Consume Now or Later? Time Inconsistency, Collective Choice and Revealed Preference*

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Abstract

We develop a revealed preference methodology that allows us to explore whether time inconsistencies in household choice are the product of individual preference nonstationarities or the result of individual heterogeneity and renegotiation within the household. An empirical application to household-level microdata highlights that an explicit recognition of the collective nature of household choice enables the observed behaviour to be rationalised by a theory that assumes preference stationarity at the individual level. The methodology created in this paper also facilitates the recovery of theory-consistent discount rates for each individual within a particular household under study.

JEL Classification: D11, D12, D13, C14.

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In this paper, we consider the extent to which patterns in household intertemporal choice can be rationalised by simple economic models that assume time consistency at the individual level. Our analysis reveals that standard models that assume exponential discounting are consistent with observed household consumption so long as they are applied to the appropriate decision making unit, i.e. the individual.

Samuelson's (1937) canonical "discounted utility" (DU) model is the standard framework by which economists conceptualise intertemporal choice. Under this model, a decision maker's time preferences are captured by a single, time invariant discount rate and are *time consistent.* Intuitively, preferences are time consistent if the choice between alternatives does not depend on when in time that choice occurs: if receiving X at t is preferred to receiving Y at $t + d$, the decision maker will always prefer X at τ to Y at $\tau + d$.

Although the DU model is applied widely because of its tractability, the predictive validity of the model is thought to be on fragile footing. In experimental settings, decision makers are often found to behave in a time inconsistent manner, with individuals acting impatiently in the current moment whilst planning to act patiently in the future. For example, an individual may prefer to receive $100 today over receiving $110 tomorrow, whilst simultaneously preferring receiving $110 in 31 days to receiving $100 in 30 days. Preference reversals such as this are well documented in the psychology and economics literature (for a survey, see Frederick, Loewenstein and O'Donoghue 2002) but can not be rationalised by a straightforward application of the DU model.

Research effort has largely focused on modelling the sources of time inconsistent behaviour at the individual level. Standard methods of modelling discounting have been a prime target of criticism, with the behavioural economics literature increasingly favouring frameworks that assume hyperbolic discount functions. Hyperbolic discount functions are characterised by a relatively high discount rate over short horizons and a relatively low rate over long horizons. This lack of constancy in the discount rate introduces a conflict between today's preferences and future preferences, and a "present bias" to decision making. Hyperbolic discounting has been offered as an explanation for many stylised facts, from under-saving and excess co-movement of income and consumption (Laibson 1997, 1998; Angeletos *et al.* 2001) to procrastination, addiction and lack of exercise (O'Donoghue and Rabin 1999, 2001; Gruber and Koszegi 2000; DellaVigna and Malmendier 2006).

In this paper we take a different approach. We consider how acknowledging the *collective* nature of choice can rationalise time inconsistencies in non-experimental household consumption patterns. The preference structure associated with the DU approach is often applied to model group behaviour without modification. Under this "unitary" approach, one assumes that the collective acts as a single decision making unit, and therefore can be treated as if a rational individual. No explicit allowance is made for the separateness of persons nor preference heterogeneity within a group.

Acknowledgement of the collective nature of choice can rationalise apparent time inconsistencies in household behaviour. Deriving a time independent discount rate from the underlying preferences of a heterogeneous population has long been recognised as problematic (Marglin 1963; Feldstein 1964). When time preferences within a group differ, the collective preference is typically time inconsistent, even when the underlying population has perfectly time consistent preferences (Zuber 2011; Jackson and Yariv 2012). In fact, Jackson and Yariv (2012) show that, with a uniform distribution of discount rates in an otherwise homogeneous population, group utility maximisation generates aggregate behaviour that corresponds to hyperbolic discounting.

Furthermore, renegotiations of the household choice rule can also generate nonstationarities in family behaviour. Relative decision making power within the collective unit can vary, and differences in time preferences can prompt periodic innovations in the intrahousehold preference weighting. Other things equal, it is optimal to relatively favour impatient group members in early periods and patient members in later periods. However, as time passes and impatient members begin to receive lower shares of the group surplus, there is an incentive for them to demand a renegotiation of allocations in their favour or threaten to leave the group. Renegotiations prompt changes in the intra-household preference weighting, generating nonstationarities in the collective preference.

Although it is true that nonstationarities at the individual level will translate into a failure of time consistency at the collective level, understanding whether the primary locus of inconsistent behaviour is at the self or group level is important from both a methodological and policy perspective. The DU preference structure is tractable and parsimonious. Thus, if it cannot be rejected on the basis of choice behaviour, there are compelling analytic reasons for its retention. Further, policy design should be influenced according to whether time inconsistent behaviour is the product of individual nonstationarities or collective aggregation issues.

Methodological contribution. This paper puts forward a nonparametric characterisation of household intertemporal choice and develops a revealed preference methodology for analysing the sources of time inconsistent collective choice. Our approach follows in the spirit of Afriat (1967), Diewert (1973), and Varian (1982) and incorporates insights gained from the extension of the revealed preference methodology to an intertemporal setting by Browning (1989) and Crawford (2010), and to the collective model by Cherchye, De Rock and Vermeulen (2007, 2009). The framework presented allows us to explore whether time inconsistencies in household choice can be rationalised by preference heterogeneity and renegotiation within the collective unit rather than individual nonstationarities. Further, our methodology allows for the recovery of theory-consistent spousal discount rates and an assessment of the degree of intrahousehold commitment. This provides the basis for additional analysis on time preference heterogeneity within the household.

Our methodology is novel in this context and has clear advantages over existing empirical tests of time consistency and household intertemporal behaviour. Current tests of dynamic collective choice models and time discounting are parametric and tend to reject the assumptions of constant discounting (see Frederick, Loewenstein and O'Donoghue 2002) and a time-independent intrahousehold preference weighting (Mazzocco 2007). However, such studies are sensitive to the parametric specification employed. The common assumption of linear consumption utility imparts an upward bias to discount rate estimates and is thought to contribute to the unrealistically high discount rates observed in the literature. Although recent developments have seen the linear-utility specification somewhat relaxed (Andersen *et al.* 2012, Andreoni and Sprenger 2012), estimates are still dependent on the set of functional form assumptions made concerning the form of the utility function. Further, the experimental nature of existing time discounting studies can be critiqued. Dohman *et al.* (2012) highlight that elicited preferences are not procedurally independent and that discount rate estimates are hugely sensitive to the experimental design employed.

The methodology and empirical application presented in this paper avoids such criticisms. Our revealed preference approach is wholly nonparametric and, thus, our results are not contingent on any particular specification of family member utility functions. Rather than directly estimate the preference parameters that best "fit" with some assumed functional specification, we ask whether there exists a non-empty feasible set to the system of inequalities that are implied by maximising household behaviour within the framework imposed by economic theory. The existence of a non-empty feasible set to these inequalities is then a necessary and sufficient condition for household behaviour and the theory in question to be consistent. Using this approach, we are able to determine whether time inconsistencies in the revealed household preference can be explained as a result of discount rate heterogeneity and imperfect commitment within the collective unit, or whether one must additionally allow for nonstationarities at the individual level.

Our tests are explicitly designed for use with household consumption data, although they can be profitably applied to an experimental setting. Our empirical application is one of the few in recent years to be fully grounded in "real world" household consumption behaviour, rather than make use of preference and choice data that has been elicited in an artificially constructed environment. This allows us to avoid many of the procedural nuances that plague experimental studies.

Empirical results. We find that accounting for the collective nature of choice allows us to rationalise time inconsistencies in aggregate household behaviour without positing nonstationarities in individual preferences. Simply allowing for some limited heterogeneity in familial discount rates allows the behaviour of 97.2% of households in our sample to be rationalised by standard models of household intertemporal behaviour. Our belief in the validity of this result is strengthened by our finding that the time consistent model performs well when applied to explain the consumption behaviour of single person households, for whom collective explanations do not apply. We find that the minimal divergence in spousal discount rates is, in general, not well explained by observable household characteristics, although older couples require a significantly smaller difference in their time preferences to rationalise their behaviour.

Outline. The paper proceeds as follows. Section I defines time consistency of the collective preference and outlines the associated revealed preference restrictions for establishing the time consistency of group choice. Section II uses these revealed preference conditions to evaluate the empirical validity of time consistency for a Spanish panel of household microdata. Our main conclusion here will be that time consistency is heavily rejected for couples, even when using nonparametric revealed preference restrictions, but that it performs significantly better for single person households. Section III then explores how a recognition of the collective nature of household choice can rationalise nonstationarities in the revealed collective preference and derives a nonparametric methodology for testing hypotheses on the sources of time inconsistent household behaviour. Section IV continues our empirical application and provides strong empirical support for this collective rationalisation of observed time inconsistencies. Here we also correlate intrahousehold time preference heterogeneity with observable household characteristics. Section V concludes. The Appendix contains the proofs of our main results and some descriptive statistics for our data.

I Time consistency and collective choice

The aim of this paper is to provide a framework for exploring the sources of time inconsistencies in household choice. Specifically, we wish to determine whether explicitly accounting for the collective nature of choice can allow one to rationalise patterns of household behaviour without positing nonstationarities at the individual level. This section formally defines the concept of time consistency tested in this paper and derives simple revealed preference conditions that can be used to determine the time consistency of observed household choice.

I.A The "collective" preference

Collective intertemporal models explicitly recognise the separateness of persons within a household h, and allow for complete heterogeneity in family member felicity functions and discount rates. For notational simplicity, we focus on a two-member household, constituted of members $m \in \{A, B\}$. The extension of results to an M-member ($M > 2$) household is straightforward.

Individual preferences are represented by a time-additive discounted utility function that is defined over private and public consumption. We assume N private goods and K public goods. At a given time t, a household h consumes private quantities $\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B} \in \mathbb{R}_+^N$ (with associated discounted prices $\mathbf{p}_t \in \mathbb{R}_{++}^N$), and public quantities $\mathbf{Q}_t^h \in \mathbb{R}_+^K$ (with associated discounted prices $\mathbf{P}_t \in \mathbb{R}_{++}^K$). Public goods are consumed jointly and non-exclusively within the household. Each household member m in household h is associated with a concave and strictly increasing felicity function $u^{h,m}$ and discount rate $r^{h,m} \in [0, \infty)$, such that they evaluate the household stream of public and private consumption $\mathbf{C}_{ij}^{h,m} = \{\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h\}_{t=i,\ldots,j}$ as:

$$U^{h,m}(\mathbf{C}_{ij}^{h,m}) = \sum_{t=i}^{j} \beta_{h,m}^{t-1} u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h),$$

where $\beta_{h,m} = 1/(1 + r^{h,m})$.

Regarding the aggregate household preference, collective models do not assume a priori that individual preferences can be aggregated into a single time-independent household felicity function nor do they specify a single intrahousehold bargaining process. Rather, collective models simply assume that some cooperative decision making process exists and that this process leads to Pareto efficient outcomes over the affordable budget set.[1] With these assumptions, one can define the relative Pareto weight ω_t^h to summarise the bargaining process within household h in period t. Then, for a household h, the "collective preference" over some lifecycle consumption profile $\mathbf{C}^h = \{\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h\}_{t \in T}$, where $T = \{1, ..., |T|\}$, is given by:

$$U^h(\mathbf{C}^h) = \sum_{t \in T} \left\{ \beta_{h,A}^{t-1} u^{h,A}(\mathbf{q}_t^{h,A}, \mathbf{Q}_t^h) + \omega_t^h \beta_{h,B}^{t-1} u^{h,B}(\mathbf{q}_t^{h,B}, \mathbf{Q}_t^h) \right\},$$

with $\omega_t^h = f(\mathbf{Z}_t^h)$, where $\mathbf{Z}_t^h$ denotes the set of relevant "distribution factors" at time t for household h. The standard theory places no restrictions on what variables count as relevant distribution factors, beyond requiring that they are independent of individual preferences (Browning *et al.* 1994). We note that this lack of structure makes our non-

[1]See, for example, Chiappori (1988, 1992) and Browning and Chiappori (1998) for detailed discussions of the Pareto efficiency assumption in collective household models. Mazzocco (2007) discusses this framework in an intertemporal context.

parametric framework especially attractive as, unlike parametric tests of intertemporal behaviour, our methodology does not require a formal specification of the factors that jointly determine ω_t^h.

I.B Time consistency

Given a household's time series of consumption choices, the first question of interest is whether there exists a time consistent household utility function that could have generated this choice pattern.

What does it mean for the collective preference to be time consistent? Note that the collective preference over the consumption stream $\mathbf{C}_{ij}^h = \{\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h\}_{t=i,\ldots,j}$ is defined as:

$$U^h(\mathbf{C}_{ij}^h) = \sum_{t=i}^{j} \left\{ \beta_{h,A}^{t-1} u^{h,A}(\mathbf{q}_t^{h,A}, \mathbf{Q}_t) + \omega_t^h \beta_{h,B}^{t-1} u^{h,B}(\mathbf{q}_t^{h,B}, \mathbf{Q}_t) \right\}.$$

Let $\mathbf{C}_{i'j'}^h$ represent an "updated" counterpart of $\mathbf{C}_{ij}^h$ $(i, j \in T,\ i < j)$, which denotes the consumption streams $\mathbf{C}_{ij}^h$ shifted forward into the future by some amount $0 \leq \tau \leq |T| - j$, and let $\{\mathbf{C}_{ij}^h, \mathbf{C}_{kl}^h\}$ be the combination of two (non-overlapping) consumption streams $\mathbf{C}_{ij}^h$ and $\mathbf{C}_{kl}^h$ $(i, j, k, l \in T,\ i < j,\ k < l$ and $j < k$ or $l < i)$. Then, we can formally define time consistency of the collective preference as follows.

