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Risk as Impediment to Privatization? The Role of Collective Fields in Extended Agricultural Households

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Abstract

As in the case of agricultural cooperatives, collective fields in extended agricultural households act as an insurance device, by redistributing income between household members. At the same time they entail inefficiencies arising from the incentives to free ride on co-workers efforts. Privatization solves the latter problem but comes at a cost of lower risk-sharing (Carter, 1987). The classic analysis of the trade-off between efficiency and risk-sharing rules out another major risk-sharing mechanism, namely voluntary interpersonal transfers. This paper is a first attempt to merge the two insurance mechanisms: collective production, which is plagued by free riding and income transfers, which are hampered by limited commitment. Privatization of land is shown to interact with incentives to abide by the insurance agreement, so that the tradeoff between risk-sharing and production efficiency may or may not be maintained with income transfers. We show that an increase in the value of the household members' exit option or a decrease in patience decreases the optimal rate of privatization. With the help of numerical simulations we argue that households of greater size are also more likely to privatize land.

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1 Introduction

When workers share an output that they collectively produce, they de facto pool production risk and therefore experience less fluctuation in their individual income than if they produce individually. Collective production thus plays an insurance role. The downside of output sharing is the temptation to free ride on other workers' efforts, resulting in a sub-optimal effort allocation. This classic trade-off between efficiency and risk-sharing has been extensively analyzed in the literature on agricultural producers cooperatives (see below). In this context, individualization (or privatization) of the production process is always depicted as detrimental to insurance. This conclusion rests on the strong assumption that collective production is the only risk-sharing technology. In particular it ignores voluntary interpersonal transfers, which are a major source of insurance in poor countries. In this paper we directly address this shortcoming and shed new light on the trade-off between privatization and risk-sharing in a context where co-workers may engage in individual transfers in addition to sharing collective production, as is the case in extended households.

Even if the existing literature on risk and mutual insurance in poor countries focuses largely on interpersonal transfers (for recent surveys, see Platteau, 2006: 854-74), a few works have nonetheless focused on the insurance role of collective property rights over productive assets or resources. Upon careful look, however, these works do not address the issue of risk sharing per se, but are concerned with social protection and redistribution. For instance, informal systems of turns are aimed at ensuring equitable access to resources when sites are heterogeneous from the viewpoint of quality and fertility (Alexander, 1982; Bromley and Chavas, 1989; Platteau, 1991b: 122-35; Platteau and Seki, 2001; Platteau, 2006: 829-46). Or, the existence of common property resources is justified by the need to guarantee their livelihoods to poor people who lack adequate private assets (Hayami, 1981; Jodha, 1986; MacKean, 1986; Agarwal, 1991; Dasgupta and Maler, 1993; Beck, 1994; Baland and Platteau, 1996; Godoy et al., 2000; Pattanayak and Sills, 2001; Wunder, 2001). Baland and François (2005) have proposed a formal analysis of the latter problem and they argue that, because of the superior, so-called insurance properties of common property resources (which tend to provide income maintenance in low states), any feasible scheme of private transfers under private property cannot ex ante Pareto-dominate allocations under the commons despite the efficiency gains from privatization (when markets are incomplete). Yet, since their game extends over a single period, the mechanism that they describe is just a one-shot redistribution that provides a minimum income to the poor.

The only strand of economic literature that genuinely looks at the insurance function of collective assets is concerned with agricultural producer cooperatives. Thus, Putterman and DiGiorgio (1985) and Carter (1987) have analyzed the role of collective fields as a way to redistribute income from lucky to unlucky members in a multi-period framework. This effect is achieved because collective output (or at least part of it) is distributed equally among members (Putterman, 1989). If collective farming is subject free-riding, a trade-off inevitably arises between efficiency and risk-sharing considerations. Complete subdivision of cooperative land has been shown by Carter to be suboptimal while intermediate forms that preserve some degree of risk sharing may prove superior. It bears emphasis that in the existing models no system of private reciprocal transfers is allowed to operate side by side with collective property. Carter's conclusion that complete privatization (of cooperative land) is not optimal does not, therefore, come as a surprise. Note that there is a perfect analogy between the context of producer cooperatives consisting of a combination of collective and private landholdings (or a combination of collective and private activities) and the context of large, extended family farms in which a common field whose output is equally shared coexists with private plots farmed by individual members (the so-called mixed farms especially prevalent in West-Africa, as attested in Fafchamps, 2001, Udry, 1996, Guirkinger and Platteau, 2011; von Braun and Webb, 1989).

In this paper, we want to consider the question as to whether collective property can survive in the presence of voluntary reciprocal transfers aimed at providing insurance. Or, to put it in the converse way: once private transfers are possible, can we expect that the risk-pooling collective activity will vanish due to the efficiency loss that it gives rise to? The answer to that question is not evident because, if the collective activity is subject to incentive problems, reciprocal transfer arrangements are vulnerable to a well-known commitment problem. There is, indeed, the temptation for each member of an informal risk-pooling network to break his or her promise to help a fellow participant who has suffered from a negative shock. Toward the purpose of shedding light on the above issue, we model a family farm of the mixed type. We assume that the family farm institution manages land allocation and risk sharing. We define the rate of privatization as the share of the farmland allocated to private plots allotted to each household member. Over these plots, holders have use rights plus the right to rent them out. In accordance with Guirkinger and Platteau's analysis of the patriarchal farm (2011), we obtain that first-best efficiency is achieved on private plots whereas production on the collective field is plagued by free-riding. Like them, also, we assume that collective output is equally shared and privatized land is equally apportioned among all members (so that privatization is equitable). Unlike them, however, but similarly to Carter (1987) or Baland and François (2005), we posit the existence of an idiosyncratic shock[1] and of risk aversion on the part of family members.

In this rather complex framework, we are able to show analytically that, under some conditions, further privatization of the land leads to a win-win situation. This implies that efficiency gains are

[1]The model allows for both idiosyncratic and covariate shocks. Obviously, insurance arrangements of the risk pooling type are only effective to deal with idiosyncratic shocks. In our model, the focus is therefore on the latter type of risk. A form of covariate risk is however introduced in order to highlight the robustness of the results.

compounded by greater insurance benefits obtained through increased decentralized income transfers. This (potentially) positive effect of privatization on insurance transfers is, to our knowledge, ignored in the existing literature. It is obtained when privatization raises the value of staying in the household in such a way that even under limited commitment lucky members are willing to transfer larger amounts to unlucky members. The key point is that insurance lost as a result of forgone collective production is outweighed by increased voluntary transfers. This situation is more likely when family size is sufficiently large, implying that the free-riding problem is serious enough. As a consequence, in large families, complete privatization is optimal. The same win-win outcome is likely to obtain if members give sufficient importance to future income flows, or if the available exit opportunities are not very attractive. Depending on the model parameters, the efficiency-insurance trade-off may well exist and persist, and whether complete privatization is optimal in that situation is unclear, indicating that the outcome highlighted by Carter and Baland/François is a distinct possibility.

The remainder of the paper is structured as follows. In Section 2, we describe the setup of assumptions on which the model is based, in particular the nature of access rights with respect to land, the market environment, the risk structure and preferences, and the risk-sharing mechanisms available inside the household. Section 3 addresses the issue of production efficiency, specifying successively the pattern of labor allocation between collective and private production, and consumption levels in each state of nature. After these preliminary steps, we characterize the first-best arrangement, complete privatization, that is obtained when income transfers are perfectly enforceable. In Section 4, the incentive compatibility condition on income transfers and the second-best vector of income transfers are derived. In Section 5, we come to the heart of our problem, and analyze the trade-off between production efficiency and risk-sharing. After setting the benchmark case in which the trade-off is unavoidable owing to the absence of income transfers, the impact of privatization on expected consumption and risk-sharing with incentive compatible income transfers is analyzed. We highlight the conditions under which the trade-off may vanish or persist when commitment is limited. Moreover, we carry out numerical simulations to fully characterize the impact of key parameters of the model on the optimal privatization rate. Section 6 summarizes the central lessons from the analysis.

2 The model setup

2.1 The land tenure regime and the market environment

We consider an extended household composed of n members $i \in N$. Each household member is endowed with L units of land of homogeneous quality and one unit of productive time, provided he is not sick in the period considered, otherwise, his time endowment is zero. The household total land endowment nL is divided between collective and individual fields. Let μ denote the fraction of the aggregate family landholdings that is individually apportioned. For ease of exposition, we call μ the privatization rate. Notice, however, that the existence of a complete set of property rights associated with land is unnecessary for our results to hold. In particular, we do not assume that land can be sold or serve as a collateral (there is no credit market in the model). We discuss later the status of private land in the case of household dissolution, namely the issue of whether an individual could leave the household along with his individual plot. For the time being, it is sufficient to point out that these considerations do not play a critical role in our model. Strictly speaking, we only assume that the individual who receives a private plot is allowed to fully appropriate its proceeds. Individual fields are of homogeneous size μL and each household member farms his plot independently. The size of the collective field is therefore $(1-\mu)\,nL$ and its proceeds are equally shared among the members. Because of prohibitive supervision costs, labor on the collective field is supplied non-cooperatively. Agricultural technology is identical across collective and individual parcels. The production function is defined on labour e and land l, and is written $f\,(e,l)$. It is subject to constant returns to scale and exhibits the usual following properties:

$$f_e > 0, f_l > 0, f_{ee} < 0, f_{ll} < 0, f_{le} = f_{el} > 0.$$

The market environment is characterized by the absence of credit and insurance markets. As already mentioned, we are agnostic about the existence of a land sale market. We nonetheless assume that land can be rented out, either inside or outside the household, at a rental rate equal to the profit-maximizing return per unit of land. Finally, we assume that labor allocated to agricultural work has a constant opportunity cost, w, per unit of time. For instance, household members may choose to undertake off-farm activities at a constant marginal return w. This implies the availability of non-agricultural income opportunities arising from the local labor market, temporary migration or handicraft activities. Technically speaking, a unit time endowment has to be allocated between three activities: work on the collective field e_C, work on the private plot e_I , and off farm work, $1 - e_C - e_I$.

