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Supposedly Strong Instruments and Good Leverage Points

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Supposedly strong instruments and good leverage points

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Abstract

In this paper, we warn on the overoptimistic conclusions led by weak instruments testing when good leverage points are present in the first stage of an IV estimation. Some simulations and an empirical application are provided to illustrate the point raised.

Keywords: Instrumental variables, Weak instruments, Outliers, Robust Statistics, Informal Sectors. *JEL classification: C3, C12, O1*

1 Introduction

Recent econometric literature on instrumental variable (IV) estimations has paid a considerable attention to the relevance of the instruments used. Indeed, it is well-known that when the instruments are weak, the hypothesis tests can exhibit large size distortions and coefficients and the standard errors can be severely biased (see Andrews and Stock, 2005).

A standard approach to assess the relevance of the instruments is to compare the first-stage F-statistic associated to the Wald test checking for the significance of the excluded

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instruments to some tabulated values (see Stock and Yogo, 2005). When several explanatory variables are endogenous and need to be instrumented, a Cragg and Donald (1993) or a Kleibergen and Paap (2006) statistic is called-on but the logic remains the same. We consider here only the case of a single troublesome variable. The generalization to more is straightforward.

What we investigate is the behavior of weak instruments testing when some observations, while following the same "endogenous regressor-instruments" relation as the others, lie isolated on the regression hyperplane (i.e. good leverage). We argue that first stage good leverage points lead to overoptimistic conclusions on the strength of the instruments and consequently on the quality of the instrumenting strategy. We present Monte Carlo simulations to support this conclusion. The paper of McKenzie and Sakho (2010) studying the effect of formality on micro-firm profits in Bolivia is revisited to illustrate how a limited number of apparently well behaving observations may substantially influence the conclusions on weak instruments.

The structure of this note is the following: in section 2 the effect of good leverage points on weak instruments inference is discussed and in section 3, some Monte Carlo simulations are run to assess the size of the problem. Section 4 highlights the relevance of the result using an economic example. Section 5 concludes.

2 Good leverage points and weak instruments

The problem induced by good leverage points on weak instrument testing can easily be explained calling on the well-known Wald test. The intuition is the following: assume a linear model of the type

$$y = X\beta + u \tag{1}$$

where y is the $n \times 1$ dependent variable vector, X is a $n \times p$ matrix of explanatory variables and u is the $n \times 1$ vector of disturbances. One column of X is assumed correlated with the error term. The least squares estimated parameters are therefore biased and inconsistent. To tackle this problem, instrumental variable estimations are generally used relying on a set of k excluded instruments (Z). The relevance of Z is usually tested by comparing the first stage F-statistic (of the Wald test checking if all parameters associated to excluded instruments are equal to zero) to tabulated critical values available from Stock and Yogo (2005). The $F-$statistic is computed as

$$F = \frac{(R\hat{\theta} - q)' \left(R(Z'Z)^{-1}R'\right)^{-1} (R\hat{\theta} - q)}{ks^2} \tag{2}$$

where R is the matrix of parameters restrictions, q is a k-dimensional vector of zeros, $\hat{\theta}$ is the vector of estimated parameters and s^2 is the estimated variance of the first stage residuals.

The effect of good leverage points on F is unambiguous. Good leverage points have a rather limited effect on the denominator of equation (2) since, by lying close to the regression hyperplane, they do not affect substantially the estimated variance of the residuals. On the other hand, their influence on the numerator is large (via $Z'Z$). As a consequence, a very limited number of individuals outlying in the space of the instruments Z but following the model relating X to Z may lead to overoptimistic conclusions about the relevance of instruments. What we suggest to do to cope with this is to identify all type of outliers in the first stage and downweight their importance (in this note we use the most drastic weighting scheme that consists in awarding a weight zero to any outlying observation but others can be envisaged). The outliers are identified by running a multivariate outlier detection tool on the matrix (X, Z). The multivariate estimator used here is Stahel (1981) and Donoho (1982), hereafter called SD: it consists in calculating the outlyingness of each point by projecting the data cloud unidimensionally in all the possible directions and estimating the distance from each observation to the centre of each projection. Then, the degree of outlyingness is defined as the maximal distance that is obtained when considering all possible projections (see Maronna and Yohai, 2006). Since this outlyingness distance is distributed as $\sqrt{\chi^2_{p+k}}$, we can choose a quantile above which we consider an observation as being outlying (we consider here the 95th percentile). Once the importance awarded to outliers is downweighted, the problem raised above disappears.

