THE TRAGEDY OF THE COMMONS IN A VIOLENT WORLD

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Abstract

Earlier research has shown that the tragedy of the commons may be resolved by Folk theorems for dynamic games. In this article we graft on a standard natural-resource exploitation game the possibility to appropriate the resource through violent means. Because conflict emerges endogenously as resources get depleted, the threat supporting the cooperative outcome is no longer subgame perfect, and thus credible. The unique equilibrium is such that players exploit non-cooperatively the resource when it is abundant and they revert to conflict when it becomes scarce. The players’ utility is shown to be lower even if conflict wastes no resources.

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1 Introduction

The worldwide depletion of natural resources has been increasingly underlined by scientists, with some researchers drawing our attention on the important consequences of environmental depletion (Homer Dixon 1999) or human-induced climate change (Stern 2007) in the coming decades. The list of resources being severely degraded is vast and it includes such different goods as fauna, forests, oil and minerals, water sources, arable land, or even carbon-free air. Economists have been concerned with the question of exhaustible resources’ conservation early on, with the main conclusions being that resources are spontaneously exploited in an economically efficient way, whereas full conservation of a resource is usually sub-optimal. Indeed, Hotelling’s rule specifying the optimal rate of extraction of an exhaustible resource in a dynamic setting dates back to 1931 (Hotelling 1931). This simple rule states that natural resources should be seen as any other asset, and that the price of an exhaustible resource must therefore grow at the rate of return of the best alternative investment.

Yet, for the economically efficient management rule to emerge spontaneously it is necessary that property rights over the resource be well defined and enforced (Coase 1960). Property rights over mineral resources for instance, are usually well defined and enforced, and one can therefore expect their apparent over-exploitation to be economically efficient. On the other hand, there exist resources known under the acronym common property resources (CPRs) whose property and exploitation are collective. Some resources falling in this category are the environment (carbon-free air, non-polluted soil or water), international river basins, or wild fish. These resources give rise to a negative externality because agents fail to internalize the damaging effects of their own exploitation on the other exploiters, thus resulting in an overexploitation of the resource, also known as the tragedy of the commons (Hardin 1968). In a seminal article, Levhari and Mirman (1980) demonstrate that in the (Markov perfect) equilibrium of a dynamic resource exploitation game the resource is over-exploited as compared to the first-best central planner solution.1

Some authors investigated institutional settings and strategies that yield in strategic games of CPRs the efficient outcome at equilibrium (Eswaran and Lewis 1984). Having identified the backbone of the problem, Hardin stated that given “the air and waters surrounding us cannot readily be fenced” the solution should be sought in “coercive laws or taxing devices” (Hardin 1968:1245). The necessary institutions are, however, absent in many settings

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1 Dutta and Sundaram (1993) generalize the setting and demonstrate that, whereas the Markov perfect equilibria are indeed always suboptimal, they may however imply an under-exploitation of the resource in some settings.
because of a lack of local (sub-national) or global (international) governance. Ostrom (1990) and Baland and Platteau (1996) narrate how local communities may succeed in overcoming the tragedy of the commons. Yet, the mechanisms identified by these authors necessitate a strong form of social capital that is absent in many CPR contexts. Extending Folk theorems for repeated games to dynamic games, economists then proposed a more general solution to the CPR management inefficiency. Cooperation on the efficient exploitation of a depletable resource may be achieved by the threat of reverting to non-cooperation in case of non-compliance to the agreed behavior (Dutta 1995, Cave 1987, Sorger 2005, Dutta and Radner 2009).

Scholars thus conclude that provided the agents are sufficiently patient, CPRs can be managed efficiently. Yet, to the extent that efficiency does not imply conservation, the dynamic depletion of the resource cannot be excluded along the equilibrium path. In the absence of strong institutions, however, the following question may be asked: in a context of dynamic resource scarcities, could the players decide to turn violent to appropriate the remaining stock of resources? And if so, how would the anticipation of a violent confrontation influence the equilibrium strategies of the players? We address these questions in the present article.

The impact of the presence and exploitation of natural resources on conflict has received much attention lately as revealed by the literature reviews on conflict (Blattman and Miguel 2010) and natural resources (Van Der Ploeg 2011). The roots of conflict in resource-rich countries seem to be better understood in recent years. Weak institutions (Mehlum et al. 2006, Besley and Persson 2010), poor economic alternatives (Angrist and Krugler 2008, Dube and Vargas 2008), the geographic concentration of the resources (Le Billon 2001, Wick and Bulte 2006), or ethnic inequality (Esteban and Ray 2011, Esteban et al. 2012) constitute some of the factors favoring the occurrence of conflict. But more important for our study is the fact that resource scarcities are viewed by many scholars as a central driver of conflict (André and Platteau 1998, Homer-Dixon 1999, Diamond 2005, Kahl 2006). One could, therefore, expect the (efficient) dynamic depletion of a stock of resources to create scarcities that will eventually spark conflict.

