Enforcing Import Tariffs (and Other Taxes)

Mirco Tonin

Economics Division,

School of Social Sciences,

University of Southampton,

Southampton SO17 1BJ,

United Kingdom

m.tonin@soton.ac.uk

February 25, 2011

Abstract

This paper derives the implications for compliance and fiscal revenues of a tax base that is the product of several factors. For instance, in the case of import tariffs, the tax base is the product of quantity and unit value, both reported to, and during an audit assessed by, the custom authority. Import tariff are particularly interesting as custom receipts represent an important share of government revenues in many developing countries and there has recently been a surge in empirical studies showing how evasion in this field is a pervasive phenomenon. I show that, with a multiplicative tax base, when the fiscal authority has an imperfect detection technology a greater declaration in one dimension actually increases the fine when evasion in the other dimension is detected. Therefore, there is an additional incentive for the taxpayer to underdeclare and a multiplicative tax base is subject to more evasion, compared to a tax base that can be assessed directly. As a result, fiscal revenues decrease with the dimensionality of the tax base. Also, voluntary compliance and fiscal revenues may be higher when the importer is required to declare only the total value of imports instead of quantity and unit value separately.

This paper provides an argument in favour of uniform or specific tariffs and a reason for why a flat tax may improve compliance.

JEL Codes: F13, H26, H27, K42, O17, O24

Keywords: tariff, tax evasion, multiplicative tax base, imperfect detection, flat tax

1 Introduction

In some cases the tax base is the product of several factors, each of them to be assessed by the tax authority during an audit. The aim of this paper is to derive the implications of this feature for compliance and fiscal revenues. For instance, in the case of import tariffs, the tax base is the product of quantity and unit value and evasion can take place through "underreporting of unit value, underreporting of taxable quantities, and the mislabelling of higher taxed products as lower-taxed products" (Fisman and Wei, 2004). Import tariff are particularly interesting as custom receipts represent an important share of government revenues in many developing countries and there has recently been a surge in empirical studies showing how evasion in this field is a pervasive phenomenon. This paper shows how the fact that the tax base is the product of several factors has important effects on the tariff evasion decision by importers and on government revenues.

The importance of tariff receipts for developing countries is highlighted by the data collected by Baunsgaard and Keen (2010). As reported in Jean and Mitaritonna (2010), "the share of trade tax revenue in total tax receipt over the period 2001-2006 amounted on average to 2.5% in high-income countries, 18.1% in middle-income countries and 22% in low-income countries". Recent empirical studies have found tariff evasion to be widespread. This literature usually exploits discrepancies in trade flows as recorded by the exporting and the importing country and interprets the correlation between these trade gaps and tariff rates as evidence of tariff evasion taking place. Fisman and Wei (2004) look at trade between Hong Kong and China in 1997-1998 and find that a "one-percentage-point increase in the tax rate is associated with a 3 percent increase in evasion". Javorcik and Narciso (2008) use data on trade between Germany and ten Eastern European countries during 1992-2003 and find that a "one-percentage-point increase in the tariff rate is associated with a 0.4% increase in the trade gap in the case of homogeneous products and a 1.7% increase in the case of differentiated products". Mishra et al. (2008) exploit a major tariff reform in India in the 1990s and find a "robust positive elasticity of evasion with respect to tariffs". Jean and Mitaritonna (2010) use a large panel dataset including observations for 75 countries in 2001 and

2004 and find that "evasion of custom duties in larger in poorer countries" and that in the poorest countries "a one percentage point higher tariff is found to be associated on average with import understatement by one percent or more". They also report the results of several studies showing how in many African countries collection of custom duties is very poor (the worst reported case is the Democratic Republic of Congo, with "80% of custom taxes *not* being collected"). Thus, tariff receipts are an important source of revenues for the public finances of developing countries and, at the same time, tariff evasion seems to be particularly significant in these countries.[1] The empirical evidence concerning the relative importance of the different channels through which tariff evasion takes place, i.e. quantity, unit price, misclassification, is rather mixed: Fisman and Wei (2004) find evidence of misclassification and underreporting of unit values, but not of quantities. Javorcik and Narciso (2008) find that evasion takes place through misrepresentation of import prices, but not through misclassification or underreporting of quantities, while Mishra et al. (2008) find evasion in quantities to be significant. Jean and Mitaritonna (2010) find that underreporting of both quantities and unit price are "widespread modalities of custom evasion, with comparable importance". The generally bad quality of quantity data in trade statistics[2] makes the identification of the different channels difficult.

Since the seminal contribution by Allingham and Sandmo (1972), the theoretical analysis of tax evasion has mainly been focused on individual decision makers dealing with personal income tax, with some more recent studies looking at tax noncompliance by businesses (for a review of this vast literature see Andreoni, Erard and Feinstein, 1998; Slemrod and Yitzhaki, 2002; Slemrod, 2007). Generally, the assumption is that the tax base is represented by a number, y, and the taxpayer's report by a possibly different number, x, with evasion being the difference between the two. I will refer to this as the "unitary tax base" case. This way of modelling the tax base is problematic when dealing with tariff evasion given that, as mentioned above, one feature of import tariffs is that the

[1] Evidence of tariff evasion has been found also for developed countries with a low level of corruption: Stoyanov (2009) analyzes trade between Canada and the US in 1989 and find that "tariff rates have a strong and significant effect on the apparent trade gap among the two".

[2] Quantities are often indicated for information only (Jean and Mitaritonna, 2010) and the unit of measurements reported by different statistical agencies may be non-convertible, e.g. units vs. kilograms (Stoyanov, 2009). See also Rozanski and Yeats (1994) for a comprehensive analysis of the reliability of trade statistics.

tax base is the product of quantity and unit value and that evasion can concern each of the two dimensions, plus the misclassification of goods.

To the best of my knowledge, no theoretical analysis of tax evasion when the tax liability is the product of several independently reported variables has been conducted so far. This overlook of the literature may be due to the fact that generally tax evasion has been modelled by assuming perfect detection of the true tax liability in case of an audit by the tax authority. In this case, whether the tax base is unitary or the product of several independently reported variables is indeed irrelevant. I will show that it does however matter under the more plausible assumption that detection in case of an audit is imperfect, i.e. when the tax authority does not discover for sure the true tax liability during an audit.[3,4] In particular, with a standard unitary tax base, a taxpayer decides how much to declare by trading-off the marginal benefit of a higher declaration, namely a reduction in the expected fine, to its marginal cost, namely a higher tax payment. In case of a multiplicative tax base, a greater declaration in one dimension entails an additional cost in that it increases the fine when evasion in *the other* dimension is detected. This additional effect is not present under the usual assumption of perfect detection.