2.2 Risk structure and preferences

In each growing season, t, a subset $H_t \subset N$ of size h_t is randomly drawn within the household. Household members belonging to this subset have one unit of productive time while the others are unable to work for the current season. This setting typically represents the case of a health shock. Risks affecting agricultural yields will essentially differ from this example since decisions are then taken before uncertainty is realized. In the case of illness, however, labor decisions are taken after the shock strikes and thus under certainty, which makes our model more tractable. Moreover, because we want to focus on intra-household risk-sharing, we need to concentrate on shocks that household members can share. The risk of illness offers a particularly appropriate example as it is at least partially uncorrelated between members. On the contrary, a drought or low market prices are highly correlated and, therefore, not insurable at the household level. Household members are homogeneous in the sense that, ex ante, they face the same probability to be healthy in any season t. More specifically, a household member's time endowment is drawn from a Bernoulli distribution:

$$P(i \in H_t) = 1 - P(i \notin H_t) = \frac{\bar{h}}{n}, \forall i \in N, \forall t,$$

where $\overline{h}/n$ is the unconditional probability of being healthy, with $\bar{h}$ denoting the expected number of healthy members in the household. Our risk structure is very general in the sense that any type of correlation between individual draws is allowed. Let us briefly mention two extreme cases, namely perfectly covariate and perfectly idiosyncratic risks. The former would result from perfectly positive correlation between individual draws, in which case everyone has the same outcome in any given period. Household members are then unable to insure one another and uncertainty is entirely borne at the household level. Perfectly idiosyncratic risk obtains if the number of healthy household members remains constant over time, in which case perfect insurance can technically be achieved if risks are pooled within the household. As these extreme cases suggest, the covariate component of uncertainty is given by the probability distribution of the number of healthy household members h_t at the household level, which we define as

$$P(h_t = h) = p_h, \forall t,$$

with

$$\sum_{h=0}^{n} p_h = 1; E(h_t) = \sum_{h=0}^{n} p_h h = \bar{h}, \forall t.$$

At this stage, we leave the probability distribution of h unspecified, thereby allowing any correlation between individual draws. However, we assume that draws are iid over time.[2] Let us therefore drop the subscript t from now on. We are now able to specify the probability to be healthy conditional on the number of healthy members in the household, which is simply given by

$$P(i \in H \mid h) = 1 - P(i \notin H \mid h) = \frac{h}{n}, \forall i \in N.$$

Given the above risk structure, the household members' per period expected utility within the

[2] The case of iid draws is obtained if h follows a binomial $\left(\frac{\bar{h}}{n}, n\right)$ distribution. On the other hand, if $h = \bar{h}$ with probability 1, risk is perfectly idiosyncratic and correlation between draws is negative.

household[3] is given by

$$V = \sum_{h=0}^{n} p_h \left[\left(1 - \frac{h}{n}\right) u\left(c_0\left(h\right)\right) + \frac{h}{n} u\left(c_1\left(h\right)\right) \right], \tag{1}$$

where $c_1(h)$ denotes a member's consumption level if he belongs to H (is healthy) and $c_0(h)$ denotes his consumption if sick, when h household members are healthy. Agents are risk averse. Their attitude toward risk is captured by the shape of the utility function: $u' > 0; u'' < 0$. Agents are infinitely lived and discount the future. The objective function of a household member i belonging to H for the current period therefore writes

$$U_{i \in H} = u\left(c_{i1}\left(h\right)\right) + \delta V_i,$$

where $\delta = \frac{\eta}{1-\eta}$, with $\eta \in [0, 1]$ the discount factor.

2.3 The risk-sharing mechanisms

In our framework, there are three channels through which sick members may be compensated for the shock they face.

1. First, they are able to rent out their private plot of land. The size of the income shock is therefore normalized to the value of the productive time lost.

2. Second, the output of the collective field is shared equally. As we make clear below, this sharing rule implies a form of transfer in kind from healthy to sick agents. More precisely, the transfer takes the form of labor applied to the collective field.

3. Third, there are direct income transfers between household members. We let the transfer depend on the state of the world at the household level h. Let $\tau_h^{ou\tau}$ and τ_h^{in} denote the amounts transferred by healthy members and received by sick members, respectively. In each period, the insurance (direct income sharing) budget constraint is

$$h\tau_h^{ou\tau} \geq (n - h)\, \tau_h^{in}. \tag{2}$$

   We assume that the budget constraint is binding in each period: no savings or borrowing is allowed. Note that precautionary savings at the household level would allow to insure against the covariate component of uncertainty. We do not allow for this possibility and concentrate on insurance against idiosyncratic shocks, which only relies on intra-period transfers. The second assumption is in accordance with our assumption of absent credit markets.

[3] Exit options are defined in a subsequent section.

In the following, we explore different assumptions regarding the level of commitment that can be attached to these reciprocal insurance transfers. We introduce two benchmark cases. First, the case of perfect commitment allows us to establish the first-best allocation. Second, we briefly explore the case of totally unenforceable transfers which is considered in the aforementioned works of Putterman (1989) and Carter (1987). This extreme situation is worth examining with our model because the trade-off between risk-sharing and production efficiency then unambiguously arises. Finally, we extensively discuss the more realistic case in which direct income transfers are impeded by limited commitment, thereby necessitating the introduction of an incentive compatibility condition.

Let us now solve the model. We proceed in two steps. The first step consists in analyzing the implications of a non-cooperative allocation of labor within the household. As pointed out in the next section, collective production entails free riding behavior, thereby creating a positive relationship between production efficiency and the privatization rate. We then characterize the first-best situation in this model. In a second step, risk and insurance implications are analyzed in detail.

3 Production efficiency

3.1 Labor allocation

In this model, risk takes the form of a random shock that is supposed to affect the agents' ability to work. It follows that the agents' decision regarding the allocation of their time takes place once uncertainty is realized and is therefore unaffected by it. Healthy household members simply maximize their current consumption level, which writes

$$c_1 = \frac{1}{n} y^C + y^I + w\left(1 - e^C - e^I\right) - \tau_h^{out}, \tag{3}$$

where y^C and y^I stand for agricultural production on the collective field and on the member's private plot, respectively. From now on, we use superscripts C and I to refer to activities on the collective and individual lands, respectively. According to our assumptions, y^C et y^I are given by

$$\begin{aligned} y^C &= f\left(E, (1-\mu)\, nL\right), \text{ where } E = \sum_{i \in H} e_i^C, \\ y^I &= f\left(e^I, \mu L\right). \end{aligned}$$

In words, equation (3) states that a healthy member's consumption is composed of a fraction $1/n$ of collective production, his entire private production, and off-farm income of w per residual unit of time (there are no savings). Besides, any healthy agent gives an income transfer τ_h^{out}. Recall that labor allocation is not contractible within the household and is hence chosen non-cooperatively.

In any Nash equilibrium, the allocation of time between the three activities is then given by the following arbitrage condition:[4]

$$\frac{1}{n} f_e^C = f_e^I = w, \tag{4}$$

As equation (4) illustrates, production on the collective field is plagued by free riding. As expected, given the equal sharing rule, labor is under-provided since its marginal productivity is n times higher that its opportunity cost w. Production on the collective field is therefore inefficient. In order to compare the rent generated on the collective field to its private field counterpart, the following Lemma is useful.

Lemma 1 *Under constant returns to scale, if labor is applied so as to maintain its marginal productivity constant, total rent is proportional to the cultivated land area l: if $\tilde{e}(l,k)$ is such that $f_e(\tilde{e}, l) = k$, then*

$$\begin{aligned} R(l,k) &= f(\tilde{e}(l,k), l) - w\tilde{e}(l,k) \propto l \\ \iff & \quad R(l,k) = lR(k). \end{aligned}$$

Proof. Provided in Appendix 1. ■

It follows that the rent per unit of land area is constant. Let then R^* and R^C denote the rent per unit of land on private plots and the collective field, respectively. Equation (4) implies that $R^C < R^*$. The rent on private land is actually maximized with respect to labor application. Notice that, due to a dilution effect, $\partial R^C / \partial n < 0$. Put differently, incentives to work worsen following an increase in the household size, because the fraction of output that workers can appropriate is thereby reduced.

3.2 Consumption levels

Let us write the two consumption levels:

$$c_1 = \frac{1}{n} y^C + y^I + w\left(1 - e_i^C - e_i^I\right) - \frac{n-h}{h}\tau_h, \tag{5}$$

$$c_0 = \frac{1}{n} y^C + \tau_h + \mu L R^*. \tag{6}$$

The three types of transfers received by sick household members clearly appear in equation (6): (1) they receive a share of collective production, (2) they benefit from a pure income transfer τ_h, (3)

[4] More precisely, we only know that aggregate provision of labor is given by: $\frac{1}{n} f_e(E^*, (1-\mu) nL) = w$. There is a continuum of Nash equilibria satisfying this condition but with different distributions of effort among healthy household members. For simplicity, we assume that the symmetric equilibrium, in which $e_{iC}^* = \frac{E^*}{h}, \forall i \in H$, is selected.

and they are able to rent out their private plot at a rate R^*. Substituting for equilibrium labor allocation, we obtain the following expressions:

$$c_1 = L\left[(1-\mu) R^C + \mu R^*\right] + w - \frac{n-h}{h}\left(\frac{wE^*}{n} + \tau_h\right), \tag{7}$$

$$c_0 = L\left[(1-\mu) R^C + \mu R^*\right] + \left(\frac{wE^*}{n} + \tau_h\right). \tag{8}$$

The consumption levels are determined by the income generated by the two factors owned by members, namely land and labor. On the one hand, the rent associated to the land endowment is a weighted average of the collective and optimal rent levels, where weights are determined by the privatization rate. On the other hand, the value of labor is simply given by its opportunity cost w. By comparing equations (7) and (8), it can be seen that the extent of the shock faced by sick agents is precisely w. This is intuitive since what they lose is the value of their productive time. Finally, we can highlight the two transfers from the h healthy to the $(n-h)$ sick agents: τ_h is the pure income transfer received, while wE/n is a transfer in kind. The latter corresponds to the value of labor devoted to the collective field, per household member.

