\beta = \frac{(1-\gamma\Delta)}{\gamma(\gamma+4)} = \frac{1-\gamma}{\gamma} \gtreqless 0 \Longleftrightarrow \gamma \lesseqgtr 1.$$

The required boundaries for β are also easily obtained as $-2/(n-1) = \underline{\beta} < \beta < \overline{\beta} = 1/(n-1)$, i.e. $-0.66 = \underline{\beta} < \beta < \overline{\beta} = 0.33$. We can now compute the equilibrium fundraising levels in any feasible alliance structure $S \in \mathcal{S}$ and the corresponding payoffs (see the Tables 1 and 2 below).

Alliance structure	**Effort of** $i \in A_k$	**Effort of** $j \in N\backslash A_k$
$S_1 = (\{4\})$	$\widetilde{y}_i = \frac{1}{2}$	$--$
$S_2 = (\{3\},\{1\})$	$\overline{y}_i = \frac{(3\beta-2)}{(8\beta+3\beta^2-4)}$	$\overline{y}_j = \frac{(2\beta-1)(3\beta+2)}{(8\beta+3\beta^2-4)}$
$S_3 = (\{2\},\{2\})$	$\overline{y}_i = \frac{(\beta-1)}{2(2\beta-1)}$	$\overline{y}_j = \frac{(\beta-1)}{2(2\beta-1)}$
$S_4 = (\{2\},\{1\},\{1\})$	$\overline{y}_i = \frac{1}{2}\frac{(\beta^2-\beta+2)}{(2-3\beta-\beta^2)}$	$\overline{y}_j = \frac{(\beta+1)(\beta-1)}{(3\beta+\beta^2-2)}$
$S_5 = (\{1\},\{1\},\{1\},\{1\})$	$\overline{y}_i = \frac{1}{2-3\beta}$	$\overline{y}_j = \frac{1}{2-3\beta}$

Table 1. Effort levels of nonprofits inside alliance A_k and in smaller (or equal size) alliances in $N\backslash A_k$.

Alliance structure	**Payoff of** $i \in A_k$	**Payoff of** $j \in N\backslash A_k$
$S_1 = (\{4\})$	$Q_i(\widetilde{y}) = \frac{(1-3\beta)(1-c)}{4}$	$--$
$S_2 = (\{3\},\{1\})$	$Q_i(\overline{y}) = \frac{(3\beta-1)^2(1-2\beta)(\beta+2)^2(1-c)}{(8\beta+3\beta^2-4)^2}$	$Q_j(\overline{y}) = \frac{(3\beta-1)^2(\beta-2)^2(1-c)}{(8\beta+3\beta^2-4)^2}$
$S_3 = (\{2\},\{2\})$	$Q_i(\overline{y}) = \frac{1}{4}\frac{(1-\beta)(3\beta-1)^2(1-c)}{(2\beta-1)^2}$	$Q_j(\overline{y}) = \frac{1}{4}\frac{(1-\beta)(3\beta-1)^2(1-c)}{(2\beta-1)^2}$
$S_4 = (\{2\},\{1\},\{1\})$	$Q_i(\overline{y}) = \frac{(3\beta-1)^2(1-\beta)(\beta+2)^2(1-c)}{4(3\beta+\beta^2-2)^2}$	$Q_j(\overline{y}) = \frac{(3\beta-1)^2(1-c)}{(3\beta+\beta^2-2)^2}$
$S_5 = (\{1\},\{1\},\{1\},\{1\})$	$Q_i(\overline{y}) = \frac{(1-3\beta)^2(1-c)}{(2-3\beta)^2}$	$Q_j(\overline{y}) = \frac{(1-3\beta)^2(1-c)}{(2-3\beta)^2}$

Table 2. Payoffs of nonprofits inside alliance A_k and in smaller (or equal size) alliances in $N\backslash A_k$.

5.2. **Omitted Proofs.**

5.2.1. *Proof of Proposition 1.* To prove proposition 1, we start by showing a necessary and sufficient condition for Nash stability under the unanimity rule. This is the property (denoted as *individual rationality*) that the payoffs of any nonprofit in any coalition are at least as good as in a situation in which all members of that coalition play as singletons (keeping unaltered the structure of remaining coalitions). For this purpose, it is convenient to introduce the following

Definition 4. *A nonprofit alliance structure $S = (A_1, A_2, ..., A_k, ..., A_m)$ is individually rational (IR) if, for every $i \in A_k$ and each coalition $A_k \in S$ (for $k = 1, 2, .., m$)*

$$Q_i(\overline{y}\,(A_1, A_2, ..., A_k, ..., A_m)) \geq Q_i\left(\overline{y}\left(A_1, A_2, ..., \{i\}_{i\in A_k}, ..., A_m\right)\right), \tag{5.1}$$

where $\{i\}_{i\in A_k}$ indicates that the nonprofits previously in A_k play now as singletons.