What I show in this paper is that a tax base that is the product of two different parameters is subject to higher underreporting compared to a tax base that can be assessed directly, and that improving compliance along one dimension has a positive feedback on compliance along the other dimension. More generally, I show that the higher the dimensionality of the tax base, the lower the amount of fiscal revenues generated by it. I also show that requiring an unitary declaration of a multiplicative tax base, i.e. requiring the importer to declare only the total value of imports, even if quantity and unit value need to be separately assessed during an audit, may increase voluntary compliance. This is because an unitary declaration mitigates the impact of the additional incentive

[3] The empirical studies by Feinstein (1991) and Erard (1997) show that nondetection is indeed a serious issue even in the US context. Feldman and Slemrod (2007) cite an IRS study that found that "for every dollar of underreported income detected by TCMP [Taxpayer Compliance Measurement Program] examiners without the aid of third-party information documents, another $2.28 went undetected". The issue of imperfect detection is plausibly even more relevant for developing countries, where administrative capacity is generally low.

[4] There is a related literature considering the implications of the fact that the tax base is not perfectly observed. See Slemrod and Traxler (2010) for a recent contribution.

to underreport described above. Clearly, it also reduces the enforcement ability of the tax authority, as less information is provided by the taxpayer. Nevertheless, an unitary declaration may actually increase fiscal revenues when the penalty faced by the taxpayer in case evasion is detected does not translate one-to-one into higher revenues.

This paper focuses on import tariffs, however, the applicability of the model is more general, as the tax liability is the product of several independently reported variables in other situations. For instance, the base for property taxes is often determined, in particular in developing countries, according to an area-based approach, where the tax base is the product of the taxable area of the property and unit values (e.g. rental or capital value per square meter) based on factors such as location, services available, and quality of the structure (Bahl et al., 2008). Similar issues arise with regard to the enforcement of individual transferable quota (ITQ) in fisheries, in which underreporting may concern quota value and weight.[5] More generally, in most tax systems there exist several tax bases to which different tax rates apply, e.g. corporate rather than personal income tax. There is then an incentive to shift income between these different bases (Gordon and Slemrod, 2000). Thus, given an income generated through a certain activity, the taxpayer may be able to choose both how much to declare and, through income shifting, the tax rate to apply, with the tax liability being the product of the two. If income shifting is not a legitimate activity and can be challenged by the tax authority, for instance because it misrepresents the source of income, then it can be analyzed in light of the model presented here.

The aim of the paper is to highlight the positive implications of a multiplicative tax base for compliance and fiscal revenues. Therefore, I take as given the parameters characterizing the fiscal environment, i.e. the tax rate, the probability of an audit, and the fine rate. The normative implications of a multiplicative tax base, regarding for instance the optimal tax rate or probability of auditing, are beyond the scope of this paper and are left for future research. Nevertheless, the results presented in this paper provide insights into important policy issues. For instance, should tariffs be uniform or differentiated? Should they be specific or ad-valorem? Will the introduction

[5]"Fishers may fail to report their harvest, or misreport its weight. When a report is filed, they may attempt to report the species taken as some other species with a lower quota value." (Johnson, 1995)

of a flat tax improve compliance? In the conclusions, I will discuss what are the implications of the results obtained in this paper for these issues.

The rest of the paper is organized as follows. Next section outlines the imperfect detection model with an unitary tax base. The following section extends the model to the multiplicative case. In section 4, I introduce an example and analyze the unitary declaration of a multiplicative tax base. The last section discusses the policy implications and concludes.

2 Unitary Tax Base

Here, I model evasion using the "standard" approach, i.e. considering that the tax base has to be declared as a single number. Beside introducing the imperfect detection technology (see also Tonin, 2010), the results obtained here will be compared to the multiplicative case, analyzed in the next section, where the tax base is declared as the product of two factors, quantity and unit value.

Consider a firm who faces an ad valorem tariff duty $t \in (0,1)$ on an exogenously given import y. The importer declares a fraction $x \in [0,1]$ of it to the custom authority. The custom authority may perform an audit to find out whether the importing firm complies with custom regulation. The probability of an audit being performed is $\gamma \in [0,1]$. A fine proportional to the amount evaded is imposed in case evasion is detected (Yitzhaki, 1974). However, the fact that an audit is performed does not imply that the authority with certainty discovers the true liability. Instead, it may find evidence to impute a fraction of $\hat{x} \in [0,1]$ of the true value of the shipment.

I assume that $\hat{x}$ is distributed over the support $[0,1]$ according to a given distribution function G, so that $G(0) = 0$ and $G(1) = 1$, and the corresponding density function is indicated as g. To simplify the discussion, I assume that $g > 0$ within the support, so that G is invertible within $[0,1]$.

Given a declaration of x and collected evidence of $\hat{x}$, the custom authority imposes, in case $\hat{x} > x$, the payment of $\theta t y \left(\hat{x} - x\right)$, consisting of tariff duties plus an additional fine proportional to the assessed evasion, thus $\theta > 1$. In case $\hat{x} \leq x$, the custom authority cannot prove any evasion, so no fine is imposed.[6] Given a true import value y and a reported fraction $x \in [0,1]$, the expected

[6]An equivalent narrative is that in an audit, the custom authority may find no evidence at all of evasion with prob-

fine in case of auditing, f, is

$$f = t\theta y \int_{x}^{1} (\hat{x} - x) g(\hat{x}) d\hat{x}. \tag{1}$$

The firm is risk-neutral and maximizes its expected profits. For simplicity, I disregard other costs unrelated to custom duties. Therefore, the firm optimally chooses to declare

$$x^* = \arg \max_{x \in [0,1]} y - \gamma f - tyx. \tag{2}$$

After substituting (1) into (2), the first-order condition is (see Appendix for details)

$$G(x^*) = 1 - \alpha, \tag{3}$$

where, to simplify notation the two enforcement parameters are summarized by $\alpha \equiv 1/(\gamma\theta)$. The second-order condition, $-t\gamma\theta y g(x) < 0$, is always satisfied. The boundary condition $x \leq 1$ is always satisfied. Notice that full compliance (i.e. $x = 1$) does not take place unless $\theta \to +\infty$. The condition $x \geq 0$ implies that full evasion will take place, i.e. $x = 0$, when enforcement is very weak, i.e. $\gamma\theta \leq 1$. To summarize, the solution to the reporting problem is given by

$$x^* = \begin{cases} G^{-1}(1 - \alpha) & \text{if} \quad \alpha < 1 \\ 0 & \text{if} \quad \alpha \geq 1 \end{cases}. \tag{4}$$

As $\partial\alpha/\partial\gamma < 0$ and $\partial\alpha/\partial\theta < 0$, in an interior solution, the declared fraction increases as enforcement improves, either because of more frequent audits or heavier penalties. The equilibrium fine, f^*, is given by substituting (4) into (1). Expected profits in equilibrium are then given by

$$\Pi^* = y - \gamma f^* - tyx^*. \tag{5}$$

ability $G(x)$, which is increasing as the liability declared to the authorities increases. Conditional on detection taking place, the density for any given fraction of import value $\hat{x} \in [x, 1]$ being discovered is given by $g(\hat{x}) / [1 - G(x)]$.

3 Multiplicative Tax Base

Here, I model a tax base that is the product of two parameters, unit value and quantity in case of trade tariffs, each of them to be reported to the tax authority. The detection technology along each dimension is the same as the one outlined in the previous section.