Definition 1 *The collective preference is time consistent if the following two conditions are satisfied:*

1. *For any $i, j \in T$ with $i < j$,*

$$U^h(\mathbf{C}_{ij}^h) > U^h(\widetilde{\mathbf{C}}_{ij}^h) \text{ if and only if } U^h(\mathbf{C}_{i'j'}^h) > U^h(\widetilde{\mathbf{C}}_{i'j'}^h).$$

2. *For any $i, j, k, l \in T$ with $i < j$, $k < l$ and, in addition, $j < k$ or $l < i$,*

$$U^h(\{\mathbf{C}_{ij}^h, \mathbf{C}_{kl}^h\}) > U^h(\{\widetilde{\mathbf{C}}_{ij}^h, \mathbf{C}_{kl}^h\}) \text{ if and only if } U^h(\{\mathbf{C}_{ij}^h, \overline{\mathbf{C}}_{kl}^h\}) > U^h(\{\widetilde{\mathbf{C}}_{ij}^h, \overline{\mathbf{C}}_{kl}^h\}).$$

The first condition in Definition 1 imposes stationarity on the collective preference; the ranking of consumption streams should not depend on when in time those streams are situated. The second condition requires that the ranking of consumption streams is independent of periods with identical consumption bundles.

Conditions for time consistency. Results originally given in Koopmans (1960), and applied to a collective setting by Jackson and Yariv (2012), imply that time consistency of the collective preference requires the ability to re-express the household preference in the following format:

$$U^h(\mathbf{C}_{ij}^h) = \sum_{t=i}^{j} \beta_h^{t-1} u^h(\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h),$$

for $\beta_h \in (0,1]$. Therefore, the existence of a single, constant household discount rate and a stationary household felicity function is sufficient for a time consistent collective preference in our setting.

If the following two conditions hold, then the collective preference can be recast in the above format.[2] First, both household members must discount the future at the same rate, $\beta_{h,A} = \beta_{h,B} = \beta_h$. Second, the intrahousehold decision making mechanism must give rise to a constant Pareto weight across the lifetime of the household, i.e. $\omega_t^h = \omega^h$ for all t. Under these conditions, the lifecycle consumption profile of household h, $\mathbf{C}^h$, is evaluated as:

$$\begin{aligned} U^h(\mathbf{C}^h) &= \sum_{t \in T} \left\{ \beta_{h,A}^{t-1} u^{h,A}(\mathbf{q}_t^{h,A}, \mathbf{Q}_t^h) + \omega_t^h \beta_{h,B}^{t-1} u^{h,B}(\mathbf{q}_t^{h,B}, \mathbf{Q}_t^h) \right\} \\ &= \sum_{t \in T} \beta_h^{t-1} u^h(\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h), \end{aligned}$$

where

$$u^h(\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h) = u^{h,A}(\mathbf{q}_t^{h,A}, \mathbf{Q}_t^h) + \omega^h u^{h,B}(\mathbf{q}_t^{h,B}, \mathbf{Q}_t^h).$$

If these two conditions hold, then the household can be modelled as a time consistent representative agent with a latently separable, time-independent felicity function and constant discount rate.

I.C Revealed preference conditions

The revealed preference approach to establishing the time consistency of household behaviour asks whether one can find necessary and sufficient conditions under which observed choices can be rationalised by a stationary collective preference subject to the lifecycle budget constraint.

Following the above, we assume $|T|$ observed consumption choices for any given household h: $T = \{1, ..., |T|\}$. For each observation t, we observe the privately consumed quantities, $\mathbf{q}_t^h$, and the publicly consumed quantities, $\mathbf{Q}_t^h$, as well as the corresponding (discounted) prices, $\mathbf{p}_t$ and $\mathbf{P}_t$. This defines a set of observations $S^h = \{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t \in T}$.

[2] As a technical remark, we indicate that these conditions are sufficient but, in general, not necessary for time consistency. Specifically, the (*only*) case where necessity does not hold is where $\beta_{h,A} \neq \beta_{h,B}$ but $\omega_t^h = (\beta_{h,A}/\beta_{h,B})^{t-1}$, which means that the more patient household member has a steadily decreasing relative bargaining weight that *exactly* offsets the difference in patience. This alternative model is empirically indistinguishable from ours, i.e. it implies the exact same testable conditions as the time consistency model (see Section I.C). Clearly, however, this is a very particular (and essentially theoretical) construction, from which we will abstract in what follows.

Note that we only assume that aggregate private quantities $\mathbf{q}_t^h$, but not the individual private quantities $\mathbf{q}_t^{h,A}$ and $\mathbf{q}_t^{h,B}$ (with $\mathbf{q}_t^{h,A} + \mathbf{q}_t^{h,B} = \mathbf{q}_t^h$), are observed. This assumption is motivated by the fact that, in most household surveys (including the one we use in our own application), information on "who gets what" is limited and the decomposition of private consumption into that consumed by household members is generally unobserved.

Then, rationalisation of the data set on household h is defined as follows.

Definition 2 *The set of observations on household h, $S^h = \{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t\in T}$, can be rationalised by the time consistency model if there exist, for all $t \in T$, private quantities $\mathbf{q}_t^{h,A}$, $\mathbf{q}_t^{h,B} \in \mathbb{R}_+^N$ (with $\mathbf{q}_t^{h,A}+ \mathbf{q}_t^{h,B} = \mathbf{q}_t^h$) and, in addition, a concave, strictly increasing felicity function u^h and discount factor $\beta_h \in (0,1]$ such that*

$$\sum_{t\in T} \beta_h^{t-1} u^h(\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h) \geq \sum_{t\in T} \beta_h^{t-1} u^h(\boldsymbol{\zeta}_t^A, \boldsymbol{\zeta}_t^B, \boldsymbol{\zeta}_t^h)$$

for all affordable consumption plans $\{\boldsymbol{\zeta}_t^A, \boldsymbol{\zeta}_t^B, \boldsymbol{\zeta}_t^h\}_{t\in T}$ (with $\boldsymbol{\zeta}_t^A, \boldsymbol{\zeta}_t^B \in \mathbb{R}_+^N$ and $\boldsymbol{\zeta}_t^h \in \mathbb{R}_+^K$) that satisfy

$$\sum_{t\in T} [\mathbf{p}_t'(\mathbf{q}_t^{h,A} + \mathbf{q}_t^{h,B}) + \mathbf{P}_t'\mathbf{Q}_t^h] \geq \sum_{t\in T} [\mathbf{p}_t'(\boldsymbol{\zeta}_t^A + \boldsymbol{\zeta}_t^B) + \mathbf{P}_t'\boldsymbol{\zeta}_t^h].$$

In words, the data can be rationalised by a time consistent household preference if observed choices maximise discounted lifetime household utility out of affordable lifetime consumption plans for a stationary collective preference.

Theorem 1 then states the revealed preference conditions for a data rationalisation as defined above. We refer the reader to Appendix A for proofs of all our main results.

Theorem 1 *The set of observations on household h, $S^h = \{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t\in T}$, can be rationalised by the time consistency model if and only if there exist, for all $t \in T$, a utility number $u_t^h \in \mathbb{R}$ and a positive constant $\beta_h \in (0,1]$ that, for any s,$t \in T$, satisfy*

$$u_s^h - u_t^h \leq \frac{1}{\beta_h^{t-1}} [\mathbf{p}_t'(\mathbf{q}_s^h - \mathbf{q}_t^h) + \mathbf{P}_t'(\mathbf{Q}_s^h - \mathbf{Q}_t^h)].$$

Theorem 1 is an equivalence result. In words, if there exists a household discount factor β_h and constants $\{u_t^h\}_{t\in T}$ such that the stated inequalities hold, then there exists a stationary household felicity function and discount rate that provide a perfect within-sample rationalisation of the choices observed for household h. The existence of a non-empty feasible set to these inequalities implies that one cannot reject the hypothesis of discount factor homogeneity and a time-invariant Pareto weight within the household. Determining the actual existence of such a non-empty feasible set is easily done empirically. Conditioning on β_h, the inequalities defined by Theorem 1 are linear in unknowns and can be easily verified using standard linear programming techniques.

II An empirical test of time consistency

In this section we test the time consistency conditions in Theorem 1 for household panel data taken from the Spanish Continuous Family Expenditure Survey (the *Encuesta Continua de Presupuestos Familiares,* ECPF). We strongly reject the hypothesis of a time consistent collective preference, as defined above.

We examine the robustness of our rejection of the time consistency model as a rationalisation of household choice by evaluating the sensitivity of our results to two crucial underlying assumptions. First, we investigate the possibility that actual behaviour is effectively time consistent but prices or quantities are measured with error. Ignoring measurement error is a well-known criticism of revealed preference tests and is often used as an argument to explain their rejection. However, we find that this alternative explanation does not convincingly rationalise the observed rejections of time consistency. Second, with respect to our conclusions in Sections III and IV, it is important to know if individual preferences are time consistent and thus that rejections are indeed driven by the collective nature of household choice. We evaluate the time consistency model for single person households, and find that the model's empirical performance is significantly better in this case than for multi person households. These results motivate our focus on individual heterogeneity within households as a source of time inconsistency.

II.A The data

Our empirical analysis uses household level expenditure information from the Spanish Continuous Family Expenditure Survey (the Encuesta Continua de Presupuestos Familiares, ECPF). The ECPF data set is a rotating panel that is conducted every quarter by the Spanish Statistics Office (INE). Participating households are surveyed in the same week of each successive quarter, with each adult family member completing an expenditure diary in which they record their spending during the survey week. Appendix B provides summary statistics on the characteristics of the demographic, expenditure and price data used in our empirical analysis, but we here document the main restrictions that we impose upon our sample of households.

The data used are drawn from the period 1985-1997. We do not consider data past 1997 because at this time the Continuous Household Budget Survey (CHBS) replaced the ECPF. The CHBS aimed to provide estimates of the level and change of the aggregate cost of the household rather than a detailed breakdown of expenditures at the good level. Participant households are randomly rotated at a rate of 12.5% each quarter, implying that a household may be observed for at most eight consecutive quarters. However, there are a sizable number of households who do not complete all eight interviews. To achieve a large sample size, whilst maintaining a long enough panel to provide the right context for our intertemporal tests, we consider households that report information for at least four consecutive quarters.

We further restrict our sample according to certain demographic and employment characteristics of the household. First, we only consider "married" couples (as opposed to households composed of a collection of adults), with and without children, with both members under 65 years old. The restriction to married couples is to ensure that we only consider collections of individuals who may reasonably be expected to act cooperatively (in line with our theoretical model). Although we consider households with different numbers of children, we do require that the number of children in the household is stable over the period of observation to prevent our results being unduly influenced by the inevitable disruption and change in household dynamics that occurs upon the birth of a child. We also require that the employment status of husbands and wives is stable over time. The requirement of stable employment status is to allow the potential nonseparability of leisure and consumption to be ignored for the time being, as our theoretical framework does not presently consider the household labour supply decision.

Regarding the household choice bundle, we consider household choice defined over a commodity bundle of eight nondurable goods for four consecutive quarters.[3] Only households with positive total expenditures on this sample of goods in each time period are considered. This final restriction leaves us with a sample of 2083 couples to work with.

Each good is classified as contributing to either "private" or "public" consumption. Our bundle of six privately consumed goods (N=6) consists of: 1) Food and non-alcoholic drinks; 2) Clothing and footwear; 3) Transport; 4) Leisure (cinema, theatre, clubs for sport); 5) Personal services and 6) Restaurants and bars. The bundle of two publicly consumed goods (K=2) consists of: 1) Household services (heating, water and furniture repair) and 2) Petrol. Prices are calculated from published prices aggregated at the household level to correspond to the listed expenditure categories. In our empirical application, these prices are discounted by the average nominal interest rate on consumer loans. Appendix B provides summary statistics on the level and variability of the budget shares and prices of these goods across the sample.

II.B Revealed preference tests

Revealed preference tests of a given model are defined by hypotheses of the following form:

$\mathbf{H}^0$: Household behaviour can be rationalised by the model.

$\mathbf{H}^1$: Household behaviour cannot be rationalised by the model.

Therefore, such tests yield "yes/no" answers; either household behaviour is consistent with the model in question or it is not. A "yes" result implies that the model cannot be

[3]Thus, we implicitly assume separability between durable and nondurable consumption.

rejected on the basis of observed behaviour. However, it does not necessarily imply that the model is "the truth". Popper (1959) points to the logical asymmetry between verification and falsification. No number of observed passes of model X allows one to derive the universal statement: "All households can be rationalised by model X". However, failure of a revealed preference test allows us to logically derive the conclusion: "The household cannot be rationalised by model X".

As explained in Section I, testing whether household h's consumption choices can be rationalised by a time consistent household utility function boils down to checking the linear conditions defined by Theorem 1 for a given value of the discount factor β_h. In our empirical application we conduct a grid search on β_h for every household $h = 1, ..., H$. We report results for a grid search of individual discount factors on $[0.9, 1]$ with a spacing of 0.005.[4] At this point, it is worth noting that all our following results are robust to alternative grid search specifications (including a grid search across the full interval $(0, 1]$). These robustness checks can be obtained from the authors on request.