Then, the following lemma holds:

Lemma 1. *Under the unanimity rule of alliance formation, an alliance structure $S = (A_1, A_2, ..., A_m)$ is Nash stable if and only if it is individually rational (IR).*

Proof. Let (5.1) hold for every coalition in S, and suppose every member of each alliance has made the same announcement. Then, the alliance structure S is Nash stable under the unanimity rule, because no player has an incentive to change its announcement. If (5.1) does not hold, the members of one or more alliances $A_k \in S$ have an incentive to deviate. Therefore, (5.1) is necessary and sufficient for the Nash stability of any alliance structure $S \in \mathcal{S}$. □

Proposition 1. *When fundraising efforts are strategic complements ($\beta > 0$), all alliance structures $S \in S$ are Nash stable under the unanimity rule of coalition formation.*

Proof. We first prove that for $\beta > 0$ the equilibrium fundraising of every nonprofit in coalition A_k is lower than when playing as singleton: $\overline{y}^i_{\{i\}} \geq \overline{y}^i_{A_k}$, keeping fixed the partition of remaining nonprofits (here simply denoted $N\backslash A_k$), independently of the organization in coalitions of the $(n - a_k)$ nonprofits. By the equilibrium property of the fundraising strategy of every $i \in A_k$, denoted $\overline{y}^i_{A_k}$, we can write

$$Q_{A_k}(\underbrace{\overline{y}^i_{A_k}, \overline{y}^i_{A_k}, ..., \overline{y}^i_{A_k}}_{a_k}, \overline{y}_{N\backslash A_k}) \geq Q_{A_k}(\underbrace{\overline{y}^i_{\{i\}}, \overline{y}^i_{\{i\}}, ..., \overline{y}^i_{\{i\}}}_{a_k}, \overline{y}_{N\backslash A_k}), \tag{5.2}$$

where $\overline{y}^i_{\{i\}}$ is the equilibrium strategy of nonprofits when playing as singletons. Similarly, in equilibrium, for every nonprofit playing as singleton,

$$Q_i(\underbrace{\overline{y}^i_{\{i\}}, \overline{y}^i_{\{i\}}, ..., \overline{y}^i_{\{i\}}}_{a_k}, \overline{y}_{N\backslash A_k}) \geq Q_i(\underbrace{\overline{y}^i_{\{i\}}, \overline{y}^i_{A_k}, ..., \overline{y}^i_{\{i\}}}_{a_k}, \overline{y}_{N\backslash A_k}). \tag{5.3}$$

Using the symmetry of all nonprofits in A_k, and (5.2) and (5.3), we obtain

$$a_k \cdot Q_i\left(\overline{y}^i_{A_k}, \overline{y}^i_{A_k}, ..., \overline{y}^i_{A_k}, \overline{y}_{N\backslash A_k}\right) \geq a_k \cdot Q_i\left(\overline{y}^i_{\{i\}}, \overline{y}^i_{A_k}, ..., \overline{y}^i_{\{i\}}, \overline{y}_{N\backslash A_k}\right). \tag{5.4}$$

By negative fundraising externalities, expression (5.4) implies that $\overline{y}^i_{\{i\}} \geq \overline{y}^i_{A_k}$, for every $i \in A_k$. Since for $\beta > 0$ fundraising activities are strategic complements, $y_{A_h}(\overline{y}^i_{\{i\}}) \geq y_{A_h}(\overline{y}^i_{A_k})$, where $y_{A_h}(.)$ indicates the best-reply of every alliance A_h in $N\backslash A_k$. Similarly, strategic complementarity implies that, for every $i \in A_k$, $y^i_{A_k}(y_{A_h}(\overline{y}^i_{\{i\}})) \geq y^i_{A_k}(y_{A_h}(\overline{y}^i_{A_k}))$.

This process of adjustments through increasing best-replies implies that, for every generic coalition A_h in $N \backslash A_k$,

$$\overline{y}_{A_h}(A_1, A_2, ..., A_k, ..., A_m) \leq \overline{y}_{A_h}\left(A_1, A_2, ..., \{i\}_{i \in A_k}, ..., A_m\right),$$

and, therefore, by negative fundraising externalities, we have that for every $i \in A_k$,

$$Q_i(\overline{y}(A_1, A_2, ..., A_k, ..., A_m)) \geq Q_i\left(\overline{y}\left(A_1, A_2, ..., \{i\}_{i \in A_k}, ..., A_m\right)\right).$$

This proves that the alliance structure $S = (A_1, ..., A_m)$ is individually rational (IR) and therefore, by Lemma 1, it is Nash stable. □