The true values of the two parameters are y_1 and y_2. For each of them, the firm has to decide the fraction to report to the custom authority, so that $x_1 \in [0, 1]$ is the declared fraction of the first parameter and $x_2 \in [0, 1]$ is the declared fraction of the second one. In case of an audit, the custom authority manages to impute a fraction $\hat{x}_1 \in [0, 1]$ and $\hat{x}_2 \in [0, 1]$ of the true value of the parameters. The probabilities of detection along the two dimensions are assumed to be independently distributed according to distribution functions $G_{\hat{x}_1}$ and $G_{\hat{x}_2}$, with the corresponding density functions indicated as $g_{\hat{x}_1}$ and $g_{\hat{x}_2}$, both strictly positive within the support. The assumption of independence is reasonable when the two parameters are import quantity and unit value, as the evidence to be produced by the custom authority to prove a higher import quantity (e.g. container inspection) is likely to be unrelated to evidence needed to prove higher unit value (e.g. comparison with listed prices).

The fine, f, depends on the detected and declared values of both parameters. In particular, if $\hat{x}_1 < x_1$ and $\hat{x}_2 < x_2$, then no evasion is discovered in either dimension and no penalty is imposed, thus $f = 0$. If $\hat{x}_1 > x_1$ and $\hat{x}_2 > x_2$, then evasion is discovered in both dimensions and the fine is, as in the unitary case, levied on the difference between the assessed tax base and the declared one, thus $f = t\theta y_1 y_2 \left(\hat{x}_1 \hat{x}_2 - x_1 x_2\right)$. There are two other possible cases: either $\hat{x}_1 < x_1$ and $\hat{x}_2 > x_2$, or $\hat{x}_1 > x_1$ and $\hat{x}_2 < x_2$. In these two cases underreporting is discovered in one dimension only and the fine is imposed on assessed underreporting in that dimension multiplied by the declared value on the other dimension so that $f = t\theta y_1 y_2 x_2 \left(\hat{x}_1 - x_1\right)$ or $f = t\theta y_1 y_2 x_1 \left(\hat{x}_2 - x_2\right)$.

Notice that when evasion is unidimensional, a greater declaration has the advantage of reducing the expected fine, but the disadvantage of increasing the tax payment. When a second dimension is involved, there is a further disadvantage in that a higher declaration in one dimension increases the fine when evasion in the *other* dimension is detected, as evident in the last two cases above.[7] This means that an importer will take into account, when deciding about the declared unit value of the goods, that, if evasion about the quantity of imported goods is detected by custom officials, while the unit value declaration goes unchallenged, then the value of discovered evasion, and therefore the fine, will be assessed using the declared unit value.

Given a declaration (x_1, x_2), the expected fine is then

$$f = y_1 y_2 t\theta \left[\begin{array}{c} \int_{x_1}^{1}\int_{x_2}^{1} (\hat{x}_1\hat{x}_2 - x_1x_2)\, g_{\hat{x}_1}(\hat{x}_1) g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_1 d\hat{x}_2 + x_2 G_{\hat{x}_2}(x_2) \int_{x_1}^{1} (\hat{x}_1 - x_1)\, g_{\hat{x}_1}(\hat{x}_1) d\hat{x}_1 + \\ + x_1 G_{\hat{x}_1}(x_1) \int_{x_2}^{1} (\hat{x}_2 - x_2)\, g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2 \end{array} \right], \tag{6}$$

If the firm chooses to declare x_1 and x_2, then expected profits are

$$\Pi = y_1 y_2 - y_1 y_2 x_1 x_2 t - \gamma f, \tag{7}$$

where f is given by (6). Therefore, the optimal declaration is given by:

$$(x_1^*, x_2^*) \qquad s.t. \max_{x_1 \in [0,1], x_2 \in [0,1]} y_1 y_2 - y_1 y_2 x_1 x_2 t - \gamma f. \tag{8}$$

At an optimum, the following two conditions have to be satisfied (see Appendix for details)

$$\begin{aligned} x_1^* &= G_{\hat{x}_1}^{-1}\left((1-\alpha) \left[G_{\hat{x}_2}(x_2^*) + \frac{1}{x_2^*} \int_{x_2^*}^{1} \hat{x}_2 g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2 \right]^{-1} \right) \\ x_2^* &= G_{\hat{x}_2}^{-1}\left((1-\alpha) \left[G_{\hat{x}_1}(x_1^*) + \frac{1}{x_1^*} \int_{x_1^*}^{1} \hat{x}_1 g_{\hat{x}_1}(\hat{x}_1) d\hat{x}_1 \right]^{-1} \right). \end{aligned} \tag{9}$$

[7] Notice that for this to be the case, it is not necessary that the two random variables are independent. Suppose, for instance, we are in the plausible situation such that $\hat{x}_1$ and $\hat{x}_2$ are positively correlated. Then the probability that either $\hat{x}_1 < x_1$ and $\hat{x}_2 > x_2$, or $\hat{x}_1 > x_1$ and $\hat{x}_2 < x_2$, is strictly positive for any pair (x_1, x_2), unless $\hat{x}_1$ and $\hat{x}_2$ are perfectly correlated, in which case the two dimensional problem reduces to an unidimensional one.

From now on, I will assume that there is an unique interior solution to (9).

Suppose we are in an environment characterized by given enforcement parameters, i.e. probability of an audit and fine rate, and a given detection technology, i.e. a distribution function for the fraction of the true value the tax authority is able to impute along each dimension. Then, is evasion higher with an unitary tax base or with a multiplicative one? By comparing (4) to (9), it is possible to show that for each dimension of a multiplicative tax base the share that is evaded is higher than in the unitary case. This is because of the additional incentive to underreport that has been underlined above. Moreover, as the share that is evaded is higher in both dimensions, then the share of the total tax base that is evaded is also higher with a multiplicative tax base than with an unitary one. I can then state the following:

Proposition 1 *In an environment with imperfect detection of tax evasion, a tax base that is the product of two different parameters is subject to higher underreporting compared to a tax base that can be assessed directly.*

Proof. See Appendix. ■

This result is consistent with what is generally found in the empirical literature about tariff evasion (Javorcik and Narciso, 2008; Mishra et al., 2008; Jean and Mitaritonna, 2010), namely that differentiated products (e.g. "Footwear"), i.e. those not having a reference price or those whose price is not quoted on organized exchanges (Rauch, 1999), are more prone to evasion compared to homogeneous products (e.g. "Lead and Lead Alloys, Unwrought"). Given that the unit price of homogeneous products is easily observable, the importer declaration decision can be modelled as concerning only the quantity, while for the case of differentiated products also the unit price is subject to manipulation. Then, the model indeed predicts higher underreporting for differentiated products compared to homogeneous ones.