We also remark here that our tests allow for unrestricted preference heterogeneity across households, as captured by the individual-specific felicity functions. The theory-consistency of each household's behaviour is tested independently and the data is not pooled at any stage.

Accounting for how "demanding" a test is. An obvious measure for the empirical performance of a behavioural model is its pass rate, i.e. the number of households in our sample that pass its testable implications. However, a model's pass rate only captures one dimension of its empirical performance. Alongside the pass rate, empirical applications of revealed preference tests typically report two additional performance metrics: discriminatory power and predictive success. Predictive success gives a holistic measure of the empirical performance of a behavioural model by simultaneously accounting for a model's pass rate and how "demanding" our test is. As such, we will primarily evaluate our various behavioural models according to the predictive success metric.

Discriminatory power. Following Bronars (1987), the discriminatory power of a revealed preference test for a particular behavioural model is defined as the probability of detecting behaviour that is not rationalisable by the model. It, therefore, provides a measure of how demanding a test is. Bronars suggests an iterative procedure to compute his power metric, which we apply to each household in our sample. At every iteration, the procedure simulates random behaviour (i.e. behaviour that is not generated by any optimising model) by drawing $|T| \times (N + K)$ random budget shares from the uniform distribution. For a given household h, these budget shares then define a new random

[4] This corresponds to a search for discount rate on $[0, 0.11]$, which given the quarterly periodicity of our data is not unreasonable.

consumption stream $\{\mathbf{q}_t^{h,R}, \mathbf{Q}_t^{h,R}\}_{t\in T}$ that exhausts their total wealth.[5] We then test the revealed preference conditions of the model under evaluation on the correspondingly defined set $\{\mathbf{q}_t^{h,R}, \mathbf{Q}_t^{h,R}; \mathbf{p}_t, \mathbf{P}_t\}_{t\in T}$. In our application, we iterate this procedure 1000 times per household and calculate the proportion of the randomly generated consumption streams that fail the revealed preference restrictions of the behavioural model.

We use the proportion of randomly generated consumption streams that fail the revealed preference restrictions to calculate a household-specific measure for discriminatory power. This proportion proxies the true probability that random household behaviour will fail the restrictions of the behavioural model for observed prices and total household expenditure. For example, if 60% of all randomly generated consumption streams fail to meet the requirements of a revealed preference test, then there is approximately a 60% chance that our tests will correctly reject random choice behaviour. Generally, high power signals a restrictive model and there is a high probability that revealed preference tests will detect irrational/random behaviour.

At this point, recall that we conduct a grid search on the discount factor (β_h) to compute the pass rate for the time consistent model. Computing power requires an analogous grid search at each iteration. We define a household specific grid size, conditional on whether a household's choices are rationalisable by the model under study. If observed behaviour is not rationalisable, then at each iteration we define the same grid size as before: the interval $[0.9, 1]$ with a spacing of 0.005. However, if household h's observed behaviour *can* be rationalised by the model, we use this information to define a finer grid when computing their power metric. Specifically, we search only over β_h no lower than the maximum value under which rationalisability is obtained. This adjusted grid search substantially limits the computational burden of our power assessment, whilst accounting for the information on individual time preferences as revealed by the observed behaviour.

Predictive success. We primarily evaluate the empirical performance of a model according to the value of its predictive success metric. This measure combines the pass rate and the power of a particular behavioural model into a single metric that can be interpreted as the power-adjusted pass rate. It was recently axiomatised by Beatty and Crawford (2011) and is based upon an original proposal of Selten (1991). For each household, predictive success is calculated by subtracting 1 minus the power measure from the pass measure (1 or 0). Therefore, the measure is always situated between -1 and 1.

The higher the average predictive success measure, the better the empirical performance of the behavioural model under evaluation. A predictive success value in the neighbourhood of -1 indicates that a household fails the rationalisability conditions, implying a pass measure equal to 0, even though the power of the test is low and relatively

[5] This simulated random behaviour corresponds to Becker's (1962) notion of irrational behaviour as behaviour that randomly exhausts the available budget.

easy to pass (i.e. discriminatory power is close to 0). Conversely, a predictive success value in the neighbourhood of 1 indicates a household that passes the model restrictions in a situation where the model has high power. This represents the ideal scenario if you will. Finally, a predictive success value equal to zero suggests that the model is not informative for the household at hand: the model does not outperform the uninformative assumption that households exhibit random consumption behaviour, for which the power is 0 and the pass measure equals 1, by construction.

II.C Test results

Table 1 reports the test results obtained from applying the revealed preference conditions in Theorem 1 to the ECPF data. We find that only 14% of the 2083 households in our sample can be rationalised by a time consistent collective preference. This result suggests that the empirical support for the time consistency model as an explanation for household choice is weak. However, the revealed preference test of the model is relatively stringent. On average, the test has an 84% probability of rejecting randomly simulated choice behaviour. This is very high in comparison to the power metrics typically calculated for the static collective model, which typically lie in the neighbourhood of zero (see, for example, Cherchye, De Rock and Vermeulen, 2009).

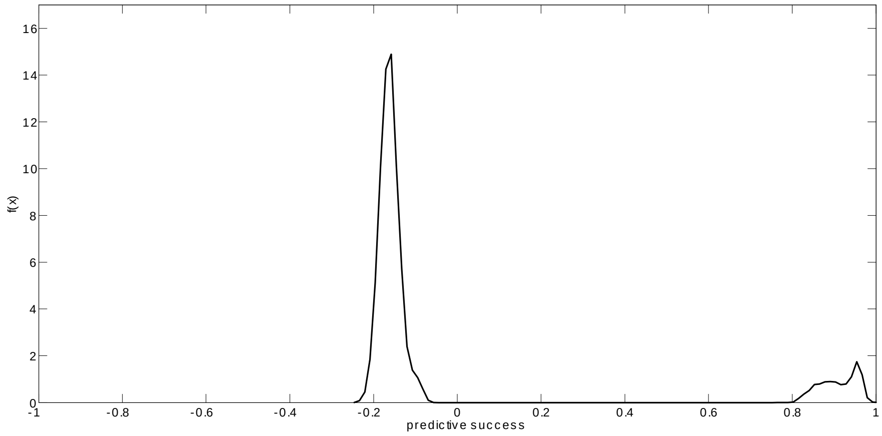
Table 1: Time consistency model

	pass rate	*power*	*predictive success*
mean	0.1402	0.8490	-0.0108
(*st. error*)	(0.0076)	(0.0008)	(0.0082)

Yet, even taking account fo the high demandingness of the test, the empirical performance of the time consistency model is poor. The predictive success measure of -0.01 highlights that, on average, households are performing on a par with a random number generator.

Figure 1 gives the distribution of predictive success that is associated with the time consistent model. The distribution is bimodal and the fit of the model is very poor for the majority of households; predictive success is negative for 86% of the households in our sample. Our predictive success findings suggest that we can safely conclude that the data does not endorse the applicability of the time consistent model (as it stands) to explain household intertemporal choice behaviour.

Figure 1: Predictive success distribution for time consistency model



II.D Robustness checks

In this section, we investigate the robustness of the rejection of the time consistency model as an explanation for household choice behaviour by investigating the impact of two crucial underlying assumptions. First, we relax the assumption that observed prices and consumption quantities are unaffected by measurement error. We find that an allowance for measurement error does not significantly improve the empirical performance of the model. Second, we examine the empirical validity of the time consistency model for single person households. This is particularly relevant in view of our discussion in the following sections, which aims at rationalising time-inconsistency of multi person households under the maintained assumption that individual household members are time consistent. Interestingly, we will find that the time consistency model indeed does perform significantly better for single person households than for multi person households.

Measurement error. The decisive rejection of the time consistency model could be the result of errors in listed expenditure data, rather than deviations of actual choice from the model's prescriptions. Revealed preference tests are "sharp" in that household behaviour is either consistent with the model in question or it is not. Thus, small deviations in observed quantities or prices away from the their true values could have a large impact on predictive success. Moreover, seasonality in our quarterly data might also explain why time consistency at the household level fails, due to the possible changes in the structure of consumption across seasons.[6] If this is the case for our sample, then introducing a sufficient amount of measurement error should pick up this effect.

To explore the impact of allowing for measurement error on our test results, we use a procedure that is based on an original idea of Varian (1985). Essentially, the procedure evaluates the predictive success of the time consistency model by considering a weaker test that accounts for possible errors in the data. We consider two exercises. Our first exercise considers errors in the quantity data, and our second exercise focuses on errors in the price data. We cannot consider the impact of allowing for price and quantity errors simulatenously because this introduces a non-linearity to the procedure, which introduces a significant computational difficulty.

For compactness, we here explain only our procedure for quantity errors. However, the reasoning for price errors is directly analogous. Consider a data set $S^h = \{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t \in T}$ for a household h that does not meet the conditions defined by Theorem 1. For a household observation t, let $q_{t,n}^h$ represent the observed quantity of the n-th private good and $Q_{t,k}^h$ the observed quantity of the k-th public good. Due to measurement error, these quantities may deviate from $q_{t,n}^{h,*}$ and $Q_{t,n}^{h,*}$, the true (but unobserved) values of the private and public quantities that are consumed within the household. The divergence between listed and true quantities are quantified by the "relative quantity errors":

$$\epsilon_{t,n}^h = \frac{q_{t,n}^{h,*} - q_{t,n}^h}{q_{t,n}^h} \text{ and } \varepsilon_{t,k} = \frac{Q_{t,k}^{h,*} - Q_{t,k}^h}{Q_{t,k}^h}$$

As we cannot observe the true values, $q_{t,n}^{h,*}$ and $Q_{t,n}^{h,*}$ (or, equivalently, the actual errors $\epsilon_{t,n}^h$ and $\varepsilon_{t,k}^h$), we approximate the necessary measurement error required for listed expenditure data to pass the revealed preference conditions. We do this by calculating the minimal adjustments of the quantity data that are required to obtain theory consistency. Specifically, we define the following perturbation to observed quantities for each particular household h:

[6] At this point, we also note that our use of broad good categories may be expected to considerably mitigate seasonality effects, by construction.

$$\widetilde{q}^h_{t,n} = \left(1+\widetilde{\epsilon}^h_{t,n}\right) q^h_{t,n} \text{ and } \widetilde{Q}^h_{t,k} = \left(1+\widetilde{\varepsilon}^h_{t,k}\right) Q^h_{t,n},$$

and minimise the sum of squared error terms for the household :

$$\min \widetilde{\mathbf{V}}^h = \sum_{t=1}^{|T|} \left(\sum_{n=1}^{N} (\widetilde{\epsilon}^h_{t,n})^2 + \sum_{k=1}^{K} (\widetilde{\varepsilon}^h_{t,k})^2 \right),$$

subject to the constraint that the perturbed data set $\widetilde{S}^h = \{\widetilde{\mathbf{q}}^h_t, \widetilde{\mathbf{Q}}^h_t; \mathbf{p}_t, \mathbf{P}_t\}_{t\in T}$ satisfies the model. This effectively calculates the smallest perturbations $\widetilde{\epsilon}^h_{t,n}$ and $\widetilde{\varepsilon}^h_{t,k}$ that are necessary for $\widetilde{S}^h$ to be rationalised by the time consistent model.

Using the estimated minimal perturbations, we can compute

$$\widetilde{\sigma}^h = \frac{\sqrt{\widetilde{\mathbf{V}}^h}}{|T| * (N+K)},$$

which gives an estimate for the average quantity error that we need to make our data satisfy the time consistency model. By construction, we will have that $\widetilde{\sigma}^h = 0$ if the original data set S^h meets the conditions in Theorem 1, while higher values of $\widetilde{\sigma}^h$ indicates that more measurement error is required to rationalise the observed behaviour as being time consistent.

If one allows for sufficient measurement error, any behaviour can be classed as consistent with the model. Therefore, we assess the average minimum measurement errors that are required to rationalise the data relative to particular benchmark upper bounds on the standard deviation of the measurement error process, $\overline{\sigma}$. Clearly, lower values for $\overline{\sigma}$ obtain more stringent tests, with $\overline{\sigma} = 0$ yielding the original conditions in Theorem 1. As previously, we account for the trade-off between pass rate and power by evaluating the predictive success of the time consistency model for alternative $\overline{\sigma}$ values.

In Table 2, we report predictive success outcomes for "low" and "high" measurement error scenarios. We define the upper bound in our low measurement error scenario as the cut-off value $\overline{\sigma}_{LO}$ that corresponds to a pass rate of one-third (i.e. $\overline{\sigma}_{LO}$ is the 33%-quantile of the empirical distribution of $\tilde{\sigma}^h$).[7] In particular, we get the cut-off value $\overline{\sigma}_{LO}^{quant} = 0.05$ for quantities and $\overline{\sigma}_{LO}^{price} = 0.005$ for prices. Our second scenario accounts for a large amount of measurement error, using a cut-off value $\overline{\sigma}_{HI}$ that is sufficient to rationalise the full sample. In this case, we obtain $\overline{\sigma}_{HI}^{quant} = 0.15$ for quantities and $\overline{\sigma}_{HI}^{price} = 0.1$ for prices. The values of these cut-offs, and of the associated predictive success, differ between measurement error in prices and measurement error in quantities because of differences in the extent, and nature, of variation of prices and quantities.