In that respect, the endogenous collapse of Easter Island’s society as documented by Diamond (2005) is very instructive. It is estimated that the Polynesians who were the first humans to have colonized the island did so circa 900 A.D. after traveling more than 1000 km on board of wooden canoes. At the peak of its development, it is estimated that this 160 square kilometer island hosted between 15 to 30 thousand inhabitants. The society was
organized in hierarchical clans that peacefully competed with each other for power supremacy by erecting stone statues weighing up to 80 tons. For this purpose, the island’s tallest trees were cut down, as a result of which a rapid deforestation occurred.\textsuperscript{2} By the time the first European expedition reached the island in 1722, deforestation of any tree measuring more than 3 meters height, including the two tallest species of trees the island used to harbor and that measured up to 15 and 30 meters, respectively, was complete. The exhaustion of this valuable natural resource implied an incapacity to build new large canoes required for high sea fishing, as a consequence of which the rate of consumption of on-land food necessarily increased. This latter fact combined with the increase in population led to a depletion of the island’s fauna that eventually forced the Easter Islanders to include in their nutritional diet a disproportionally high rate of rats. The erosion sparked by the deforestation accentuated the water supply problems of the island, hence further degrading the islanders’ nutrition. Analyses of the settlements’ middens, and of oral traditions conclude that the food scarcities became so important as to open the way for cannibalism. In 1680, as the situation must have reached dramatic levels of deprivation of all kinds, and with the elites proving unable to deliver their promises to their people, a sort of military coup occurred, followed by a prolonged period of wars.

This historical example points at the violent consequences of resource depletion. According to Diamond, Easter Island’s society plunged into chaos because of resource depletion that was provoked by their clans’ permanent quest for prestige. Consistent with this approach is the analysis of Brander and Taylor who state that “rather than being the cause of decline, violent conflict is commonly the result of resource degradation and occurs after the civilization has started to decline, as on Easter Island” (Brander and Taylor 1998: 132). Yet, one can reverse this argument and explore the extent to which depletion of the island’s resources has been a consequence of anticipated conflicts over the resources.

In this article we introduce the possibility of violently appropriating the stock of resources in a dynamic game of CPR exploitation. The main finding of the article is that when resources are dynamically depleted along the cooperative extraction path, conflict will occur with certainty when resources are sufficiently depleted. When the resource becomes scarce, should the players decide to fight over the remaining stock of resources, they will be acquiring few armaments. Consequently, conflicts would then be of low intensity, which would also mean that they

\textsuperscript{2}A controversy on the real causes of the Island’s deforestation is still open among scientists (Hunt and Lipo 2011)
would be less destructive. As a consequence, in the presence of depleted stocks of resources, players may violently appropriate at a low cost part of the goods which they will then manage privately. Since the first best solution fails to be stable, no alternative path of play to the conflict-reversion strategy can be an equilibrium either. As a consequence, the punishment threat for not respecting a cooperative agreement stops being credible (i.e. subgame perfect), thus implying that cooperation itself breaks down. Interestingly, even if conflict intensity is extremely low at equilibrium, the resulting breakdown of cooperation implies that the players’ utility will always be lower as compared to a conflict-free setting. This follows from the over-exploitation of the resource in the periods where the stock is sufficiently abundant for players not to fight over its control.

This article contributes to the expanding field of conflict theory. Whereas the initial writings mainly focused on static properties of conflict models, the dynamic dimension has received increased attention lately. Some interesting issues that have been explored are the timing of conflict when the players’ strength evolves either exogenously (Bester and Konrad 2004), or endogenously yet in a deterministic way (Powell 2012), deterrence and preventive motives for conflict (Jackson and Morelli 2009, Chassang and Padro i Miquel 2009), as well as dynamic incentives to pursue fighting over time (Leventoglu and Slantchev 2007). A common dynamic incentive in conflict settings is to attempt modifying the sharing of the contested pie in the short run through violent means in order to enhance one’s continuation value, although the more valuable the prize, the more intense and thus costly will the conflict be (McBride and Skaperdas 2007, Esteban et al. 2012).

Garfinkel (1990) and Yared (2010) both investigate the scope for reaching peaceful agreements in repeated games of resource exploitation and conflict. Garfinkel (1990) was the first to study Folk theorems for conflict models and established that in a repeated prisoner-dilemma type of setting, peace can be supported as an equilibrium for sufficiently patient players. More recently, Yared (2010) extended the analysis to a context involving information imperfections, while also considering the harshest existing subgame perfect punishments. In Yared’s model, because of informational imperfections, temporary punishments under the form of non-permanent wars may occur along the equilibrium path. Common to these two articles is the repeated nature of the game, which implies that the resource at stake is a constant flow of wealth. Such models are well suited therefore to formalize potential conflicts over the control of economic sectors and territories. In our setting, the resource is instead conceptualized as a stock that regenerates at some non-negative rate, thus implying that our model captures better the dynamics of finite
resources, whether they regenerate or not.