3.3 The first-best arrangement

We are now set to analyze the impact of privatization on production efficiency. This is the aim of the following lemma.

Lemma 2 *The expected aggregate household income is increasing in privatization:*

$$\partial E_h(Y)/\partial\mu > 0.$$

Proof. In the absence of savings, aggregate household income is simply given by aggregate consumption, which writes

$$Y(h) = hc_1 + (n-h)c_0 = hw + nL\left[\mu R^* + (1-\mu) R^C\right], \tag{9}$$

where use has been made of equations (7) and (8). Recalling that h is a random variable, the derivative of expected aggregate income is then

$$\frac{\partial E_h(Y)}{\partial\mu} = \sum_{h=0}^{n} p_h \frac{\partial Y(h)}{\partial\mu} = \left(R^* - R^C\right) nL > 0.$$

■

This result is straightforward. For each unit of land withdrawn from the collective field and reallocated to private parcels, the rent increases by the difference between R^* and R^C. Privatization strengthens the incentives to work, thereby improving labor allocation. In the absence of economies of scale, the usefulness of collective production can only come from insurance considerations. However, if pure income transfers are perfectly enforceable, they should be preferred to collective production as a risk-coping device. This result appears in the next proposition, which establishes the first-best arrangement, understood as the optimal rule governing production and risk-sharing within the household.

Proposition 1 ***First best:*** *Under non-cooperative labor allocation, the optimal institution in terms of land tenure and income transfers* $\left(\mu^{FB}, \tau_1^{FB}...\tau_{n-1}^{FB}\right)$ *is characterized by*

1. *Complete privatization:* $\mu^{FB} = 1$*,*

2. *Perfect insurance against idiosyncratic risk:* τ_h^{FB} *are such that* $c_1 = c_0, \forall h \in \{0, ...n\}$*. More precisely,* $\tau_h^{FB} = wh/n$*.*

Proof. Maximizing expected utility (1) with respect to any given transfer τ_h and using (7) and (8), the first order condition is as follows:

$$\begin{aligned} \frac{\partial V}{\partial \tau_h} &= p_h \left(1 - \frac{h}{n}\right) \left[u'\left(c_0\left(h\right)\right) - u'\left(c_1\left(h\right)\right)\right] = 0 \\ \iff & \quad c_1 = c_0 \iff \tau_h^*\left(\mu\right) = \frac{w}{n}\left(h - E^*\right). \end{aligned}$$

Then, for each h and for a given privatization rate, there is an optimal income transfer $\tau_h^*\left(\mu\right)$ that equalizes the two consumption levels. Substituting for this optimal insurance scheme $\left(\tau_1^*...\tau_{n-1}^*\right)$ in the objective function gives

$$V = \sum_{h=0}^{n} p_h u\left(\frac{Y\left(h\right)}{n}\right),$$

where the household's aggregate income $Y\left(h\right)$ is given by equation (9). Since, according to Lemma 2, the aggregate household income Y increases with privatization, V immediately appears to be itself increasing in privatization:

$$\frac{\partial V}{\partial \mu} = \sum_{h=0}^{n} p_h u'\left(\frac{Y\left(h\right)}{n}\right) \frac{1}{n} \frac{\partial Y\left(h\right)}{\partial \mu} > 0.$$

The privatization rate is therefore at a corner $\mu^{FB} = 1$. Finally, $\tau_h^{FB} = \tau_h^*\left(1\right) = wh/n$, since no effort is spent on the collective field ($E = 0$). ■

Proposition 1 implies that, if perfect risk-sharing through income transfers is feasible, land should be completely privatized to maximize household income. To be more precise, the first best is actually defined by the combination of (1) a labor allocation that maximizes aggregate income and (2) equal levels of consumption between household members. The way to achieve the first-best outcome clearly hinges on enforceability issues. If labor allocation is enforceable, its optimal application on the collective field can be specified and the status of land becomes irrelevant. If, on the contrary, labor is not contractible while income transfers are fully enforceable, complete privatization allows to reach the first best. As already mentioned, however, commitment issues may limit the scope for risk-sharing through income transfers. The coexistence of incentive constraints on both labor and transfers is at the heart this paper and its implications are examined below.

Before turning to this issue, two additional points are worth mentioning. First, as intuition would suggest, the optimal insurance transfer received, τ_h^{FB}, is increasing in the number of healthy members and in the value of labor w (since the shock is then higher). Second, while risk-sharing through transfers offers a perfect coverage against idiosyncratic shocks, the household remains obviously uninsured against fluctuations of its aggregate income $Y(h)$.

4 Risk-sharing under limited commitment

If perfect risk-sharing is desirable for everyone ex ante, it might be in a healthy agent's interest to renege on his income transfer ex post. The benefit of doing so is simply given by the amount he was required to transfer. As regards the cost of such a deviation, we need to define the sanctions that the household can implement in order to deter deviation. We assume that two sanctions are used. First, an agent who deviates can be excluded from the household. Assuming that the threat of exclusion is credible is an extreme assumption, but the important thing to note is that the stronger the sanction the more efficient risk-sharing. This actually tilts the argument against collective production as a way of enforcing risk-sharing. Second, a member who deviates does not receive his share of collective production, which is reasonable to assume since this sanction, taking place afterwards, is simple and easy to implement. The following time structure follows:

1. Nature draws a subset $H \subset N$ of size h. The household members belonging to H are endowed with one unit of productive time.

2. Household members $i \in H$ choose either to stay within the family farm and to abide by the insurance agreement $(\tau_1 ... \tau_{n-1})$, or to leave with the output of their private parcel at the end of the growing season.

3. Household members $i \in H$ non-cooperatively allocate their work effort (e_C, e_I).

4. Household members who had chosen so in stage 2 leave the household forever with the output of their private parcel. The other members consume the sum of their private output and their share of collective output adjusted for the transfers they make or receive.

A series of points deserve a brief discussion here. First, there is no relevant distinction between being excluded and leaving the household voluntarily. Second, even if the household member leaves at the end of the season, the decision to leave is taken beforehand so that it can have an impact on the agent's optimal allocation of labor. Indeed, anticipating that he will not receive his share of the collective output, an agent who leaves should not take part in collective production. Third, in order to write the incentive compatibility condition on transfers, we need to define the agents' exit option, that is their per period reservation utility outside the household, which we call $\bar{V}$. The latter critically depends on prevailing rules of access to land. Three options are available here: (1) the departing member leaves without land. This would be the case if migration turns out to be the best opportunity or if land is under the corporate ownership of the family unit; (2) The departing member leaves with his total land endowment L; (3) he leaves with his private plot, μL. The latter possibility implies the existence of complete and well defined property rights, which are not assumed in our model. Moreover, it will become clearer below that, by generating a tradeoff between production efficiency and risk-sharing, case (3) would naturally tilt the argument against privatization. This is because privatization improves the household members' exit option, worsens the commitment issue and hence impedes income-sharing. Case (3) nevertheless remains relevant and we therefore briefly explore it at the end of the paper. For the main analysis, we assume that $\bar{V}$ is exogenously given, which encompasses cases (1) and (2).

We are now able to write the incentive compatibility condition on transfers $(\tau_1 ... \tau_{n-1})$. In stage 2, for a given state of the world h, healthy household members have to decide whether to stay within the household. It will be optimal for them to pay their transfer and to stay provided that

$$u(c_d) + \delta\bar{V} \leq u(c_1) + \delta V,$$

where c_d is their current consumption level if they deviate. The definition of c_d is given by

$$c_d = w + \mu L R^*. \tag{10}$$

This consumption level is achieved if the departing member allocates his labor force optimally between farm activities on his private land and off-farm activities. He therefore reneges on the two types of transfers, namely the pure income transfer and the labor contribution to the collective field. The second best vector of income transfers $(\tau_1 ... \tau_{n-1})$ must achieve the highest possible level of risk-sharing, while satisfying the following incentive compatibility condition

$$u(c_d) - u(c_1(\tau_h)) - \delta\left(V - \bar{V}\right) \leq 0, \forall h \in \{0, ...n\}. \tag{11}$$

It is noteworthy that neither c_d (see equation (10)) nor V depend on a particular state of the world h. Indeed, V is the expected utility over all possible realizations of h. It follows that the incentive compatibility constraint simply imposes a minimum consumption level $\tilde{c}_1$, such that condition (11) holds with equality. In other words, to deter deviation, the household should at least provide healthy members with a consumption of $\tilde{c}_1$. This, in turn, imposes a maximum amount of transfer. We define κ as the highest incentive compatible amount of transfers in both income and labor:

$$\kappa = \frac{n-h}{h}\left(\frac{wE}{n} + \tilde{\tau}_h\right) \iff \tilde{\tau}_h = \frac{h}{n-h}\kappa - \frac{wE}{n}, \tag{12}$$

such that

$$u(c_d) - u(c_1(\kappa)) - \delta\left(V - \bar{V}\right) = 0. \tag{13}$$

Thus, κ measures the aggregate willingness to transfer of a healthy household member.

Lemma 3 *The incentive compatibility condition on income transfers will be binding if the number of healthy household members is lower than* $\tilde{h} = \left(1 - \frac{\kappa}{w}\right)n$*, otherwise perfect insurance will be incentive compatible.*

Proof. As already mentioned, the income transfer that equalizes the consumption levels is given by

$$\tau_h^* = \frac{w}{n}(h - E), \tag{14}$$

while the incentive compatible transfer writes

$$\tilde{\tau}_h = \frac{h}{n-h}\kappa - \frac{wE}{n}. \tag{15}$$

Hence, the optimal income transfer will be incentive compatible provided that

$$\tau_h^* < \tilde{\tau}_h \iff h > \tilde{h} = \left(1 - \frac{\kappa}{w}\right)n.$$

■

This lemma tells us that there exist two types of states of the world: states where the incentive compatibility condition is binding and risk-sharing incomplete, on the one hand, and states where the condition is not binding and risk-sharing is therefore complete, on the other hand. The result is very intuitive. Indeed, when there are few healthy workers, insurance needs are important and cannot be met by the agents' willingness to transfer, which is constant across states. On the contrary, when h is high, the required transfer is reduced and is therefore more likely to be incentive compatible.