Table 2: Measurement error

		predictive success	
		mean	*st. error.*
no measurement error (time consistency)		-0.0108	0.0076
measurement error in prices:	- low (cut-off $\overline{\sigma}_{LO}^{price}$)	0.0426	0.0127
	- high (cut-off $\overline{\sigma}_{HI}^{price}$)	0.0482	0.0038
measurement error in quantities:	- low (cut-off $\overline{\sigma}_{LO}^{quant}$)	-0.0898	0.0192
	- high (cut-off $\overline{\sigma}_{HI}^{quant}$)	0.0060	0.0214

Neither the predictive success values for measurement error in prices nor quantities provide strong support for the hypothesis that these errors are primarily responsible for the poor empirical performance of the time consistency model. All of the predictive success values in Table 2 remain in the neighbourhood of zero. In our opinion, one may safely argue that measurement error does not lie behind the poor empirical performance of the time consistency model.

Single person households. Our data set contains information on a small sample of single person households. As with our sample of couples, we impose the requirements of stable employment status, positive expenditures on the considered goods categories, and that they are aged under 65 for the full length of observations. This leaves us with

[7] We define this cut-off because it effectively doubles the pass rate of the time consistency model, thereby providing sufficient margin for the predictive success of the model with low measurement error to dominate that of the time consistency model.

a sample of 189 individuals. Our robustness check verifies the conditions in Theorem 1 for each of these 189 individuals to assess whether it is reasonable to assume that singles indeed behave time consistently.

Interestingly, Table 3 shows that the time consistency hypothesis fits significantly better for singles than for couples. In particular, for singles we obtain a predictive success rate that is substantially above zero. This indicates that the model effectively provides a more informative explanation of the singles' behaviour in our sample. Furthermore, this value for predictive success is impressive compared to those associated with other revealed preference tests. For example, Beatty and Crawford (2010) calculate an average predictive success measure of 0.045 for the static utility maximisation model on a sample also drawn from the ECPF. Adams (2011) calculates a predictive success measure of 0.000 for the static collective model.

Table 3: Single person households

		predictive success	
		mean	*st. error.*
Time consistency	- couples	-0.0108	0.0076
	- singles	0.1444	0.0355

We conclude that individual nonstationarities alone cannot fully account for the poor empirical performance of households when evaluated in line with the time consistency model. There must be some other force at work. This directly motivates our following analysis, in which we investigate our core hypothesis that the observed time inconsistency of couples' behaviour is in large part due to the multi person nature of household decisions.

III Collective choice and time inconsistency

The collective preference cannot be recast in the format required for time consistency in the presence of either heterogeneity in individual time preferences or periodic innovations in the Pareto weight. The failures of these assumptions manifest themselves in observed choice behaviour differently. This section explores these sources of time inconsistency in greater depth and utilises Mazzocco's (2007) theoretical framework to develop an empirical strategy for distinguishing between the different sources of time inconsistent household behaviour.

III.A Individual heterogeneity

Within household h, individual heterogeneity and innovations in the Pareto weight, ω_t^h, can introduce time inconsistencies to household consumption patterns, even if individuals within the household have perfectly time consistent preferences.

With intrahousehold discount rate heterogeneity, one cannot collapse individual discount rates into a single household rate of time preference, $\beta_A^t + \beta_B^t \neq (\beta_A + \beta_B)^t$. This implies that members' preferences are weighted differently in the household allocation problem at different points in time, even if the Pareto weight remains constant. Other things equal, the preferences of the more patient member become relatively more important in future time periods, introducing a time inconsistency to the collective preference. Jackson and Yariv (2012) prove that for a group of otherwise homogeneous individuals choosing a common consumption stream, any heterogeneity in time preferences necessitates a present biased collective preference, and that with a uniform distribution of discount rates in a population, the collective utility function is hyperbolic. Thus, time inconsistencies in group behaviour need not be derivative of nonstationarities at the individual level. Rather, they can arise from the aggregation of heterogeneous time preferences.

The full efficiency model. Individual heterogeneity is the only source of time inconsistency in Mazzocco's (2007) "full efficiency" model. The model assumes the existence of a perfect commitment mechanism, removing the possibility of intrahousehold renegotiation. This implies the existence of a single, fixed Pareto weight to summarise the household decision making process.

For a given data set on household h, $S^h = \{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t \in T}$, the full efficiency model corresponds to the following rationalisation condition.[8]

[8] One important difference between our theoretical framework and that of Mazzocco's (2007) "full efficiency" model is the assumption of perfect foresight. Revealed preference tests of martingale processes lack content as, without a specification of the expectation process, one can always posit an unexpected shock to rationalise behaviour. Following Mazzocco, our framework also assumes perfect capital markets.

Definition 3 *The set of observations on household h,* $S^h = \{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t\in T}$*, can be rationalised by the full efficiency model if there exist, for all* $t \in T$*, private quantities* $\mathbf{q}_t^{h,A}$, $\mathbf{q}_t^{h,B} \in \mathbb{R}_+^N$ *(with* $\mathbf{q}_t^{h,A} + \mathbf{q}_t^{h,B} = \mathbf{q}_t^h$*) and, in addition, concave, strictly increasing felicity functions* $u^{h,A}$ *and* $u^{h,B}$*, discount factors* $\beta_{h,A}$, $\beta_{h,B} \in (0,1]$ *and a Pareto weight* $\omega^h > 0$ *such that*

$$\sum_{t\in T}\left\{\beta_{h,A}^{t-1}u^{h,A}(\mathbf{q}_t^{h,A},\mathbf{Q}_t^h)+\omega^h\beta_{h,B}^{t-1}u^{h,B}(\mathbf{q}_t^{h,B},\mathbf{Q}_t^h)\right\} \geq \sum_{t\in T}\left\{\beta_{h,A}^{t-1}u^{h,A}(\boldsymbol{\zeta}_t^A,\boldsymbol{\zeta}_t^h)+\omega^h\beta_{h,B}^{t-1}u^{h,B}(\boldsymbol{\zeta}_t^B,\boldsymbol{\zeta}_t^h)\right\}$$

for all affordable consumption plans $\{\boldsymbol{\zeta}_t^A, \boldsymbol{\zeta}_t^B, \boldsymbol{\zeta}_t^h\}_{t\in T}$ *(with* $\boldsymbol{\zeta}_t^A, \boldsymbol{\zeta}_t^B \in \mathbb{R}_+^N$ *and* $\boldsymbol{\zeta}_t^h \in \mathbb{R}_+^K$*), which satisfy*

$$\sum_{t\in T}[\mathbf{p}_t'(\mathbf{q}_t^{h,A}+\mathbf{q}_t^{h,B})+\mathbf{P}_t'\mathbf{Q}_t^h] \geq \sum_{t\in T}[\mathbf{p}_t'(\boldsymbol{\zeta}_t^A+\boldsymbol{\zeta}_t^B)+\mathbf{P}_t'\boldsymbol{\zeta}_t^h].$$

As for the time consistent model, the constant Pareto weight ω^h incorporates the combined impact of all changes in distribution factors over time; it can be considered as the average relative power of family members across the lifetime of the household. In this setting, the only source of time inconsistent aggregate behaviour is discount rate heterogeneity. Recall that if $\beta_{h,A} = \beta_{h,B} = \beta_h$, then the collective preference could be recast in a time consistent format with a stationary household felicity function.

Revealed preference conditions. How can we test for the importance of individual heterogeneity as a source of time inconsistency? Discount rate heterogeneity negates the possibility of representing the collective preference in representative-consumer format. Given this, the composition of household consumption and its distribution between family members plays a central role in revealed preference tests of the full efficiency model. This has two important implications for the revealed preference conditions associated with the full efficiency model. First, for privately consumed goods, the information on $\mathbf{q}_t^{h,A}$ and $\mathbf{q}_t^{h,B}$ is relevant. Second, for publicly consumed goods, the relevant "prices" for an

We recognise that these assumptions are very strong. Still, in our empirical application we will find that nearly all observed household behaviour in our sample can be rationalised even when maintaining these assumptions. See Section IV for a further discussion.

individual family member will be so-called Lindahl prices, $\mathbf{P}_t^{h,A}$ and $\mathbf{P}_t^{h,B}$. These prices coincide with a family member's marginal willingness to pay and, given the maintained assumption of cooperative decision making, sum to observed prices, $\mathbf{P}_t^{h,A} + \mathbf{P}_t^{h,B} = \mathbf{P}_t$.

Theorem 2 gives the conditions under which household choice can be rationalised by the full efficiency model. If choices can be rationalised by the model, we cannot reject the hypothesis that time inconsistencies in aggregate behaviour are the result of discount rate heterogeneity within the family.

Theorem 2 *The set of observations on household* h, $S^h = \{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t\in T}$ *can be rationalised by the full efficiency model if and only if there exist, for all* $t \in T$, *private quantities* $\mathbf{q}_t^{h,A}$, $\mathbf{q}_t^{h,B} \in \mathbb{R}_+^N$ *(with* $\mathbf{q}_t^{h,A} + \mathbf{q}_t^{h,B} = \mathbf{q}_t^h$*), Lindahl prices* $\mathbf{P}_t^{h,A}$, $\mathbf{P}_t^{h,B} \in \mathbb{R}_+^N$ *(with* $\mathbf{P}_t^{h,A} + \mathbf{P}_t^{h,B} = \mathbf{P}_t^h$*), utility numbers* $u_t^{h,A}$, $u_t^{h,B} \in \mathbb{R}$ *and constants* $\beta_{h,A}$, $\beta_{h,B} \in (0,1]$ *that, for any* $s, t \in T$, *satisfy*

$$
\begin{aligned}
u_s^{h,A} - u_t^{h,A} &\leq \frac{1}{\beta_{h,A}^{t-1}} \left[\mathbf{p}_t'(\mathbf{q}_s^{h,A} - \mathbf{q}_t^{h,A}) + \mathbf{P}_t^{h,A\prime}(\mathbf{Q}_s^h - \mathbf{Q}_t^h) \right], \\
u_s^{h,B} - u_t^{h,B} &\leq \frac{1}{\beta_{h,B}^{t-1}} \left[\mathbf{p}_t'(\mathbf{q}_s^{h,B} - \mathbf{q}_t^{h,B}) + \mathbf{P}_t^{h,B\prime}(\mathbf{Q}_s^h - \mathbf{Q}_t^h) \right].
\end{aligned}
$$

In words, if we can find a discount factor $\beta_{h,m}$ and constants $\{u_t^{h,m}\}_{t\in T}$ for each member $m \in \{A, B\}$ in household h, along with feasible private quantities and Lindahl prices, such that the inequalities defined by Theorem 2 hold, then there exists a pair of felicity functions and constant discount rates that provide a perfect within-sample rationalisation of the household data. Conversely, if we cannot find values of all relevant variables such that these inequalities hold, then there does not exist a theory-consistent specification of household member preferences and a constant Pareto weight that rationalise the observed consumption stream. This allows us to test whether time inconsistencies in choice can be explained by appeal to discount rate heterogeneity alone. If a non-empty feasible set is associated with the full efficiency constraints, one cannot reject the hypothesis that time inconsistency in household choice is simply the product of individual heterogeneity within the collective unit; one does not necessarily require nonstationarities in individual preferences or renegotiations of the household choice rule over time to rationalise the observed behaviour.

III.B Renegotiation

If household behaviour is inconsistent with the full efficiency model, an appeal to more than just discount rate heterogeneity is required. The second condition for time consistency of the collective preference is the existence of a constant Pareto weight across the full lifetime of the household. Whether this is necessarily attained depends upon the existence of a perfect intrahousehold commitment mechanism. Without a perfect commitment device, the Pareto weight can vary over time to reflect renegotiations of the household choice rule. These renegotiations open up an additional mechanism for time inconsistent behaviour.

The no-commitment model. Mazzocco's (2007) "no-commitment" model weakens the assumption of perfect intrahousehold commitment. Instead, the household solves the lifetime bargaining problem subject to additional incentive compatibility constraints. Mazzocco (2007) classes a consumption stream as incentive compatible if it does not provide an incentive for any family member to quit the household at some point to take their "outside option". An individual's outside option is defined as the utility they could derive from divorcing and continuing in the world alone.[9]

Using the method developed by Marcet and Marimon (1998), the no-commitment model can be formulated as a recursive saddle point problem and theory consistent behaviour corresponds to the following rationalisation condition.[10]

Definition 4 *The set of observations on household h,* $S^h = \{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t\in T}$ *can be rationalised by the no-commitment model if there exist, for all* $t \in T$*, private quantities*

[9] This definition is akin to Shaked and Sutton's (1984) formulation of outside options.

[10] This formulation implicitly embodies the requirement that a particular consumption profile must be incentive compatible, i.e. for each member $m = \{A, B\}$ and $t \in \{1, ..., |T|\}$

$$\sum_{s=0}^{|T|-t} \beta_{h,m}^s u^{h,m}(\mathbf{q}_{t+s}^{h,m}, \mathbf{Q}_{t+s}^h) \geq \underline{u}_t^{h,m}.$$

In words, by remaining within the household each household member must achieve welfare at least as great as when exiting via divorce.