Acemoglu, Golosov, Tsyvinski, and Yared (AGTY, 2012) is perhaps the closest contribution to ours since they consider a dynamic game of trade and conflict over an exhaustible resource whose value is a function of its scarcity. In exploring the conditions favoring the peaceful dynamic trade as opposed to an invasion by a resourceless player, AGTY show that if the depletable resource, which is located in one country, is exploited competitively, individual firms fail to internalize the negative externality of their individual extraction on the increased likelihood of foreign intervention. The high prices that this exploitation generates in the future, boosts the future incentives for conflict, thus feeding back in the firms’ short run incentives to exploit the resource, which eventually triggers immediate conflict. Whereas both our models are dynamic and feature an exhaustible resource, they differ along several dimensions.

These two articles feature very different institutional frameworks, so that our research question - the possibility of cooperative behavior in a CPR - cannot be answered with AGTY’s model. In AGTY, the resource is public for a set of players (producers of exporting country - possibly a singleton), but the end-user of the resource does not belong to this set. The feasibility of cooperation among producers and exporters in a CPR cannot be addressed since the agents are not given the tools to cooperate through resource-extraction/production. Similarly, any potential cooperation among producers is assumed away by imposing a competitive setting whenever the resource is exploited by more than one player. Second, we impose log-utility functions that in AGTY would translate in war-incentives not depending on the amount of resources. Third, unlike AGTY, in our framework arming is not a separable cost, hence implying that conflict generates a double inefficiency: it destroys part of the stock of resources, and it diverts otherwise productive resources to fighting activities, which further contributes to the depletion of the stock. The two previous points link closely to a fourth distinction. In AGTY the cost of conflict and its outcome are both exogenous (assailant wins with unit probability). Instead, we enable both sides to decide their armament levels, hence also making the winning probabilities endogenous. This also reflects the differences in the mechanics conducive to conflict in AGTY and in this article. In AGTY conflict results from the market price of resources being an increasing price of their scarcity, while the cost of violently appropriating the resources through war remains constant. In our framework instead, as resources become scarce, the investments in armaments decrease as well, thus reducing the burden of the conflict while equally making it less destructive. Lastly, we extend the analysis to
the concept of subgame perfection instead of focusing on the subset of Markov perfect equilibria.

In the next section we develop a standard benchmark model of dynamic resource exploitation and we identify sufficient conditions for the efficient solution to be sustained as an equilibrium. In Section 3 we introduce the possibility of reverting to violence in this same game. Lastly, Section 4 concludes.

2 The model: peaceful world

We consider an infinite time horizon game of a renewable resource exploitation. Time is discrete and denoted by \( t = 0, 1, \ldots, \infty \). Two players labeled 1 and 2 simultaneously decide at each time period the amount of resources to exploit from a common pool of renewable resources. In time zero the world is endowed with a stock of \( r_0 \) resources that grows at a linear rate \( \gamma \).\(^3\) Players costlessly invest effort in resource-use, so that player \( i \)'s appropriation effort of renewable resources in time \( t \) is denoted by \( e_{i,t} \), with \( e_{i,t} \in [0, \bar{e}] \), \( \bar{e} > r_0 \). Player \( i \)'s associated consumption is given by \( x_{i,t} \) such that:

\[
x_{i,t} = \begin{cases} 
  e_{i,t} & \text{if } e_{1,t} + e_{2,t} \leq r_t \\
  \frac{e_{i,t}}{e_{1,t} + e_{2,t}} r_t & \text{otherwise}
\end{cases}
\]  

(1)

The resources available in period \( t \), \( r_t \), equal:

\[
r_t = (1 + \gamma)(r_{t-1} - x_{1,t-1} - x_{2,t-1})
\]  

(2)

The instantaneous utility of any player \( i \) in time \( t \) is given by:

\[
u_{i,t} = \ln(x_{i,t})
\]

And the discounted life-time utility of player \( i \) in time period 0 equals:

\[
U_{i,0} = \sum_{t=0}^{\infty} \delta^t \ln(x_{it})
\]  

(3)

Where \( \delta \) stands for the discount rate.

\(^3\)The growth rate of the resources could equally be capturing the (constant) growth of productivity resulting from technological advances.
We define a strategy for player $i$ as $e_i = \{e_{i,t}\}_{t=0}^{\infty}$.

The efficient solution

The efficient solution of this resource exploitation game is given by the solution to the central planner’s following problem:

$$\max_{e_1, e_2} \sum_{t=0}^{\infty} \delta^t \ln(x_{i,t})$$  \hspace{1cm} (4)

subject to (1) and (2)

We denote by $V^c(r_t)$ the value function of this problem given the resource stock $r_t$, meaning that the indirect aggregate utility can be expressed as a Bellman equation:

$$V^c(r_t) = \arg \max_{e_{1,t}, e_{2,t}} \left[ \sum_{i=1,2} \ln (x_{i,t}) + \delta V^c((1 + \gamma)(1 - 2\lambda^c(r_t))) \right]$$  \hspace{1cm} (5)

Next, given the assumed regeneration rule, the above expression can be written as:

$$V^c(r_t) = \arg \max_{e_{1,t}, e_{2,t}} \left[ \sum_{i=1,2} \ln (x_{i,t}) + \delta V^c((1 + \gamma)(r_t - x_{1,t} - x_{2,t})) \right]$$  \hspace{1cm} (6)