5.2.2. *Proof of Proposition 3.* **Proposition 3.** *Under the unanimity rule of coalition formation, the alliance structures of the form $S = (\{A\}, \{j\}_{j \in N \backslash A})$ fail to be Nash stable if $\underline{\beta} < \beta < \widehat{\beta}(n, a)$, with $\widehat{\beta}(n, a) < 0$.*

Proof. The individual rationality of an alliance structure of the form $S = (\{A\}, \{j\}_{j \in N \backslash A})$ requires that

$$Q_i(\overline{y}(\{A\}, \{j\}_{j \in N \backslash A})) - Q_i\left(\overline{y}\left(\{i\}_{i \in N}\right)\right) \geq 0,$$

which holds if and only if

$$\frac{(1-c)(\delta - \beta(a-1))(\beta(n-1) - \delta - \alpha)^2 (\beta + 2\delta)^2}{\left(2\beta\delta(3-n-a) + 4\delta^2 + \beta^2(2 - 2n + na - a^2)\right)^2} - \frac{(1-c)\delta(\delta + \alpha - \beta(n-1))^2}{(2\delta - \beta(n-1))^2} \geq 0.$$

Straightforward calculations show that, for any n and a there exist a (negative) value $\beta = \widehat{\beta}(n, a)$ that respects the above inequality. Depending on the range of the model parameters and the values of n and a, the level of $\widehat{\beta}(n, a)$ may or may not be contained in $\beta \in (\underline{\beta}, 0)$. For instance, for a set of parameters such that $\alpha = 0$, $\delta = 1$ and $c = 0$, we obtain that, $\widehat{\beta}(3, 2) = -0.55$, $\widehat{\beta}(4, 2) = -0.31$, $\widehat{\beta}(5, 2) = -0.21$, $\widehat{\beta}(5, 3) = -0.48$, $\widehat{\beta}(6, 2) = -0.16$, $\widehat{\beta}(6, 3) = -0.33$, $\widehat{\beta}(7, 2) = -0.13$, $\widehat{\beta}(7, 3) = -0.25$, and so on. All these values are contained in the admissible interval for β. It follows that, when β is sufficiently negative and alliance A sufficiently small, the corresponding alliance structures $S = (\{A\}, \{j\}_{j \in N \backslash A})$ fails to be Nash stable. □

5.2.3. *Proof of Proposition 4.* For the analysis of Nash stability of alliance structures under the aggregative rule, we need to introduce a property that extends the concept of *stand-alone stability* (Yi, 1997, 2003) to any alliance structure.

Definition 5. *An alliance structure $S = (A_1, A_2, ..., A_k, ..., A_m)$ is stand-alone stable (SAS) if, for every member $i \in A_k$ and every coalition A_k in S,*

$$Q_i(\overline{y}(A_1, A_2, ..., A_k, ..., A_m)) \geq Q_i(\overline{y}(A_1, A_2, ..., A_k \backslash \{i\}, \{i\}, ..., A_m)). \tag{5.5}$$

Stand-alone stability in our model is stronger than individual rationality and implies that there is no incentive for a nonprofit to leave its alliance while expecting in response that the members of the alliance remain together (again, keeping unaltered the structure of remaining coalitions). It is easy to prove that this condition is necessary and sufficient for the Nash stability of an alliance structure under the *aggregative* rule.

Lemma 2. *Under the aggregative rule of alliance formation, any alliance structure $S = (A_1, ..., A_m)$ is Nash stable if and only if it is stand-alone stable.*

Proof. If (5.5) holds for every alliance A_k in S, then S is Nash stable under the aggregative rule, as no nonprofit has an incentive to split away from its alliance. If (5.5) does not hold for some $A_k \in S$, then at least one nonprofit in A_k has an incentive to deviate. Therefore, (5.5) is necessary and sufficient for the stability of S. □

Proposition 4. *Let $n \geq 3$ and fundraising efforts be strategic complements. The grand coalition of nonprofits $\{N\}$ is Nash stable under the aggregative rule of coalition formation if and only if strategic complementarity is strong enough, i.e. $\beta \geq \beta^*$, with*

$$\beta^* = \frac{\delta\left(8n - 10 - 2n^2\right) + 2\delta\sqrt{28 - 44n + 27n^2 - 8n^3 + n^4}}{n-1}.$$