$\mathbf{q}_t^{h,A}$, $\mathbf{q}_t^{h,B} \in \mathbb{R}_+^N$ *(with* $\mathbf{q}_t^{h,A} + \mathbf{q}_t^{h,B} = \mathbf{q}_t^h$*) and, in addition, concave, strictly increasing felicity functions* $u^{h,A}$ *and* $u^{h,B}$*, discount factors* $\beta_{h,A}$*,* $\beta_{h,B} \in (0,1]$*, Pareto weights* $\omega_t^{h,A}, \omega_t^{h,B} > 0$*, multipliers* $\varphi_t^{h,A}$ *and* $\varphi_t^{h,B}$ *and outside utilities* $\underline{u}_t^{h,A}$ *and* $\underline{u}_t^{h,B}$ *such that:*

$$\sum_{t\in T}\left\{\left[\omega_t^{h,A}\beta_{h,A}^{t-1}u^{h,A}(\mathbf{q}_t^{h,A},\mathbf{Q}_t^h) - \varphi_t^{h,A}\underline{u}_t^{h,A}\right] + \left[\omega_t^{h,B}\beta_{h,B}^{t-1}u^{h,B}(\mathbf{q}_t^{h,B},\mathbf{Q}_t^h) - \varphi_t^{h,B}\underline{u}_t^{h,B}\right]\right\} \geq$$
$$\sum_{t\in T}\left\{\left[\omega_t^{h,A}\beta_{h,A}^{t-1}u^{h,A}(\boldsymbol{\zeta}_t^A,\boldsymbol{\zeta}_t^h) - \varphi_t^{h,A}\underline{u}_t^{h,A}\right] + \left[\omega_t^{h,B}\beta_{h,B}^{t-1}u^{h,B}(\boldsymbol{\zeta}_t^A,\boldsymbol{\zeta}_t^h) - \varphi_t^{h,B}\underline{u}_t^{h,B}\right]\right\}$$

for all affordable consumption plans $\{\boldsymbol{\zeta}_t^A,\boldsymbol{\zeta}_t^B,\boldsymbol{\zeta}_t^h\}_{t\in T}$ *(with* $\boldsymbol{\zeta}_t^A,\boldsymbol{\zeta}_t^B \in \mathbb{R}_+^N$ *and* $\boldsymbol{\zeta}_t^h \in \mathbb{R}_+^K$*), which satisfy*

$$\sum_{t\in T}[\mathbf{p}_t'(\mathbf{q}_t^{h,A} + \mathbf{q}_t^{h,B}) + \mathbf{P}_t'\mathbf{Q}_t^h] \geq \sum_{t\in T}[\mathbf{p}_t'(\boldsymbol{\zeta}_t^A + \boldsymbol{\zeta}_t^B) + \mathbf{P}_t'\boldsymbol{\zeta}_t^h].$$

In this definition, $\varphi_t^{h,m}$ represents the Lagrange multiplier on m's incentive compatibility constraint, and $\underline{u}_t^{h,m}$ gives the utility associated with m's outside option, that is, the stream of utility that m could receive when leaving household h at period t. The household choice rule, summarised by the set of intrahousehold preference weights, $\omega_t^{h,m}$, is sequentially renegotiated to reflect changes in the slackness of the incentive compatibility constraints: $\omega_1^{h,A} = 1$, $\omega_t^{h,A} = \omega_{t-1}^{h,A} + \varphi_t^{h,A}$ and $\omega_1^{h,B} = \omega^h$, $\omega_t^{h,B} = \omega_{t-1}^{h,B} + \varphi_t^{h,B}$. Assuming positive gains to marriage continuation for at least one spouse in every time period, there will always be at least one individual who is strictly better off if the marriage continues rather than dissolving through divorce. Thus, the incentive compatibility condition can only bind for one family member at any point in time, i.e. if $\varphi_t^{h,A} \neq 0$ then $\varphi_t^{h,B} = 0$, and vice versa. In periods where the incentive compatibility constraint binds for some member, the weight assigned to her preferences is increased until she is indifferent between taking their outside option and staying within the household. This new weighting of family member preferences then prevails in subsequent time periods until an incentive constraint again binds and another reweighting of preferences is implemented.

Stationarity of the household per-period felicity function can now also be undermined by renegotiations of the household choice rule, which take place whenever an incentive compatibility constraint binds. That allocations are sensitive to outside option heterogeneity is clear from the family maximisation problem; $\underline{u}_t^{h,m}$ appears explicitly in the household objective function. However, the interaction between discount rate heterogeneity and incentive compatibility is more subtle. Consider a couple who are identical in every respect except for their patience, $\beta_{h,A} < \beta_{h,B}$. The optimal lifecycle plan would see a more present-weighted consumption profile for A than B. Thus in early periods, A would receive a greater relative share of per-period expenditure, and the opposite in later periods. However, without a commitment mechanism, this plan may be infeasible. In some period, as her per-period resource share drops, A could conceivably do better by quitting the household, especially given the low weight she attaches to future marital surpluses. The Pareto weight will then be renegotiated to reemphasise A's preferences in the household allocation problem to prevent her from dissolving the household.

Revealed preference conditions. The no-commitment model implies the existence of a set of mutually exclusive subsets within there is no renegotiation and thus, the same Pareto weight is applied.[11] To introduce the potential for renegotiation into our revealed preference set-up, consider a partition of the set T into Υ mutually exclusive subsets T_τ of the form

$$\mathbf{T} = \{T_1, ..., T_\Upsilon\},$$

with

$$T = \cup_{\tau=1}^{\Upsilon} T_\tau \qquad \text{and} \qquad T_{\tau_s} \cap T_{\tau_t} = \emptyset \qquad \text{if} \qquad \tau_s \neq \tau_t,$$

such that

$$\tau_1 < \tau_2 \text{ implies } t_1 < t_2 \text{ for all } t_1 \in T_{\tau_1} \text{ and } t_2 \in T_{\tau_2}.$$

Each subset represents a distinct "Pareto weight regime", thus $\omega_s^m = \omega_t^m$ for all $s, t \in T_\tau$. Let the Pareto weight in subset T_τ thus be denoted ω_τ^m. We then have the following testability result.

[11] For rich enough data sets we can define these subsets by using information on outside options. In our own application (in Section IV), however, we do not have such information and, therefore, we consider all possible partitions $\mathbf{T}^h$.

Theorem 3 *Given a partition* $\mathbf{T}$*, the set of observations on household* h*,* $S^h = \{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t\in T}$*, can be rationalised by the no-commitment model only if there exist, for all* $t \in T$*, private quantities* $\mathbf{q}_t^{h,A}$*,* $\mathbf{q}_t^{h,B} \in \mathbb{R}_+^N$ *(with* $\mathbf{q}_t^{h,A}+ \mathbf{q}_t^{h,B} = \mathbf{q}_t^h$*), Lindahl prices* $\mathbf{P}_t^{h,A}$*,* $\mathbf{P}_t^{h,B} \in \mathbb{R}_+^N$ *(with* $\mathbf{P}_t^{h,A}+ \mathbf{P}_t^{h,B} = \mathbf{P}_t$*), utility numbers* $u_t^{h,A}$*,* $u_t^{h,B} \in \mathbb{R}$ *and constants* $\beta_{h,A}$*,* $\beta_{h,B} \in (0,1]$ *that, for any* $s,t \in T_\tau$ *(*$\tau \in \{1,...,\Upsilon\}$*), satisfy*

$$\begin{aligned} u_s^{h,A} - u_t^{h,A} &\leq \frac{1}{\beta_{h,A}^{t-1}} \left[\mathbf{p}_t'(\mathbf{q}_s^{h,A} - \mathbf{q}_t^{h,A}) + \mathbf{P}_t^{h,A\prime}(\mathbf{Q}_s^h - \mathbf{Q}_t^h)\right], \\ u_s^{h,B} - u_t^{h,B} &\leq \frac{1}{\beta_{h,B}^{t-1}} \left[\mathbf{p}_t'(\mathbf{q}_s^{h,B} - \mathbf{q}_t^{h,B}) + \mathbf{P}_t^{h,B\prime}(\mathbf{Q}_s^h - \mathbf{Q}_t^h)\right]. \end{aligned}$$

The interpretation is similar to before. In this particular case, innovations in the household's Pareto weight define the partitions of the set T. Thus, within sub-periods T_τ, household h's Pareto weight is constant and choices must satisfy the revealed preference inequalities associated with the full efficiency model (in Theorem 2).

IV Rationalising observed time inconsistency

In this section, we resume our empirical application using the ECPF data. We find that simply allowing for limited intrahousehold heterogeneity in the discount rate allows the behaviour of 97.2% of families to be rationalised without recourse to individual deviations from exponential discounting. Moreover, and more importantly, the predictive success of the model amounts to no less than 56.1%. Given this positive result, we conduct a detailed investigation of the theory-consistent differences in spousal discount rates.

Although the vast majority of household behaviour can be explained without any mention of intrahousehold renegotiation, we provide results for a strengthened version of the conditions in Theorem 3, which assumes equal discount factors for the individual household members A and B within a particular household h, $\beta_{h,A} = \beta_{h,B}$. We consider this strengthened version of Theorem 3 to allow for an assessment of the relative importance of time preference heterogeneity and renegotiation in generating observed time inconsistency. Our results suggest that discount rate heterogeneity is the more relevant channel for explaining patterns of household choice in our sample.

IV.A Test results

We first discuss some specific methodological issues related to testing and computing discriminatory power of the behavioural models that we consider here. Subsequently, we present our results for the full efficiency model, and find that this model provides a good empirical fit of the household behaviour in our sample. Finally, we turn to the renegotiation model, and we conclude that the empirical support for this model is much weaker than that for the full efficiency model.

Testing procedure. Our empirical metrics (pass rate, power and predictive success) have the same interpretations as in Section II but our testing procedure is slightly modified from the one that we used previously to account for the collective nature of choice. In particular, we test the conditions in Theorem 2 using a two-dimensional grid search for each household h over individual discount factors $\beta_{h,A}$ and $\beta_{h,B}$. This grid search is again defined on $[0.9, 1]^2$, with a spacing of 0.005. To test the conditions in Theorem 3 (with $\beta_{h,A} = \beta_{h,B}$), we consider alternative scenarios defined by the maximum number of renegotiations that are permissible in the one-year period that a household is observed: this maximum can range from 0 (i.e. time consistent behaviour) to 3 (i.e. the Pareto weight changes in each different consumption quarter).

Our main focus will be on the predictive success of the alternative behavioural models under evaluation. As in Section II.B, we compute the power of our tests for each household by conducting a grid search at each of 1000 iterations of random choices that exhaust the total budget for full period of consideration. To account for the time preferences revealed by a household's observed behaviour, we again define a household specific grid size depending on whether the observed household choices are rationalisable by the model under study. In particular, for the full efficiency model, if household h's observed behaviour is not rationalisable by the model, then we define the grid size as in our basic testing procedure (i.e. the interval $[0.9, 1]^2$ with a spacing of 0.005). By contrast, if observed behaviour can be rationalised, we incorporate this information by searching only over $(\beta_{h,A}, \beta_{h,B})$ with the difference $(\beta_{h,A} - \beta_{h,B})$ not exceeding the minimum difference under which household h's observed behaviour is rationalisable.[12] For the power calculation of the renegotiation model, we proceed exactly as in Section II.B: for nonrationalisable behaviour, we consider $\beta_{h,A} = \beta_{h,B}$ in the interval $[0.9, 1]$, with a spacing of 0.005; and for rationalisable behaviour, our finer grid contains all $\beta_{h,A} = \beta_{h,B}$ that are not situated below the maximum value that can rationalise the observed behaviour.

Full efficiency model. Table 4 presents summary results for the full efficiency model, which allows for β-heterogeneity within the household but imposes a single Pareto weight for the period of consideration. Figure 2 gives the distribution of predictive success for couples in our sample. The results on the full efficiency model stand in stark contrast to those reported in Section II for the time consistency model. We can explain the behaviour of the overwhelming majority of households using the framework of this extremely simple intertemporal collective model, under which the only source of time inconsistency in household revealed preferences is variation in the time preferences of family members. No further recourse to nonstationarities at the individual level is required.

We find this result surprising given the strong assumptions that the theoretical framework incorporates. However, we cannot reject the hypothesis that these assumptions are valid in the short term. For 97.2% of households (2025 out of 2083), we are able to find a well-behaved felicity function and a constant discount rate for each family member that provide a perfect within-sample rationalisation of their choice behaviour.[13] Moreover, and more importantly, the predictive success of 0.57 is a significant improvement over the time consistent model, showing that the full efficiency model has much explanatory power for this relatively short panel.[14] In fact, as is clear from Figure 2, approximately 75

[12]In Section IV.B we explain our procedure to recover this minimum difference $(\beta^h_A - \beta^h_B)$ that is consistent with rationalisability.

[13]Here it is worth to remark that Mazzocco (2007) actually rejected the full efficiency model in his empirical application. Our findings suggest that this rejection could be the result of biases introduced by misspecification, omitted relevant distribution factors or the synthetic nature of the panel used, rather than a failure of commitment itself.

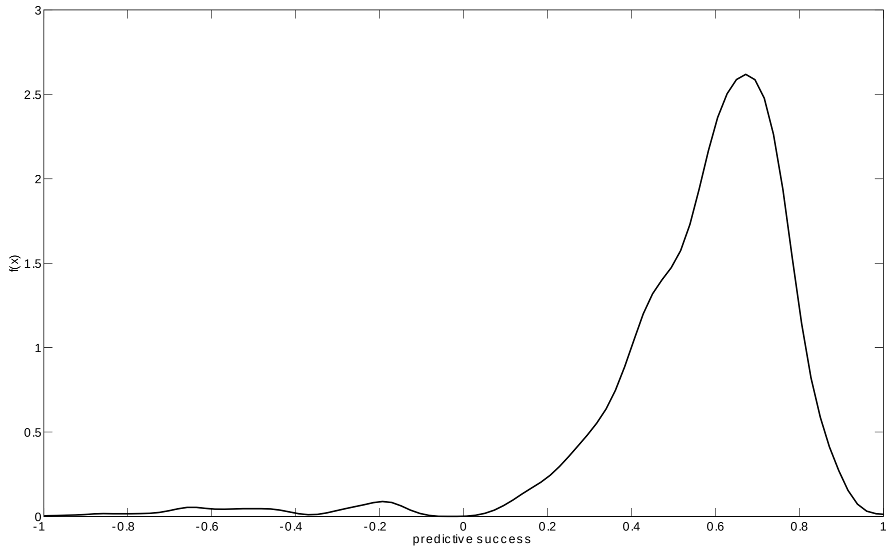
[14]It is not a problem that the predictive success of the full efficiency model for couples exceeds that of the time consistency model for singles. Although both models assume time consistency at the individual level, the full efficiency model is tested on aggregate household data and we are unable to perfectly assign

% of the data has a predictive succes rate exceeding 0.5 and the distribution is unimodal, indicating that the empirical performance of this model is exceptional.