Proposition 2 ***Second best insurance arrangement.*** *Under limited commitment, the vector of income transfers* $\left(\tau_1^{SB}...\tau_{n-1}^{SB}\right)$ *is such that*

$$\begin{aligned} \tau_h^{SB} = \min\{\tilde{\tau}_h, \tau_h^*\} &= \tilde{\tau}_h \text{ if } h < \tilde{h}, \\ &= \tau_h^*, \text{ otherwise,} \end{aligned}$$

where $\tilde{\tau}_h$ *and* τ_h^* *are given by equation (15) and (14), respectively.*

Proof. This is a direct corollary of Lemma 3. ■

It follows that the level of risk-sharing achieved within the household is positively correlated with its aggregate income. When there are few workers, this income is lower and risk-sharing is incomplete. Household members are then more vulnerable to idiosyncratic shocks when the shock is important at the household level, or equivalently, when more people are hit at the same time. This result is fully compatible with previous work on risk-sharing (Coate & Ravallion (1993)). Risk-sharing is clearly impeded by limited commitment, especially if the covariate shock is important.

5 The trade-off between production efficiency and risk-sharing

In this section, we explore the trade-off that might exist between production efficiency and risk-sharing. While we have already highlighted the positive relationship between production efficiency and privatization (see Lemma 2), we now need to examine how privatization affects risk-sharing. This question is answered in two steps. In a first step, we briefly assume that income transfers are totally unenforceable. In a second step, we tackle the general case in which income transfers are allowed but are also required to be incentive compatible. The latter case corresponds to the second best risk-sharing mechanism described in the preceding section.

5.1 The impact of privatization in the absence of income transfers

The aim of this subsection is to highlight that, absent income transfers, a trade-off between production efficiency and risk-sharing automatically arises. Our setting thus allows for the result of Carter (1987) as a particular case.

For ease of exposition, we keep a similar notation[5] and simply assume that $\tau_h = 0, \forall h$ in equation (12). Transfer from healthy to sick household members can be only in kind, taking the form of the value of labor on the collective field. We have

$$\kappa_h = \frac{n-h}{h}\frac{wE}{n}.$$

Insurance being provided through the distribution of the collective production only, privatization will necessarily result in a reduction of the transfer: $\partial \kappa_h / \partial \mu < 0, \forall h$. This is because, if the size of the collective parcel is reduced, the equilibrium level of collective labor also decreases: $\partial E / \partial \mu < 0$. We can then easily derive the following proposition.

[5]While κ denotes the maximal incentive compatible amount of transfers (both labor and income transfers) and is constant accross the different states of the world h, let κ_h in this subsection stand for the transfer in labor, which will differ accross states.

Proposition 3 ***Impact of privatization on expected consumption and risk-sharing in the absence of income transfers*** $(\tau_h = 0, \forall h)$*:*

1. Privatization increases the expected consumption of household members:

$$\frac{\partial}{\partial \mu} \sum_{h=0}^{n} p_h \frac{Y(h)}{n} > 0.$$

2. Privatization increases the gap between the consumption levels of healthy and sick household members in each state of the world:

$$\frac{\partial}{\partial \mu} (c_1 - c_0) \geq 0, \forall h.$$

3. Consequently, there is a tradeoff between production efficiency and risk-sharing.

Proof. The first point is a direct corollary of Lemma 2.

Concerning the second point, from equations (8) and (7), it is evident that $\tau_h = 0$ implies

$$c_1 - c_0 = w\left(1 - \frac{E}{h}\right), \tag{16}$$

which is increasing in μ as $\partial E / \partial \mu < 0$.

Finally, the trade-off results from the preceding points. For the sake of completeness, we derive the first-order condition with respect to the privatization rate: in the absence of income transfers, the optimization program is

$$\underset{\mu}{Max} V = \sum_{h=0}^{n} p_h \left[\left(1 - \frac{h}{n}\right) u(c_0) + \frac{h}{n} u(c_1)\right].$$

Bearing in mind that $\tau_h = 0, \forall h$ and making use of (8), (7) and Lemma 2, we end up with the following condition, which determines the (constrained) optimal privatization rate:[6]

$$E(u')\left(R^* - R^C\right) L = E_h\left[u'(c_0) - u'(c_1)\right] \frac{n-h}{n} \frac{w}{n} \left|\frac{\partial E^*}{\partial \mu}\right|. \tag{17}$$

■

In the absence of income transfers, privatization necessarily comes at the expense of risk-sharing, as the gap between the consumption of the sick and the healthy increases with privatization (equation 16). A trade-off therefore arises between risk and expected consumption. As condition (17) illustrates, privatization cannot be complete if both effects are to be balanced.

[6] where $E(u') = \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \left[\left(1 - \frac{h}{n}\right) u'(c_0(\kappa)) + \frac{h}{n} u'(c_1(\kappa))\right] + \sum_{\lfloor \tilde{h} \rfloor + 1}^{n} p_h u'\left(\frac{Y}{n}\right)$.

5.2 The impact of privatization with incentive compatible income transfers: analytical results

We are now ready to analyze the most interesting case of limited commitment. In other words, we want to determine the impact of privatization on household production and risk-sharing when incentive compatible income transfers are allowed.

First, it is useful to write the expected utility of household members that is obtained under the above assumption. Incorporating the second best vector of income transfers described in Proposition 2, we end up with the following expression:

$$V=\sum_{h=0}^{\lfloor\tilde{h}\rfloor}p_h\left[\left(1-\frac{h}{n}\right)u\left(c_0\right)+\frac{h}{n}u\left(c_1\right)\right]+\sum_{\lfloor\tilde{h}\rfloor+1}^{n}p_h u\left(\frac{Y}{n}\right), \tag{18}$$

where

$$\begin{aligned} c_0 &= L\left[(1-\mu)R^C+\mu R^*\right]+\frac{h}{n-h}\kappa, \\ c_1 &= L\left[(1-\mu)R^C+\mu R^*\right]+w-\kappa. \end{aligned}$$

Bear in mind that limited commitment may or may not impede insurance through income transfers depending on the number of contributing (healthy) household members. More precisely, perfect insurance against idiosyncratic risk can be achieved provided healthy household members are numerous enough $\left(h\geq\tilde{h}\right)$. Two types of states of the world then exist. When $h<\tilde{h}$, limited commitment is binding, income sharing is incomplete and household members remain exposed to a certain amount of idiosyncratic risk as they either consume c_0 or c_1. When $h\geq\tilde{h}$, by contrast, sick household members are fully compensated for their shock and any agent's consumption level is equal to Y/n.[7] Note also that the willingness to transfer of the contributing members, κ, is itself a function of expected utility V (see its definition (13)). A higher expected utility within the household, indeed, provides an incentive to transfer. It follows that equation (18) does not give an explicit expression of expected utility. What we have is an implicit definition of the objective function $V(\mu)$, written as follows

$$\Psi\left(\mu,V\right)=\sum_{h=0}^{\lfloor\tilde{h}\rfloor}p_h\left[\left(1-\frac{h}{n}\right)u\left(c_0\right)+\frac{h}{n}u\left(c_1\right)\right]+\sum_{\lfloor\tilde{h}\rfloor+1}^{n}p_h u\left(\frac{Y}{n}\right)-V=0$$

In this configuration, the first-order condition with respect to the privatization rate μ is given by $\Psi_\mu=0$, where Ψ_μ is the partial derivative of Ψ with respect to μ. V is therefore taken as a constant

[7] Note that $\tilde{h}$ being a continuous function, the last constrained state (where limited commitment is binding) is given by $\lfloor\tilde{h}\rfloor$, that is, the entire part of $\tilde{h}$. For the same reason, the first unconstrained state is $\lfloor\tilde{h}\rfloor+1$.

in computing the following derivatives.[8] Moreover, the sign of the marginal utility of privatization $dV/d\mu$ is given by the sign of Ψ_μ.

The household is assumed to implement the second best vector of income transfers (Proposition 2) and to select the privatization rate that maximizes expected utility. In the following, we explore the implications of this optimization program.

Lemma 4 *With incentive compatible income transfers, the (constrained) optimal privatization rate should satisfy the following Kuhn-Tucker conditions*

$$\begin{aligned} \Psi_\mu &= \frac{\partial V}{\partial \mu} + \frac{\partial V}{\partial \kappa}\frac{\partial \kappa}{\partial \mu} = 0 \text{ and } \mu^* < 1, \\ &= \frac{\partial V}{\partial \mu} + \frac{\partial V}{\partial \kappa}\frac{\partial \kappa}{\partial \mu} > 0 \text{ and } \mu^* = 1. \end{aligned} \tag{19}$$

Proposition 4 ***Impact of privatization on expected consumption and risk-sharing with incentive compatible income transfers****:*

1. *Privatization increases the expected consumption level of household members:*

$$\frac{\partial}{\partial \mu}\sum_{h=0}^{n} p_h \frac{Y(h)}{n} > 0.$$

Holding constant the willingness to transfer of contributing members, κ, the direct effect of privatization on expected utility is positive:

$$\frac{\partial V}{\partial \mu} = E(u')\left(R^* - R^C\right)L > 0.$$

2. *In equilibrium, the impact of privatization on risk-sharing is indeterminate: κ, the willingness to transfer of a healthy household member might either decrease or increase with privatization:*

$$\frac{\partial \kappa}{\partial \mu} < 0 \iff \frac{u'(c_d)}{u'(c_1)} > 1 - \frac{R^C}{R^*},$$

where κ and c_d are given by equations (13) and (10), respectively.

Proof. The first point directly follows from Lemma 2.