Proof. By straightforward manipulations of the coalitional payoffs and of (2.16), the stand-alone stability of the grand coalition is satisfied only if

$$Q_i(\{N\}) - Q_i\left(\{i\}, \{N \backslash \{i\}\}\right) =$$

$$= \frac{\left(\alpha + \delta - \beta\left(n-1\right)\right)^2}{4\left(\delta - \beta\left(n-1\right)\right)} - \frac{\delta\left(\alpha + \delta - \beta\left(n-1\right)\right)^2\left(n\beta - 2\delta - 3\beta\right)^2}{\left(4n\beta\delta - 8\beta\delta - \beta^2 - 4\delta^2 + n\beta^2\right)^2} \geq 0,$$

which can be shown to hold for

$$\left(20\beta\delta - 16n\beta\delta - \beta^2 + 12\delta^2 + n\beta^2 - 4n\delta^2 + 4n^2\beta\delta\right) \geq 0,$$

requiring, for any $n \geq 3$, that

$$\beta \geq \frac{\delta\left(8n - 10 - 2n^2\right) + 2\delta\sqrt{28 - 44n + 27n^2 - 8n^3 + n^4}}{(n-1)} > 0.$$

Concerning the existence of a value of β such that $\overline{\beta} > \beta \geq \beta^*$, note that for $n \geq 3$ the difference $B = \left(\overline{\beta} - \beta^*\right)$ is monotonically increasing in δ and monotonically decreasing in α. For $\alpha = 0$, B turns out to be positive for any $n \geq 3$ and $\delta \geq 0$ and, hence, a value of $\beta^* < \overline{\beta}$ always exists. In this specific case, $\lim_{n \to \infty} B(n) = 0$, hence, the higher the number of nonprofits competing for funds, the more β^* has to approach the upper bound $\overline{\beta}$ for the grand coalition to be stand-alone stable. For $\alpha > 0$, the condition $B > 0$ requires instead a ratio $\alpha/\delta \leq x(n)$, where $x(n) \in (0, 1)$ is decreasing in the number of existing nonprofits: $x(n) = 21\%$ for $n = 5$, $x(n) = 1\%$ for $n = 10$, $x(n) = 0.23\%$ for $n = 100$, and so on. This shows that high levels of f and ω, both reducing α, facilitate the existence of a $\beta^* < \overline{\beta}$. □

5.2.4. *Proof of Proposition 6.* We first prove the two following Lemmata. The first shows that, at the fundraising equilibrium, regardless of the form of the alliance structure, a smaller coalition always exerts a higher (lower) fundraising effort than a bigger coalition under negative (positive) fundraising externalities. The second uses this result to prove that at the fundraising equilibrium the members of a smaller coalition receive a higher payoff than the members of a bigger coalition.

Lemma 3. *Let the fundraising activities of nonprofits be strategic complements (substitutes). Then, at the fundraising equilibrium associated with a generic alliance structure $S = (A_1, ..., A_m)$, for any two alliances A_k and A_h with $a_k \geq a_h$, any nonprofit in alliance A_k exerts a weakly lower (higher) fundraising effort than any nonprofit in A_h.*

Proof. Suppose, by contradiction, that if $\beta > 0$, for $i \in A_k$ and $j \in A_h$, $y_i > y_j$. By the first-order condition for a maximum of every $i \in A_k$, we have

$$(1-c)\delta\,(1-y_i) - (1-c)D_i(y_i, y_{-i}) - \beta\,(a_k-1)\,(1-y_i)\,(1-c) = 0, \tag{5.6}$$

where, by the symmetry of every $i \in A_k$,

$$D_i(y_i, y_{-i}) = (\alpha + \delta y_i - \beta \sum_{l \neq k} a_l y_l - \beta(a_k-1)y_i) \tag{5.7}$$

is the donors' revenue raised by nonprofit $i \in A_k$ in alliance structure $S = (A_1, ..., A_m)$ and y_l is the fundraising effort of every nonprofit in a generic alliance $A_l \neq A_k$. Similarly, for every $j \in A_h$, the first-order condition is

$$(1-c)\delta\,(1-y_j) - (1-c)D_j(y_j, y_{-j}) - \beta\,(a_h-1)\,(1-y_j)\,(1-c) = 0, \tag{5.8}$$

where

$$D_j(y_j, y_{-j}) = (\alpha + \delta y_j - \beta \sum_{l \neq h} a_l y_l - \beta(a_h-1)y_j). \tag{5.9}$$

Rearranging the expressions (5.6)-(5.9), it follows that

$$\delta - \alpha - 2\delta y_i + \beta a_1 y_1 + ... + \beta a_m y_m - \beta y_i - \beta\,(a_k-1) = 0$$

and

$$\delta - \alpha - 2\delta y_j + \beta a_1 y_1 + ... + \beta a_m y_m - \beta y_j - \beta\,(a_h-1) = 0,$$

which, for $a_k > a_h$ and $y_i > y_j$, leads to a contradiction. Similarly, for $\beta < 0$ it can be proven that $a_k > a_h$ is in contradiction with $y_j > y_i$. □

Lemma 4. *In any alliance structure $S = (A_1, A_2, ..., A_m)$, and for every $i \in A_k$ and $j \in A_h$ such that the size of alliances are $a_k \geq a_h$, the equilibrium outputs are such that $Q_j\,(\overline{y}) \geq Q_i\,(\overline{y})$.*