Table 4: Full efficiency and β-heterogeneity

	pass rate	*power*	*predictive success*
mean	0.9722	0.5937	0.5659
(*st. error*)	(0.0036)	(0.0033)	(0.0050)

Figure 2: Predictive success distribution for full efficiency model with β-heterogeneity



No-commitment model. Table 5 reports the results for the model that admits renegotiation but assumes β-homogeneity. Unsurprisingly, allowing for more frequent renegotiations of the Pareto weight is associated with an increase in the pass rate.[15] For the extreme scenario that allows innovations in the Pareto weight between any two consecutive periods, the pass rate amounts to 98.4%, which is almost identical to the pass rate of the full efficiency model. However, and importantly, once we correct for how demanding the test is, by taking into account the discriminatory power, the average predictive success of the no-commitment model with homogeneous discount rates falls below zero,

consumption to individual family members. However, in the case of single people, we perfectly observe their consumption. The data environments under which the two models are tested would be equivalent only if all consumption within multi-person households were assignable. The fact that this is not the case contributes to the high pass rate associated with the full efficiency model. Surprisingly, the power of the full efficiency model is not much decreased, which leads to the much higher predictive success score in this case.

[15] This is unsurprising since increasing the number of (possible) renegotiations generally obtains less stringent rationalisability conditions.

indicating that the model is not informative for household choice. As soon as we require a stable Pareto weight over two periods or more, the pass rate drops steadily, and the predictive success always remains negative and close to zero.

We conclude that accounting for the collective nature of household choice allows the intertemporal behaviour of families in our sample to be explained using simple models that assume constant discounting at the individual level. For the given data set, our results provide particularly strong empirical support for a model which locates the primary source of time inconsistent family behaviour with intrahousehold β-heterogeneity. The full efficiency model seems plausible given the short time span of our sample.

Table 5: Renegotiation and β-homogeneity

renegotiations (max.)		*pass rate*	*power*	*predictive success*
0	*mean*	0.1402	0.8490	-0.0108
	(*st. error*)	(0.0076)	(0.0008)	(0.0082)
1	*mean*	0.6279	0.3347	-0.0374
	(*st. error*)	(0.0106)	(0.0019)	(0.0104)
2	*mean*	0.9073	0.0696	-0.0231
	(*st. error*)	(0.0064)	(0.0008)	(0.0063)
3	*mean*	0.9842	0.0130	-0.0028
	(*st. error*)	(0.0027)	(0.0002)	(0.0027)

IV.B Time preference recovery

The analysis above suggests that the full efficiency model, which allows for time preference heterogeneity, performs well for the data at hand. Given this implied importance of discount rate heterogeneity in accounting for household consumption behaviour, we now explore the nature of the theory-consistent set of household discount rates, and consider whether the necessary degree of unobservable preference heterogeneity correlates with observable family characteristics.

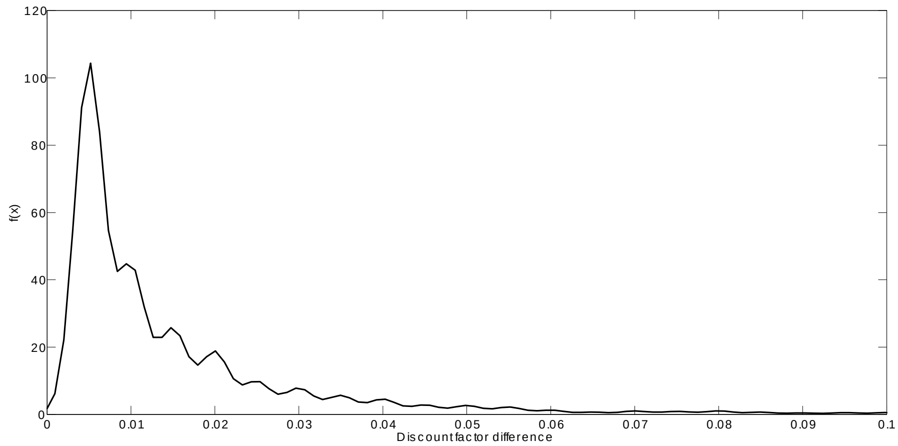
Minimum heterogeneity. For each household that can be rationalised by the full efficiency model, we recover the discount rates that make observed consumption behaviour consistent with the rationalisability conditions in Theorem 2. One drawback of our revealed preference methodology is that the identification of these discount rates is necessarily weakened by the lack of structure our framework imposes on individual preferences and the household choice problem. Our recovery problem is thus underdetermined and preferences are only set identified in the sense of Manski (2007). We refer to this set of potential time preferences as the set of “theory consistent discount rates”.

We deal with this non-uniqueness by reporting results for the *minimal* amount of discount rate heterogeneity that is necessary to rationalise the household consumption stream. To determine the minimum difference in household h's discount factors, $(\beta_{h,A} - \beta_{h,B})$, over the discount factor grid, we iterate the testing procedure by considering the allowable grid points in a specific order. We initially set $\beta_{h,A} - \beta_{h,B} = 0$, which corresponds to the time consistent model, and thus is rejected for about 86% of the households in our sample. For the remaining households, we ask if behaviour can be rationalised for $\beta_{h,A} - \beta_{h,B} = 0.005$, by testing for a non-empty set to the inequalities defined by Theorem 2 with $\beta_{h,A} \in [1, 0.995, ..., 0.905]$ and $\beta_{h,B} = \beta_{h,A} - 0.005$. For the remaining non-rationalisable households, the difference is then set at 0.01, and so on until the maximum difference is reached when $\beta_{h,A} = 1$ and $\beta_{h,B} = 0.90$.

We are able to assume that $\beta_{h,A} \geq \beta_{h,B}$ without loss of generality because the data set does not contain assignable goods, whose consumption can be tied to a particular household member. Therefore, the revealed preference conditions for $m = \{A,B\}$ are fully symmetric. Sadly this implies that we cannot test for whether the husband or wife is the more patient household member, and we choose not to make a prior assumption on this dimension. If assignable information were available, analysis using our methodology could be easily extended to address gender-related questions.

The distribution of the minimum discount factor heterogeneity for all 2025 rationalisable is shown in Figure 3. Only limited heterogeneity is required to rationalise the behaviour of most households. About three quarters of the households that are consistent with Theorem 2 only require $\beta_A - \beta_B \leq 0.020$ to rationalise their behaviour. The kernel plot in Figure 3 confirms that the distribution of minimal heterogeneity shows a high and relatively narrow peak for limited amounts of within-household heterogeneity in time preferences. We also note that the maximum allowable heterogeneity level of 0.10 is observed only for a very small fraction of rationalisable households.

Figure 3: Distribution of β-heterogeneity



Appeal to observed heterogeneity. We investigate whether patterns in minimum unobservable time preference heterogeneity ($\beta_{h,A} - \beta_{h,B}$) are related to observable heterogeneity via a simple regression of the log of the minimum discount rate distance on recorded household characteristics.[16] The ECPF contains information on a number of household characteristics, including the age of each spouse, schooling and employment status of the head, employment status of the wife ("wife working"), job skill level of both spouses, presence and number of children in the household, and housing tenure. The "husband skilled job" dummy equals 1 if the husband is a specialized worker but his wife is not, with analogous interpretations for the "wife skilled job" and "both skilled job" dummies. We also consider the level of the log of total consumption expenditures as independent variables.

We note at the outset that the aim of this reduced form analysis is not to lay bare structural relations between discount rate heterogeneity and household characteristics. We mainly intend to conduct an -admittedly- fairly rudimentary exploratory exercise. One significant drawback of our analysis is that we are unable to investigate gender differences in time preferences because we lack assignable consumption information in our sample. This makes it impossible to know the identity of members A and B without

[16] As we are interested in the direction of correlation between particular variables and the difference in discount rates, rather than the precise magnitude of the coefficient, we consider the dependent variable $\log(0.1 + \beta_{h,A} - \beta_{h,B})$ to account for the skew in the distribution of differences, while keeping those households who could be rationalised with $\beta_{h,A} = \beta_{h,B}$ in our sample.

invoking additional heroic assumptions. Furthermore, it should be noted that we only consider one element of the set of theory-consistent discount rates in our regression (that giving rise to the minimal divergence between spousal discount rates). These factors help to explain why the R-squared of our regression is very low.

Despite the limitations of our analysis, we are still able to provide some informal evidence that relates our results to the recent literature on marriage matching and female bargaining power. For example, Chiappori and Reny (2005) consider marriage matching in the presence of heterogeneous risk preferences.[17] In their model, marriage acts as a device to share exogenous income risk. In their unique stable matching equilibrium, there is negative assortative matching on the risk preferences, i.e. more risk averse men will always be matched with less risk averse women. However, there is no reason to expect negative assortative matching on time preferences. In related literature, match quality is closely aligned to similarity in time preferences. For example, Schaner (2014) classes couples as either "well" or "badly" matched purely on the basis of differences in their elicited discount rates. Moreover, an individual's level of patience (measured by the discount factor) is likely to be correlated with other personal characteristics such as education level. The couples in our sample have already self-selected into marriage, presumably because of a high match quality, making it more likely that these couples will also have similar time preferences, i.e. $\left|\beta_{h,A} - \beta_{h,B}\right|$ will be small. Actually, our empirical results seem to confirm this intuition, as they suggest that little intrahousehold heterogeneity is required to rationalise the consumption behaviour in our sample. In fact, this also makes that there is not much heterogeneity left to explain in our OLS regression. However, as we discuss below, we do find some significant results that are noteworthy.

[17] In a related study, Dupuy and Galichon (2014) investigate the empirical relationship between sorting on the marriage market and individual risk attitudes in interaction with other personal characteristics.

Table 6: β-heterogeneity and household characteristics

$\log(0.1 + \beta_A - \beta_B)$	*coefficient*	*st. err.*
Young household head (dummy)	0.016*	0.007
Age difference	0.001	0.001
Children (dummy)	-0.002	0.007
High school degree	-0.011*	0.004
Log expenditures (logexp)	0.002	0.015
Homeowner (dummy)	0.010	0.007
Wife working	0.016**	0.007
Husband skilled job	-0.001	0.007
Wife skilled job	-0.004	0.016
Both skilled job	-0.004	0.021
R-squared	0.086	

* = significant at 5%-level, ** = significant at 10%-level.

Table 6 shows the regression results. We find that intrahousehold heterogeneity in discount factors correlates with some household characteristics: the age of the household head (captured by a dummy, equal to one if the household head is younger than the median age of household heads), the education of the household head (captured by the "high school degree" dummy) and the employment status of the wife.

First let us consider possible rationalisations of the inverse relationship between age of the household head and time preference heterogeneity. We hypothesise that a positive association between age and match quality generates this correlation. We see two alternative explanations for the positive association between the age of the household head and match quality. First, the older people are, the more likely it is for them to have met and married someone similar to them. Second, only well-matched couples stay together in the long term. Before considering these explanations in more detail, we must first comment on the place of divorce in our framework. Our empirical test assumes an invariant intrahousehold decision making rule for the four quarters a family is observed. However, this is not to say that renegotiation and divorce cannot occur in the long run. Clearly, some couples do divorce in reality but the probability of divorce is declining in match quality (Becker, Landes and Michael, 1977; Weiss and Willis, 1997). The higher the quality of a match, the more marital surplus is available to be shared by a couple and the less likely it is for marriage dissolution to dominate for either spouse in any time period.

Now, on the first proffered explanation for the association between match quality and age, consider the search process leading to marriage. If this process is costly, individuals will accept imperfect matches even if it is known that differences in time preferences will create inefficiencies in the new household (see Burdett and Coles (1999) for a formal framework). Over time, if some search continues during marriage, individuals will continue to acquire new information on other potential matches. If a spouse meets a high enough quality match, it can be worth dissolving an existing marriage to take up a new opportunity. The older someone is, the longer they will have been a participant in the marriage market and therefore, the more likely it is that they will have met someone similar to themselves and thus be a member of a "high quality" match. In this way, costly search can create a negative association between age and discount rate heterogeneity.

Alternatively, on the second explanation, it is not unreasonable to assume that older couples have been married longer. Some model match quality as an experience good (Nelson 1970; Jovanovic 1979; Weiss and Willis 1997; Chiappori and Weiss 2006). In contrast to the above, these explanations assume that one cannot perfectly assess the suitability of a potential mate until marriage occurs and a match is experienced. Given that young couples have lower marriage experience, one can expect a greater variation in match quality (i.e. heterogeneity in discount rates) amongst this group, as these couples are still assessing the degree to which they are suited and not all poor matches will have been terminated. However, only well suited couples will remain married for a long period of time, again creating a negative association between the age of a couple and discount rate heterogeneity.