Regarding the second point, recall that

$$\begin{aligned} c_1 &= w + L\left[(1-\mu) R^C + \mu R^*\right] - \kappa, \\ c_d &= w + \mu L R^*. \end{aligned}$$

[8] The interested reader can refer to Appendix 2 for a technical discussion.

Applying the implicit function theorem to equation (13), and holding V constant[9], we find

$$\frac{\partial \kappa}{\partial \mu} = -\frac{u'(c_d)}{u'(c_1)} LR^* + \left(R^* - R^C\right) L,$$

so that

$$\frac{\partial \kappa}{\partial \mu} < 0 \iff \frac{u'(c_d)}{u'(c_1)} > 1 - \frac{R^C}{R^*}. \tag{20}$$

■

The first part of Proposition 4 states that, under limited commitment, holding the level of transfers (κ) constant, privatization continues to have a positive impact on household production. Privatization therefore increases expected utility: $\partial V/\partial \mu > 0$. This term is the first term that appears in the first order condition (equation 19). The second term $\partial V/\partial \kappa$ is positive. Indeed, when income-sharing is incomplete, an increase in the amount transferred by healthy household members reduces the difference between the consumption levels of healthy and sick members, thereby reducing the extent of idiosyncratic risk. The third term, namely $\partial \kappa/\partial \mu$, precisely represents the impact of privatization on risk-sharing. As highlighted in the second point of Proposition 4, this effect is ambiguous under limited commitment. In other words, we cannot rule out the possibility that an increase in privatization raises the level of risk-sharing. This is why, in Lemma 4, we allow for corner solutions with respect to the optimal privatization rate, μ^* (the whole family landholding is distributed in the form of private parcels).

A positive impact of privatization on risk-sharing is not readily understandable. Indeed, the mechanical effect by which privatization reduces the size of the collective production remains. Put differently, the transfer in kind, measured by the labor productivity of healthy members on the collective field, is unambiguously smaller. However, income transfers may increase so much as to offset this effect. The potential increase in income transfers would be caused by the change in the incentives to transfer. To understand this effect, let us analyze condition (20) in detail.

To begin with, notice that the left hand side is lower than one since $c_d > c_1$. Recall that κ is determined by the incentive compatibility condition (ICC) (13), which we reproduce here:

$$u(c_d) - u(c_1(\kappa)) = \delta\left(V - \bar{V}\right).$$

In the light of this condition, one can immediately infer that $c_d > c_1 \iff V > \bar{V}$, which must be true for the ex ante participation constraint to be satisfied. In other words, since we allow household members to leave the household once they are informed about their type (healthy or sick), we must also allow them to leave ex ante, so that we must have $V > \bar{V}$. A member should be better off inside the household, ex ante. Since exclusion implies a future sanction, in the current period, insurance transfers can drive c_1 below c_d so that a departing member would consume more outside

[9]See Appendix 2.

the household in the current period. Looking back at condition (20), it can be seen that $\partial\kappa/\partial\mu$ is more likely to be negative if c_d is close to c_1, that is, if V is close to $\bar{V}$. Loosely speaking, income transfers are more difficult to enforce if the exit option is too high. In this instance, privatization reduces risk-sharing, implying that the role of collective production as an insurance mechanism becomes more important. Besides, the ICC also tells us that c_d is closer to c_1 if δ is low. Therefore, privatization reduces risk-sharing when the degree of the members' patience is low. The reasoning is exactly the same as before. The threat of exclusion is stronger if agents are patient, which helps to enforce the income transfers.

Let us now focus on the effect of n on inequality (20). Household size increases the wedge between the private benefit of collective production (proportional to $1/n$) and private cost, and therefore decreases the rent on the collective field R^C (R^* is independent of n). A decrease in R^C tightens inequality (20), as it increases the RHS and decreases the LHS. The later effect is driven by the decrease in c_1, which decreases the ratio $\frac{u'(c_d)}{u'(c_1)}$. The intuition is straightforward: in large households, collective production entails important efficiency losses, which pushes towards privatization. Stated differently, collective production is less costly as an insurance device in small households where privatization is then more likely to reduce the extent of risk-sharing.

In conclusion, the analysis of the partial effect of privatization on the incentives to transfer reveals that a household is more likely to maintain some form of collective production (μ^* is more likely to be interior) if:

1. its size n is small,
2. the household members' exit option $\bar{V}$ is high,
3. the discount factor δ is low (agents are impatient).

To fully characterize the impact of these parameters on the optimal privatization rate, we need to take into account additional partial effects. Appendix 3 presents the complete comparative statics for $\bar{V}$ and δ.

Proposition 5 ***Patience, exit opportunities and the optimal privatization rate***. *Under constant absolute risk aversion (CARA), the optimal privatization rate*

1. decreases with $\bar{V}$, and

2. increases with δ.

Proof. Provided in Appendix 3. ■

To provide some intuition about this result, let us examine the partial effects of $\bar{V}$ on the first order condition of the problem (the same analysis applies to δ). Using an envelope argument, appendix X establishes that:

$$sign\left(\frac{d\mu}{d\bar{V}}\right) = sign\left(\frac{\partial^2 V}{\partial\mu\partial\kappa}\frac{\partial\kappa}{\partial\bar{V}} + \frac{\partial^2 V}{\partial\kappa^2}\frac{\partial\kappa}{\partial\mu}\frac{\partial\kappa}{\partial\bar{V}} + \frac{\partial V}{\partial\kappa}\frac{\partial^2\kappa}{\partial\mu\partial\bar{V}}\right)$$

.

The last term of this sum ($\frac{\partial^2\kappa}{\partial\mu\partial\bar{V}}$) captures the partial effect on incentive compatible transfers analyzed above, which is negative since higher exit opportunities tighten the incentive compatibility constraint: $\frac{\partial^2\kappa}{\partial\mu\partial\bar{V}} < 0$ (note that $\frac{\partial V}{\partial\kappa} > 0$, insurance transfers increases the expected utility of current consumption, everyting else held constant). The first two terms are second order effects, linked the concavity of the utility function. The first one is about the impact of κ on the efficiency gain induced by privatization. The term ($\frac{\partial^2 V}{\partial\mu\partial\kappa}$) is negative since decreasing marginal utility implies that better off individuals are less sensitive to an increase in expected consumption. It is multiplied by the negative effect of $\bar{V}$ on incentive compatible transfers, so that overall this first partial effect goes in opposite direction to the last term. Conversely the second partial effect reinforces the last one since for an interior solution $\frac{\partial\kappa}{\partial\mu} < 0$ (see equation 20). This second partial effect captures the increasing cost of lower transfers when marginal utility is decreasing. We have now signed each term separately:

$$\left(\overbrace{\frac{\partial^2 V}{\partial\mu\partial\kappa}}^{<0}\overbrace{\frac{\partial\kappa}{\partial\bar{V}}}^{<0} + \overbrace{\frac{\partial^2 V}{\partial\kappa^2}}^{<0}\overbrace{\frac{\partial\kappa}{\partial\mu}}^{<0}\overbrace{\frac{\partial\kappa}{\partial\bar{V}}}^{<0} + \overbrace{\frac{\partial V}{\partial\kappa}}^{>0}\overbrace{\frac{\partial^2\kappa}{\partial\mu\partial\bar{V}}}^{<0}\right),$$

In the case of CARA, it is easy to show that the first term is dominated by the last ones, so that increasing exit opportunties reduce privatization. In the case of decreasing absolute risk aversion, we could not unambiguously sign the expression but the simulation results presented in the next section reveal the negative impact of $\bar{V}$ on privatization holds with a DARA utility function.

The analysis of the impact of household size on privatization is more complex, as n is a discrete variable that determine the number of states of the world $h \in \{0, 1, ..., n\}$ and the probability distribution of these states (each p_h will be affected). In the following we discuss the various effects of n in our problem and present simulations results for different household sizes.

5.3 The effect of exit option, patience and family size on privatization: simulation results

Proposition (5) establishes the role of key parameters (discount rate and exit option) on the optimal rate of privatization in the case of constant absolute risk aversion. In the following simulations, we relax this asumption and confirm that the result holds with a decreasing absolute risk aversion utility function. In addition we examine the effect of family size on the optimal privatization rate. Proposition (4) shows that family size n decreases incentives to transfer (at the optimum level of privatization). As argued below, this negative effect is reinforced by the partial effects of n on efficiency and on the value of household membership (V). Finally our simulation results confirm that family size reduces the optimal privatization rate.

The simulation uses the following functional forms. The utility is assumed logarithmic, $u(x) = ln(x)$ so that it exhibits decreasing absolute risk aversion and unitary constant *relative* risk aversion. The production function is a constant return to scale Cobb-Douglas function, $f(e,l) = e^{\beta} l^{1-\beta}$. Finally, we take a binomial distribution of h, $h \sim B\left(\frac{\bar{h}}{n}, n\right)$, and thus assume that shocks are independently and identically distributed.

After setting parameter values, the optimal privatization rate is obtained by a grid search over the range $[0-1]$.[10] [11]

Figures 1 and 2 plot the optimal privatization rate for a range of $\bar{V}$ and δ, and for various n. The figures clearly show that an increase in the value of the exit option decreases the optimal privatization rate, while an increase in patience has the opposite effect. In fact, if members exhibit very little patience or have good exit opportunities, the optimal allocation is a purely collective farm. This results holds across a range of household sizes.

Let us now turn to the effect of the number household members n on privatization. Family size has a direct effect on transfers but also on efficiency (through the free-riding problem) and on the scope for risk sharing through its impact on the number of states of the world and their distribution. The previous section establishes the direction of the first partial effect: privatization is more likely to have a positive impact on the size of voluntary transfers in large than in small households. Second, the direct effect of n on efficiency is negative. As the size of the team increases, so does the wedge between the private benefit of collective production (proportional to $1/n$) and private cost. As a result, the incentive to work on the collective field worsens and the gain from privatization increases. Finally, there is more scope for risk sharing in large families. Intuitively, the probability that all members fall sick simultaneously decreases with n (except in the extreme and unrealistic case of perfect correlation of risks mentioned in section 2.2). While the probability to be healthy is independent of family size ex ante, ex post, there are more healthy individuals in a position to help a sick individual in a large family. Strictly speaking, the probability of intermediate (extreme) outcomes increases (decreases) in the binomial incentive compatibility participation constraint, making privatization more desirable. To conclude, the three partial effects of household size go in the same direction: privatization increases. Figure 1 and 2 illustrates this result: as n is raised, the optimal rate of privatization increases from zero to one, for any level of $\bar{V}$ or n.