What happens when enforcement along one of the two dimensions improves? Suppose for instance that $G_{\hat{x}_2}$ is replaced by a new distribution function, $G'_{\hat{x}_2}$, that first-order stochastically dominates $G_{\hat{x}_2}$. From (9), it is evident that for a given compliance along the first dimension, compliance along the second dimension improves. But what happens to compliance along the first dimension? It is possible to show that

Proposition 2 *Given a tax base that is the product of two different parameters in an environment with imperfect detection of tax evasion, an increase in compliance along one dimension induces an increase in compliance also along the other dimension.*

Proof. See Appendix. ■

Consider the implications of this result for the issue of crime displacement, i.e. the tendency of higher enforcement along one dimension to increase criminal activity in alternative dimensions. For instance, in the context of tariffs, Yang (2008) has shown that in the Philippinnes increased enforcement through preshipment inspection of imports led to an increase in an alternative method of duty evasion (shipping via duty-exempt export processing zones), so that total duty avoidance did not change. What I have shown is that with a multiplicative tax base, the interdependence between the different dimensions actually generates a positive feedback instead of displacement. This is because improving compliance along one dimension, say unit price, through better enforcement, increases the expected fine when evasion along the other dimension, quantity, is detected, thus encouraging a higher declaration.

Notice that the tax evasion problem faced by the taxpayer in case of an unitary tax base (2) is equivalent to the tax evasion problem when there is a multiplicative tax base (8) with the additional constraint that one of the two dimensions is fully declared, e.g. $x_2 = 1$. Therefore, the taxpayer cannot be better off, i.e. profits cannot be higher, in an environment with an unitary tax base compared to an environment with a multiplicative one. Notice that this argument can be extended to a tax base that is the product of several independently reported variables: an increase in the number of factors gives more flexibility to the taxpayer to minimize fiscal payments and, therefore, as the dimensionality of the tax base decreases, the taxpayer cannot be better off. What is then the impact of a multiplicative tax base on public finances? To look at this issue, consider that net revenues include both revenues due to voluntary compliance and revenues due to enforcement. While it may be reasonable to assume that 1 USD paid by the importing firm through voluntary compliance translates into 1 USD net revenue for the custom authority, in the case of revenues due to enforcement this is less likely to be the case. Indeed, part of the costs that are labelled as a fine may be of nonpecuniary nature, like, for instance, reputational costs (Gordon, 1989), or part of the payments in case some evasion is detected may not translate into increased revenues for the tax authority, e.g. when bribes are paid to custom inspectors to reduce or eliminate the fine. Therefore, I will assume that only a portion $\beta \in [0,1]$ of the fine represents a net revenue for the custom authority.[8] Alternatively, the authority may have an objective function such that revenues due to voluntary compliance may be preferred to revenues due to enforcement, due for instance to some underlying government social welfare function that emphasize the "carrot" over the "stick" (Frey and Holler, 1998). Therefore, net revenues for the custom authority are given by

$$T = txy + \beta\gamma f \text{ or } T = tx_1x_2y_1y_2 + \beta\gamma f \tag{10}$$

for an unitary and a multiplicative tax base, where f is given by (1) and (6) respectively. When $\beta = 1$ net revenues for the custom authority are given by the tax base minus profits. To compare the unitary and multiplicative tax bases in terms of their budgetary implications, consider that voluntary compliance is lower with a multiplicative tax base than with an unitary one. Moreover, as underlined above, profits for the importer are higher with a multiplicative tax base than with an unitary one, so, even if $\beta = 1$ and fines increase, they do not increase enough to compensate for the decrease in voluntary compliance in terms of net revenues. Therefore, in general net revenues are lower with a multiplicative tax base than with an unitary one. This argument extends to more than two dimensions, with an increase in dimensionality associated with a decrease in net revenues. To summarize, I can state the following

Proposition 3 *In an environment with imperfect detection of tax evasion, a tax base generates*

[8] Of course, costs associated with compliance, like time spent on filling returns and money spent on professional assistance may be relevant (Slemrod and Sorum, 1984). However, evasion may require these activities to even a greater extent. For what follows, it would be immaterial to consider that only a portion of voluntary compliance translates into net revenues for the custom authority, while $1 - \beta$ is the *additional* loss associated with collecting revenues through enforcement.

lower revenues if it is the product of two different parameters compared to the case when it can be assessed directly. More generally, other things being equal, the higher the dimensionality of the tax base, the lower the amount of fiscal revenues generated by it.

From the previous analysis it emerges that a tax whose base is the product of different factors, each of them declared to and assessed by the tax authority, is more difficult to enforce compared to a tax whose base can be directly assessed. This should be taken into consideration when deciding on which base to levy a tax. An alternative possibility, however, is to require an unitary declaration of a multiplicative tax base. This amounts to ask the taxpayer to report not the different factors, but only their product. For the case of import tariffs, the custom authority may require only the declaration of the total value of the merchandise instead of asking the importer to declare both the unit value and the taxable quantities, . Clearly, an unitary declaration of a multiplicative tax base reduces the amount of information available to the tax authority and, thus, should make enforcement less effective. However, it also counter the disincentive to declare represented by the fact that a higher declaration in one dimension increases the fine when evasion in the other dimension is detected. In what follows, I will provide an example that shows that indeed an unitary declaration of a multiplicative tax base may increase voluntary compliance and fiscal revenues.

4 An Example

In this example I will assume that the probability of detection is uniformly distributed. To simplify the discussion, I will also assume that enforcement is sufficiently strong to avoid full evasion. First, I will obtain closed form solutions to the evasion problem in both the unitary and multiplicative case and compare them. Then, I will analyze the implications of requiring an unitary declaration of a multiplicative tax base.

4.1 Unitary Tax Base

For an unitary tax base, the expression for the expected fine (1) when $g(\cdot)$ is uniform in the support $[0,1]$, i.e. $\hat{x} \sim U_{[0,1]}$, becomes

$$\gamma f = \gamma t\theta y(1-x)^2/2. \tag{11}$$

The optimal reporting behavior given by (4) becomes

$$x^* = 1-\alpha. \tag{12}$$

So, the model implies that what is revealed to the authorities is a fraction of the true import value that depends on the enforcement parameters. Using (11), the expected fine is given in equilibrium by

$$\gamma f^* = yt\alpha/2 \tag{13}$$

and thus, substituting (12) and (13) into (5), I get the equilibrium expected profits

$$\Pi^* = y(1-t+t\alpha/2). \tag{14}$$

4.2 Multiplicative Tax Base

For a multiplicative tax base, the expression for the expected fine (6) when the probabilities of detection are uniformly distributed over the relevant intervals, so that $g_{\hat{x}_1}(\hat{x}_1) = 1$ and $g_{\hat{x}_2}(\hat{x}_2) = 1$, becomes

$$\gamma f = t\gamma\theta y_1 y_2 \left[(1-x_1x_2)^2 + (x_1-x_2)^2\right]/4, \tag{15}$$

where it is evident how the fine depends on the total share of the tax base that is evaded, $(1-x_1x_2)$, and on the difference between the share of evasion in the two dimensions, (x_1-x_2). Given the total amount of evasion, it is evident from (15) and (7) that the only effect of declaring an unequal portion along the two dimensions is to increase the expected fine and, therefore, the optimal behavior is to equalize them. The first-order conditions are indeed simultaneously satisfied if and only if