The household head having a high school degree and the wife *not* working are also associated with smaller heterogeneity in time preferences. If one is willing to make the strong assumption that the husband has a lower discount factor than the wife, then we can label the wife as member A and the husband as B, since our dependent variable $\beta_{h,A} - \beta_{h,B}$ is defined to be (weakly) positive.[18] Additionally, one may posit that more

[18] In the literature, it is often found that males are less patient than females, supporting our assumption; see for example Kirby and Maracovic (1996), Warner and Pleeter (2001), Ubfal (2013), and Dittrich and Leipold (2014). This might be the case because men have a shorter life expectancy than women, making them less patient on average, all else equal.

patient individuals are more likely to invest more in their education (i.e. to have a high school degree) and also therefore more likely to be employed later in life. Hence, all else equal, a husband (defined to be the household head for our sample) with a high school degree will have a higher patience level $\beta_{h,B}$, leading to a lower level of time preference heterogeneity. Similarly, a working wife will have a higher $\beta_{h,A}$, causing a higher level of time preference heterogeneity.

As a different explanation, working wives may have more bargaining power or better outside options, which makes it more likely that their participation constraint becomes binding at some point, so creating a need to renegotiate the intra-household Pareto weights. As a result, because our full efficiency model excludes such renegotiation, it might be harder to satisfy its empirical restrictions and, thus, a larger set of grid points $(\beta_{h,A}, \beta_{h,B})$ needs to be considered in order to rationalise the data. Since our dependent variable $(\beta_{h,A} - \beta_{h,B})$ is defined as the minimum required discount rate heterogeneity needed to explain the data, the positive coefficient on the "working wife" dummy would then be a mechanical result of our testing procedure.

Follow-up research, potentially using richer data sets, may focus on better explaining these patterns. For example, as indicated above, data sets that include information on assignable goods can allow for more robust identification of gender-specific effects, which seem to be relevant here.

V Conclusion

We have provided a revealed preference analysis of time inconsistencies in household consumption. Adopting a collective perspective, we focused on rationalising these inconsistencies as the product of individual time preference heterogeneity and renegotiation within the household unit. An empirical application to a Spanish consumption panel highlights that an explicit recognition of the collective nature of choice allows us to rationalise the vast majority of time inconsistent household behaviour. Almost all observed household behaviour turns out to be consistent with a simple model that assumes perfect intrahousehold commitment (i.e. no renegotiation) in the one-year period under consideration together with exponential discounting at the individual level, so long as time preferences heterogeneity is allowed for.

We have also shown that revealed preference restrictions can be fruitfully applied to recover individual time preferences. One can then relate these results to specific individual or household characteristics. For example, for our application we found that the intrahousehold heterogeneity in discount factors correlates with the age and educational level of the household head and the wife's employment status. This application demonstrates the potential usefulness of revealed preference methodology to address this type of questions in an effective manner. It is our belief that richer household data sets (e.g. including assignable goods and more information on observable characteristics) may yield additional and more refined insights. In principle, for long enough panels with detailed information, our framework also enables one to investigate how variation in specific (individual and household) characteristics relates to patterns of intrahousehold renegotiation.

References

[1] Adams, Abi. 2011. "Household Intertemporal Choice: A Nonparametric Characterisation". Mimeo.

[2] Afriat, Sydney. 1967. "The Construction of Utility Functions from Expenditure Data". *International Economic Review* 8(1): 67-77.

[3] Andersen, Steffen, Glenn W. Harrison, Arne Risa Hole, Morten Lau and E. Elisabet Rutström. 2012. "Non-linear Mixed Logit". *Theory and Decision* 73(1): 77-96.

[4] Andreoni, James and Charles Sprenger. 2012. "Estimating Time Preferences from Convex Budget Sets". *American Economic Review* 102(7): 3333-3356.

[5] Angeletos, George-Mario, David Laibson, Andrea Repetto, Jeremy Tobacman and Stephen Weinberg. 2001. "The Hyperbolic Consumption Model: Calibration, Simulation and Empirical Evaluation". *Journal of Economic Perspectives* 15(3): 47-68.

[6] Beatty, Timothy K. and Ian Crawford. 2011. "How Demanding is the Revealed Preference Approach to Demand?". *American Economic Review* 101: 2782-2795.

[7] Becker, Gary. 1962. "Irrational Behavior and Economic Theory". *Journal of Political Economy* 70: 1-13.

[8] Becker, Gary, Elizabeth Landes and Robert Michael. 1977. "An Economic Analysis of Marital Instability". *Journal of Political Economy* 85: 1141-1187.

[9] Bronars, Stephen G.. 1987. "The Power of Nonparametric Tests of Preference Maximization". *Econometrica* 55: 693-698.

[10] Browning, Martin. 1989. "A Nonparametric Test of the Life-Cycle Rational Expectations Hypothesis". *International Economic Review* 30(4): 979-992.

[11] Browning, Martin, Francois Bourguignon, Pierre-Andre Chiappori and Valerie Lechene. 1994. "Income and Outcomes: A Structural Model of Intrahousehold Allocation". *Journal of Political Economy* 102: 1067-1096.

[12] Browning, Martin and Pierre Andre Chiappori. 1998. "Efficient Intra-household Allocations: A General Characterisation and Empirical Tests". *Econometrica* 66(6): 1241-1278.

[13] Burdett, Kenneth and Melvyn Coles. 1999. "Long Term Partnership Formation: Marriage and Employment". *Economic Journal* 109: 307-334.

[14] Cherchye, Laurens, Bram De Rock and Frederic Vermeulen. 2007. "The Collective Model of Household Consumption: A Nonparametric Characterization". *Econometrica* 75(2): 553-574.

[15] Cherchye, Laurens, Bram De Rock and Frederic Vermeulen. 2009. "Opening the Black Box of Intra-household Decision-making: Theory and Nonparametric Empirical Tests of General Collective Consumption Models". *Journal of Political Economy* 117: 1074-1104.

[16] Chiappori, Pierre-André. 1988. "Rational Household Labor Supply". *Econometrica* 56: 63-89.

[17] Chiappori, Pierre-André. 1992. "Collective Labor Supply and Welfare". *Journal of Political Economy* 100: 437-467.

[18] Chiappori, Pierre-André and Philip J. Reny. 2005. "Matching to Share Risk". Mimeo.

[19] Chiappori, Pierre-André and Yoram Weiss. 2006. "Divorce, Remarriage and Welfare: A General Equilibrium Approach". *Journal of the European Economic Association, Papers and Proceedings of the Twentieth Annual Congress of the European Economic Association* 4(2/3): 415-426.

[20] Choi, James, David Laibson, Brigitte Madrian and Andrew Metrick. 2001. "For Better or for Worse: Default Effects and 401(k) Savings Behavior". In *Perspectives on the Economics of Ageing*, ed. by David Wise, University of Chicago Press for NBER.

[21] Crawford, Ian. 2010. "Habits Revealed". *Review of Economic Studies* 77(4): 1382-1402.

[22] DellaVigna, Stefano and Ulrike Malmendier. 2006. "Paying Not To Go To The Gym". *American Economic Review* 96: 694-719.

[23] Diewert, Erwin. 1973. "Afriat and Revealed Preference Theory". *Review of Economic Studies* 40(3): 419-425.

[24] Dittrich, Marcus and Kristina Leipold. 2014. "Gender Differences in Time Preferences". *Economics Letters* 122: 413-415.

[25] Dohman, Thomas, Armin Falk, David Huffman and Uwe Sunde. 2012. "Interpreting Time Horizon Effects in Inter-Temporal Choice". CESifo Working Paper No. 3750.

[26] Dupuy, Arnaud and Alfred Galichon. 2014. "Personality Traits and the Marriage Market". *Journal of Political Economy* forthcoming.

[27] Feldstein, Martin S.. 1964. "The Social Time Preference Discount Rate in Cost Benefitt Analysis". *Economic Journal* 74: 360-379.

[28] Frederick, Shane George Loewenstein and Ted O'Donoghue. 2002. "Time Discounting and Time Preference: A Critical Review". *Journal of Economic Literature* 40 (2): 351-401.

[29] Gruber, Jonathan and Botond Koszegi. 2000. "Is Addiction 'Rational?' Theory and Evidence". *Quarterly Journal of Economics* 116: 1261-1305.

[30] Jackson, Matthew and Leeat Yariv. 2012. "Collective Dynamic Choice: The Necessity of Time Inconsistency". Mimeo.

[31] Jovanovic, Boyan. 1979. "Job Matching and the Theory of Turnover". *Journal of Political Economy* 87(5): 972-990.

[32] Kirby, Kris. N. and Nino M. Marakovic. 1996. "Delay-discounting Probabilistic Rewards: Rates decrease as amounts increase". *Psychonomic Bulletin and Review* 3(1): 100-104.

[33] Koopmans, Tjalling. 1960. "Stationary Ordinal Utility and Impatience". *Econometrica* 28: 287-309.

[34] Laibson, David. 1997. "Golden Eggs and Hyperbolic Discounting". *Quarterly Journal of Economics* 112: p443-477.

[35] Laibson, David. 1998. "Life-Cycle Consumption and Hyperbolic Discount Functions". *European Economic Review* 42: 861-871.

[36] Marcet, Albert and Ramon Marimon. 1998. "Recursive Contracts". Mimeo, Universitat Pompeu Fabra.

[37] Marglin, Stephen. 1963. "The Social Rate of Discount and The Optimal Rate of Investment". *Quarterly Journal of Economics* 77(1): 95-111.

[38] Manski, Charles. 2007. *Identification for Prediction and Decision.* Cambridge, MA: Harvard University Press.

[39] Mazzocco, Maurizio. 2007. "Household Intertemporal Behaviour: A Collective Characterization and a Test of Commitment". *Review of Economic Studies* 74(3): 857-895.

[40] Nelson, Phillip. 1970. "Information and Consumer Behavior". *Journal of Political Economy* 78(2): 311-329.

[41] O'Donoghue, Ted and Matthew Rabin. 1999. "Doing It Now or Later". *American Economic Review* 89(1): 103-124.

[42] O'Donoghue, Ted and Matthew Rabin. 2001. "Choice and Procrastination". *Quarterly Journal of Economics* 116(1): 121-160.

[43] Popper, Karl. 1959. *The Logic of Scientific Discovery.* London: Hutchinson and Company.

[44] Samuelson, Paul. 1937. "A Note on the Measurement of Utility". *Review of Economic Studies* 4: 155-161.

[45] Schaner, Simone G.. 2014. "Do Opposites Detract? Intrahousehold Preference Heterogeneity and Inefficient Strategic Savings". *American Economic Journal: Applied Economics* forthcoming.

[46] Selten, Reinhard. 1991. "Properties of a Measure of Predictive Success". *Mathematical Social Sciences* 21: 153-167.

[47] Shaked, Avner and John Sutton. 1984. "Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model". *Econometrica* 52(6): 1351-1364.

[48] Thaler, Richard and Shlomo Benartzi. 2004. "Save More Tomorrow: Using Behavioral Economics to Increase Employee Saving". *Journal of Political Economy* 112(1): 164-187.

[49] Ubfal, Diego. 2013. "How General Are Time Preferences? Eliciting Good-Specific Discount Rates". Working Papers 473, Bocconi University.

[50] Varian, Hal. 1982. "The Nonparametric Approach to Demand Analysis". *Econometrica* 50: 945-973.

[51] Varian, Hal. 1985. "Nonparametric Analysis of Optimizing Behavior with Measurement Error". *Journal of Econometrics* 30: 445-458.

[52] Warner, John T. and Saul Pleeter. 2001. "The Personal Discount Rate: Evidence From Military Downsizing Programs". *American Economic Review* 91(1): 33-53.

[53] Weiss, Yoram and Robert Willis. 1997. "Match Quality, New Information and Martial Dissolution". *Journal of Labor Economics* 15(1): 293-329.

[54] Stephane Zuber. 2011. "The Aggregation of Preferences: Can We Ignore the Past?". *Theory and Decision* 70: 367-384.

Appendix A: Proofs

This appendix gives the proofs of our Theorems 1-3.

Proof of Theorem 1

(Necessity) Suppose the set of observations S^h can be rationalised by the.time consistency model. Let η denote the Lagrange multiplier associated with the household budget constraint. We get the following first order constraints for the household optimisation problem (with $\frac{\partial u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h)}{\partial \mathbf{q}_t^{h,m}}$ $(m \in \{A,B\})$ and $\frac{\partial u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h)}{\partial \mathbf{Q}_t^h}$ the subgradients for the function u^h at $(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h)$):

$$\begin{aligned}
\beta_h^{t-1}\frac{\partial u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h)}{\partial \mathbf{q}_t^{h,m}} &\leq \eta \mathbf{p}_t \ (m \in \{A,B\}) \text{ and} \\
\beta_h^{t-1}\frac{\partial u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h)}{\partial \mathbf{Q}_t^h} &\leq \eta \mathbf{P}_t.
\end{aligned}$$

Under concavity of the function u^h, we have

$$\begin{aligned}
u^h(\mathbf{q}_s^{h,A},\mathbf{q}_s^{h,B},\mathbf{Q}_s^h) - u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h) \leq\ & \left(\frac{\partial u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h)}{\partial \mathbf{q}_t^{h,A}}\right)'(\mathbf{q}_s^{h,A}-\mathbf{q}_t^{h,A}) + \\
& \left(\frac{\partial u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h)}{\partial \mathbf{q}_t^{h,B}}\right)'(\mathbf{q}_s^{h,B}-\mathbf{q}_t^{h,B}) + \\
& \left(\frac{\partial u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h)}{\partial \mathbf{Q}_t^h}\right)'(\mathbf{Q}_s^h-\mathbf{Q}_t^h).
\end{aligned}$$

Combining the different inequalities leads to

$$u^h(\mathbf{q}_s^{h,A},\mathbf{q}_s^{h,B},\mathbf{Q}_s^h) - u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h) \leq \frac{\eta}{\beta_h^{t-1}}\left[\mathbf{p}_t'(\mathbf{q}_s^h-\mathbf{q}_t^h)+\mathbf{P}_t'(\mathbf{Q}_s^h-\mathbf{Q}_t^h)\right].$$

By using $\frac{u^h(\mathbf{q}_s^{h,A},\mathbf{q}_s^{h,B},\mathbf{Q}_s^h)}{\eta} = u_s^h$, we obtain the inequalities in Theorem 1.