Differentiating (6) with respect to the two decision variables, $e_{1,t}$ and $e_{2,t}$, and making use of (1) we obtain the following system of equations:

$$\begin{cases}
\frac{\partial V^c(r_t)}{\partial e_{1,t}} = \frac{1}{x_{1}^c(r_t)} - \delta(1 + \gamma) \sum_{i=1,2} V_{i}^{c'}((1 + \gamma)(r_t - x_{1}^c(r_t) - x_{2}^c(r_t))) = 0 \\
\frac{\partial V^c(r_t)}{\partial e_{2,t}} = \frac{1}{x_{2}^c(r_t)} - \delta(1 + \gamma) \sum_{i=1,2} V_{i}^{c'}((1 + \gamma)(r_t - x_{1}^c(r_t) - x_{2}^c(r_t))) = 0
\end{cases}$$  \hspace{1cm} (7)

Where these equations hold because the constraint $e_{1,t} + e_{2,t} \leq r_t$ will never be binding, since $\lim_{r_t \to 0} V_{i}^{c'} = +\infty$.

From (7) we deduce that $x_{1}^c(r_t) = x_{2}^c(r_t) = x^c(r_t)$. To derive the efficient equilibrium, we inquire whether $x^c(r_t)$ may be a linear function of its argument so that $x^c(r_t) = \lambda^c r_t$. This assumption implies that the stock of resources in time period $t + 1$ can be expressed as $r_{t+1} = (1 + \gamma)(1 - 2\lambda^c) r_t$. Replacing in equation (4), together with using the regeneration rule gives us:

$$V^c(r_t) = 2 \left[ \ln (\lambda^c r_t) + \delta \ln (\lambda^c(1 + \gamma)(1 - 2\lambda^c) r_t) + \delta^2 \ln (\lambda^c(1 + \gamma)^2(1 - 2\lambda^c)^2 r_t) + \ldots \right]$$  \hspace{1cm} (8)
Rearranging the terms of (8) gives us:

\[ V^c(r_t) = \frac{2 \ln (\lambda^c r_t)}{1 - \delta} + 2 \sum_{\tau=0}^{\infty} \delta^\tau \ln ((1 + \gamma)^\tau (1 - 2\lambda^c)^\tau) \]  

(9)

Thus implying that \( V^c'(r_t) = \frac{2}{(1-\delta)r_t} \). Substituting in (7) for \( V^c'(r_t) \) yields:

\[
\frac{1}{\lambda^c r_t} - \frac{2\delta(1+\gamma)}{(1-\delta)(1+\gamma)(1-2\lambda^c)r_t} \Rightarrow \lambda^c = \frac{1-\delta}{2}
\]

This last expression stands for the share of available resources consumed by each player under the efficient solution. The proportion of the stock of resources which is preserved from one period to another therefore equals \( (1 + \gamma)\delta \). The optimal consumption for any individual \( i \) is therefore given by:

\[ x^c_i = \frac{(1 - \delta)r_t}{2} \]  

(10)

Hence implying that any player’s life time utility in time \( t \) can be written as:

\[
V^c_{i,t} = \frac{1}{1 - \delta} \left[ \ln \left( \frac{(1 - \delta)r_t}{2} \right) + \frac{\delta}{1 - \delta} \ln (\delta(1 + \gamma)) \right]
\]  

(11)

Expression (11) gives us the life time utility of players in what we term the “cooperative scenario”.

For this first-best solution to be a SPE, it is necessary that no profitable deviation exists for either player. The instantaneous utility players obtain by deviating from the efficient solution can straightforwardly be shown to be higher than the instantaneous utility of cooperating. Hence, for the first-best solution to be a SPE, it needs to be sustained by some form of decentralized dynamic punishment. Any such threat oughts to be subgame perfect itself, however. In what follows we consider the harshest subgame perfect threats.

**Harshest punishment**

We denote by \( \mathcal{H} \) the set of strategies generating the harshest punishments of this game. At time \( \tau \) any strategies satisfying the following conditions belong to \( \mathcal{H} \).

\[
e_{i,\tau} \geq r_{\tau}, \quad e_{i,\tau+t} \in [0,\bar{e}] \quad \forall t > 0, \forall i \in \{1,2\}
\]

\[\text{Notice that the resource is dynamically depleted if } (1 + \gamma)\delta < 1 \iff \gamma < \frac{1-\delta}{\delta}. \text{ For the problem to be salient, in the remainder of the article we assume that this condition is satisfied.}\]
Since there exists no profitable unilateral deviation from such strategy profiles, they describe a SPE where the players’ associated utility is infinitely negative. From this stems the following result:

**Proposition 1.** The first-best solution to the resource exploitation game is always supported as a subgame perfect equilibrium.

**Proof.** Consider the following punishment: if \((e_{1,t}, e_{2,t}) \neq (e_{1,t}^*, e_{2,t}^*)\) for some \(t\), then \(e_{t+1} \in \mathcal{H}\). Since \(V_i,t\) as given by equation (11) is finite, deviating and obtaining infinitely negative utility can never be profitable.