$$x_1^* = x_2^* = \sqrt[2]{1 - 2\alpha}, \tag{16}$$

The expected fine is then given by

$$\gamma f^* = y_1 y_2 t\alpha, \tag{17}$$

giving expected profits of

$$\Pi^* = y_1 y_2 (1 - t + t\alpha). \tag{18}$$

As underlined in the previous sections, with a multiplicative tax base there is an additional incentive to underreport along one dimension as a higher declaration increases the fine when evasion in the other dimension is detected. Too see how this is indeed the case, consider what would happen if the taxpayer disregarded the fact that the two dimensions of the tax base are linked and instead considered them in isolation. Then, evasion along each dimension would equal evasion in the unitary case, giving a total declaration of $(1-\alpha)^2 y_1 y_2$. As $(1-\alpha)^2 > 1 - 2\alpha$, it is evident that taking into account the fact that the two dimensions are related increases underreporting.

4.3 Comparison

Table 1 summarizes the comparison between the two cases.

Table 1: Comparison

	Unitary	Multiplicative
Voluntary compliance rate	$1-\alpha$	$1-2\alpha$
Fine rate	$\alpha/2$	α
Effective tax rate/Statutory tax rate	$1-\alpha/2$	$1-\alpha$

The proportion of the tax liability that is declared, the voluntary compliance rate, is clearly higher in case of an unitary tax base. The proportion of tax liability that is paid through enforcement, the fine rate, is instead higher for a multiplicative tax base. This does not imply that a multiplicative tax base is easier to detect than an unitary one, maybe because two parameters instead of one have to be reported to the tax authority. Actually, the opposite is true. In the unitary case, from (11) we have

$$\frac{f}{y} = \frac{t\theta}{2}(1-x)^2 .$$

Compare it with (15) when the same proportion is evaded along the two dimensions

$$\frac{f}{y_1 y_2} = \frac{t\theta}{4}(1-x_1 x_2)^2 .$$

It is evident that for any given proportion of undeclared income, more is uncovered in the unitary case compared to the multiplicative one. The higher fine rate in the multiplicative case is simply due to higher evasion. Taking both voluntary compliance and enforcement into account, a taxpayer manages to reduce total payments to the fiscal authority more effectively with a multiplicative tax base than with an unitary one. Indeed, with a multiplicative tax base, the taxpayer succeeds through tax evasion to reduce the proportion of the tax base that is paid to fiscal authorities, the effective tax rate, by a factor of α compared to what he should have paid, the statutory tax rate. In the unitary case this reduction is only by a factor of $\alpha/2$.

4.4 Unitary Declaration

Consider a tax base that is the product of two parameters and their true values are y_1 and y_2, so that $y = y_1 y_2$. However, it is not the case anymore that each of the two parameters has to be reported to the tax authority. Instead, the taxpayer decides the proportion of the total tax base to report, $x \in [0,1]$, and declares xy. In case of an auditing, detection still takes place along the two dimensions, so that the tax authority manages to impute $\hat{x}_1 \in [0,1]$ and $\hat{x}_2 \in [0,1]$, as in section 3. Instead of (6), the fine is now given by

$$f = y_1 y_2 t\theta \int_0^1 \int_0^1 \max\left[\hat{x}_1 \hat{x}_2 - x, 0\right] g_{\hat{x}_1}(\hat{x}_1) g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_1 d\hat{x}_2. \tag{19}$$

This captures the fact that, given the assessed values along the two dimensions, the tax authority imposes a fine only if their product is greater than the declared amount. The optimal declaration is given by solving (2), with the expression for the fine now given by (19). The first order condition is (see Appendix for details)

$$x - x(\ln x) = (1-\alpha). \tag{20}$$

The second order condition, $(\ln x) < 0$, is always satisfied. The expression above does not have an analytical solution. It is easy to show (see Appendix for details) that in this case the share of the tax base that is declared is less than in the unitary case. It is also possible to show (see Appendix for details) that in an environment with low enforcement, i.e. high α, and a multiplicative tax base, voluntary compliance is higher when only the product of the two parameters needs to be declared compared to the case when both parameters need to be declared. As a numerical example, consider the case with $\alpha = 0.4$. Then, declaration in the standard multiplicative case is given by $1 - 2\alpha = 0.2$, while declaration in case of an unitary declaration of a multiplicative tax base is approximately 0.25.

I can also compare net revenues, as defined in (10), or equivalently, the ratio of net revenues over the true tax liability, $T/(ty)$, provided by the two possible institutional arrangements. Figure 1 plots $\frac{T}{ty}$ with an unitary declaration minus $\frac{T}{ty}$ with a multiple declaration and reveals that for high enough α and low enough β, net revenues are indeed greater with an unitary declaration compared to a multiple declaration. For instance, when $\alpha = 0.4$ and $\beta = 0.1$, 24% of the true tax liability translates into tax revenues with a multiple declaration, while this is 27.5% in case of an unitary declaration.

I summarize the results in the following proposition

Proposition 4 *Given a tax base that is the product of two different parameters, voluntary compliance may be higher when only the product of the two parameters needs to be declared compared to the case when both parameters need to be declared. Moreover, if penalties paid by the taxpayer*

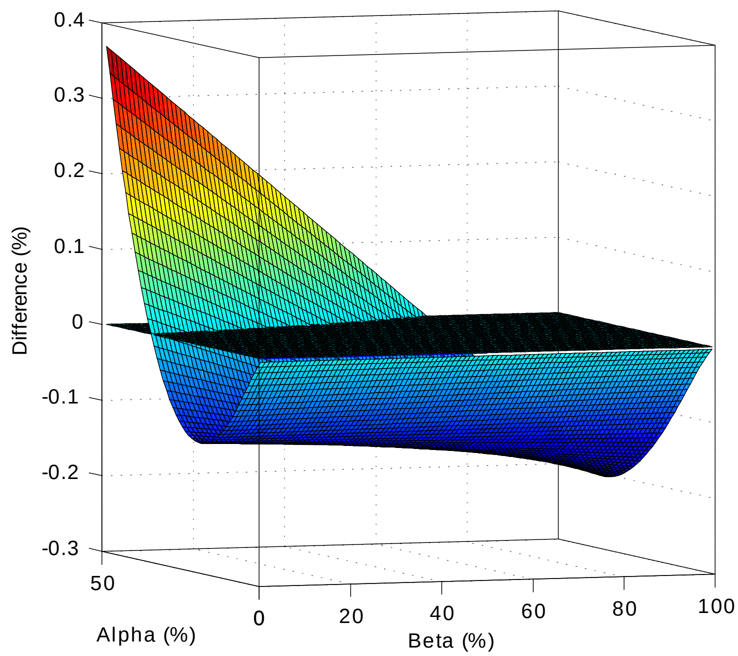

Figure 1: Difference in Revenues

translate into budgetary revenues at a low enough rate, then net revenues may be higher with an unitary declaration.