In the next section we illustrate the effect of a limited number of first stage good leverage points on the F-statistic and investigate the effectiveness of the procedure we propose by running some Monte Carlo simulations.

3 Monte Carlo results

In the simulations, we use a data generating process $y = X_1 + X_2 + u$ and $X_1 = \theta Z + X_2 + v$ where X_2 and Z are generated from two independent $N(0,1)$ and disturbances are generate from a bivariate normal,

$$\begin{pmatrix} u \\ v \end{pmatrix} \sim N\left((0,0), \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$$

The degree of "endogeneity" ρ is set to 0.75 and θ to $\sqrt{2}$. In this way the first stage F-statistic is equal to 3. The sample size n considered is 1000. The simulation setup is the following: we contaminate the dataset by replacing 1% of the X (and Z is modified accordingly to keep the relation unchanged) by adding an integer that ranges from 1 to 10. For each contamination we run 2000 simulations and calculate the bias of F and the percentage of rejection. The resulting curbs are presented in Figure 1.

[INSERT FIGURE I HERE]

The plain line in the left panel shows that the bias of the F-statistic increases quickly when good leverage points are present, leading to overoptimistic conclusions. The plain line in the right panel of Figure 1 shows that the percentage of (incorrect) rejections of the null of weak instrument increases quickly to 100%. These results highlight the extreme sensitivity of the first stage F-statistic to good leverage points with relatively low contamination levels. The dashed line in Figure 1 also shows that the weighed IV estimator we propose (SDIV from now on) behaves well since the F-statistic and the percentage of rejection are not sensible to good leverage points. We simulated alternative scenarios increasing the number of instruments and/or the percentage of contamination and the results are similar.

4 Example: The effect of formality on micro-firm profits

In McKenzie and Sakho (2010) the effect of formality on micro-firms profits in Bolivia is investigated. The authors use the logarithm of the distance to a tax office as an instrument for firms' formality level, their main explanatory variable. Furthermore they use a set of control variables to remove confounding effects. For the sake of brevity we do not discuss them here and just refer to them as included instruments. Their main finding is that increasing formality in Bolivia raises firm's profits. Their instrumenting strategy seems to perform well since their first stage F-statistic is approximately 12.68 which leads to the conclusion that the bias of least squares has been substantially reduced (about 88%) when instrumenting.

To identify influential outliers in the first stage estimation, we run the projection-based SD estimator on all the variables entering the first stage equation (i.e. the explanatory troublesome variable, the included and the excluded instruments). If we run a simple (trimmed) linear regression excluding the identified outliers (23 observations), the estimated coefficients do not vary substantially, which seems to indicate that outliers were not an issue in their dataset. Therefore, all evidence point towards a powerful instrumenting strategy.