Because of the logarithmic specification we have chosen, the harshest punishments consist in fully depleting the resource. Since the players’ utility would then become infinitely negative, any path of play generating some non-infinitely negative payoff to both players may be sustained as a SPE. By extension therefore, the efficient solution is equally supported as an equilibrium. The full-depletion SPE supporting the efficient solution do not survive stronger equilibrium refinements (as the extensive form trembling hand perfect equilibrium). It can be shown, however, that there exist milder punishments sustaining the efficient solutions of some \(1 > \delta > \bar{\delta} > 0\). Since our theory will be shown to be true for any subgame perfect equilibrium, we do not further explore these punishments.

### 3 The model: violent world

We now introduce in the game the possibility for players to revert to violence to establish private property rights over the common pool resource.

The players begin by unilaterally deciding whether or not to initiate a contest over the establishment of property rights on the common pool resource. Conflict ensues if either or both players opt for conflict. In case of indifference, we assume that players refrain from initiating conflict. In a second stage, players simultaneously decide the amount of effort to devote to resource extraction to be used for building weapons \((\hat{e}_{1,t}, \hat{e}_{2,t})\). Player \(i\)’s associated amount of weapons acquired in \(t\) is given by \(g_{i,t}\) such that:

\[
g_{i,t} = \begin{cases} \hat{e}_{i,t} & \text{if } \hat{e}_{1,t} + \hat{e}_{2,t} \leq r_t \\ \frac{\hat{e}_{i,t}}{\hat{e}_{1,t} + \hat{e}_{2,t}} & \text{otherwise} \end{cases}
\]  

(12)
If conflict occurs in time \( t \), a share \( \phi(g_{1,t},g_{2,t}) \) of the remaining stock of resources is destroyed, and player \( i \) eventually retains control over a share \( p(g_{i,t},g_{j,t}) = \frac{g_{i,t}}{g_{i,t} + g_{j,t}} \) of the remaining stock of resources. We make the following assumptions on the function \( \phi(g_{i,t},g_{j,t}) \):

**Assumption 1.** 
\( \phi(g_{i},g_{j}) \geq 0 \)  
\( \phi(0,0) = \phi(0,0) = 0 \)

**Assumption 2.** 
\( \phi(g_{i},g_{j}),\phi(g_{i},g_{j}),\phi(g_{i},g_{j}) \geq 0 \)

We are therefore assuming that war is increasingly destructive for higher (individual or aggregate) levels of the strength of the contestants involved in the conflict.\(^5\)

Lastly, in stage 3, players decide the extraction efforts \( e_{1,t} \) and \( e_{2,t} \) for producing consumables given that the pool is either commonly owned if no conflict took place in period \( t \) or before, or else it is partitioned into private properties.

If conflict is decided in time period \( \tau \), player 1’s utility equals:

\[
U_{w}^{w} = \ln \left( (1-\delta)p(g_{1,\tau},g_{2,\tau})(1-\phi(g_{1,\tau},g_{2,\tau})) \left( r_{\tau} - \sum_{i=\{1,2\}} g_{i,\tau} \right) \right) + \frac{\delta \ln ((1+\gamma)\delta)}{(1-\delta)^2} \tag{13}
\]

After conflict occurred period \( \tau \) player \( i \) becomes the sole manager of a share \( p(g_{1,\tau},g_{2,\tau})(1-\phi(g_{1,\tau},g_{2,\tau})) \) of the stock of resources \( r_{\tau} \), out of which \( g_{1,\tau} + g_{2,\tau} \) have been extracted for building weapons.\(^6\) Since the players will afterwards manage the resource efficiently, it is easy to show using the previous section’s techniques that the optimal extraction rates over their private properties will then be \((1-\delta)\).

The timing of the resource extraction game in a violent world at each point in time is the following:

1. Players simultaneously decide whether to initiate conflict

2. Players simultaneously decide the amount of consumables to dedicate to weapon-building \( (g_{1,t},g_{2,t}) \)

3. Players simultaneously decide their extraction efforts \( (e_{1,t},e_{2,t}) \) for consumption purposes \( (x_{1,t},x_{2,t}) \)

\(^5\)These assumptions are sufficient for deriving the results in this article. The (weak) convexity of \( \phi(.,.) \) guarantees the monotonicity of the players’ optimal weapons as a function of the stock of resources. Sufficiently concave functions would lead to non-monotonic relationships.

\(^6\)While we are assuming that after conflict takes place, private property rights are irreversibly implemented, the *exact same results* would obtain if the players were given the possibility to re-initiate conflicts.
Optimization in a violent world

Optimizing (13) for player 1 with respect to \( g_{1,\tau} \) subject to (12), and simplifying we obtain the following F.O.C.:

\[
\frac{g_{2,\tau}}{(g_{1,\tau} + g_{2,\tau})^2} (1 - \varphi(g_{1,\tau}, g_{2,\tau}))(r_{\tau} - g_{1,\tau} - g_{2,\tau}) - p(g_{1,\tau}, g_{2,\tau})\varphi_{g_{1,\tau}}(g_{1,\tau}, g_{2,\tau})(r_{\tau} - g_{1,\tau} - g_{2,\tau}) \\
-p(g_{1,\tau}, g_{2,\tau})(1 - \varphi(g_{1,\tau}, g_{2,\tau})) = 0
\] (14)

Re-arranging, simplifying, and dropping the time subscripts for notational reasons, we obtain:

\[
g_{2}(1 - \varphi(g_{1,\tau}, g_{2,\tau}))(r - g_{1} - g_{2}) - \varphi_{g_{1}}(g_{1,\tau}, g_{2,\tau})(r - g_{1} - g_{2})g_{1}(g_{1} + g_{2}) - (1 - \varphi(g_{1,\tau}, g_{2,\tau}))g_{1}(g_{1} + g_{2}) = 0
\] (15)

In Appendix A.1 we show that the S.O.C. is satisfied at optimality, so that objective function is quasi-concave in \( g_{1} \).