5 Discussion and conclusions

In this paper, I have analyzed the implications of a feature of import tariffs that has not been considered so far, i.e. the fact that the tax base is the product of two factors, unit value and quantity, each of them reported by the importer and assessed by the custom authority in case of an audit. What I have shown is that when the tax base presents this characteristic instead of being assessed directly, as in the unitary case, then evasion is higher. I have also shown under which circumstances net revenues may be higher when the custom authority does not require the importer to declare both factors, but only their product. The model can be easily reinterpreted as studying evasion through mislabelling and either underreporting of quantity or underreporting of unit price.[9]

Moreover, it has been shown that fiscal revenues decrease as the dimensionality of the tax base increases. This should be taken into consideration when designing the tax system. Thus, in the context of tariffs, this paper provides an argument in favour of an uniform or a specific tariff, as opposed to a differentiated ad-valorem one. Indeed, with an uniform tariff there is not anymore an incentive to mislabel products, while a specific tariff removes the need to report unit prices. In both cases, the dimensionality of the tax base decreases, thus increasing revenues. More generally, tariffs have been used by developing countries to raise revenues because they benefit from low collection costs compared to other taxes (Burgess and Stern, 1993) and it has been argued that they may be part of the optimal tax structure in developing countries, where tax enforcement is more difficult (Gordon and Li, 2009). However, what has been underlined in this paper is that custom duties may be actually more difficult to enforce than other taxes and this should be taken into account when thinking about the optimal tax structure for developing countries, where administrative capacity is generally low and, thus, issues of enforcement are very prominent.

Beside tariffs, the paper has implications for other aspects of tax policy. For instance, one of the arguments usually put forward in favour of a flat tax is that it improves compliance. There is empirical evidence consistent with this argument (Gorodnichenko et al., 2009), but very little analysis about the implication of flatness per se on compliance (Keen et al., 2006). The mechanism proposed in this paper suggests a reason why flatness per se may indeed increase compliance. If the flat tax is implemented in a way that eliminates the incentives for income-shifting across different bases, for instance because it equalizes the applicable tax rate, it will reduce the dimensionality of the tax base and, therefore, induce taxpayers to declare more of their true income. Notice that the prediction is that there will not be a simple rearrangement across bases, so that one tax base increases at the expenses of the other, but an overall increase in compliance.

[9] Clearly, mislabelling is better modelled as a discrete rather than continuous choice. This would require some more cumbersome notation, but would not change the results in a significant way.

References

[1] Allingham, M.G. and Sandmo, A. (1972), "Income tax evasion: A theoretical analysis", Journal of Public Economics vol. 1(3-4), pp. 323–338.

[2] Andreoni, J., Erard, B., and Feinstein, J. (1998), "Tax compliance", Journal of Economic Literature, Vol. 36, pp. 818-860.

[3] Bahl, R., Martinez-Vazquez, J., and Youngman, J. (2008), “The Property Tax in Practice”, in: Bahl, R., Martinez-Vazquez, J., and Youngman, J. “Making the Property Tax Work”, Lincoln Institute of Land Policy, Cambridge (MA).

[4] Baunsgaard, T., and Keen, M. (2010), "Tax revenue and (or?) trade liberalization", Journal of Public Economics, vol. 94(9-10), pp. 563-577.

[5] Burgess, R., and Stern, N. (1993), "Taxation and Development", Journal of Economic Literature, vol. 31(2), pp. 762-830.

[6] Erard, B. (1997), "Self-selection with measurement errors. A microeconometric analysis of the decision to seek tax assistance and its implications for tax compliance", Journal of Econometrics, vol. 81, pp. 319-356.

[7] Feinstein, J. (1991), "An econometric analysis of income tax evasion and its detection", RAND Journal of Economics, vol. 22, pp. 14-35.

[8] Feldman, N., and Slemrod, J. (2007), "Estimating tax noncompliance with evidence from unaudited tax returns", The Economic Journal, vol. 117(518), pp. 327-352.

[9] Fisman, R., & Wei S. (2004). Tax Rates and Tax Evasion: Evidence from “Missing Imports” in China, Journal of Political Economy, vol. 112(2), pp. 471-496.

[10] Frey, B., and Holler, M. (1998), "Tax Compliance Policy Reconsidered," Homo Oeconomicus, vol. 15, pp. 27-45.

[11] Gordon, J. (1989), "Individual morality and reputation costs as deterrents to tax evasion", European Economic Review, vol. 33(4), pp. 797-805.

[12] Gordon, R., and Li, W. (2009), "Tax structures in developing countries: Many puzzles and a possible explanation", Journal of Public Economics, vol. 93(7-8), pp. 855-866.

[13] Gorodnichenko, Y., Martinez-Vazquez, J., and Sabirianova Peter, K. (2009), "Myth and Reality of Flat Tax Reform: Micro Estimates of Tax Evasion Response and Welfare Effects in Russia", The Journal of Political Economy, vol. 117(3), pp. 504-554.

[14] Javorcik, B. and Narciso, G. (2008), “Differentiated products and evasion of import tariffs”, Journal of International Economics 76, 208–222.

[15] Jean, S. and Mitaritonna, C. (2010), "Determinants and pervasiveness of the evasion of custom duties", mimeo.

[16] Johnson, R. (1995), "Implications of Taxing Quota Value in an Individual Transferable Quota Fishery", Marine Resource Economics, vol. 10, pp. 327-340.

[17] Mishra, P., Subramanian, A., and Topalova, P. (2008), "Tariffs, enforcement, and customs evasion: Evidence from India", Journal of Public Economics, vol. 92(10-11), pp. 1907-1925.

[18] Rauch, J., (1999), "Networks versus markets in international trade", Journal of International Economics, vol. 48 (1999), pp. 7–35.

[19] Rozanski, J., Yeats, A. (1994), "On the (in)accuracy of economic observations: An assessment of trends in the reliability of international trade statistics", Journal of Development Economics, vol. 44(1), pp. 103-130.

[20] Slemrod, J. (2007), "Cheating Ourselves: The Economics of Tax Evasion", The Journal of Economic Perspectives, vol. 21(1), pp. 25-48.

[21] Slemrod, J., and Sorum, N. (1984), "The Compliance Cost of the U.S. Individual Income Tax System," National Tax Journal, vol. 37, pp. 461-74.

[22] Slemrod, J., and Traxler, C. (2010), "Optimal observability in a linear income tax", Economics Letters, vol. 108, pp. 105-108.

[23] Slemrod, J., and Yitzhaki, S. (2002), "Tax avoidance, evasion, and administration", in: Auerbach, A., Feldstein, M. (eds.) Handbook of Public Economics, vol. III, Elsevier, Amsterdam.

[24] Stoyanov, A. (2009), “Tariff Evasion Under Free Trade Agreement: Empirical Evidence From Canada-U.S. Free Trade Agreement”, mimeo.

[25] Tonin, M. (2010), "Minimum Wage and Tax Evasion: Theory and Evidence", WP 101 International Policy Center, Gerald R. Ford School of Public Policy, University of Michigan.