Proof. By the definition of an equilibrium strategy, we can write

$$Q_j\,(\overline{y}) \geq Q_j\left(\overline{y}_{A_k}, \overline{y}_{A_h\backslash\{j\}}, \overline{y}_i, \overline{y}_{N\backslash\{A_k\cup A_h\}}\right), \tag{5.10}$$

expressing the simple fact that if we let a nonprofit $j \in A_h$ switching its fundraising level with that of any nonprofit in alliance $i \in A_k$, whereas the remaining nonprofits in $N\backslash\{A_k \cup A_h\}$ continue to play as before, its payoff, by definition, will not improve. By Lemma 3 we know that if the size of the two alliances are such that $a_k \geq a_h$ and fundraising activities are strategic complements, then $\overline{y}_i \leq \overline{y}_j$ for every $i \in A_k$ and $j \in A_h$ (and, contrarily, $\overline{y}_i \geq \overline{y}_j$, when fundraising activities are strategic substitutes). Therefore, regardless of the sign of fundraising externalities, if we let a nonprofit in A_k to play $\overline{y}_j$ instead of $\overline{y}_i$, for every nonprofit in A_h we obtain

$$Q_j\left(\overline{y}_{A_k}, \overline{y}_{A_h\backslash\{j\}}, \overline{y}_i, \overline{y}_{N\backslash\{A_k\cup A_h\}}\right) \geq Q_j\left(\overline{y}_j, \overline{y}_{A_k\backslash\{i\}}, \overline{y}_{A_h\backslash\{j\}}, \overline{y}_i, \overline{y}_{N\backslash\{A_k\cup A_h\}}\right). \tag{5.11}$$

Next, by the symmetry of all nonprofits, switching strategies implies switching payoffs, and we can write

$$Q_j\left(\overline{y}_j, \overline{y}_{A_k\backslash\{i\}}, \overline{y}_{A_h\backslash\{j\}}, \overline{y}_i, \overline{y}_{N\backslash\{A_k\cup A_h\}}\right) = Q_i\left(\overline{y}_i, \overline{y}_{A_k\backslash\{i\}}, \overline{y}_{A_h\backslash\{j\}}, \overline{y}_j, \overline{y}_{N\backslash\{A_k\cup A_h\}}\right) = Q_i\,(\overline{y})\,.$$

Therefore, by (5.10) and the last inequality we obtain that, for every $i \in A_k$ and $j \in A_h$,

$$Q_j(\overline{y}) \geq Q_i(\overline{y}).$$

$\square$

Finally we can prove

Proposition 6. *If the strategic complementarity in fundraising activities is sufficiently strong (i.e.* $\overline{\beta} > \beta \geq \beta^* > 0$*), the grand coalition is the unique alliance structure of the form* $S = (\{A\}, \{j\}_{j\in N\backslash A})$ *to be coalitionally stable under the unanimity rule of coalition formation.*

Proof. For $\overline{\beta} > \beta > \beta^* > 0$, we can write

(5.12)
$$Q_i(\widetilde{y}(\{N\})) > Q_j(\overline{y}(\{j\}, \{N\backslash\{j\}\})) > Q_j\left(\overline{y}\left(\{j\}_{j\in N\backslash A}, \{A\}\right)\right) > Q_i\left(\overline{y}\left(\{A\}, \{j\}_{j\in N\backslash A}\right)\right),$$

where the first inequality follows from the stand-alone stability (SAS) of the grand coalition holding for $\beta > \beta^* > 0$. The second inequality can be easily proven if we show that, in the new alliance structure $S = \left(\{j\}_{j\in N\backslash A}, \{A\}\right)$, the fundraising effort $(\overline{y}_i)_{i\in A} > (\overline{y}_i)_{i\in N\backslash\{j\}}$for every $i \in A$ and $i \in N\backslash\{j\}$. By simple calculations, in any arbitrary $S = \left(\{j\}_{j\in N\backslash A}, \{A\}\right)$ the effect of the size of alliance A is negative, since

$$\begin{aligned} \frac{d(\overline{y}_i)_{i\in A}}{da} &= \frac{d\left(\frac{\delta(3\beta-2\alpha)+2\delta^2-\beta(\alpha+2\delta a)+\beta^2(1+n(a-1)-a^2)}{2\beta\delta(3-n-a)+4\delta^2+\beta^2(2-2n+na-a^2)}\right)}{da} = \\ &= -(2\delta - n\beta + 2\beta a_k)(\alpha + \beta + \delta - n\beta)(\beta + 2\delta)\beta < 0 \end{aligned}$$