However, when we have a closer look to the first stage equation, some concerns emerge. To start with, we plot the standardized residuals of the trimmed regression against the outlyingness distance obtained by running the SD-estimator only on the (included and excluded) instruments. Since residuals are assumed normally distributed in a linear regression (for Gaussian data), we consider a point as potentially outlying in the vertical dimension if the (absolute) Standardized Residuals (SR from now on) are larger than the 97.5th percentile of the normal distribution (i.e. 1.96). As far as the instruments outlyingness distance is concerned, it is well-known to be distributed as a $\sqrt{\chi^2}$ with degrees of freedom equal to the total number of (included and excluded) continuous instruments plus the number of endogenous regressors. We decided to consider a point as potentially outlying in the space of the instruments if its Outlyingness Distance (OD) is larger than the 95th percentile of a $\sqrt{\chi^2_{10}}$ distribution (i.e. 4.28). These limits are plotted respectively as horizontal and vertical lines. The resulting graphical tool, described thoroughly in Rousseeuw and Van Zomeren (1990), allows to recognize the type of outliers present. Indeed, standard observations should be associated with absolute $SR < 1.96$ and instruments $OD < 4.28$. On the other hand, vertical outliers should have instruments OD comparable to the bulk of data but should be associated to absolute $SR > 1.96$. Bad leverage points should be associated to instruments $OD > 4.28$ and absolute $SR > 1.96$. Finally, good leverage points (the one we are interested in here) should be associated to absolute $SR < 1.96$ and $OD > 4.28$. Good leverage points, by lying close to the regression hyperplane, do not distort the slope estimations and are therefore generally considered as desirable in the literature.

[INSERT FIGURE II HERE]

From Figure 2 it is clear that, as explained above, no bad leverage points and only very mild vertical outliers are present. The slope coefficients should therefore be trustworthy. In the first column of Table 1 here below we reproduce the original results of McKenzie and Sakho (2010) obtained using all the data since apparently no influential outlier was present. However, from the 23 points identified as potentially outlying, 14 of them are good leverage points. As explained in the previous section, these could false the weak instrument test even if their effect on the slope coefficient estimation is negligible. We therefore re-estimate the key specification of McKenzie and Sakho (2010) removing all potential outliers and present the results in column two of Table 1. In column three we apply a similar trimmed IV estimator but where only the 14 good leverage points identified in Figure 2 are removed. The results obtained by both outlier-free estimations are coherent and lead to the conclusion that, contrarily to what stated in the original paper, the instruments are weak since the F-statistic is more than halved and falls well below the acceptable critical levels. Even if the instrumenting strategy seemed appealing, the apparent strength of the instruments was due to a limited number of firms and once controlling for this, the instrument turned-out to be weak.

[INSERT TABLE I HERE]

5 Conclusions

In the econometric literature, it is well established that bad outliers affect IV estimations (see Desbordes and Verardi, Forthcoming). On the contrary the effect of "good" outliers is not well documented. In this paper, we investigate the latter focusing on their effect on the quality of testing for weak instruments in a IV setup. We present some simulations and show that the first stage F-statistic increases substantially when even a limited number of good leverage points are present. We propose a simple method to correct for this using Stahel (1981) and Donoho (1982) multivariate location and scatter estimator.

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Table 1: Robust against outliers IV estimations

	(1)	(2)	(3)
Dependent: lnProfits	Original	No outlier	No good leverage
First stage			
Instrument: lnDistance	-0.13***	-0.11**	-0.10**
	(0.04)	(0.05)	(0.04)
F-statistic	12.68	5.8	5.99
Second stage			
Endogenous: Formality	1.45**	0.81	0.89
	(0.68)	(0.89)	(0.88)
Observations	369	346	355