[10] For each privatization rate, κ is itself found by grid search using the incentive compatibility constraint

[11] The parameter values used for the graphs are $\delta = 0.5$,$L = 12$, $w = 3$, $\frac{\bar{h}}{n} = 0.7$, $n = 5$, $\bar{V} = 0.4$.

Figure 1: Optimal privatization rate as a function of reservation utility $\bar{V}$, for various n

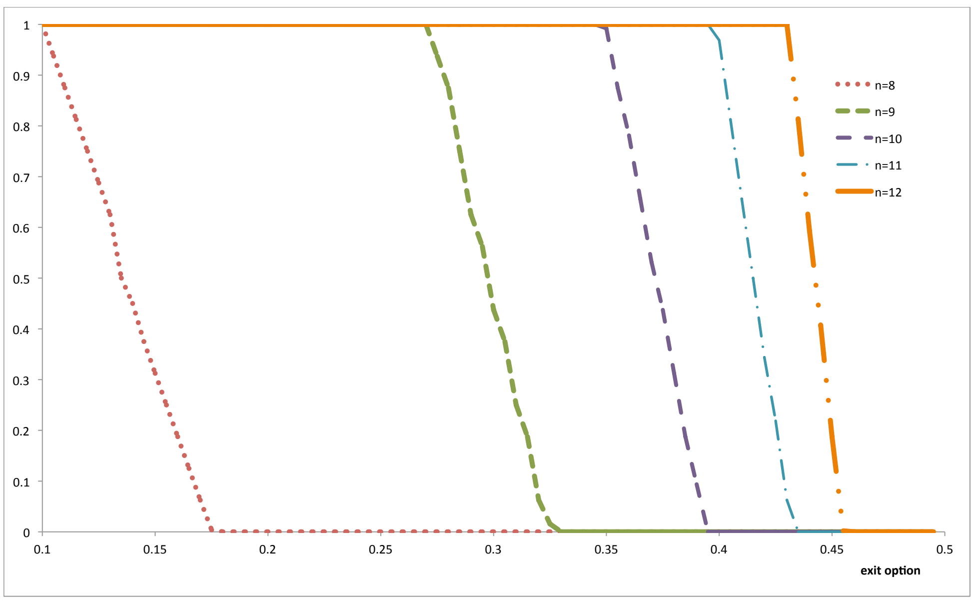
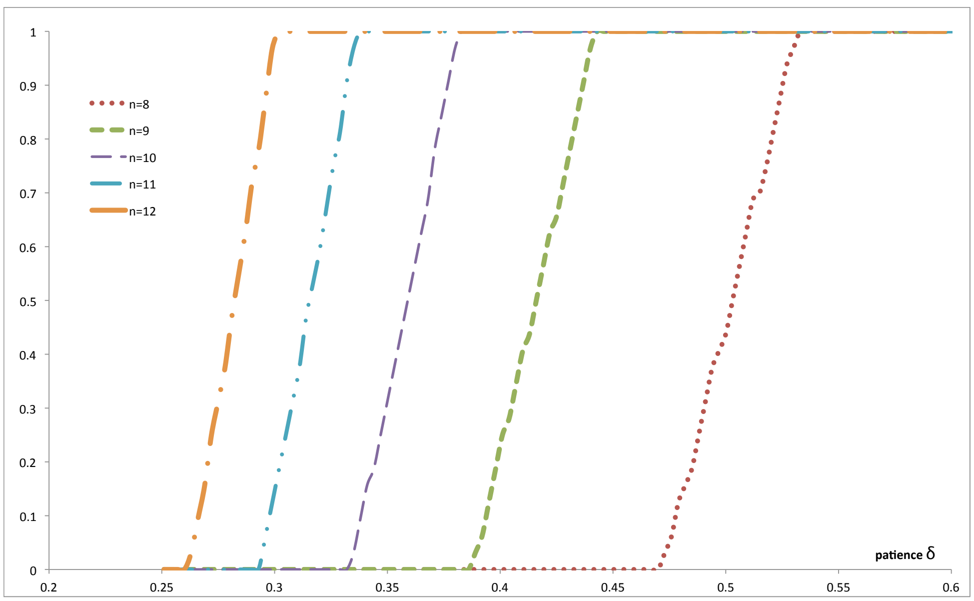


Figure 2: Optimal privatization rate as a function of patience δ, for various n



5.4 The impact of complete property rights over individual land on the trade-off between efficiency and risk-sharing

The issue of a precise definition of the exit option $\bar{V}$ in terms of land ownership has been discussed earlier. Let us briefly come back to this question in the light of our results, which have been derived for the case of exogenous $\bar{V}$. The implication of this assumption in terms of land ownership is that either the departing household member leaves with his total land endowment L, or without land, since in these two instances the exit option is independent of the privatization rate. One could imagine that the holder of what we call a private parcel actually enjoys complete property rights over it, in which case the departing member would leave with a parcel of size μL. Under this alternative assumption, we would have a positive effect of privatization on the exit option: $\partial\bar{V}/\partial\mu > 0$. It is easy to see that this situation would be detrimental to risk-sharing since it would negatively affect the willingness to transfer, κ. The trade-off between production efficiency and risk-sharing would then be more likely to arise.

6 Conclusion

When a risk-pooling, collective mechanism is available side by side with a private activity providing high efficiency but no insurance, we expect an efficiency-insurance trade-off to exist. Removal of the collective mechanism in the context of incomplete (insurance and credit) markets would therefore be suboptimal as has been illustrated in the case of agricultural producer cooperatives.

If private transfers are feasible inside the household, it is not clear whether this trade-off subsists and therefore, what is the implication in terms of the desirability of full privatization. Indeed, to the extent that risk can be shouldered through voluntary, reciprocal transfers, we cannot rule out the possibility that further privatization will enhance both insurance and efficiency, thus creating a win-win outcome. This paper has precisely shown that, indeed, the trade-off between efficiency and insurance can disappear when agents are allowed to make income transfers. Complete privatization may thus become optimal when some conditions are satisfied. Among these conditions is a sufficiently large size of the social unit managing the collective mechanism (in our particular setup, the family farm) so that free-riding impedes collective production enough, or the low attractiveness of exit options available to household members.

In reality, since the above two conditions favorable to land division and privatization may not coexist -reduced mortality leads to larger households but growing market integration raises the level of exit options-, the empirical prediction following from our model seems to be strongly ambiguous. However, market integration may involve other effects than just creating new outside income opportunities. In particular, it typically gives rise to new patterns of demand for agricultural products and the new, higher value-added products (vegetables, fruits,...) often require a shift to more care-intensive production techniques. Because the use of such techniques is especially vulnerable to the free-riding problem (it results in so-called "management diseconomies of scale"), we should expect collective production to become even less efficient when farm output mixes are tilted in favor of the new products demanded by urban consumers.

The central lesson from our theoretical foray is the following: it cannot be assumed that collective production is justified as soon as insurance markets are incomplete and agents are risk averse. When private transfers are possible, the efficiency-insurance trade-off is no more certain to exist, and land tenure individualization might bring both efficiency and insurance benefits. Conversely, it is not because intra-household private transfers are possible that complete division of a family landholding among members is necessarily optimal.

References

Agarwal, B. (1991). *Social Security in Developing Countries*, chapter Social Security and the Family: Coping with Seasonality and Calamity in Rural India, (pp. 171–244). Clarendon Press, Oxford.

Alexander, P. (1982). Sri lankan fishermen: rural capitalism and peasant society. Monographs on South Asia (no. 7). Canberra: ANU.

Baland, J. & Platteau, J. (1996). *Halting Degradation of Natural Resources: Is There a Role for Rural Communities?* Oxford: Clarendon Press.

Baland, J.-M. & Francois, P. (2005). Commons as insurance and the welfare impact of privatization. *Journal of Public Economics*, 89(2-3), 211– 231.

Beck, T. (1994). Common property resource access by the poor and class conflict in west bengal. *Economic and Political Weekly*, 29, 187–197.

Bromley, D. W. & Chavas, J.-P. (1989). On risk, transactions, and economic development in the semiarid tropics. *Economic Development and Cultural Change*, (pp. 719–736).

Carter, M. (1987). Risk sharing and incentives in the collectivization of agriculture. *Oxford Economic Papers*, 39, 577–595.

Coate, S. & Ravallion, M. (1993). Reciprocity without commitment, characterization and performance of informal insurance arrangements. *Journal of Development Economics*, 40, 1–24.

Dasgupta, P. & Maler, G. (1993). *Handbook of Development Economics, Vol. 3A*, chapter Poverty, Institutions, and the Environmental-Resource Base, (pp. 2371–2463). Oxford University Press.

Fafchamps, M. (2001). Intrahousehold access to land and sources of inefficiency: Theory and concepts. In J. P. A. DeJanvry, G. Gordillo & E. Sadoulet (Eds.), *Access to Land, Rural Poverty and Public Action* (pp. 68–96). Oxford: Oxford University Press.

Godoy, R., Wilke, D., Overman, H., Cubas, A., Cubas, G., Demmer, J., McSweeney, K., & Brokaw, N. (2000). Valuation of consumption and sale of forest goods from a central american rain forest. *Nature*, 16406, 62–63.

Guirkinger, C. & Platteau, J. (2011). Transformation of the family under rising land pressure: A theoretical essay. University of Namur. Department of Economics Working Paper Series. WP 1108.