for $\beta < \frac{2\delta}{(n-1)}$. Thus, given that $(\overline{y}_i)_{i\in A} > (\overline{y}_i)_{i\in N\backslash\{j\}}$, by the strategic complementarity of fundraising efforts (i.e. $\beta > 0$) also the nonprofits at the fringe (singletons) will exert more fundraising than before and this makes everybody worse-off, as compared to the previous alliance structure. Finally, the third inequality follows from Lemma 2. Overall, expression (5.12) proves two major things: (i) when $\beta > \beta^* > 0$, the per-capita payoff of every nonprofit in the grand coalition is higher than the one obtained by nonprofits in any other alliance structure S of the form $S = \left(\{j\}_{j\in N\backslash A}, \{A\}\right)$; and (ii) the payoff obtained in the grand coalition is even higher of that of nonprofits playing as singletons against an alliance of any size. Therefore, when nonprofits can deviate in coalitions (including all set of players), they will always prefers to form the grand coalition. This proves that $\{N\}$ is the unique coalitionally stable alliance structure of the form $S = \left(\{j\}_{j\in N\backslash A}, \{A\}\right)$ under the unanimity rule of coalition formation. $\square$

5.2.5. *Proof of Proposition 7.* **Proposition 7.** *Regardless of the rule of alliance formation, no partition of nonprofits* $S^E = \{A_1, A_2, ..., A_m\}$, *such that every alliance possesses the same size* $a_1 = a_2 = ... = a_m$, *can be coalitionally stable.*

Proof. In every alliance A_k, each member receives the equal-split payoff $Q_i = Q_{A_k}(\overline{y})/a_k$. Since in the symmetric alliance structure S^E the unique fundraising equilibrium profile must be symmetric, $Q_{A_k}(\overline{y}) = Q_{A_h}(\overline{y})$ for every A_k, $A_h \in S^E$ and $Q_i(\overline{y}) = Q_j(\overline{y})$ for every $i \in A_h$

and $j \in A_h$, i.e., every nonprofit obtains the same payoff. The efficiency of the profile y^e associated to the grand coalition $\{N\}$ implies, for every nonprofit $i \in N$,

$$Q_i(y^e) \geq Q_i(\overline{y}),$$

and, for at least one $j \in N$,

$$Q_j(y^e) > Q_j(\overline{y}).$$

Hence, $y^e \neq \overline{y}$, and since at $\overline{y}$ every nonprofit receives the same payoff, it must be that

$$\sum_{i \in N} Q_i(y^e) > \sum_{A_k \in S^E} \sum_{i \in A_k} Q_i(\overline{y}),$$

Therefore, since every nonprofit in S^E would gain by announcing $\sigma_i = \{N\}$ and forming the grand coalition, every symmetric alliance structure S^E different from N can be improved upon and can never be coalitionally stable. □

5.2.6. *Proof of Proposition 8.* **Proposition 8.** *If β is sufficiently small (i.e. $\beta^* > \beta^{**} > \beta > \underline{\beta}$), all alliance structures of the form $S = \left(\{A\}, \{j\}_{j \in N \backslash A}\right)$ with $A \subset N$, are coalitionally stable under the unanimity rule of coalition formation.*

Proof. From the proof of Proposition 6, we know that when fundraising efforts are strategic complements ($\beta > 0$) the payoffs of nonprofits in alliance $A \in S = \left(\{A\}, \{j\}_{j \in N \backslash A}\right)$ are monotonically increasing in the size of the alliance A. This occurs because the level of fundraising of all nonprofits at the fringe as well as that of nonprofits within the alliance A increase with the reduction of a and, by negative externality, this implies a reduction in the equilibrium payoff of every $i \in A$. As a consequence, the payoffs of nonprofits in the grand coalition are higher, by definition, than those of nonprofits in any alliance $A \in S = \left(\{A\}, \{j\}_{j \in N \backslash A}\right)$. Therefore, to prove that an alliance structure of the form $S = \left(\{A\}, \{j\}_{j \in N \backslash A}\right)$ is coalitionally stable under the unanimity rule, we only need to show that $S = \left(\{A\}, \{j\}_{j \in N \backslash A}\right)$ respects the additional requisite that the nonprofits at the fringe, when acting as singletons in competition with the alliance A, would abstain from forming the grand coalition with all remaining nonprofits. In other words, we need to show that

$$Q_j(\{A\}, \{j\}_{j \in N/A}) \geq Q_i(\{N\}),$$

which, in particular, is respected for

$$\frac{\delta(\alpha+\delta-\beta(n-1))^2(2\delta-\beta(a_k-2))^2(1-c)}{\left(2\beta\delta(3-n-a_k)+4\delta^2+\beta^2(2-2n+na_k-a_k^2)\right)^2} > \frac{1}{4}\frac{(\alpha+\delta-\beta(n-1))^2(1-c)}{\delta-\beta(n-1)}. \tag{5.13}$$