[26] Yang, D. (2008), "Can Enforcement Backfire? Crime Displacement in the Context of Custom Reform in the Philipinnes", The Review of Economics and Statistics, vol. 90(1), pp. 1-14.

[27] Yitzhaki, S. (1974),“A Note on ‘Income Tax Evasion: A Theoretical Analysis.”’, Journal of Public Economics, vol. 3 (May), pp. 201–2.

6 Appendix

Unitary Tax Base: Derivation of the First Order Condition Take the first derivative with respect to x of

$$y - \gamma t\theta y \int_x^1 (\hat{x} - x) g(\hat{x}) d\hat{x} - tyx \tag{21}$$

to get

$$-\gamma\theta ty \frac{d}{dx} \int_x^1 (\hat{x} - x) g(\hat{x}) d\hat{x} - ty.$$

Using Leibniz integral rule,

$$\frac{d}{dx} \int_x^1 (\hat{x} - x) g(\hat{x}) d\hat{x} = \int_x^1 \frac{d}{dx} (\hat{x} - x) g(\hat{x}) d\hat{x} = \int_x^1 - g(\hat{x}) d\hat{x} = -1 + G(x).$$

Then, the f.o.c. is

$$\gamma\theta ty - \gamma\theta tyG(x) - ty = 0$$

$$\gamma\theta - 1 = \gamma\theta G(x)$$

$$G(x) = 1 - \frac{1}{\gamma\theta} = 1 - \alpha$$

Multiplicative Tax Base: Derivation of the First Order Condition Considering (7), the two first order conditions are given by

$$\begin{aligned}\frac{\partial \Pi}{\partial x_1} &= 0 \Leftrightarrow -y_1 y_2 x_2 t - \gamma \frac{\partial f}{\partial x_1} = 0\\ \frac{\partial \Pi}{\partial x_2} &= 0 \Leftrightarrow -y_1 y_2 x_1 t - \gamma \frac{\partial f}{\partial x_2} = 0.\end{aligned}$$

To derive $\frac{\partial f}{\partial x_1}$, rewrite the first term within square brackets in (6) as

$$\int_{x_1}^{1}\int_{x_2}^{1} (\hat{x}_1\hat{x}_2)\, g_{\hat{x}_2}(\hat{x}_2) g_{\hat{x}_1}(\hat{x}_1) d\hat{x}_2 d\hat{x}_1 - (x_1 x_2) \int_{x_1}^{1} g_{\hat{x}_1}(\hat{x}_1) \int_{x_2}^{1} g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2 d\hat{x}_1$$

or

$$\int_{x_1}^{1} \hat{x}_1 g_{\hat{x}_1}(\hat{x}_1) \left[\int_{x_2}^{1} \hat{x}_2 g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2\right] d\hat{x}_1 - (x_1 x_2)\left[1 - G_{\hat{x}_2}(x_2)\right]\left[1 - G_{\hat{x}_1}(x_1)\right].$$

Applying Leibniz Integral Rule to the first term, the derivative with respect to x_1 is given by

$$-x_1 g_{\hat{x}_1}(x_1) \left[\int_{x_2}^{1} \hat{x}_2 g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2\right] - x_2\left[1 - G_{\hat{x}_2}(x_2)\right]\left[1 - G_{\hat{x}_1}(x_1) - x_1 g_{\hat{x}_1}(x_1)\right] \tag{22}$$

Then, apply Leibniz Integral Rule to the second term in (6) to get the first order condition with respect to x_1

$$x_2 G_{\hat{x}_2}(x_2)\left[G_{\hat{x}_1}(x_1) - 1\right], \tag{23}$$

while, for the third term in (6) the first order condition with respect to x_1 is given by

$$\left[G_{\hat{x}_1}(x_1) + x_1 g_{\hat{x}_1}(x_1)\right] \int_{x_2}^{1} \left(\hat{x}_2 - x_2\right) g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2. \tag{24}$$

Putting together (22), (23), and (24) and simplyfing, we get the expression for the derivative of the fine with respect to x_1.

$$\frac{df}{dx_1} = y_1 y_2 t\theta \left[G_{\hat{x}_1}(x_1) \int_{x_2}^{1} \hat{x}_2 g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2 - x_2 + x_2 G_{\hat{x}_2}(x_2) G_{\hat{x}_1}(x_1)\right].$$

The same steps can be done to obtain the derivative with respect to x_2. Then, the two first order conditions are given by

$$(1-\alpha) = G_{\hat{x}_1}(x_1) \left[G_{\hat{x}_2}(x_2) + \frac{1}{x_2} \int_{x_2}^{1} \hat{x}_2 g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2\right]$$

$$(1-\alpha) = G_{\hat{x}_2}(x_2) \left[G_{\hat{x}_1}(x_1) + \frac{1}{x_1} \int_{x_1}^{1} \hat{x}_1 g_{\hat{x}_1}(\hat{x}_1) d\hat{x}_1\right].$$

Then, x_1^* and x_2^* have to simultaneously satisfy

$$x_1^* = G_{\hat{x}_1}^{-1} \left(\frac{(1-\alpha)}{\left[G_{\hat{x}_2}(x_2^*) + \frac{1}{x_2^*} \int_{x_2^*}^{1} \hat{x}_2 g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2\right]} \right) \tag{25}$$

$$x_2^* = G_{\hat{x}_2}^{-1} \left(\frac{(1-\alpha)}{\left[G_{\hat{x}_1}(x_1^*) + \frac{1}{x_1^*} \int_{x_1^*}^{1} \hat{x}_1 g_{\hat{x}_1}(\hat{x}_1) d\hat{x}_1\right]} \right).$$

Proof of Proposition 1 Recall from (4) that an interior solution in the unidimensional case is characterized by

$$x^* = G^{-1}(1-\alpha)$$

where G is an increasing function. Then G^{-1} is also an increasing function. The solution in the bidimensional case is characterized by (25). To compare the uni- and bi-dimensional cases, the assumption is that the distributions of the probability of detection are the same, i.e. $G = G_{\hat{x}_1} = G_{\hat{x}_2}$, and enforcement is also the same in the two environments. Then,

$$\begin{aligned} & x^* > x_1^* \\ & \Leftrightarrow 1-\alpha > \frac{(1-\alpha)}{\left[G_{\hat{x}_2}(x_2^*) + \frac{1}{x_2^*}\int\limits_{x_2^*}^{1} \hat{x}_2 g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2 \right]} \\ & \Leftrightarrow \int\limits_{x_2}^{1} \hat{x}_2 g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2 > x_2^* \left[1 - G_{\hat{x}_2}(x_2^*)\right]. \end{aligned}$$

Using the fact that

$$\left[1 - G_{\hat{x}_2}(x_2^*)\right] = \int\limits_{x_2^*}^{1} g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2,$$

the condition reduces to

$$\Leftrightarrow \int\limits_{x_2^*}^{1} \left(\hat{x}_2 - x_2^*\right) g_{\hat{x}_2}(\hat{x}_2) d\hat{x}_2 > 0$$

that is always satified. The same steps can be done regarding x_2^* to show that $x^* > x_2^*$. As $x_1^* \in (0,1)$ and $x_2^* \in (0,1)$, then $x^* > x_1^* x_2^*$.