Using the equivalent expression for player 2 implies that at optimality the following equality must hold:

\[
\frac{g_{2}}{g_{1}(g_{1} + g_{2})}(1 - \varphi(g_{1,\tau}, g_{2,\tau})) - \varphi_{g_{2}}(g_{1,\tau}, g_{2,\tau}) = \frac{g_{1}}{g_{2}(g_{1} + g_{2})}(1 - \varphi(g_{1,\tau}, g_{2,\tau})) - \varphi_{g_{1}}(g_{1,\tau}, g_{2,\tau})
\]

From this last expression we deduce that \( g_{1} = g_{2} \), for otherwise the equality is necessarily violated.\(^7\) We can therefore implicitly express the optimal amount of weapons \( g_{1,\tau}^{\ast} = g_{2,\tau}^{\ast} = g_{\tau}^{\ast} \):

\[
(1 - \varphi(g_{\tau}^{\ast}, g_{\tau}^{\ast}))(r_{\tau} - 4g_{\tau}^{\ast}) - 2g_{\tau}^{\ast}\varphi_{g_{\tau}^{\ast}}(r_{\tau} - 2g_{\tau}^{\ast}) = 0
\] (16)

Where \( \varphi_{g_{\tau}^{\ast}} \) stands for a short notation of \( \varphi_{g}(g_{\tau}^{\ast}(r_{\tau}), g_{\tau}^{\ast}(r_{\tau})) \). The utility of player \( i \) in time period \( \tau \) thus equals:

\[
V_{i,\tau}^{w} = \ln \left( \frac{(1 - \delta)}{2} (1 - \varphi(g_{\tau}^{\ast}, g_{\tau}^{\ast})) (r_{\tau} - 2g_{\tau}^{\ast}) \right) + \frac{\delta \ln ((1 + \gamma)\delta)}{1 - \delta} \frac{(1 - \delta)^2}{(1 - \delta)^2}
\] (17)

The implicit description of \( g_{\tau}^{\ast} \) as given by (16) allows us to deduce the following result:

**Lemma 1.** Militarization increases monotonically in the stock of resources.

This is a standard result in the conflict literature: the larger the prize at stake, the more effort the contestants will invest in attempting to grab the resource.

**Lemma 1** is very useful in showing the following lemma:

\(^7\)If, for instance, \( g_{1} > g_{2} \), both terms of the LHS are individually smaller than their counterpart of the RHS.
Lemma 2. For sufficiently low levels of resources, deviating from the cooperative strategy and going to conflict in the subsequent period is profitable for both players.

For the proof see Appendix A.3.

This lemma constitutes an important building block of our main result because it implies that if the punishment inflicted on deviators from the efficient agreement was the reversion to conflict, then the efficient solution would not be an equilibrium outcome for low levels of resources. The intuition behind this result lies in the increasingly small cost of conflict when the resources are sufficiently depleted: as the value of the prize at stake diminishes the players will extract less resources for weapon-building if conflict was to occur, hence implying that the damage generated by conflict will equally be contained. This implies that players eventually prefer to free-ride on their opponent’s cooperative effort in the short run and then to revert to a low-intensity conflict. For sufficiently low stocks of resources, therefore, the short run benefits of reneging on the cooperative behavior coupled with low conflict intensity, and thus low dynamic inefficiencies from destroyed production, outmatch the dynamic foregone future consumption from eternal cooperation.

In Section 2 we characterized the set of SPE constituting the harshest subgame perfect punishments, and equally mentioned the existence of alternative SPE that could support the cooperative solution as an equilibrium. In light of these strategies, we need to determine whether in a conflictual world cooperation is still sustainable at equilibrium. By the very definition of the efficient solution, it is not Pareto-dominated, hence implying that compared to any potential alternative SPE at least one player is strictly better-off under the efficient solution. The very fact that conflict dominates the cooperative strategy for both players when \( r \) becomes sufficiently small equally implies therefore that no alternative SPE can exist.