Hayami, Y. (1981). Agrarian problems of india: An east and south asian perspective. *Economic and Political Weekly*, 16, 707–712.

Jodha, N. (1986). Common property resources and rural poor in dry regions of india. *Economic and Political Weekly*, 21, 1169–1182.

McKean, M. (1986). *National Research Council, Proceedings of the Conference on Common Property Resource Management*, chapter Management of Traditional Common Lands (Iriaichi) in Japan, (pp. 533–589). Washington, D.C. : National Academy Press.

Pattanayak, S. & Sills, E. (2001). Do tropical forests provide natural insurance? the microeconomics of non-timber forest product collection in the brazilian amazon. *Land Economics*, 77, 595–612.

Platteau, J. (1991). *Social Security in Developing Countries*, chapter Traditional Systems of Social Security and Hunger Insurance: Past Achievements and Modern Challenges, (pp. 112–170). Clarendon Press, Oxford.

Platteau, J. (2006). *Handbook on Gift-Giving, Reciprocity and Altruism Vol.1*, chapter Solidarity Norms and Institutions in Agrarian Societies : Static and Dynamic Considerations, (pp. 819–886). Amsterdam: North-Holland and Elsevier.

Platteau, J. & Seki, E. (2001). *Community and Market in Economic Development*, chapter Coordination and Pooling Arrangements in Japanese Coastal Fisheries, (pp. 344–402). Clarendon Press, Oxford.

Putterman, L. (1989). Agricultural producer co-operatives. In P. Bardhan (Ed.), *The Economic Theory of Agrarian Institutions* (pp. 319–339). Oxford: Clarendon Press.

Putterman, L. & Digiorgio, M. (1985). Choice and efficiency in a model of democratic semi-collective agriculture. *Oxford Economic Papers*, 37, 1–21.

Udry, C. (1996). Gender, agricultural production and the theory of the household. *Journal of Political Economy*, 5, 1010–1046.

Wunder, S. (2001). Poverty alleviation and tropical forests -what scope for synergies? *World Development*, 29, 1817–1833.

Appendix 1: Proof of Lemma 1

Suppose $\tilde{e}(l,k)$ is such that $f_e(\tilde{e},l) = k$. Under this allocation rule, the rent generated on a field of size l is then given by

$$R(l,k) = f(\tilde{e}(l,k), l) - w\tilde{e}(l,k).$$

We will proceed in two steps. We will show that, under constant returns to scale, $\tilde{e}(l,k)$ is proportional to l and that, consequently $f(\tilde{e}(l,k), l)$ is also proportional to l.

First step: $\tilde{e}(l,k) \propto l$.

First, note that $d^2\tilde{e}/dl^2 = 0$. Indeed, making use of the implicit function theorem,

$$\frac{d\tilde{e}}{dl} = -\frac{f_{el}}{f_{ee}}.$$

Besides, under constant returns to scale, we have

$$ef_{ee} + lf_{el} = 0.$$

Rearranging, we obtain

$$\frac{d\tilde{e}}{dl} = -\frac{f_{el}}{f_{ee}} = \frac{\tilde{e}(l,k)}{l}.$$

We now show that $\tilde{e}/l$ does not depend on l anymore:

$$\frac{d^2\tilde{e}}{dl^2} = \frac{d}{dl}\frac{\tilde{e}(l,k)}{l} = \frac{\frac{d\tilde{e}}{dl}l - \tilde{e}}{l^2} = 0.$$

The fact that $d^2\tilde{e}/dl^2 = 0$ tells us that $\tilde{e}(l,k)$ is linear in l, which does not automatically imply that it is proportional to l. However we can show that $\lim_{l\to 0}\tilde{e}(l,k) = 0$, so that we can exclude the existence of a constant in this linear relationship. To examine this limit, we need the following additional assumpion: $f(0,l) = f(e,0) = 0$. With constant return to scale, we have $f(e,l) = ef_e(e,l) + lf_l(e,l)$, which implies:$f_e(e,0) = 0$ and $f_l(0,l) = 0$.

To show $\lim_{l\to 0}\tilde{e}(l,k)=0$, we use a proof by contradition. Suppose there exists a strictly positive lower bound to $\tilde{e}(l,k)$ and call it $\gamma(k)$. We then have: $\lim_{l\to 0}\tilde{e}(l,k)=\gamma(k)>0$.

Since f_e is continuous in l and varies between 0 and ∞,there exists l_γ such that $f_e(\gamma(k),l_\gamma)=k$. We thus have: $\gamma(k)=\tilde{e}(l_\gamma,k)$.

Since $f_{el}>0$ for any $l<l_\gamma$,

$$f_e(\tilde{e}(l_\gamma,k),l)<f_e(\tilde{e}(l_\gamma,k),l_\gamma)=k.$$

.

Since $f_{ee}<0$, the above inequality implies,

$$\widetilde{e}(l,k)<\tilde{e}(l_\gamma,k)=\gamma(k).$$

This contradicts the existence of a strictly positive lower bound to $\tilde{e}(l,k)$ and proves $\lim_{l\to 0}\tilde{e}(l,k)=0$. Thus $\tilde{e}\left(l,k\right)$ is proportional to l.

Second step

The preceding step allows us to write $\tilde{e}\left(l,k\right)=l\tilde{e}\left(1,k\right)$. Combining this relationship with the property of constant returns to scale,

$$f\left(\tilde{e}\left(l,k\right),l\right)=lf_l\left(\tilde{e}\left(1,k\right),1\right).$$

It follows that

$$\begin{aligned}R\left(l,k\right) &= f\left(\tilde{e}\left(l,k\right),l\right)-w\tilde{e}\left(l,k\right)=l\left[f_l\left(\tilde{e}\left(1,k\right),1\right)-w\tilde{e}\left(1,k\right)\right]\\ &= lR^*\left(1,k\right).\end{aligned}$$

Appendix 2: First order condition with respect to privatization under limited commitment

As we mention in the text, the function to maximize $V\left(\mu\right)$ is implicitly given by

$$\Psi\left(\mu,V\right)=\sum_{h=0}^{\lfloor\tilde{h}\rfloor}p_h\left[\left(1-\frac{h}{n}\right)u\left(c_0\right)+\frac{h}{n}u\left(c_1\right)\right]+\sum_{\lfloor\tilde{h}\rfloor+1}^{n}p_hu\left(\frac{Y}{n}\right)-V=0. \quad (21)$$

Making use of the implicit function theorem, the first order condition with respect to the privatization rate is then as follows:

$$\frac{dV}{d\mu}=-\frac{\Psi_\mu}{\Psi_V}=0\iff\Psi_\mu=0.$$

So the first order condition simply requires that $\Psi_\mu = 0$. At this stage, one may realize that, by an application of the implicit function theorem, the numerator Ψ_μ corresponds to the partial derivative of Ψ with respect to μ, while maintaining V constant. This is why V is held constant in the analysis of the first order condition given by Lemma 19.

For the purpose of making comparative statics, however, we need to check that Ψ_μ has the same sign as $dV/d\mu$. This will be the case if and only if $\Psi_V < 0$, which is shown below. The idea is to show that, even if the objective is implicitly defined, the numerator Ψ_μ contains the usual information in the sense that it has the sign of the expected utility gain from a marginal increase in the argument, namely the privatization rate μ.

$$\begin{aligned} \Psi_V &= \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \left[\left(1 - \frac{h}{n}\right) u'(c_0) \frac{\partial c_0}{\partial \kappa} + \frac{h}{n} u'(c_1) \frac{\partial c_1}{\partial \kappa} \right] \frac{\partial \kappa}{\partial V} - 1 \\ &= \delta \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n} \left(\frac{u'(c_0)}{u'(c_1)} - 1 \right) - 1, \end{aligned}$$

where use has been made of the following relationships:

$$\frac{\partial c_0}{\partial \kappa} = \frac{h}{n-h}; \frac{\partial c_1}{\partial \kappa} = -1; \frac{\partial \kappa}{\partial V} = \frac{\delta}{u_1'}.$$

The latter equation comes from an application of the implicit function theorem on equation (13). Define[12]

$$f(V) = \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \left[\left(1 - \frac{h}{n}\right) u(c_0) + \frac{h}{n} u(c_1) \right] + \sum_{\lfloor \tilde{h} \rfloor + 1}^{n} p_h u\left(\frac{Y}{n}\right).$$

What we want to obtain is

$$\Psi_V < 0 \iff \delta \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n} \left(\frac{u'(c_0)}{u'(c_1)} - 1 \right) = f'(V) < 1.$$

The implicit definition of the objective (21) simply becomes $f(V) = V$. Graphically, V is then located at the intersection between $f(V)$ and the 45-degree line. We now show that the shape of $f(.)$ implies that, at this point $f'(V) < 1$, which is precisely what we want to demonstrate (see condition **??**). Notice that $f(.)$ is strictly concave:

$$f''(V) = \delta \sum_{h=0}^{\lfloor \tilde{h} \rfloor} p_h \frac{h}{n} \frac{u_0'' \frac{h}{n-h} u_1' + u_0' u_1''}{u_1'^2} \frac{\partial \kappa}{\partial V} < 0.$$

It follows that $f(V)$ and V intersect at most twice and that V will be the highest at the second intersection, where $f'(V) < 1$. Since, at this second intersection, the incentive compatibility

[12] Recall that V is among the determinants of κ.

condition is satisfied, we argue that the household should select, given its constraints, the point where the expected utility is the highest. This completes the proof that $\Psi_V < 0$, implying that $dV/d\mu$ has the same sign as Ψ_μ.

Appendix 3: Proof of Proposition 5

In this appendix, we provide the developments of the comparative statics exercise.