Proof of Proposition 2 Using the expression for x_1^* in (25), we have that

$$\frac{\partial}{\partial x_2^*}(1-\alpha)\left[G_{\hat{x}_2}(x_2^*)+\frac{1}{x_2^*}\int_{x_2^*}^{1}\hat{x}_2 g_{\hat{x}_2}(\hat{x}_2)d\hat{x}_2\right]^{-1}=$$

$$-(1-\alpha)\left[G_{\hat{x}_2}(x_2^*)+\frac{1}{x_2^*}\int_{x_2^*}^{1}\hat{x}_2 g_{\hat{x}_2}(\hat{x}_2)d\hat{x}_2\right]^{-2}\left[g_{\hat{x}_2}(x_2^*)-\frac{1}{(x_2^*)^2}\int_{x_2^*}^{1}\hat{x}_2 g_{\hat{x}_2}(\hat{x}_2)d\hat{x}_2-\frac{1}{x_2^*}x_2^* g_{\hat{x}_2}(x_2^*)\right]=$$

$$(1-\alpha)\frac{1}{(x_2^*)^2}\int_{x_2^*}^{1}\hat{x}_2 g_{\hat{x}_2}(\hat{x}_2)d\hat{x}_2\left[G_{\hat{x}_2}(x_2^*)+\frac{1}{x_2^*}\int_{x_2^*}^{1}\hat{x}_2 g_{\hat{x}_2}(\hat{x}_2)d\hat{x}_2\right]^{-2}>0$$

As $G_{\hat{x}_1}^{-1}$is an increasing function, it follows that as x_2^* increases, x_1^* also increases.

Unitary Declaration: Expression for Fine Using (19), an equivalent expression for the fine is

$$y_1y_2t\theta\int_{x}^{1}\left[\int_{\frac{x}{\hat{x}_1}}^{1}(\hat{x}_1\hat{x}_2-x)\,g_{\hat{x}_2}(\hat{x}_2)d\hat{x}_2\right]g_{\hat{x}_1}(\hat{x}_1)d\hat{x}_1$$

$$=y_1y_2t\theta\int_{x}^{1}\left[\int_{\frac{x}{\hat{x}_1}}^{1}(\hat{x}_1\hat{x}_2-x)\,g_{\hat{x}_2}(\hat{x}_2)d\hat{x}_2\right]g_{\hat{x}_1}(\hat{x}_1)d\hat{x}_1$$

$$=y_1y_2t\theta\int_{x}^{1}\left[\left[\frac{\hat{x}_1(\hat{x}_2)^2}{2}-x\hat{x}_2\right]_{\frac{x}{\hat{x}_1}}^{1}\right]g_{\hat{x}_1}(\hat{x}_1)d\hat{x}_1$$

$$=y_1y_2t\theta\int_{x}^{1}\frac{\hat{x}_1}{2}-x-\frac{\hat{x}_1\left(\frac{x}{\hat{x}_1}\right)^2}{2}+x\frac{x}{\hat{x}_1}d\hat{x}_1$$

$$=y_1y_2t\theta\int_{x}^{1}\frac{\hat{x}_1}{2}-x+\frac{x^2}{2\hat{x}_1}d\hat{x}_1$$

$$=y_1y_2t\theta\left[\frac{(\hat{x}_1)^2}{4}-x\hat{x}_1+\frac{x^2}{2}\ln\hat{x}_1\right]_{x}^{1}$$

$$=y_1y_2t\theta\left[\frac{1}{4}-x-\frac{x^2}{2}\ln x+\frac{3}{4}x^2\right]$$

Using this expression for the fine in (2) gives rise to the following expression for the first order condition , where y_1y_2 has been replaced by y,

$$-ty-\gamma\theta ty\left[-1-x\left(\ln x\right)-\frac{x}{2}+\frac{3}{2}x\right]=0$$
$$1+x\left(\ln x\right)-x=\alpha$$
$$x-x\left(\ln x\right)=1-\alpha$$

Unitary Tax Base vs Unitary Declaration of a Multiplicative Tax Base Recall from (12) that in case of an unitary tax base $x=(1-\alpha)$. As $-x\left(\ln x\right)>0$, then the solution to (20) has to be strictly smaller than $(1-\alpha)$

Multiplicative Tax Base vs Unitary Declaration of a Multiplicative Tax Base Indicate the solution to (20) as x^{**} and notice how the left-hand side of expression (20) is increasing in x at a decreasing rate. Recall that in case of a standard two dimensional tax base $x=1-2\alpha$. To know how this compare to x^{**}, we can calculate the value of the left-hand side of expression (20) when $x=1-2\alpha$ and see if it is smaller or bigger than $(1-\alpha)$. If it is smaller, it means that $x^{**}>1-2\alpha$, if it is bigger it means that $x^{**}<1-2\alpha$. The comparison

$$1-2\alpha-(1-2\alpha)\ln\left(1-2\alpha\right)>1-\alpha$$

gives the following condition

$$\alpha<-\left(1-2\alpha\right)\ln\left(1-2\alpha\right)$$

In figure (2), the two sides are plotted.

Let's indicate as $\alpha^{*}\simeq 0.36$ the point on the x-axis corresponding to the intersection between the two curves. Then,

$$\begin{array}{l} x^{**}<1-2\alpha \quad if \quad \alpha<\alpha^{*} \\ x^{**}\geq 1-2\alpha \quad if \quad \alpha\geq\alpha^{*} \end{array}.$$

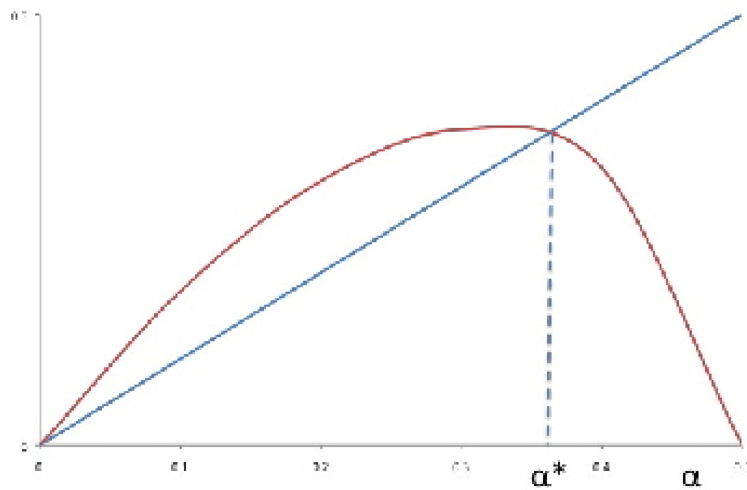

Figure 2: Comparison to a multiplicative tax base