A relevant question is whether the cooperative equilibrium may still be implemented in the short run by a threat of reversion to non-cooperation for a finite number of periods in case of non-compliance, before declaring conflict. The following Proposition establishes that cooperation can never be sustained as an equilibrium strategy in this game:

Proposition 2. In a depletable resource exploitation game where players can revert to violence to appropriate the common pool resource, the equilibrium is such that players exploit non-cooperatively the resource when the stocks of resource are abundant, whereas they revert to conflict if the resource becomes scarce.
Proof. The proof of this Proposition is immediate: Since for some finite $t$ conflict will occur, after which the resource becomes a private good, cooperation can only be followed by conflict or by a period of non-cooperation followed by conflict. In the former case, designating by $\tau$ the time period when it is profitable to deviate from the cooperative path of play, and then initiating conflict in $\tau + 1$, it is direct to deduce that in $\tau$ both players will behave non-cooperatively. Therefore in $\tau - 1$, cooperation cannot be sustained either. Applying backwardly the argument, we deduce that cooperation will never be sustained.

Proposition 2 highlights the important implications of introducing conflict in a dynamic game of CPR management. Interestingly, the prospect of conflict makes off-the-equilibrium-path threats non-credible, when these same threats would have supported the first-best outcome in the absence of conflict. Thus, conflict may be expected not to occur for a long period of time, but the very expectation of conflict induces the players not to cooperate. Because the conflict decision is endogenous, when opting for conflict the (symmetric) players necessarily fare better than by exploiting the resources non-cooperatively. This implies that the unique (Pareto-dominated) non-cooperative equilibrium of this game ceases being an equilibrium with the introduction of conflict. Yet, while conflict removes from the game its worst equilibria, it equally eliminates the Pareto-superior ones as the cooperative equilibrium can no longer be sustained. It is therefore noteworthy to emphasize that irrespectively of the amount of weapons invested in the - possibly distant - conflict, the following corollary holds true:

Corollary 1. The anticipation of conflict accelerates resource depletion as compared to a cooperative path of play, and reduces the players’ equilibrium utility.

Proof. This follows from three facts. Assume conflict takes place in time $t$. Then, (i) from that time period on the players behave similarly to the cooperative scenario since they privately own part of the resources. Hence for the same amount of resources $r_t$, conservation would be equivalent under either scenario. Next, (ii) in period $t$, under conflict resources are devoted to building weapons, and conflict destroys part of the stock of resources. These two forces reduce the post-conflict stock of resources as compared to the cooperative path of play. Lastly, (iii) in any period preceding conflict, under the conflict scenario the players play non-cooperatively, hence depleting faster the stock of resources as compared to the cooperative path of play.

This finding must not be mis-interpreted as conflict reducing the players’ utility because of either the opportunity
cost of conflict, or the direct inefficiencies tied to fighting activities. The reduction in the players’ utility is primarily linked to the inability to sustain the cooperative Pareto-superior equilibrium, because the punishments supporting this equilibrium are no longer credible.

4 Concluding remarks

We introduced in a standard dynamic game of common pool resource management the possibility of privatizing the common resource by reverting to conflict. If conflict is waged in the presence of high stocks of resources, the players invest important amounts in conflict. The high opportunity cost of this operation coupled with the potentially destructive nature of heavily armed conflicts induces the players to refrain from initiating conflict. For low levels of resources, however, conflict becomes a profitable option. The implications of this finding are profound because the strategies that are traditionally used to sustain cooperation are no longer subgame perfect. As a consequence, the cooperative equilibrium breaks down. In the presence of abundant resources, the players exploit non-cooperatively the CPR in expectation of conflict occurring after the stock has been sufficiently depleted at some point in the future. Importantly, compared to the cooperative equilibrium, the unique equilibrium in this CPR game with conflict involves a faster depletion of the resource, and lower utility levels for the players. This is true even when the players expend minimal resources in conflict.

The theory developed in this article assumes away the existence of a third party able to enforce peaceful Pareto-superior agreements. It is crucial to keep in mind that it is in such contexts that the “tragedy of the commons” emerges. Indeed, Coase’s theorem teaches us that if property rights are well defined and enforced in the absence of transaction costs, then the first best solution is implemented. In the presence of strong institutions efficient solutions are always implementable, and in the absence of transaction costs there always exist transfers between players leaving everyone better-off. Interestingly in such settings, whether ownership is private or public would make no difference in terms of efficiency. The “tragedy of the commons” is at play in the presence of weak institutions. Yet, it is precisely in such contexts that the theory developed in this paper becomes relevant. Our theory therefore underlines the importance of improving institutions when attempting to achieve more efficient economic outcomes.

In this article we have assumed a perfect information setting. Lifting this hypothesis and endowing a third-party with private information over the stock of resources or its regeneration rate would enable the third party
to potentially improve the outcome’s efficiency by strategically selecting the information to transmit, even in the presence of weak institutions. This interesting question is a fruitful topic to explore in future research.
A Appendix

A.1 Quasi-concavity of $U_{1,t}^w(g_1)$

Proof. By implicitly computing $\frac{\partial^2 U_{1,t}^w(g_1)}{\partial g_1 \partial g_1}$ using equation (15), we obtain:

$$\frac{\partial^2 U_{1,t}^w(g_1)}{\partial g_1 \partial g_1} = -g_2 \varphi_{g_1}(r - g_1 - g_2 - (2g_1 + g_2)) \varphi_{g_1}(r - g_1 - g_2) - \varphi_{g_1}(r - g_1 - g_2)g_1(g_1 + g_2) + 2 \varphi_{g_1}g_1(g_1 + g_2) - 2(g_1 + g_2)(1 - \varphi)$$

The only positive term in this expression is $2 \varphi_{g_1}g_1(g_1 + g_2)$. Yet, since by equation (15) we deduce that $\varphi_{g_1}g_1(g_1 + g_2) < g_2(1 - \varphi)$, it follows that the sign of the above expression is negative.