When $\beta < \beta^*$, we know from Proposition 4 that

$$Q_j(\{j\}, \{N \backslash j\},) > Q_i(\{N\}),$$

and, therefore, whenever a $\beta^* > \beta > 0$ exists, the inequality (5.13) certainly holds and the alliance structure $S = (\{A\}, \{j\}_{j \in N/A})$ is coalitionally stable under the unanimity rule for $a = (n-1)$. It is easy to see that when the size of the alliance decreases, the payoffs of nonprofits in the fringe decrease and the inequality (5.13) can be respected only if $0 < \beta < \beta^{**}(a)$, where

$$\beta^{**}(a) = \frac{2\delta\left(4n-2-2a-na-2n^2+4a^2-a^3-na^2+n^2a\right)+2\sqrt{\delta^2a^2(11a^2-4a-10a^3+3a^4+4n-10na+8na^2-2na^3)}}{4na-8n+4n^2-4a^2+a^4+4na^2-4n^2a-2na^3+n^2a^2+4}.$$

Numerical simulations show that $\beta^{**} \in \left(0, \overline{\beta}\right)$ only if the alliance $A \in S = \left(\{A\}, \{j\}_{j \in N \setminus A}\right)$ is such that $a < \frac{3n}{4}$, i.e. the fringe of nonprofits covers more than 25% of the total number of nonprofits. $\square$

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Figure 1. Nash stable alliance structures under unanimity rule