Robust standard errors in parentheses
** p<0.01, ** p<0.05, * p<0.1

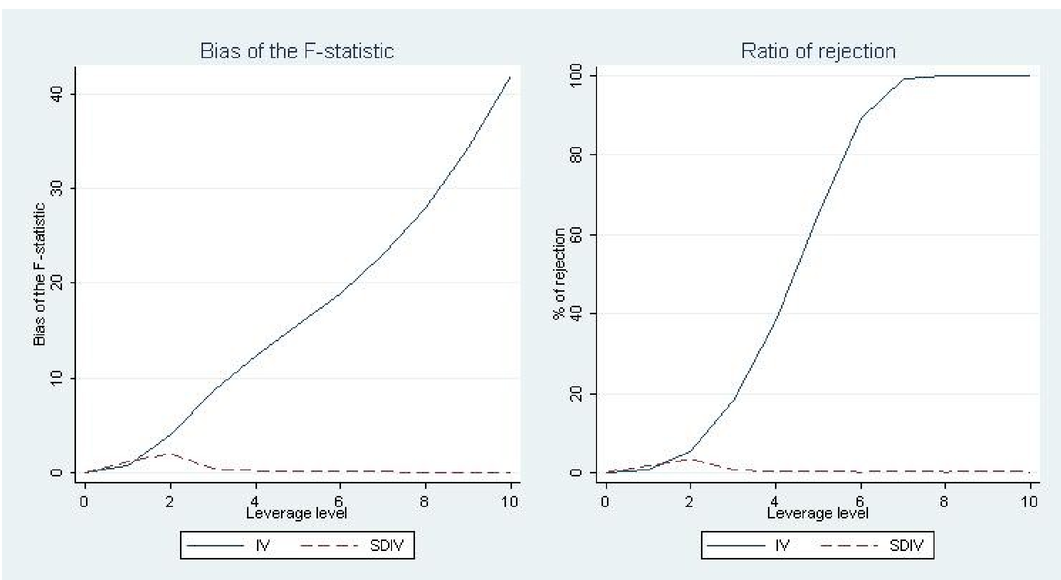

Figure 1: Bias and rate of rejection of the F-statistic

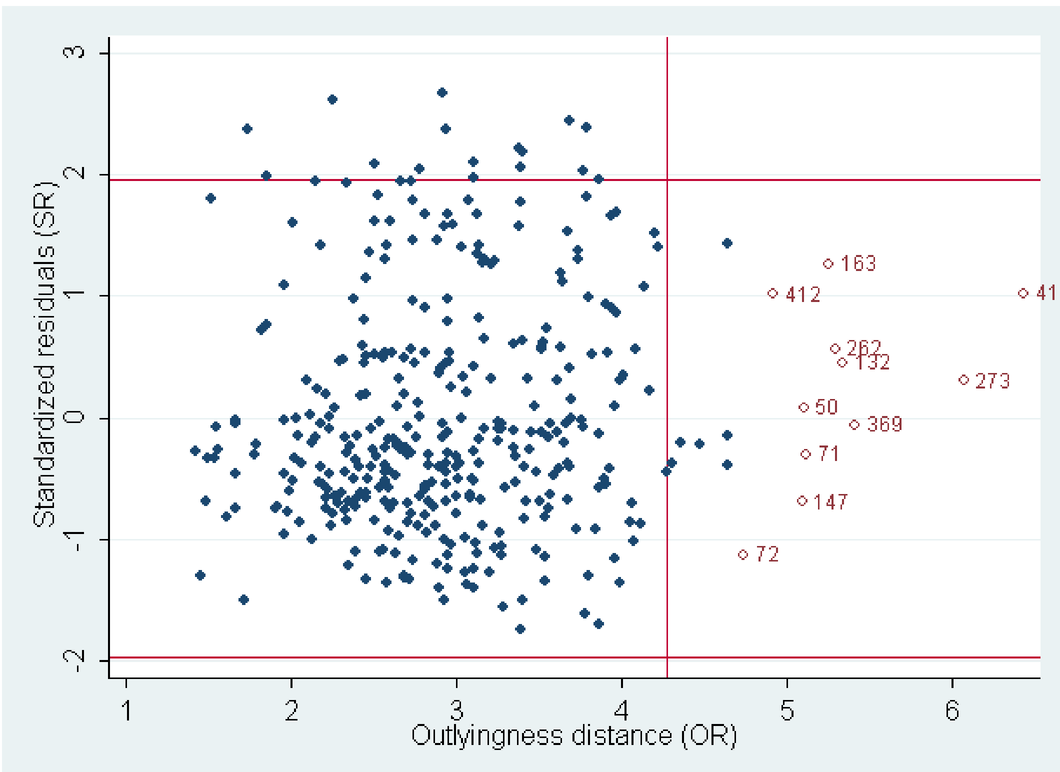

Figure 2: Type of outliers

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