A.2 Proof of Lemma 1

Proof. Applying the implicit functions’ theorem on expression (16) we obtain:

$$\frac{\partial g^*}{\partial r} = \frac{(1 - \varphi) - 2 \varphi g^*}{-2 \varphi g^*(r - 4g^*) - 4(1 - \varphi) - 2 \varphi g^*(r - 2g^*) + 4g^* \varphi g^* - 2g^* \varphi g^* g^*(r - 2g^*)} \quad (A-1)$$

Since from (16) we deduce that $(1 - \varphi) > 2g^* \varphi g^*$, the numerator of (A-1) is positive, while the denominator is negative, hence making the whole expression strictly positive for any value of $r$.

A.3 Proof of Lemma 2

Proof. Our aim is to demonstrate that $\exists r > 0$ such that $\forall r_t < r$

$$\ln(x_{1}^{dev}(x_2(r_t))) + \delta V_{1,t}^w(r_t) > V_{1,t+1}^w(r_t) \quad (A-2)$$

Replacing by the optimal values of the variables we have already derived, this expression can be re-written as:

$$\ln\left(x_1^{dev}\left(\frac{1 - \delta}{2} r_t\right)\right) + \frac{\delta \ln\left(\frac{(1 - \delta)(1 - \varphi(g_{t+1}, g_{t+1}^*))}{2} (r_{t+1} - 2g_{t+1}^*)\right)}{1 - \delta} + \frac{\delta^2 \ln\left((1 + \gamma)\delta\right)}{(1 - \delta)^2} > \frac{1}{1 - \delta} \left[ \ln\left(\frac{(1 - \delta)r_t}{2}\right) + \frac{\delta}{1 - \delta} \ln(\delta(1 + \gamma)) \right] \quad (A-3)$$

We proceed in two steps.

Assume first that in some time $t$, $\varphi(\cdot) = 0$ and $\varphi_g = 0$. If that was the case, from expression (16) we would deduce that $g_t^* = r_t/4$. Replacing in (17) would then yield:
\[ V_{1,t+1}^w = \frac{1}{1-\delta} \ln \left( \frac{(1-\delta) r_{t+1}}{4} \right) + \frac{\delta}{(1-\delta)^2} \ln((1+\gamma)\delta) \]

Replacing in (A-3) would give us:

\[
\ln \left( x_1^{\text{dev}} \frac{1-\delta}{2} r_t \right) + \frac{\delta \ln \left( \frac{(1-\delta) r_{t+1}}{4} \right) + \frac{\delta^2 \ln ((1+\gamma)\delta)}{(1-\delta)^2}}{1-\delta} + \frac{\delta}{1-\delta} \ln \left( \frac{(1-\delta) r_{t+1}}{2} \right) + \frac{\delta}{1-\delta} \ln \left( \delta(1+\gamma) \right) \]  

(A-4)

Optimizing the LHS of this inequality w.r.t. \( x_1^{\text{dev}}(x_2^c) \) we obtain the next F.O.C.:

\[
\frac{1}{x_1^{\text{dev}}} = \frac{\delta}{1-\delta} \frac{1}{(1+\delta) r_t / 2 - x_1^{\text{dev}}} \]  

(A-5)

Solving for \( x_1^{\text{dev}} \) gives us:

\[
x_1^{\text{dev}}(x_2^c; r_t) = \frac{(1-\delta)(1+\delta)}{2} r_t \]  

(A-6)

Substituting in (A-3) gives the following expression:

\[
\ln \left( \frac{(1-\delta)(1+\delta)}{2} r_t \right) + \frac{\delta \ln \left( \frac{(1-\delta)}{2} \left( \frac{(1+\gamma)\delta(1+\delta)}{2} r_t \right) \right) + \frac{\delta^2 \ln ((1+\gamma)\delta)}{(1-\delta)^2}}{1-\delta} + \frac{\delta}{1-\delta} \ln \left( \delta(1+\gamma) \right) \]  

(A-7)

Simplifying yields:

\[
\ln(1+\delta) + \delta \ln(1/2) > 0 \]  

(A-8)

And this last inequality is always true with strict inequality for \( \delta \in [0,1] \).

Second, we know that \( g(0) = 0 \), so that \( \lim_{r_t \to 0} \varphi = \lim_{r_t \to 0} \varphi_g = 0 \). Hence, we deduce that:

\[
\lim_{r_t \to 0} \left( \ln(x_1^{\text{dev}}(x_2^c(r_t))) + \delta V_{1,t+1}^w(r_{t+1}) \right) > \lim_{r_t \to 0} (V_{1,t+1}^c(r_t)) \]

We can therefore deduce that there exists a strictly positive \( r \) such that both players prefer deviating from the cooperative path and reverting to conflict \( \forall r_t \leq r \).
References