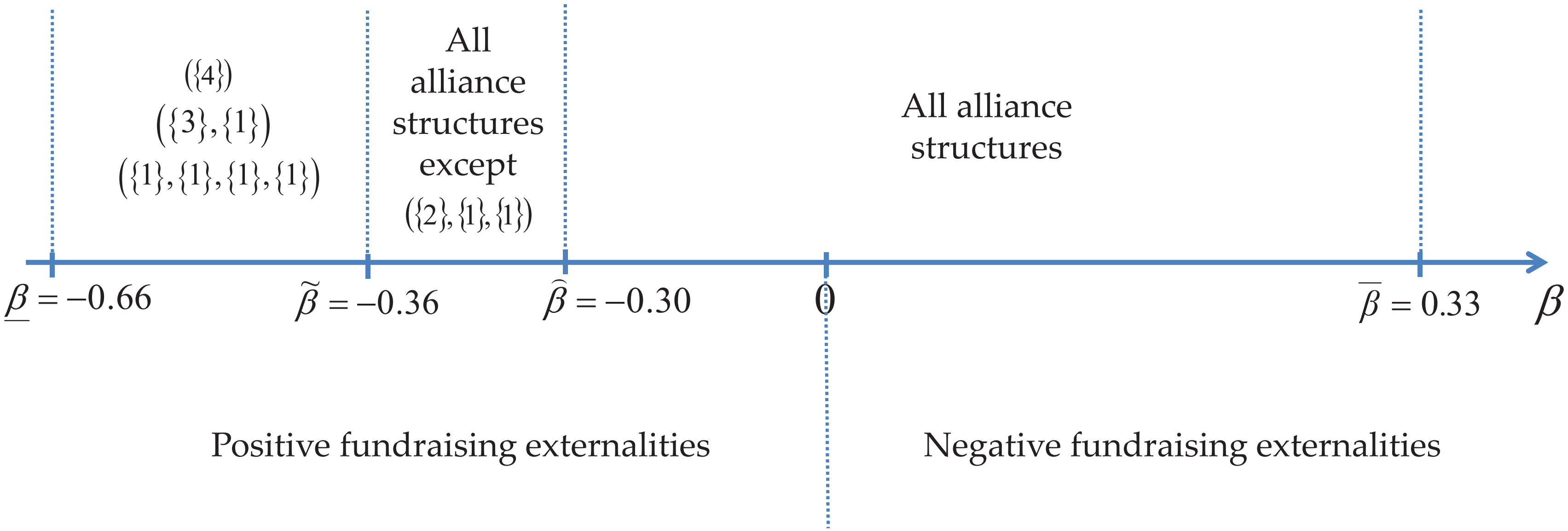

Figure 2. Nash stable alliance structures under aggregative rule

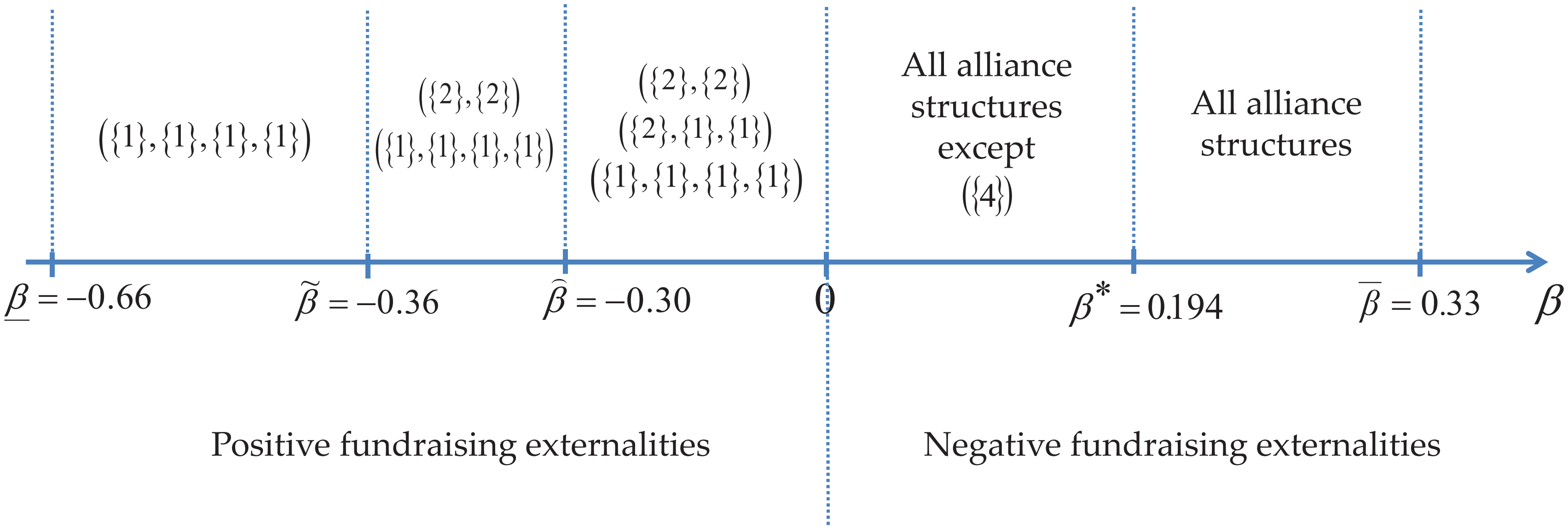

Figure 3. Coalitionally stable alliance structures under unanimity rule

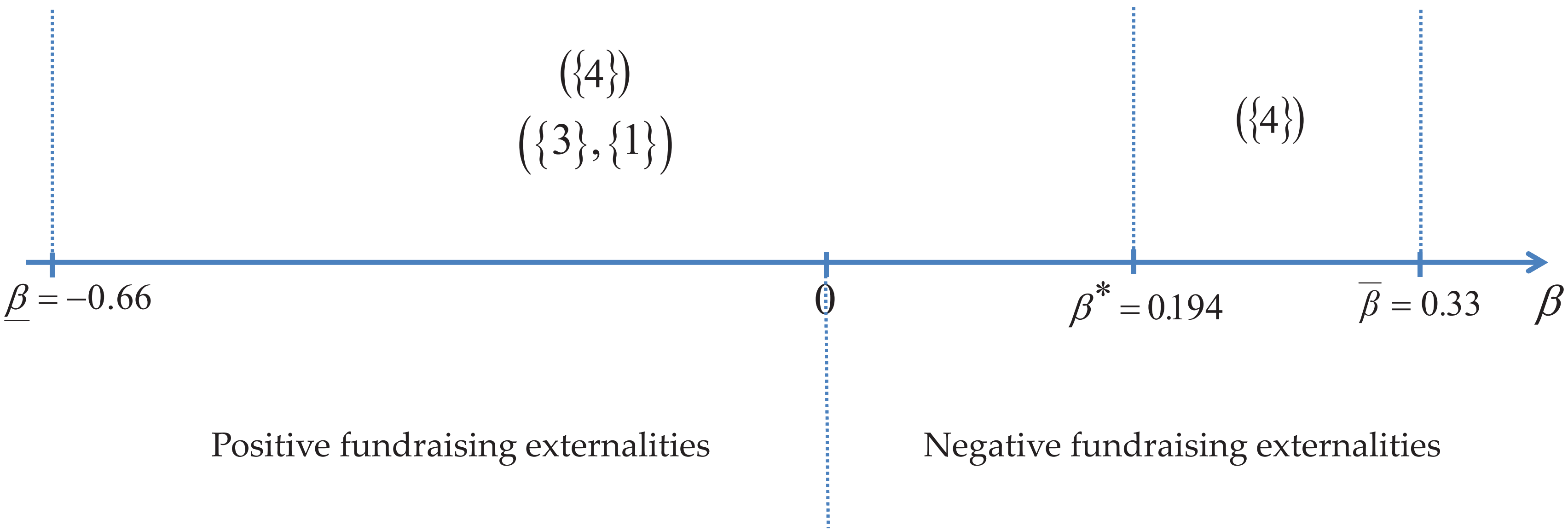

Figure 4. Coalitionally stable alliance structures under aggregative rule