According to the previous appendix, the marginal impact of privatization on per period expected utility has the following form:

$$\frac{dV}{d\mu} = -\frac{\Psi_\mu}{\Psi_V},$$

where the function Ψ is defined by equation (21). One can then make use of the implicit function theorem to find that the marginal impact of some exogenous parameter θ on an interior solution for the privatization rate μ^* has the sign of

$$\frac{d^2V}{d\theta d\mu} = -\frac{\Psi_{\theta\mu}\Psi_V - \Psi_\mu \Psi_{\theta V}}{\Psi_V^2}.$$

Since the first order condition implies that $\Psi_\mu = 0$, the sign of $d\mu^*/d\theta$ is simply given by the sign of $-\Psi_{\theta\mu}\Psi_V$. As we have shown in Appendix 2, $\Psi_V < 0$. As a consequence, we only need to determine the sign of Ψ_V. In this appendix, our attention is restricted to a couple of parameters $\left(\bar{V}, \delta\right)$ which alter the incentives to transfer, and hence the value of κ, but have nothing to do with production efficiency. Put differently, $\bar{V}$ and δ only affect Ψ_μ indirectly, through κ. We can then use the chain rule and write the derivative of expression (19) with respect to $\theta \in \left\{\bar{V}, \delta\right\}$ as[13]

$$\Psi_{\theta\mu} = \left(\frac{\partial^2 V}{\partial\kappa\partial\mu} + \frac{\partial^2 V}{\partial\kappa^2}\frac{\partial\kappa}{\partial\mu} + \frac{\partial V}{\partial\kappa}\frac{\partial^2\kappa}{\partial\kappa\partial\mu}\right)\frac{d\kappa}{d\theta}.$$

It follows that $d\mu^*/d\theta$ has the sign of $d\kappa/d\theta$ if and only if

$$\Lambda = \frac{\partial^2 V}{\partial\kappa\partial\mu} + \frac{\partial^2 V}{\partial\kappa^2}\frac{\partial\kappa}{\partial\mu} + \frac{\partial V}{\partial\kappa}\frac{\partial^2\kappa}{\partial\kappa\partial\mu} > 0.$$

This is generally true as we demonstrate below. This case is the most intuitive. Indeed, this tells us that, if for some exogenous reason the contributing members' willingness to transfer is reduced, then the privatization rate should decrease to allow the household to insure its members through a

[13]The term $\frac{\partial^2\kappa}{\partial\kappa\partial\mu}$ may beconfusing as κ appears in the denominator. Recall that the function $\kappa\left(\mu;\bar{V},\delta\right)$ is implicitly defined (see equation 13). The implicit function theorem allows us to calculate $\frac{\partial\kappa}{\partial\mu}$. However, κ still appears in the latter expression through c_1. Actually, one would desire to have $\frac{\partial\kappa}{\partial\mu}$ as a function of μ,$\bar{V}$ and δ only, which is impossible precisely because of the implicit definition of κ. While we calculate $\frac{\partial^2\kappa}{\partial\theta\partial\mu}$ ($\theta \in \left\{\bar{V},\delta\right\}$), we then need to take their indirect impact into account. Applying the chain rule, we have indeed that $\frac{\partial^2\kappa}{\partial\theta\partial\mu} = \frac{\partial^2\kappa}{\partial\kappa\partial\mu}\frac{d\kappa}{d\theta}$.

higher collective production. In other words, privatization increases with the *equilibrium value* of its members' willingness to transfer κ. This is intuitive as, in this case, the household can substitute income transfers to collective production, which is costly in terms of production efficiency, to achieve a certain level of risk-sharing. Let us analyze the sign of Λ. Afterward, we will turn to the analysis of $d\kappa/d\theta$, $\theta \in \left\{\bar{V}, \delta\right\}$.

Bearing in mind that the function V is given by equation (18), the first term of Λ is[14]

$$\frac{\partial^2 V}{\partial\kappa\partial\mu} = \left(R^* - R^C\right) L \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n} \left[u''\left(c_0\right) - u''\left(c_1\right)\right].$$

The other useful elements are

$$\begin{aligned}
\frac{\partial V}{\partial\kappa} &= \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n} \left[u'\left(c_0\right) - u'\left(c_1\right)\right], \\
\frac{\partial^2 V}{\partial\kappa^2} &= \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n} \left[u''\left(c_0\right)\frac{h}{n-h} + u''\left(c_1\right)\right], \\
\frac{\partial\kappa}{\partial\mu} &= -\frac{u'\left(c_d\right)}{u'\left(c_1\right)} LR^* + \left(R^* - R^C\right) L, \\
\frac{\partial^2\kappa}{\partial\kappa\partial\mu} &= -\frac{u'\left(c_d\right)}{u'\left(c_1\right)}\frac{u''\left(c_1\right)}{u'\left(c_1\right)} LR^*.
\end{aligned}$$

Assembling those terms, we end up with

$$\begin{aligned}
sign\left\{\Lambda\right\} &= sign\left\{\left(R^* - R^C\right) L \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n-h} u_0'' - \frac{u_d'}{u_1'} LR^* \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n} u_0' \left(\frac{u_0''}{u_0'}\frac{h}{n-h} + \frac{u_1''}{u_1'}\right)\right\} \\
&= sign\left\{-\underbrace{\left[\left(R^* - R^C\right) L - \frac{u_d'}{u_1'} LR^*\right]}_{=\frac{\partial\kappa}{\partial\mu}<0} \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n-h} u_0' \eta\left(c_0\right) - \frac{u_d'}{u_1'} LR^* \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n} u_0' \left(\eta\left(c_0\right) - \eta\left(c_1\right)\right)\right\},
\end{aligned}$$

where $\eta\left(c\right) = -u''\left(c\right)/u'\left(c\right)$ is the coefficient of absolute risk aversion. Therefore, under CARA (constant absolute risk aversion),

$$sign\left\{\Lambda\right\} = sign\left\{-\frac{\partial\kappa}{\partial\mu} \sum_{h=0}^{\lfloor\tilde{h}\rfloor} p_h \frac{h}{n-h} u_0' \eta\left(c_0\right)\right\},$$

and $\Lambda > 0$. Indeed, in the light of the first order condition with respect to the privatization rate (19), one may realize that, at any interior solution, $\partial\kappa/\partial\mu < 0$. Alternatively, under DARA (decreasing

[14]See $\frac{\partial V}{\partial\mu}$ as given in Proposition 4.

absolute risk aversion), $\eta(c_0) - \eta(c_1) > 0$ and we have an effect in the opposite direction. In our simulation results, we have tested a large set of parameters values and have precisely assumed that agents' preferences were characterized by DARA. $\Lambda > 0$ appears to be a robust result as we never encountered a negative sign for Λ in any of the tested parameters combinations.

We now turn to the analysis of $d\kappa/d\theta$, $\theta \in \left\{\bar{V}, \delta\right\}$.

It is worth reproducing here the implicit definition of κ:

$$\chi = u(c_d) - u(c_1(\kappa)) - \delta\left(V^*\left(\bar{V}, \delta\right) - \bar{V}\right) = 0, \tag{22}$$

where $V^*\left(\bar{V}, \delta\right)$ is indirect expected utility, that is V evaluated at the optimal privatization rate and for equilibrium behaviors.[15] The above equation therefore gives us a function of the form $\kappa\left(\bar{V}, \delta, V^*\left(\bar{V}, \delta\right)\right)$. We thus have that

$$\begin{aligned}
\frac{d\kappa}{d\bar{V}} &= \frac{\partial\kappa}{\partial\bar{V}} + \frac{\partial\kappa}{\partial V^*}\frac{\partial V^*}{\partial\bar{V}}, \\
\frac{d\kappa}{d\delta} &= \frac{\partial\kappa}{\partial\delta} + \frac{\partial\kappa}{\partial V^*}\frac{\partial V^*}{\partial\delta},
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial V^*}{\partial\bar{V}} &= \frac{\partial V^*}{\partial\kappa}\frac{\partial\kappa}{\partial\bar{V}}, \\
\frac{\partial V^*}{\partial\delta} &= \frac{\partial V^*}{\partial\kappa}\frac{\partial\kappa}{\partial\delta}.
\end{aligned}$$

Indeed, V^* gets affected by $\theta \in \left\{\bar{V}, \delta\right\}$ only through their impact on incentives to transfer and hence on κ. Substituting and rearranging, we obtain

$$\begin{aligned}
\frac{d\kappa}{d\bar{V}} &= \frac{\partial\kappa}{\partial\bar{V}}\left(1 + \frac{\partial\kappa}{\partial V^*}\frac{\partial V^*}{\partial\kappa}\right), \\
\frac{d\kappa}{d\delta} &= \frac{\partial\kappa}{\partial\delta}\left(1 + \frac{\partial\kappa}{\partial V^*}\frac{\partial V^*}{\partial\kappa}\right).
\end{aligned}$$

Finally, applying the implicit function theorem on equation (22), the partial derivatives are given by

$$\begin{aligned}
\frac{\partial\kappa}{\partial V^*} &= -\frac{\partial\kappa}{\partial\bar{V}} = \frac{\delta}{u_1'} > 0, \\
\frac{\partial\kappa}{\partial\delta} &= \frac{V^* - \bar{V}}{u_1'} > 0.
\end{aligned}$$

[15] While calculating the first order condition with respect to μ, we were allowed to treat V as a constant by an application of the implicit function theorem (see Appendix 2). Here, however, we cannot do that anymore as we are precisely taking the derivative one the first order condition. The equilibrium value of V contains indeed the parameters of interest. Nevertheless their indirect impact through μ^* can be neglected by an envelop argument.

It follows that the total derivative is equal to the partial derivative multiplied by a coefficient $\left(1 + \frac{\partial \kappa}{\partial V^*}\frac{\partial V^*}{\partial \kappa}\right) > 1$. Indeed, recall that $\partial V^*/\partial \kappa > 0$. The reason is that risk-sharing is incomplete in constrained states of the world. Therefore, an increase in the aggregate transfer κ increases expected utitity. This multiplicative coefficient is due to a feedback effect by which a partial effect of a parameter θ on κ gets reinforced by its effect on indirect utility V^*, which itself affects κ.

We then conclude that
$$\frac{d\kappa}{d\bar{V}} < 0; \frac{\partial \kappa}{\partial \delta} > 0.$$
Combining this with the fact that $\Lambda > 0$ leads the result.