(Sufficiency) Suppose the inequalities in Theorem 1 hold. Then, define the following felicity function:

$$u^h(\mathbf{x}^A,\mathbf{x}^B,\mathbf{X}) = \min_s\left(u_s^h + \frac{1}{\beta_h^{s-1}}[\mathbf{p}_s'(\mathbf{x}^A-\mathbf{q}_s^{h,A})+\mathbf{p}_s'(\mathbf{x}^B-\mathbf{q}_s^{h,B})+\mathbf{P}_s'(\mathbf{X}-\mathbf{Q}_s^h)]\right).$$

Using a straightforwardly similar argument as Varian (1982), we can derive $u^h(\mathbf{q}_t^{h,A},\mathbf{q}_t^{h,B},\mathbf{Q}_t^h) = u_t^h$.

Consider any consumption plan $\{\mathbf{x}_t^A, \mathbf{x}_t^B, \mathbf{X}_t\}_{t\in T}$ such that

$$\sum_{t\in T}\left[\mathbf{p}_t'(\mathbf{x}_t^A - \mathbf{q}_t^{h,A}) + \mathbf{p}_t'(\mathbf{x}_t^B - \mathbf{q}_t^{h,B}) + \mathbf{P}_t'(\mathbf{X}_t - \mathbf{Q}_t^h)\right] \leq 0,$$

i.e. the consumption plan $\{\mathbf{x}_t^A, \mathbf{x}_t^B, \mathbf{X}_t\}_{t\in T}$ is affordable given the outlay associated with $\{\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h\}_{t\in T}$. Then, we need to show that

$$\sum_{t\in T}\beta_h^{t-1}u^h(\mathbf{x}_t^A, \mathbf{x}_t^B, \mathbf{X}_t) \leq \sum_{t\in T}\beta_h^{t-1}u^h(\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h).$$

Using the utility function defined above, we obtain

$$\begin{aligned}
& \sum_{t\in T}\beta_h^{t-1}u^h(\mathbf{x}_t^A, \mathbf{x}_t^B, \mathbf{X}_t) \\
\leq\ & \sum_{t\in T}\beta_h^{t-1}\left(u_t^h + \frac{1}{\beta_h^{t-1}}[\mathbf{p}_t'(\mathbf{x}_t^A - \mathbf{q}_t^{h,A}) + \mathbf{p}_t'(\mathbf{x}_t^B - \mathbf{q}_t^{h,B}) + \mathbf{P}_t'(\mathbf{X}_t - \mathbf{Q}_t^h)]\right) \\
=\ & \sum_{t\in T}\beta_h^{t-1}u_t^h + \sum_{t\in T}[\mathbf{p}_t'(\mathbf{x}_t^A - \mathbf{q}_t^{h,A}) + \mathbf{p}_t'(\mathbf{x}_t^B - \mathbf{q}_t^{h,B}) + \mathbf{P}_t'(\mathbf{X}_t - \mathbf{Q}_t^h)] \\
\leq\ & \sum_{t\in T}\beta_h^{t-1}u_t^h,
\end{aligned}$$

so that $u^h(\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h) = u_t^h$ gives the wanted conclusion.

Proof of Theorem 2

(Necessity) Suppose the set of observations S^h can be rationalised by the full efficiency model. Let η denote the Lagrange multiplier associated with the household budget constraint. We get the following first order constraints (with $\frac{\partial u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h)}{\partial \mathbf{q}_t^{h,m}}$ and $\frac{\partial u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h)}{\partial \mathbf{Q}_t^h}$ ($m \in \{A, B\}$) the subgradients for the function $u^{h,m}$ at bundle $(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h)$):

$$\begin{aligned}
\beta_{h,A}^{t-1}\frac{\partial u^{h,A}(\mathbf{q}_t^{h,A}, \mathbf{Q}_t^h)}{\partial \mathbf{q}_t^{h,A}} &\leq \eta\mathbf{p}_t, \\
\omega^h\beta_{h,B}^{t-1}\frac{\partial u^{h,B}(\mathbf{q}_t^{h,B}, \mathbf{Q}_t^h)}{\partial \mathbf{q}_t^{h,B}} &\leq \eta\mathbf{p}_t, \\
\beta_{h,A}^{t-1}\frac{\partial u^{h,A}(\mathbf{q}_t^{h,A}, \mathbf{Q}_t^h)}{\partial \mathbf{Q}_t^h} + \omega^h\beta_{h,B}^{t-1}\frac{\partial u^{h,B}(\mathbf{q}_t^{h,B}, \mathbf{Q}_t^h)}{\partial \mathbf{Q}_t^h} &\leq \eta\mathbf{P}_t.
\end{aligned}$$

Under concavity of each felicity functions $u^{h,m}$, we have

$$u^{h,m}(\mathbf{q}_s^{h,m}, \mathbf{Q}_s^h) - u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h) \leq \left(\frac{\partial u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h)}{\partial \mathbf{q}_t^{h,m}} \right)' (\mathbf{q}_s^{h,m} - \mathbf{q}_t^{h,m}) + \left(\frac{\partial u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h)}{\partial \mathbf{Q}_t^h} \right)' (\mathbf{Q}_s^h - \mathbf{Q}_t^h).$$

Now define, for each $t \in T$,

$$\lambda^{h,A} = \eta, \; \lambda^{h,B} = \frac{\eta}{\omega^h},$$

$$\mathbf{P}_t^{h,A} = \frac{\beta_{h,A}^{t-1}}{\lambda^{h,A}} \frac{\partial u^{h,A}(\mathbf{q}_t^{h,A}, \mathbf{Q}_t^h)}{\partial \mathbf{Q}_t^h},$$

$$\mathbf{P}_t^{h,B} = \mathbf{P}_t - \mathbf{P}_t^{h,A}.$$

Combination yields

$$u^{h,m}(\mathbf{q}_s^{h,m}, \mathbf{Q}_s^h) - u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h) \leq \frac{\lambda^{h,m}}{\beta_{h,m}^{t-1}} \mathbf{p}_t'(\mathbf{q}_s^{h,m} - \mathbf{q}_t^{h,m}) + \frac{\lambda^{h,m}}{\beta_{h,m}^{t-1}} (\mathbf{P}_t^{h,m})'(\mathbf{Q}_s^h - \mathbf{Q}_t^h).$$

By using $u^{h,m}(\mathbf{q}_s^m, \mathbf{Q}_s)/\lambda^{h,m} = u_s^{h,m}$, we obtain the inequalities in Theorem 2.

(Sufficiency) Suppose the inequalities in Theorem 2 hold. Then, define the following felicity function for each member $m \in \{A, B\}$:

$$u^{h,m}(\mathbf{x}^m, \mathbf{X}) = \min_s \left(u_s^{h,m} + \frac{1}{\beta_{h,m}^{s-1}} [\mathbf{p}_s'(\mathbf{x}^m - \mathbf{q}_s^{h,m}) + \mathbf{P}_s^{h,m\prime}(\mathbf{X} - \mathbf{Q}_s^h)] \right).$$

Using a straightforwardly similar argument as Varian (1982), we can derive $u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h) = u_t^{h,m}$.

Consider any consumption plan $\{\mathbf{x}_t^A, \mathbf{x}_t^B, \mathbf{X}_t\}_{t \in T}$ such that

$$\sum_{t \in T} \left[\mathbf{p}_t'(\mathbf{x}_t^A - \mathbf{q}_t^{h,A}) + \mathbf{p}_t'(\mathbf{x}_t^B - \mathbf{q}_t^{h,B}) + \mathbf{P}_t'(\mathbf{X}_t - \mathbf{Q}_t^h) \right] \leq 0,$$

i.e. the consumption plan $\{\mathbf{x}_t^A, \mathbf{x}_t^B, \mathbf{X}_t\}_{t \in T}$ is affordable given the outlay associated with

$\{\mathbf{q}_t^{h,A}, \mathbf{q}_t^{h,B}, \mathbf{Q}_t^h\}_{t\in T}$. Then, for $\omega^A = 1$ and $\omega^B = \omega^h$ we need to show that

$$\sum_{m\in\{A,B\}} \sum_{t\in T} \omega^m \beta_{h,m}^{t-1} u^{h,m}(\mathbf{x}_t^m, \mathbf{X}_t) \leq \sum_{m\in\{A,B\}} \sum_{t\in T} \omega^m \beta_{h,m}^{t-1} u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h).$$

Without losing generality, we can assume $\omega^m = 1$. As such, we obtain

$$\begin{aligned}
& \sum_{m\in\{A,B\}} \sum_{t\in T} \omega^m \beta_{h,m}^{t-1} u^{h,m}(\mathbf{x}_t^m, \mathbf{X}_t) \\
\leq \quad & \sum_{m\in\{A,B\}} \sum_{t\in T} \omega^m \beta_{h,m}^{t-1} \left(u_t^{h,m} + \frac{1}{\beta_{h,m}^{t-1}} [\mathbf{p}_t'(\mathbf{x}_t^m - \mathbf{q}_t^{h,m}) + \mathbf{P}_t^{h,m\prime}(\mathbf{X}_t - \mathbf{Q}_t^h)] \right) \\
= \quad & \sum_{m\in\{A,B\}} \sum_{t\in T} \omega^m \beta_{h,m}^{t-1} u_t^{h,m} + \sum_{m\in\{A,B\}} \sum_{t\in T} [\mathbf{p}_t'(\mathbf{x}_t^m - \mathbf{q}_t^{h,m}) + \mathbf{P}_t^{h,m\prime}(\mathbf{X}_t - \mathbf{Q}_t^h)] \\
\leq \quad & \sum_{m\in\{A,B\}} \sum_{t\in T} \omega^m \beta_{h,m}^{t-1} u_t^{h,m},
\end{aligned}$$

so that $u^{h,m}(\mathbf{q}_t^{h,m}, \mathbf{Q}_t^h) = u_t^{h,m}$ gives the wanted conclusion.

Proof of Theorem 3

The result uses that, for a given partition $\mathbf{T}$, consistency with the no-commitment model requires that, for each subset T_τ ($\tau \in \{1, ..., \Upsilon\}$), that the corresponding subset of observations $\{\mathbf{q}_t^h, \mathbf{Q}_t^h; \mathbf{p}_t, \mathbf{P}_t\}_{t\in T_\tau}$ can be rationalized by the full efficiency model. Then, the result follows directly from Theorem 2.

Appendix B: Summary statistics

The following table provides summary statics of the data we use in our empirical application. For the different (public and private) goods it gives average expenditure shares and discounted prices over all observations (i.e. 4 observations for 2083 households), together with the corresponding standard deviations.

	Private goods	
	Mean share (st. dev: *overall* / *between* / *within*)	**Mean price** (st. dev: *overall* / *between* / *within*)
Food and non-alcoholic drinks	0.4564 (0.1893 0.1499 0.1156)	1.4330 (0.2684 0.2500 0.9783)
Clothing and footwear	0.1632 (0.1350 0.0809 0.1081)	1.4546 (0.3055 0.2845 0.1086)
Transportation	0.0461 (0.0757 0.0458 0.0604)	1.5647 (0.4406 0.3961 0.1930)
Leisure (cinema, theatre, clubs for sport)	0.0569 (0.0734 0.0501 0.0536)	1.4417 (0.2878 0.2598 0.1239)
Personal services	0.0254 (0.0441 0.0264 0.0353)	1.4359 (0.2964 0.2600 0.1425)
Restaurants and bars	0.1346 (0.1303 0.1001 0.0834)	1.5511 (0.3770 0.3473 0.1469)
	Public goods	
	Mean share (st. dev: *overall* / *between* / *within*)	**Mean price** (st. dev: *overall* / *between* / *within*)
Home Services (heating, water and furniture repair)	0.0455 (0.0658 0.0547 0.0366)	1.3815 (0.2511 0.2304 0.0999)
Petrol	0.0719 (0.0786 0.0575 0.0536)	1.0222 (0.1137 0.1106 0.0263)

The following table records summary statistics on household characteristics considered in the course of our empirical exploration of time preference heterogeneity.

Characteristic	**Mean**	**St. Dev.**
Average age of couple	35.8	8.97
Husband age	37.5	9.25
Wife age	34.2	8.91
High school degree dummy	0.57	0.49
Children dummy	0.79	0.41
Couple + 1 child	0.27	0.46
Couple + 2 children	0.42	0.49
Couple + 3 or more children	0.10	0.30
Home owner dummy	0.72	0.45
Wife working dummy	0.36	0.48
Husband skilled job dummy	0.30	0.46
Wife skilled job dummy	0.08	0.28
Both skilled job dummy	0.05	0.21